Equilibrium code consistent with field line chaos.

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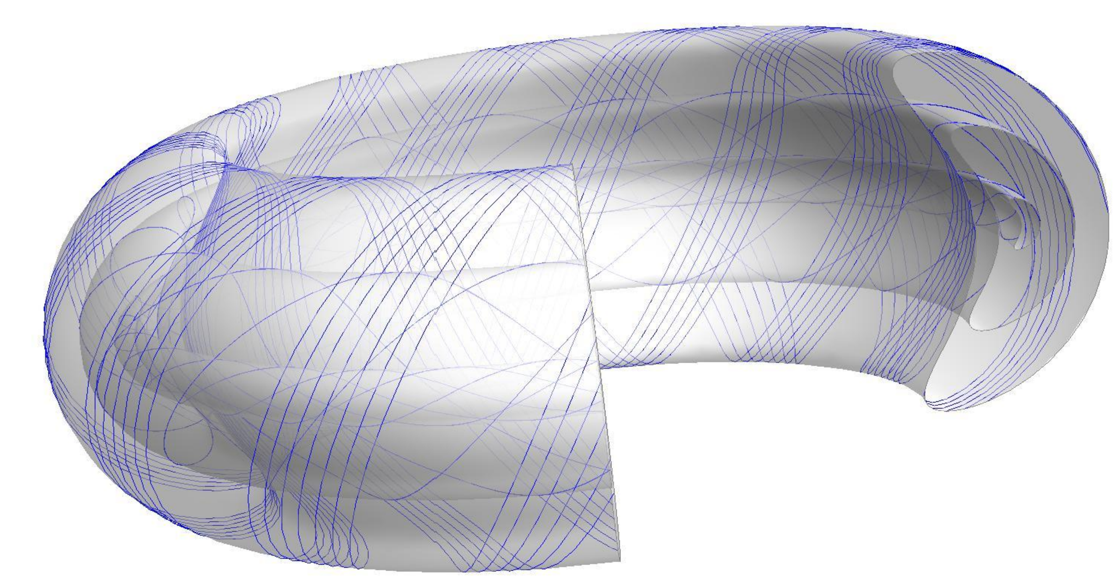
Old codes

Theory and limitations

Relax plasma energy in entire plasma volume.

Assume nested flux surfaces so problem is analytically solvable.

In reality, depending on deformation of plasma boundary, plasma has a fractal structure of flux surfaces and regions of chaos.



SPEC - new code

Stepped Pressure Equilibrium Code

Select a few surfaces most likely to survive perturbations to act as *interfaces* and solve for magnetic field between.

No requirement that field be regular - chaos in these regions allowed.

Fractal structure approached as more interfaces are inserted inside plasma.

Theory

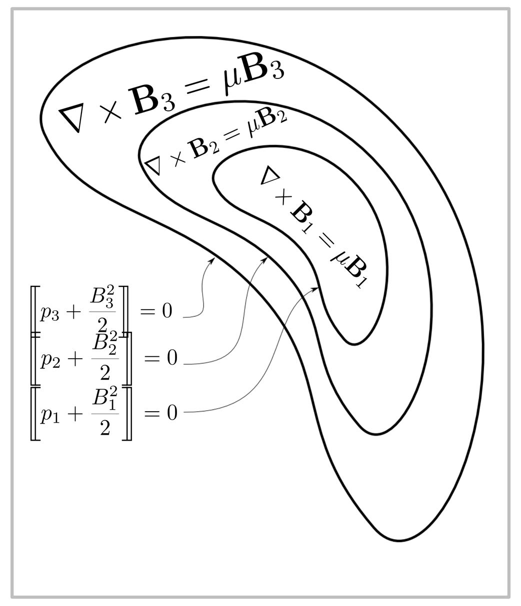
Minimise

$$W = \int_{\mathcal{P}} \frac{B^2}{2\mu_0} + \frac{pV}{\gamma - 1} dV$$

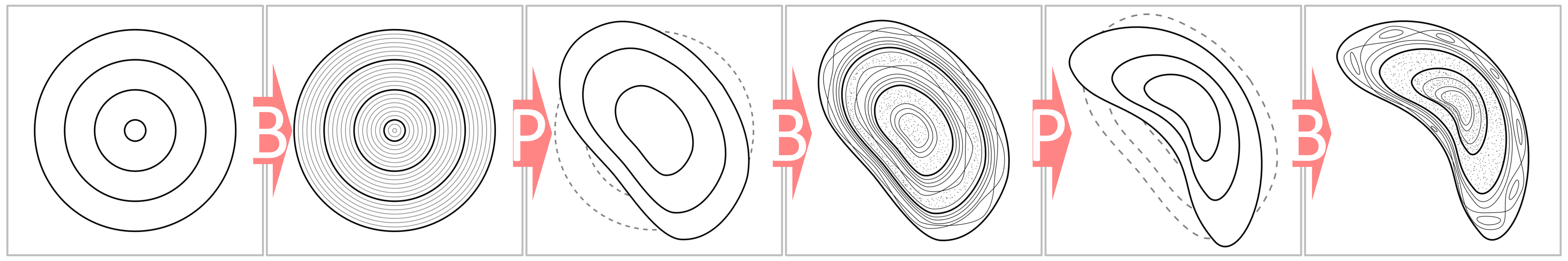
Under constraint of constant helicity

$$K = \int_{\mathcal{P}} \mathbf{A} \cdot \mathbf{B} dV \ .$$

Euler Lagrange Equation for minimisation



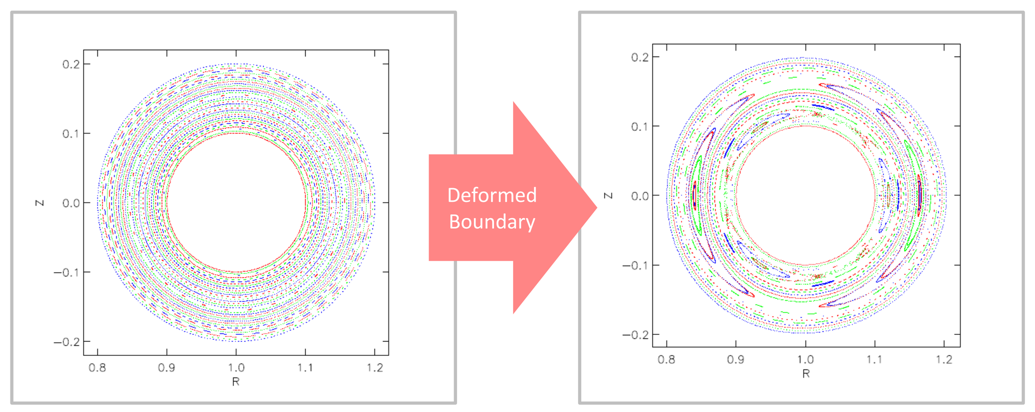
How it will work



B Solve for magnetic field $(\mathbf{B})$ between interfaces

P Warp boundaries to satisfy force balance

Current results



Solves for Beltrami field (Euler-Lagrange equation for energy minimisation) within volumes.

Fully parallelised - the field within each volume calculated simultaneously - increasing number of interfaces does not significantly increase computation time.

To do

Allow for coordinate singularity at magnetic axis

Directly warp Fourier components of decomposed surface to satisfy pressure balance.

Identify flux surfaces to use as interfaces.

References

[1] S. Hudson, M. Hole, R. Dewar. *Phys Plasmas* 14 2007 052505
[2] M. Hole, R. Dewar, S. Hudson. *J. Plasma Physics* 72 2006 p.1167
[3] R. Dewar, M. Hole, M. McGann, R. Mills, S. Hudson. *Entropy* 10 2008 p.621
[4] S. Hudson. *Phys Plasmas* 11 2004 p.667

Critical behaviour in toroidal plasma confinement

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Aim

Investigate the breakup of flux surfaces to determine their usefulness as interfaces in SPEC, and to learn more of the nature of flux surfaces.

The flux surfaces must withstand:

1. the deformation of the plasma, which introduces islands and chaotic volumes and
2. the pressure discontinuity across the surface, which we require in SPEC.

This can be done by investigating the Hamiltonian formulation of

1. The magnetic field line configuration, called the magnetic field line Hamiltonian, and
2. The pressure discontinuity condition, called the pressure jump Hamiltonian

respectively.

Hamiltonian relevence

Do Hamiltonian (phase space) trajectories map directly to field lines (configuration space)?

The relationship between the Hamiltonian formulation and the physical configuration problem is not well understood.

Kaiser and Salat [1] suggested a purely configurational version of the problem, but little information can be extracted from it.

Magnetic field line Hamiltonian

Theory

$\theta =$ angle short way round torus.

$\zeta =$ angle long way round torus.

The magnetic field within a plasma volume can be written as

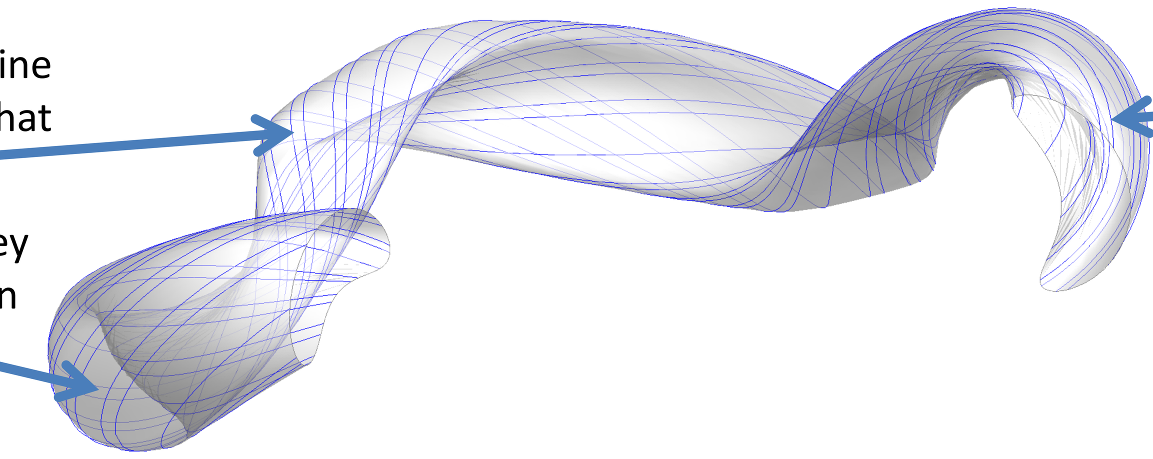
$$\mathbf{B} = \nabla\psi_t \times \nabla\theta + \nabla\zeta \times \nabla\psi_p$$

using $\frac{d\mathbf{r}}{d\zeta} = \mathbf{B}$ gives [2] $\frac{d\theta}{d\zeta} = \frac{\partial\psi_p}{\partial\psi_t}$, $\frac{d\psi_t}{d\zeta} = -\frac{\partial\psi_p}{\partial\theta}$ which are of Hamiltonian form.

Use

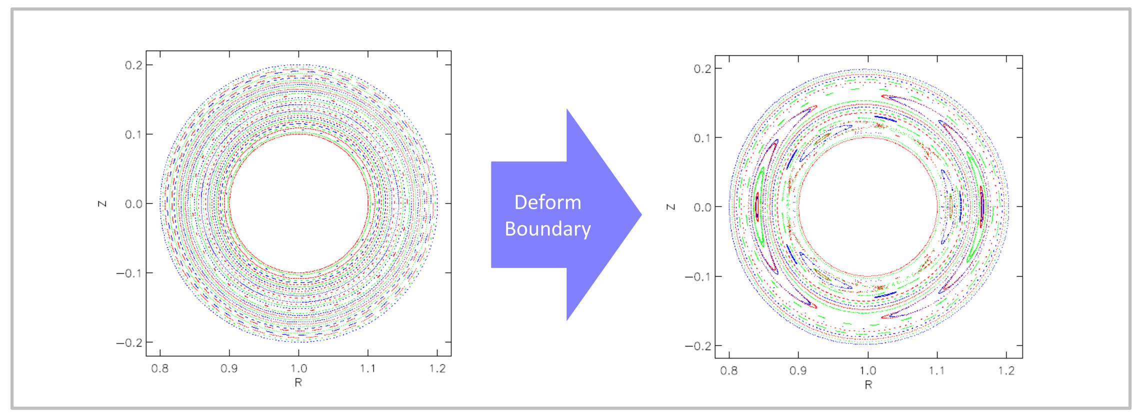
Trajectories of the field line Hamiltonian are the field lines that permeate the plasma.

If the field lines lie on a torus, they draw out a flux surface, which can act as a barrier to transport.



Existence

Increasing deformation tends to destroy flux surfaces.



Pressure jump Hamiltonian

Theory

Consider flux surface has finitely different pressure either side.

The condition of force balance due to pressure discontinuity is [3]

$$\left[\!\left[p + \frac{B^2}{2} \right]\!\right] = 0$$

Which can be shown to be a Hamiltonian Jacobi Equation with Hamiltonian

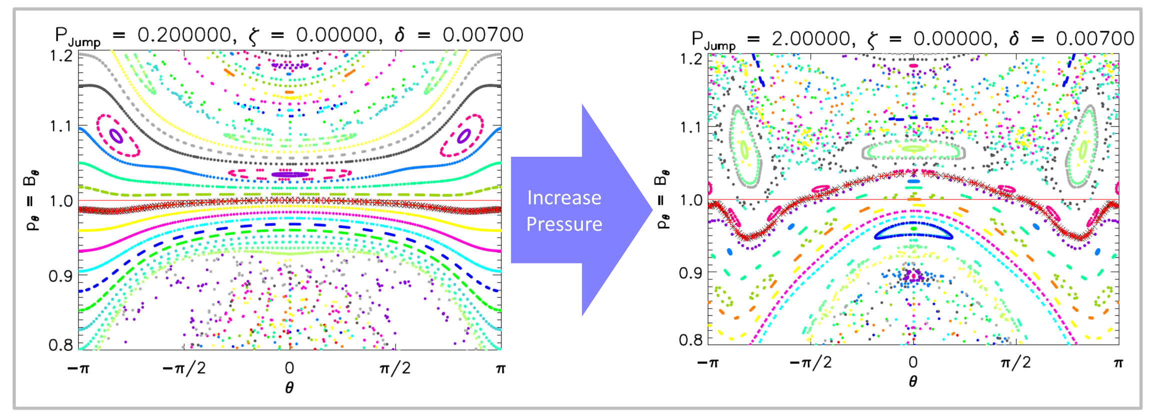
$$H = \sum_{i,j \in [\theta,\zeta]} \frac{1}{2} g^{ij} p_i p_j + V(\theta,\zeta)$$

Use

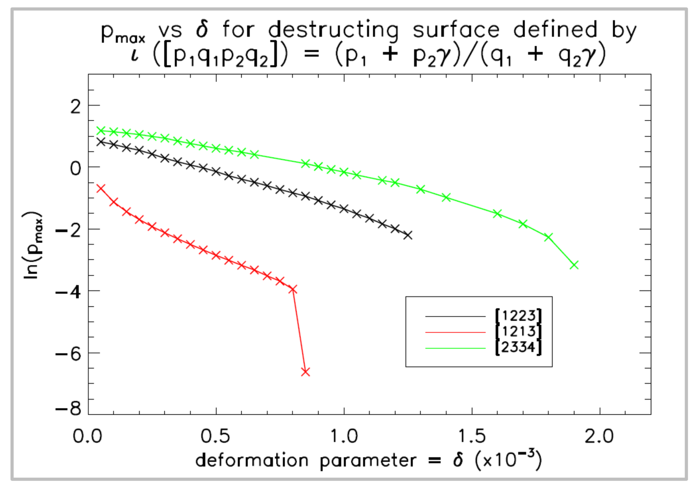
Trajectories of the pressure jump Hamiltonian correspond to the field lines on a given surface that survive a given pressure discontinuity.

Existence

Increasing pressure tends to destroy flux surfaces.



Combine



While MFLH destroys surface, PJH tests it for the maximum pressure discontinuity it can withstand.

Highly dependent on form of deformation and twist of field line.

Different flux surfaces support different pressure for different deformations.

Future Work

Is resilience to pressure shared between classes of surfaces?

Find more natural deformation parameter.

References

[1] R. Kaiser, S. Salat. *Phys Plasmas* 1994
[2] S. Hudson. *Phys Plasmas* 11 2004 p.667
[3] S. Hudson, M. Hole, R. Dewar. *Phys Plasmas* 14 2007 052505