Multiple Nested Beltrami regions as a Solution to the 3-D Toroidal MHD Equilibrium Problem

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A generalized energy principle for finite-pressure, toroidal magnetohydrodynamic (MHD) equilibria in general three-dimensional configurations is proposed. The full set of ideal-MHD constraints is applied only on a discrete set of toroidal magnetic surfaces (invariant tori), which act as barriers against leakage of magnetic flux, helicity and pressure through chaotic field-line transport. It is argued that a necessary condition for such invariant tori to exist is that they have fixed, irrational rotational transforms. In the toroidal domains bounded by these surfaces, full Taylor relaxation is assumed, thus leading to Beltrami fields: $\nabla \times B = \lambda B$, where λ is constant within each domain. Such equilibria and their stability have been studied in cylindrical geometry [1], generalizing the single Beltrami region study of Kaiser and Uecker [2]. Two distinct eigenvalue problems for λ arise in this formulation, depending on whether fluxes and helicity, or boundary rotational transforms, are fixed [3]. Beltrami states have been constructed in a three-dimensional toroidal region of annular cross section [3], a residue criterion being used to determine the threshold for connected chaos.

References

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