The MHD equilibrium problem in nonaxisymmetric toroidal plasma confinement systems

R.L. Dewar,* M.J. Hole, and M. McGann

Department of Theoretical Physics and Plasma Research Laboratory,

Research School of Physical Sciences and Engineering,

The Australian National University, ACT 0200 Australia

S.R. Hudson

Princeton University Plasma Physics Laboratory,

PO Box 451, Princeton NJ 08543, USA

(Dated: August 1, 2007)

The existence and nature of solutions to the magnetohydrodynamic (MHD) equilibrium problem $\nabla p = \mathbf{j}\times\mathbf{B}$, where $\mathbf{B}(\mathbf{r})$ is the magnetic field at position $\mathbf{r}$, $\mathbf{j} = \nabla\times\mathbf{B}$ is the current density in suitable units, and $p(\mathbf{r})$ is the plasma pressure, has long been recognized as difficult to establish [1]. A number of authors [2] have sought to formulate this problem in terms of systems where nonzero pressure gradients are restricted to toroidal surfaces on which a sheet current counterbalances a step-function change in p, and encounter a Kolmogorov–Arnol'd–Moser (KAM) style problem. These approaches seek to construct the magnetic field by integrating from the inside of the torus outward, which is an ill-posed problem. Instead we [3] propose a generalized energy principle for finite-pressure, toroidal magnetohydrodynamic (MHD) equilibria in general three-dimensional configurations with specified outer boundaries. The full set of ideal-MHD constraints is applied only on a discrete set of toroidal magnetic surfaces (invariant tori), which act as barriers against leakage of magnetic flux, helicity and pressure through chaotic field-line transport. It is argued that a necessary condition for such invariant tori to exist is that they have fixed, irrational rotational transforms. In the toroidal domains bounded by these surfaces, full Taylor relaxation is assumed, thus leading to Beltrami fields: $\nabla\times\mathbf{B} = \lambda\mathbf{B}$, where λ is constant within each domain. Two distinct eigenvalue problems for λ arise in this formulation, depending on whether fluxes and helicity are fixed, or boundary rotational transforms.

These are studied in cylindrical geometry and in a three-dimensional toroidal region of annular cross section. In the latter case, an application of a residue criterion is used to determine the threshold for connected chaos.

[1] H. Grad, Phys. Fluids **10**, 137 (1967).

[2] O. P. Bruno and P. Laurence, Commun. Pur. Appl. Math. **49**, 1717 (1996).

[3] S. R. Hudson, M. J. Hole, and R. L. Dewar, Phys. Plasmas **14**, 052505 (2007).

*Electronic address: `robert.dewar@anu.edu.au`