Temperature gradients are supported by cantori in chaotic fields

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With the tantalizing prospect that localized regions of chaotic magnetic field can be used to suppress ideal instabilities in fusion devices, as suggested by the resonant magnetic perturbation (RMP) experiments on DIIID [1], it becomes necessary to understand the impact of chaotic fields on confinement, particularly so considering that RMP fields are being considered as an ELM mitigation strategy for ITER. Using a model of heat transport for illustration, this paper will show that chaotic fields can support significant temperature gradients, despite the fact that flux surfaces may be destroyed by applied error fields. The remnants of the irrational flux surfaces, the cantori [2], present extremely effective *partial*-barriers to field-line transport, and thus present effective barriers to any transport process that is dominantly parallel to the field. We extend the concept of magnetic coordinates to *chaotic* fields, and show that the temperature, generally a function of three-dimensional space, takes the simple form $T(s)$, where s labels the chaotic-coordinate surfaces.

We consider heat transport, as described by

$$\frac{\partial T}{\partial t} = \nabla \cdot \left(\kappa_\parallel \nabla_\parallel T + \kappa_\perp \nabla_\perp T \right), \tag{1}$$

where T is the temperature, t is time, and $\kappa_\parallel$, $\kappa_\perp$ are the (constant) parallel and perpendicular diffusion coefficients. The parallel derivative is given $\nabla_\parallel T = \mathbf{b}\,\mathbf{b}\cdot\nabla T$, where $\mathbf{b} = \mathbf{B}/|B|$, and the perpendicular derivative is $\nabla_\perp T = \nabla T - \nabla_\parallel T$. Steady state solutions forced by inhomogeneous boundary conditions, for a given chaotic magnetic field $\mathbf{B}$, are obtained numerically using finite-differences.

For fusion plasmas the ratio $\kappa_\parallel/\kappa_\perp$ may exceed 10^{10}. The parallel transport dominates and the temperature adapts to the fractal structure of the magnetic field, and is thus difficult to resolve numerically. On KAM surfaces, we may expect that the temperature will be constant. Also, the temperature will flatten across island chains whose width exceeds a critical value, $\Delta w \sim (\kappa_\perp/\kappa_\parallel)^{1/4}$. Within the stochastic sea, where field lines wander *seemingly* randomly over a finite volume, it is tempting to conclude that the strong parallel transport results in a flat temperature profile. For near-threshold chaos however, this is an oversimplification. Irregular trajectories (with finite Lyapunov exponent) may take an impractically long time to leak through the partial barriers. Attempts to determine transport by averaging [3] must also take into account that within the stochastic sea there exists a finite volume of regular motion (the magnetic islands), but what the relative volume of irregular versus regular motion is remains an open question in non-linear dynamics. The point is, chaos is not random.

To show the connection between the fractal structure of the magnetic field and the *near*-fractal structure of the temperature, we develop chaotic magnetic coordinates: coordinates adapted to the invariant structures of the field line flow, the periodic orbits and cantori [4]. These coordinates naturally align with the barriers to transport in chaotic fields and partition space into (i) rational zones, with large islands and temperature flattening, and (ii) irrational zones, the cantori and smaller islands, which support gradients.

Coordinate curves are adapted to the most important (noble-irrational) cantori by constructing a set of ghost-surfaces, surfaces defined by an action-gradient flow between the action-minimax (stable) and action-minimizing (unstable) periodic orbits that approximate the cantori. Shown on the left in Fig.1 are

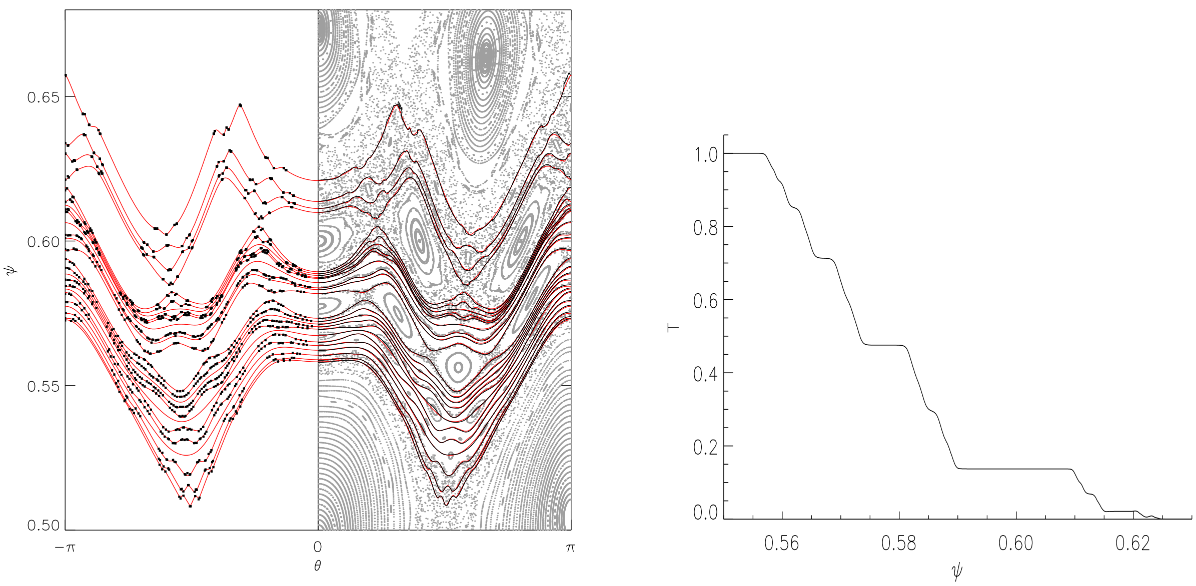

FIG. 1: Left: (Color) For $\theta < 0$: The selected ghost curves (red lines) and cantori (black square dots). For $\theta > 0$: Poincaré plot (gray dots), ghost curves (red lines) and the temperature contours (black lines) for $\kappa_\parallel/\kappa_\perp = 10^{10}$. Right: Temperature profile along line $\theta = 0$.

the selected cantori (black square dots) and the corresponding ghost surfaces (red lines). Just beyond the point at which chaos has destroyed the flux surface, the cantorus is termed near-critical: these cantori present the greatest impediment to field line transport in chaotic fields, and the corresponding ghost-surfaces are almost indistinguishable from the isotherms (superimposed black lines). As the degree of chaos increases, the cantori develop large gaps, through which field lines may easily pass through. Remarkably however, even the strongly super-critical cantori (eg. the top 4 surfaces in Fig.1) and their corresponding ghost-surfaces closely coincide with isotherms.

The parallel diffusion gives a relaxation that is dominantly tangential to the ghost-surfaces, which are "almost-invariant" under the field line flow. Given an optimal selection of ghost-surfaces, labeled by s, the temperature may be written in chaotic coordinates as $T = T_o(s) + \delta T(s, \theta, \phi)$, where $T_o(s)$ is generally a smoothed devil's staircase (flat across rationals with gradients on irrationals; see for example the profile in Fig.1), and δT is small compared to T_o for small $\kappa_\perp/\kappa_\parallel$. Such an expression serves as the basis for simplified theoretical and numerical descriptions of heat transport in chaotic fields.

Despite that all flux surfaces are destroyed for the case considered, the heat flux required to sustain the temperature gradient is enhanced only by a factor of two compared to the integrable case. We conclude that chaotic fields are capable of supporting temperature gradients, and the isotherms coincide with cantori.

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