Partially-relaxed, partially-constrained MHD equilibria.

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The commonly used equation of ideal force balance, $\nabla p = \mathbf{j} \times \mathbf{B}$, is pathological when the magnetic field, $\mathbf{B}$, is chaotic. Any continuous non-trivial pressure that satisfies $\mathbf{B} \cdot \nabla p = 0$ with a chaotic field will have an infinity of discontinuities in the pressure gradient. The perpendicular current $\mathbf{j}_\perp = \mathbf{B} \times \nabla p / B^2$ is either zero or discontinuous, and $\nabla \cdot \mathbf{j}_\perp$ is zero or not defined. This pathological structure causes problems for the so-called Spitzer iterative approach, which is fundamentally ill-posed as it depends on inverting magnetic differential equations, e.g. $\mathbf{B} \cdot \nabla (j_\parallel / B) = -\nabla \cdot \mathbf{j}_\perp$, and such equations have a dense set of singularities. We suggest instead a well-posed equilibrium construction based on an extension of Taylor relaxation: a weakly-resistive plasma will relax to minimize the plasma energy subject to the constraint of conserved helicity. To obtain non-trivial pressure profiles we add additional topological constraints on a selection of KAM surfaces on which the constraints of ideal MHD are imposed.

Consider a plasma region comprised of a set of N nested annular regions which are separated by a discrete set of toroidal interfaces, $\mathcal{I}_l$. In each volume, $\mathcal{V}_l$, bounded by the $\mathcal{I}_{l-1}$ and $\mathcal{I}_l$ interfaces, the plasma energy, U_l, the global-helicity, H_l, and the "mass", M_l, are given by the integrals:

$$U_l = \int_{\mathcal{V}_l} \left(\frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv, \qquad H_l = \int_{\mathcal{V}_l} \mathbf{A} \cdot \mathbf{B} \, dv, \qquad M_l = \int_{\mathcal{V}_l} p^{1/\gamma} \, dv, \tag{1}$$

where $\mathbf{A}$ is the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$.

The equilibrium states that we seek minimize the total plasma energy, subject to the constraints of conserved helicity and mass in each annulus. Arbitrary variations in both the magnetic field in each annulus and the geometry of the interfaces are allowed, except that we assume the magnetic field remains tangential to the interfaces which act as ideal barriers and coincide with pressure gradients. The Euler-Lagrange equations show that in each annulus the magnetic field satisfies $\nabla \times \mathbf{B} = \mu_l \mathbf{B}$, and across each interface the total pressure is continuous, $[[p + B^2/2]] = 0$.

We have implemented this model in a code, the Stepped Pressure Equilibrium Code (SPEC), which uses a mixed Fourier, finite-element representation for the vector potential. Quintic polynomial basis functions give rapid convergence in the radial discretization, and the freedom in the poloidal angle is exploited to minimize a "spectral-width". For given interface geometries the Beltrami fields in each annulus are constructed in parallel, and a Newton method (with quadratic-convergence) is implemented to adjust the interface geometry to satisfy force-balance. Convergence studies and three-dimensional equilibrium solutions with non-trivial pressure and islands and chaotic fields will be presented.