Verification of the CAS3D-perturbed equilibrium code in the cylindrical limit

WENDELSTEIN 7-X

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IPP

Perturbed equilibria

- computation of toroidal plasma equilibria
  - tokamak: GS-solver not suitable if
    - ▷ error-fields turn 2d into 3d equilibrium
  - stellarator
    - ▷ equilibrium codes assuming nested surfaces neglect the magnetic islands
    - ▷ equilibrium codes NOT assuming nested surfaces are time-consuming
- use the perturbed-equilibria method to study
  - boundary-shape perturbation
  - plasma-pressure change
  - external perturbations produce islands
  - improve approximate equilibria
- ideal MHD stability: response of plasmas to small perturbations
  - ideal MHD stability codes can be used to determine perturbed equilibria

Background

- ideal MHD framework: equilibrium equation
  - $\nabla p = \vec{j} \times \vec{B}$
- scalar pressure: p is surface function
  - $\nabla p = p' \nabla s$
- equilibrium magnetic field $\vec{B}$

$$\vec{B} = I\nabla\phi + J\nabla\theta + \tilde{\beta}\nabla s = -\frac{F_{\rm T}'}{\sqrt{g}}\vec{r}_{,\phi} - \frac{F_{\rm P}'}{\sqrt{g}}\vec{r}_{,\theta}$$

- ▷ use of magnetic coordinates (s, θ, ϕ)
- ▷ I and J: poloidal, toroidal currents
- ▷ $F_{\rm P}$ and $F_{\rm T}$: poloidal, toroidal fluxes
- ▷ V': specific volume
- ▷ $\sqrt{g}\tilde{\beta}_{\rm metric} = F_{\rm T}' g_{s\phi} + F_{\rm P}' g_{s\theta}$
- ▷ $\sqrt{g}\vec{B}\cdot\nabla\tilde{\beta}_{\rm mde} = p'(\sqrt{g} - V')$

- MHD displacement vector $\vec{\xi}$ with

$$\xi^s = \vec{\xi}\cdot\nabla s$$
$$\eta = -F_t'\,\vec{\xi}\cdot(\nabla\theta - \iota\nabla\phi)$$
$$\mu = \sqrt{g}F_t'\,\vec{\xi}\cdot(\nabla\phi + \iota\nabla\theta)$$

Discontinous normal displacement in a cylinder

- resonant error fields produce islands
  - ideal MHD: surface current on the rational surface prevents island from opening
  - strength of surface current is related to the height of the jump that is allowed in resonant normal displacement harmonics on the respective rational surface
- EITHER: code for the ideal cylindrical stability
  - $\frac{1}{r}\frac{\rm d}{{\rm d}r}\left(r\frac{{\rm d}\xi}{{\rm d}r}\right) - \frac{m^2}{r^2}\xi - \frac{j_z' R}{rB^2(\iota - n/m)}\xi = 0$
    - ▷ exterior tearing equation with singular points at the rationals and the origin
    - ▷ is numerically solved with a shooting and matching technique
- OR: 3D ideal MHD stability code CAS3D
  - ▷ resonant normal displacement harmonics may be discontinuous
  - ▷ solve the homogeneous problem $\mathcal{F}\vec{\xi} = 0$ with given boundary condition

Ideal MHD energy principle

- plasma equilibrium and stability: minimize δW through second order

$$\delta W = \int(\nabla p - \vec{j}\times\vec{B})\cdot\vec{\xi}\,{\rm d}^3r - \frac{1}{2}\int\vec{\xi}\cdot\mathcal{F}\left[\vec{\xi}\right]{\rm d}^3r$$

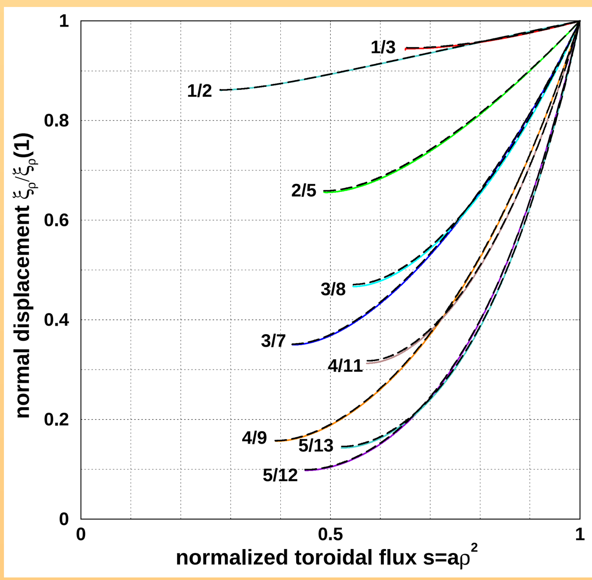
- first order term

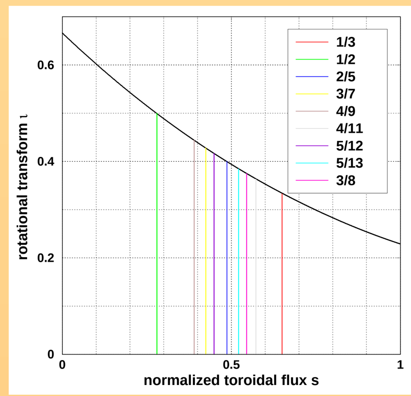
$$\delta^1 W = \int \xi^s\,\left[(p' - p'_{\rm new}) + \vec{B}\cdot\nabla(\tilde{\beta}_{\rm mde} - \tilde{\beta}_{\rm metric})\right]{\rm d}^3r$$

- second order term: $\mathcal{F}$ is the MHD force operator

$$\delta^2 W = \frac{1}{2}\int\left\{\left|\vec{B}_1 + \frac{\vec{j}\times\nabla s}{|\nabla s|^2}\xi^s\right|^2 + \gamma p(\nabla\cdot\vec{\xi})^2 - \mathcal{A}(\xi^s)^2\right\}{\rm d}^3r$$

- COMPARISON: cylinder code — CAS3D
  - apply error-field on boundary of a perfect cylinder equilibrium
  - resonant surfaces shield off the respective error-field component

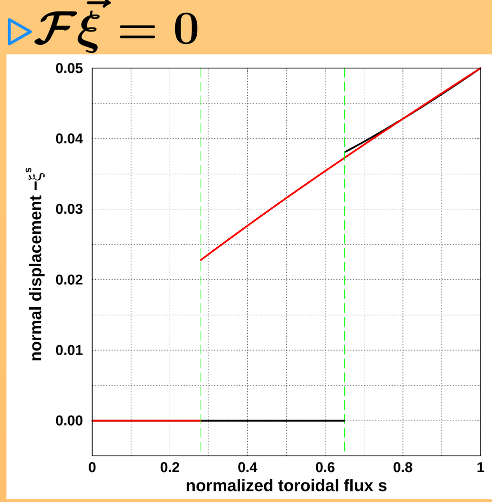




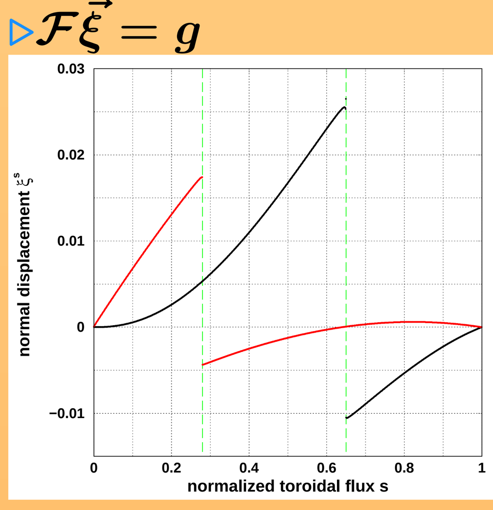
- △ aspect ratio 10
- △ plasma-$\beta \approx 0$
- ◁ cylinder code (black)
- ◁ CAS3D code (coloured)

- COMPARISON: change of boundary shape
  - study helically distorted equilibrium

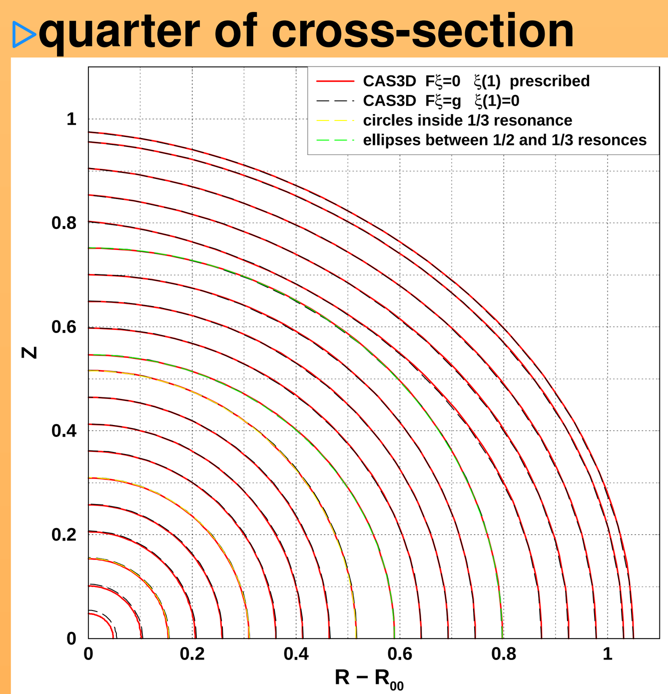
▷ $\mathcal{F}\vec{\xi} = 0$



▷ $\mathcal{F}\vec{\xi} = g$



▷ quarter of cross-section



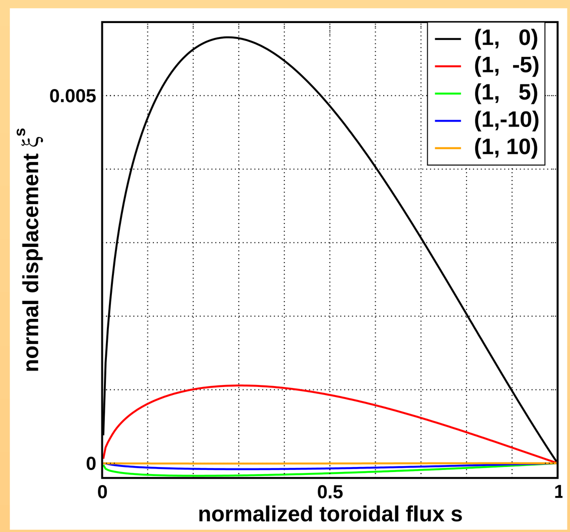
▷ $\vec{r}_1 = \vec{r}_0 + (\vec{\xi}\cdot\vec{n})\vec{n}$

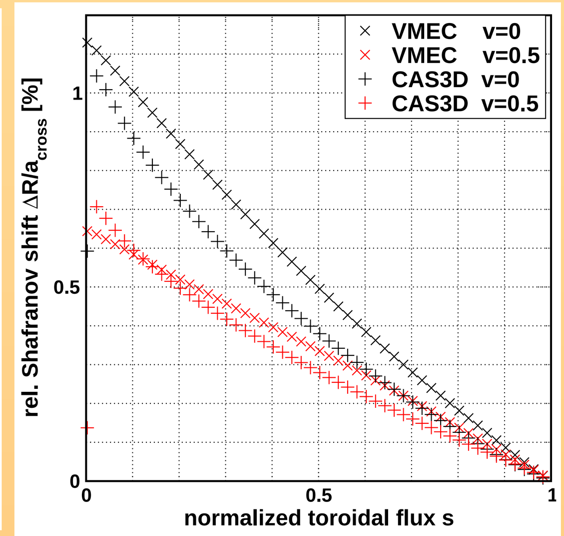
▷ jumps

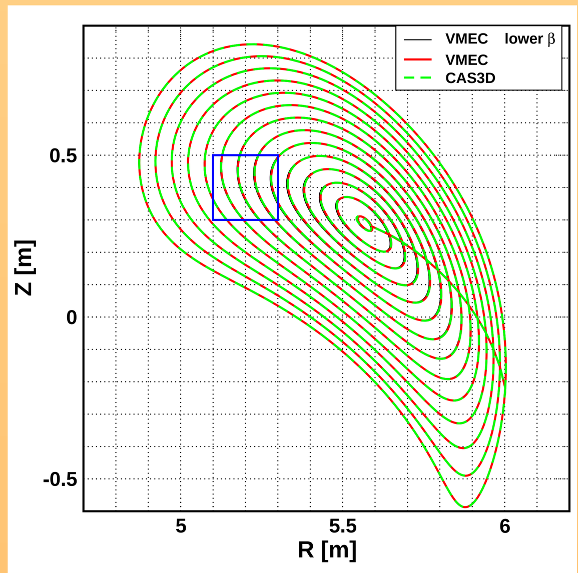
m	$\mathcal{F}\vec{\xi} = 0$	$\mathcal{F}\vec{\xi} = g$
3	0.038	0.036
2	0.023	0.022

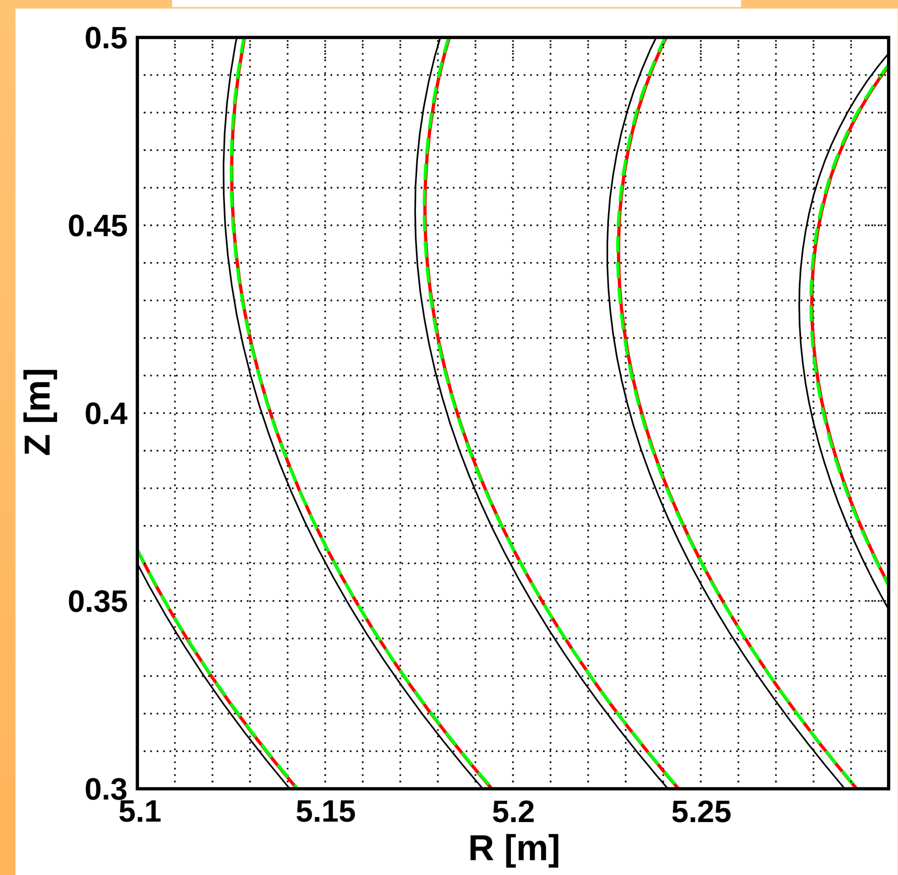
Plasma-pressure change

- W7-X with $\langle\beta\rangle = 0.045$ perturbed to 0.048
- $m = 1$ $n = -10, -5, 0, 5, 10$ perturbation









Summary

In the context of perturbed equilibria an alternative method, that employs the linearized ideal MHD stability theory, has been implemented by extending the global, ideal MHD stability code CAS3D. • In cylinder geometry the perturbed equilibrium as determined by the CAS3D code (which can treat arbitrary geometry) has been compared successfully to the exterior solution of the cylindrical tearing equation. • It has been verified that coincident results are obtained when calculating (i) a resonant normal displacement with a prescribed boundary value in a perfect cylinder equilibrium, and (ii) the resonant normal displacement vanishing on the plasma boundary in the equivalent helical equilibrium with nested surfaces. • A W7-X equilibrium has been perturbed from $\langle\beta\rangle = 0.045$ to 0.048. This calculation has been benchmarked with the VMEC equilibrium code. •

References

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