Nonlinearly perturbed MHD equilibria, with or without magnetic islands.

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Toroidal magnetic plasma confinement devices, even when nominally axisymmetric (e.g. tokamaks), unavoidably have small nonaxisymmetric imperfections. In this paper we compute the nonlinear response of an axisymmetric plasma to a small nonaxisymmetric perturbation of the boundary which is chosen to resonate with an internal rational surface. This task is equivalent to computing the three-dimensional (3D) equilibrium consistent with the perturbed plasma boundary and the given pressure, p, and rotational-transform (inverse-safety-factor), $\iota\!\!\!-$, profiles.

Non-symmetric perturbations destroy the smoothly nested family of flux surfaces present in perfectly axisymmetric configurations, and 3D magnetic fields, $\mathbf{B}$, are generally nonintegrable. However, the topology of the magnetic fields consistent with 3D plasma equilibrium solutions are not known *a priori* because they are partially self-generated by plasma currents, which are only determined as part of a consistent equilibrium calculation. In computing the 3D equilibrium, the constraints of ideal MHD *may*, or *may not*, be applied, either locally or globally.

The most elegant approaches for computing MHD equilibria begin with an energy principle. Our model is based on a constrained energy principle that combines ideal MHD and Taylor relaxation theory. The plasma region is comprised of a set of N nested annular regions which are separated by a discrete set of toroidal interfaces, $\mathcal{I}_l$. In each volume, $\mathcal{V}_l$, bounded by the $\mathcal{I}_{l-1}$ and $\mathcal{I}_l$ interfaces, the plasma energy, U_l, the global-helicity, H_l, and the "mass", M_l, are given by the integrals

$$U_l = \int_{\mathcal{V}_l} \left(\frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv, \qquad H_l = \int_{\mathcal{V}_l} \mathbf{A} \cdot \mathbf{B} \, dv, \qquad M_l = \int_{\mathcal{V}_l} p^{1/\gamma} \, dv, \tag{1}$$

where $\mathbf{A}$ is the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$.

The equilibrium states that we seek minimize the total plasma energy, subject to the constraints of conserved helicity and mass in each annulus. Arbitrary variations in both the magnetic field in each annulus and the geometry of the interfaces are allowed, except that we assume the magnetic field remains tangential to the interfaces which act as ideal barriers and coincide with pressure gradients. The Euler-Lagrange equations show that in each annulus the magnetic field satisfies $\nabla \times \mathbf{B} = \mu_l \mathbf{B}$, and across each interface the total pressure is continuous, $[[p + B^2/2]] = 0$.

We have implemented this model in a code, the Stepped Pressure Equilibrium Code (SPEC), which uses a mixed Fourier, finite-element representation for the vector potential. Quintic polynomial basis functions give rapid convergence in the radial discretization, and the freedom in the poloidal angle is exploited to minimize a "spectral-width". For given interface geometries the Beltrami fields in each annulus are constructed in parallel, and a Newton method (with quadratic-convergence) is implemented to adjust the interface geometry to satisfy force-balance. Convergence studies and three-dimensional equilibrium solutions with non-trivial pressure and islands and chaotic fields will be presented.

We present calculations of perturbed, ITER relevant equilibria. The ideal interfaces may be chosen to coincide with rational surfaces, in which case surface currents arise to shield out the perturbation and prevent island formation. If, instead, the ideal constraint is relaxed at the rational surfaces, the plasma will relax and reconnect and an island chain will form.