Action-based definitions of almost-invariant tori in close-to-integrable Hamiltonian systems

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Abstract Summary (35 words)

Transport in partially chaotic Hamiltonian systems, such as electrons moving along 3D magnetic fields, is limited by "almost-invariant" tori, which act as transport barriers. Techniques for identifying such surfaces by minimizing action gradients are presented.

Keywords-nonlinear dynamics, chaotic transport, toroidal plasma confinement, passive advection

I. INTRODUCTION

One and a half degree-of-freedom (d.o.f.) Hamiltonians (or Lagrangians) are explicitly periodic in time *t*. Physical systems where they are arise are periodically stirred incompressible planar flows and motion along toroidal magnetic fields in non-axisymmetric plasma confinement systems.

A frequently used 1½ d.o.f. "toy model" is the kicked rotor, the dynamics of which can be reduced to an iterated area-preserving map.

An important problem in such systems is calculating the transport caused by the partially chaotic dynamics. For example, the transport of a passively advected scalar in the stirred fluid, or the transport of heat along magnetic field lines in a toroidally confined plasma [1].

In near-integrable but chaotic Hamiltonian systems, invariant (KAM) and "almost-invariant" tori act as transport barriers where the temperature, for instance, changes very rapidly. This paper discusses the optimal way to calculate these barriers.

II. ACTION GRADIENT APPROACH

A statement of Hamilton's Principle is that the infinite-dimensional action gradient, in the space of all conceivable paths, vanishes on physical paths. Thus the action gradient is zero on an invariant torus, and is small on an almost-invariant torus. This observation gives rise to two strategies for defining an almost-invariant torus: the *quadratic flux minimization* approach, which minimizes the action gradient in least squares as a trial surface is varied [2,3], and the *ghost surface* approach [3], which joins action-minimax and action-minimizing periodic orbits via an action-gradient flow.

Recent work [4] indicates these two approaches can be reconciled by appropriate choice of angle coordinate. Progress on this research will be reported.

REFERENCES

[1] S.R. Hudson and J. Breslau, Phys. Rev. Lett. **100**, 095001 (2008)

[2] R.L. Dewar and J.D. Meiss, Physica D **57**, 476 (1992)

[3] S.R. Hudson and R.L. Dewar, Phys. Lett. A **373**, 4409 (2009)

[4] R.L. Dewar and S.R. Hudson and A.M. Gibson, *Unified Theory of Ghost and Quadratic-Flux-Minimizing Surfaces*, accepted for publication in J. Plasma Fusion Research SERIES (2010), http://arxiv.org/abs/1001.0483