Analysis Methods for Trim Coil Design in NCSX

Neil Pomphrey[1], Art Brooks[1], Allen Boozer[2], S.Hudson[1], Michael Zarnstorff[1]

[1]Princeton Plasma Physics Laboratory, Princeton, NJ 08543

[2]Columbia University, New York, NY 10027

Trim coils are included in the National Compact Stellarator Experiment (NCSX) primarily to control resonant field perturbations from primary coil assembly errors, to compensate for field errors generated in ferromagnetic material embedded in the concrete floor of the assembly hall, and to compensate for eddy currents in the 1.5" Modular Coil Winding Form (MCWF) – $\tau_{MCWF} \approx 27$ms. A design requirement for the trim coils is that after compensation the toroidal flux in the island regions should not exceed 10% of the total plasma flux. The trim coils will provide added control of 3D plasma shape and flexibility in physics performance and provide some divertor strike-point control.

Design constraints include a trim coil current rating of 20kA-turns with a requirement that compensation of maximum expected field errors draws at most 10kA-turns. Minimizing coil currents minimizes the coil cross section and cost and minimizes the potential impact of non-resonant fields. For physics flexibility, trim coil geometry was constrained to be stellarator symmetric. However each coil can be independently powered to generate periodicity breaking fields. The total number and shape of trim coils was reduced as much as possible, and the coils were required to fit between the MCWF and TF coils, close to the plasma.

Assembly errors were modeled by applying random displacements and rotations to the Modular coils (18), Toroidal Field coils (18) and Poloidal Field coils (6), constraining monuments to lie within specified assembly tolerances (M coils: ± 1.5 mm, TF&PF coils ± 3.0 mm). Magnetic fields were calculated for up to 10^6 assembly realizations and resonant radial B_{mn}'s calculated at low-order rational surfaces of selected VMEC equilibria. Fields from the magnetized steel and MCWF eddy currents after $2\tau_{MCWF}$ were evaluated and act as known background sources. Response matrices relating candidate trim coil-set currents to B_{mn}'s were evaluated and coil currents, I_j, sought which minimize the resonant field components and hence island size.

Two analysis algorithms were adopted for obtaining coil current solutions. The first is linear, using Singular Value Decomposition (SVD). Here, the residual targeted B_{mn}'s are minimized without active constraints on the I_j. If $I_{max} = \mathrm{Max}_j\{I_j\}$ exceeds 10kA, all I_j are scaled by $10/I_{max}$ and island size is re-calculated. In general, the SVD solution will therefore have a single I_j equal to 10kA. The second method uses non-linear optimization, minimizing the B_{mn}'s with $I_{max} \leq 10$kA as an active constraint. In general the solution will have several I_j equal to 10kA. Total kA-turns is larger than the SVD solution but residual island size is less.

Several trim coil-sets were considered with the design evolving to a final system (see Fig. 1) comprising 48 planar coils with two unique coil shapes, arranged in a top/bottom symmetric half-period pattern, each coil having 11x11=121 turns. The 48 coil configuration meets the design objectives with 100% margins at the 95% level of the Monte-Carlo assembly simulations – total island size is reduced to 3.1%, achieved with a max coil current of 10.6kA-turns. Fig. 2 illustrates field error compensation for one member of the Monte Carlo ensemble. Residual islands with m/n=7/3 and 10/6 were not targeted in the calculations.

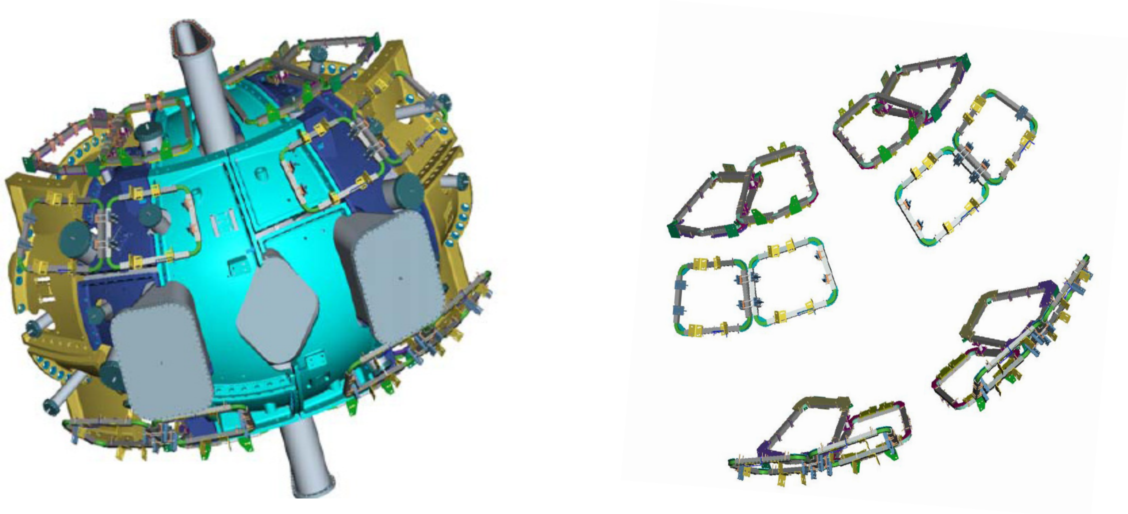

Fig.1: Final trim coil design for NCSX: 48 planar coils, 2 types.

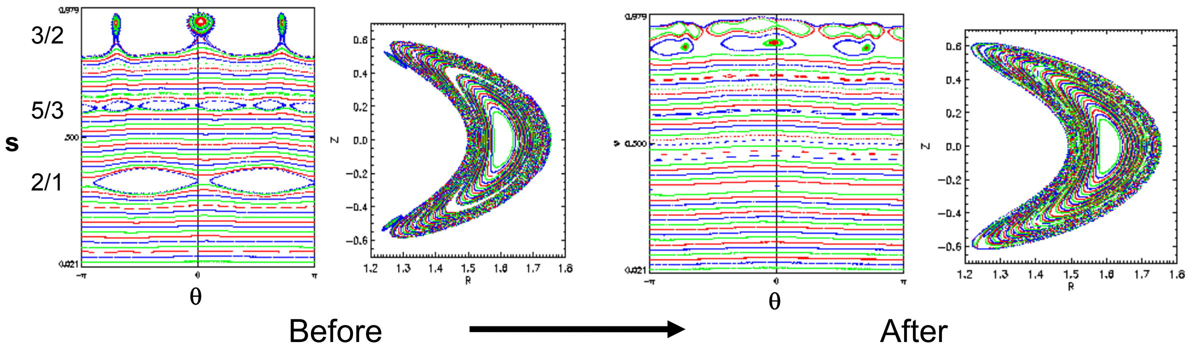

Fig. 2: Field line following before and after applying trim currents

The above approach for designing trim coils for minimizing magnetic perturbations that drive islands on low order rational surfaces is traditional in the sense that it is a superposition analysis that does not take into account the modification to the plasma equilibrium by the applied fields. A comparison of theory and experiment by J-K Park et al[1] has shown that omitting perturbed plasma currents from analysis can be problematic. In the remainder of this paper we investigate the ability of the NCSX trim coils to minimize deformations of VMEC flux surfaces caused by field errors, thereby maintaining the designed physics. The method, based on free-boundary equilibrium calculations, takes account of perturbed currents.

For the trim coil analysis it is useful to consider three response matrices – $\overleftrightarrow{D}$, $\overleftrightarrow{B}$ and $\overleftrightarrow{T}$. Matrix $\overleftrightarrow{D}$ gives the normal displacements of some plasma surface produced by distributions of normal magnetic field at that surface, $\vec{\delta}_p = \overleftrightarrow{D} \cdot \vec{b}$. Columns of $\overleftrightarrow{D}$ are calculated from VMEC output and therefore take account of perturbed equilibrium currents. Matrix $\overleftrightarrow{B}$ measures the normal fields on the plasma surface produced by equally likely displacements, $\vec{\delta}_c$, of the currents in the primary coils due to construction errors. Construction errors give a magnetic perturbation on the plasma surface of $\vec{b}_c = \overleftrightarrow{B} \cdot \vec{\delta}_c$. Matrix $\overleftrightarrow{T}$ gives the normal field distributions that can be produced by a candidate trim coil-set. The trim coils produce normal fields on the plasma surface of $\vec{b}_T = \overleftrightarrow{T} \cdot \vec{I}$ where $\vec{I}$ are the currents in the coil-set. The displacement of the plasma surface is $\vec{\delta}_p = \overleftrightarrow{D}\cdot(\overleftrightarrow{B}\cdot\vec{\delta}_c + \overleftrightarrow{T}\cdot\vec{I})$ and we seek to minimize $\vec{\delta}_p^{\,\dagger}\cdot\vec{\delta}_p$. The answer is $\vec{I} = -\overleftrightarrow{\Lambda}^{-1}\cdot(\overleftrightarrow{D}\cdot\overleftrightarrow{T})^{\dagger}\cdot(\overleftrightarrow{D}\cdot\overleftrightarrow{B})\cdot\vec{\delta}_c$ where $\overleftrightarrow{\Lambda} = (\overleftrightarrow{D}\cdot\overleftrightarrow{T})^{\dagger}\cdot(\overleftrightarrow{D}\cdot\overleftrightarrow{T})$ and $\overleftrightarrow{\Lambda}^{-1}$ is calculated by truncated SVD to trade between I_{max} and displacement residual.

No. Eigenmodes of $D{\cdot}T$ retained	I_{max} [kA-t]	I_{rms}[kA-t]	Fraction of max surface displacement compensated for
1	6.3	2.8	0.40
2	6.7	3.0	0.63
3	8.1	3.9	0.70
4	9.1	4.1	0.99

Table 1: Max and RMS trim coil currents, and fraction of the VMEC surface displacement successfully compensated. Values at 95% level in ensemble of 10^4 assembly realizations.

Table 1 shows results for error compensation of 10^4 assembly realizations of modular coils using a 1.5mm assembly tolerance. Compensation of the assembly errors, including the effect of perturbed currents, is essentially complete with 4 modes of $D{\cdot}T$ retained, and achieved with coil currents less than 10kA-turns. Figure 3 is a plot of the dominant eigen-displacement (1^{st} column of $\overleftrightarrow{U}$ in the decomposition $\overleftrightarrow{D} = \overleftrightarrow{U}\cdot\overleftrightarrow{W}\cdot\overleftrightarrow{V}^{T}$) for the ι = 0.5 surface for a perturbed full beta, full current NCSX equilibrium. The regions most sensitive to deformation by the perturbed fields are near the tips, as one might expect from simple $\nabla\psi$ arguments.

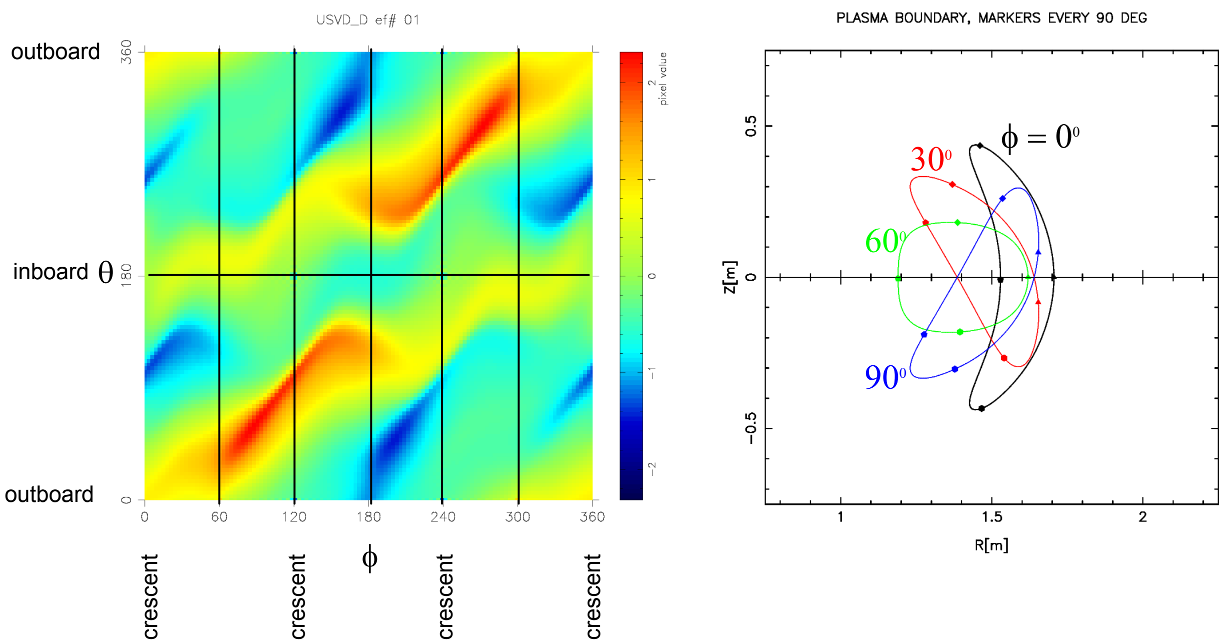

Fig. 3: Dominant eigen-displacement for ι = 0.5 surface as function of θ, ϕ. The right hand plot is an aid for determining the location of peaks and valleys of the displacement: marks appear at 90^0 poloidal θ increments at 4 toroidal ϕ sections. θ=0 for the VMEC angle is marked with a triangle.

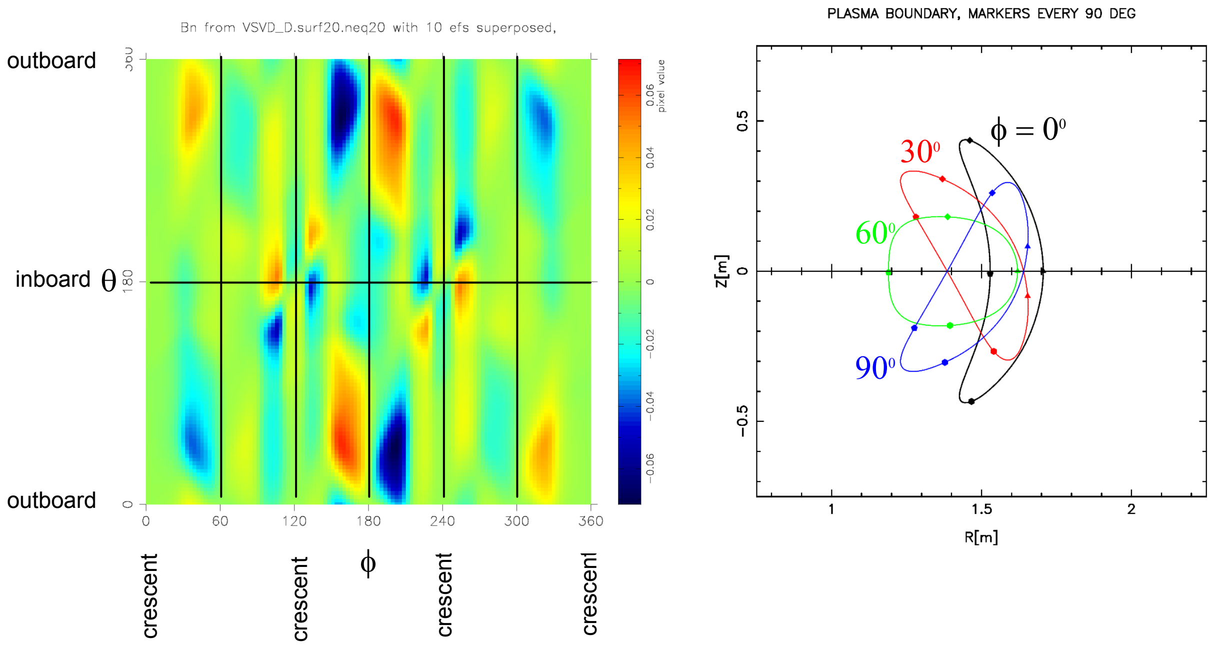

Fig. 4: Dominant eigen-Bfield pattern for ι = 0.5 surface as function of θ, ϕ.

A plot of the dominant eigen-Bfield on the plasma surface (1st column of $\vec{V}$) shows the B-normal distribution most responsible for the deformation – see Fig. 4. Regions of large amplitude (red and blue) suggest efficient regions for placing proximate coils to minimize the surface deformation. Prominent features are seen near the bullet cross sections off the outboard midplane. From the success of nulling the surface deformations these are adequately "covered" by the outboard inclined trim coils (Fig.1). Smaller features appear on the inboard side of the plasma especially near the crescent cross section. These are in agreement with some interesting recent perturbed equilibrium studies by C. Nuehrenberg (personal communication) using the CAS3D stability code, a useful tool for addressing the effects of perturbed currents on island suppression[2].

The NCSX trim coils have no inboard or outboard members which straddle the midplane (Fig. 1). Midplane coils were considered as variants in the design studies. They were found to contribute a minor improvement in the efficiency of nulling magnetic islands. The same conclusion was found in the context of the SVD analysis of VMEC surface deformation: The efficacy of a candidate coil-set is determined by its ability to replicate the important eigen-displacements, $\vec{u}_k$, of response matrix $\overleftrightarrow{D}\cdot\overleftrightarrow{B}$ ($\vec{u}_k$ is the kth column of $\overleftrightarrow{U}$). Candidate coil-sets produce eigen-displacements, $\vec{u}_k^{(c)}$ of $\overleftrightarrow{D}\cdot\overleftrightarrow{T}$. The question is whether they form an adequate basis for the $\vec{u}_k$. We can expand $\vec{u}_j = \sum_k \alpha_k^j \, \vec{u}_k^{(c)} + \vec{\varepsilon}_j^{(c)}$ with $\vec{\varepsilon}_j^{(c)}$ a remainder representing the inability of the coil-set to represent the desired eigenfunction. By construction $\vec{\varepsilon}_j^{(c)}$ is orthogonal to all the $\vec{u}_j^{(c)}$. Orthogonality of the $\vec{u}_j$ provides $|\,\vec{\varepsilon}_j^{(c)}\,| = \sqrt{1-\sum_k (\alpha_k^j)^2}$ where $\alpha_k^j = \vec{u}_j \cdot \vec{u}_k^{(c)}$ and $|\,\vec{\varepsilon}_j^{(c)}\,| \ll 1$ is a requirement for the coil-set to be adequate. Comparing $|\,\vec{\varepsilon}_j^{(c)}\,|$ with and without midplane coils gives an indication of the importance of those coils. Little difference is found.

Work sponsored by DOE grants DE-AC02-76CH03073 and DE-FG02-03ERS496

[1]J-K Park et al., Phys. Rev. Lett. **99**, 195003 (2007)

[2]C. Nuehrenberg et al., 34th EPS Conf. on Plasma Physics, Warsaw, ECA **31F**, P4.065 (2007).