Partially-relaxed, topologically-constrained MHD equilibria with chaotic fields.

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Motivation and Outline

→ The simplest model of approximating global, macroscopic force balance in toroidal plasma confinement with arbitrary geometry is magnetohydrodynamics (MHD)

→ Toroidal magnetic fields are analogous to 1-1/2 Hamiltonians, are generally ***not*** foliated by continuous family of flux surfaces, so we need an MHD equilibrium code that allows for *non-integrable* fields.

→ Ideal MHD equilibria with non-integrable magnetic fields (i.e. fractal phase space) are infinitely discontinuous. This leads to an ill-posed numerical algorithm for computing numerically-intractable, *pathological* equilibria.

→ A new partially-relaxed, topologically-constrained MHD equilibrium model is described and implemented numerically. Results demonstrating convergence tests, benchmarks, and non-trivial solutions are presented.

An ideal equilibrium with non-integrable (*chaotic*) field and *continuous* pressure, is *infinitely discontinous*

ideal MHD theory = $\nabla p = \mathbf{j} \times \mathbf{B}$, gives $\mathbf{B} \cdot \nabla p = 0$

→ transport of pressure along field is "infinitely" fast
→ no scale length in ideal MHD
→ pressure adapts exactly to structure of phase space

chaos theory = *nowhere are flux surfaces continuously nested*

*for non-symmetric systems, nested family of flux surfaces is destroyed;

*islands & irregular field lines appear where transform is rational (n/m); rationals are dense in space;

Poincare-Birkhoff theorem $\rightarrow$ periodic orbits, (e.g. stable and unstable) guaranteed to survive into chaos

*some irrational surfaces survive if there exists an $r, k \in \Re$ s.t. for all rationals, $|\iota - n/m| > r\, m^{-k}$,

i.e. rotational-transform, ι, is *poorly approximated* by rationals,

Kolmogorov, Arnold and Moser, Diophantine condition

but nowhere are smooth flux surfaces continuously nested, i.e. nowhere foliate space;

ideal MHD theory + chaos theory $\rightarrow$ infinitely discontinuous equilibrium

*iterative method for calculating equilibria is ill-posed;

1) $\mathbf{B}_n \cdot \nabla p = 0 \rightarrow \nabla p$ is everywhere discontinuous, or zero;

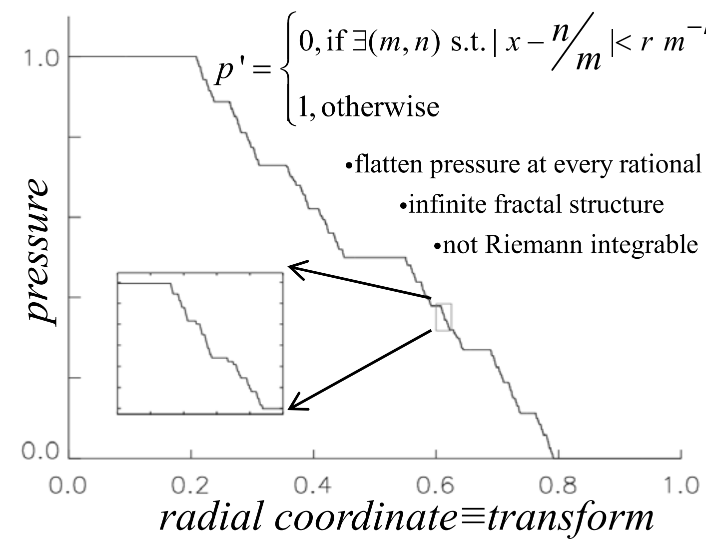
2) $\mathbf{j}_\perp = \mathbf{B}_n \times \nabla p / B_n^{\,2} \quad \rightarrow \mathbf{j}_\perp$ everywhere discontinuous or zero;

3) $\mathbf{B}_n \cdot \nabla \sigma = -\nabla \cdot \mathbf{j}_\perp$; $\quad \mathbf{B} \cdot \nabla$ is *densely and irregularly* singular;

condition that σ be single valued $\delta\sigma = -\oint_C \nabla \cdot \mathbf{j}_\perp dl / B = 0$;

pressure must be flat on every closed field line, or parallel current is not single-valued;

4) $\nabla \times \mathbf{B}_{n+1} = \mathbf{j} \equiv \sigma \mathbf{B}_n + \mathbf{j}_\perp$;

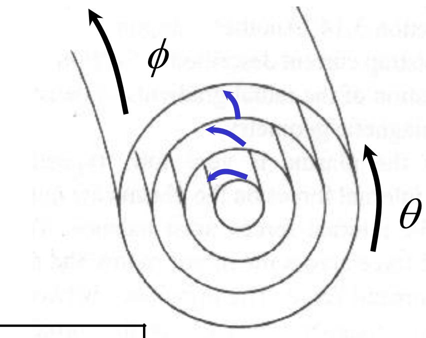


To have a well-posed equilibrium with chaotic **B** need to extend beyond ideal MHD.

e.g. introduce non-ideal terms, such as resistivity, η, perpendicular diffusion, $\kappa_\perp$, [*HINT*, *M3D*, *NIMROD*, ..],

$\rightarrow$ or can relax infinity of ideal MHD constraints

Instead, a multi-region, relaxed energy principle for MHD equilibria with non-trivial pressure and chaotic fields



Energy, helicity and mass integrals (defined in nested annular volumes)

$$\underbrace{W_l = \int_{V_l} \left(\frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv}_{\text{plasma energy}}, \quad \underbrace{H_l = \int_{V_l} (\mathbf{A} \cdot \mathbf{B})\, dv}_{\text{helicity}}, \quad \underbrace{M_l = \int_{V_l} p^{1/\gamma} dv}_{\text{mass}}$$

Seek extrema of plasma energy with constraints : $\boxed{F = \sum_{l=1}^{N} \left(W_l - \mu_l H_l / 2 - \nu_l M_l \right)}$

First variation due to *unconstrained* variations in pressure, fields and geometry

except ideal constraint $\delta \mathbf{B} = \nabla \times (\xi \times \mathbf{B})$ *imposed discretely* at interfaces

$$\delta F = \sum_{l=1}^{N} \Big\{ \int_{V_l} \underbrace{\left(\frac{1}{\gamma - 1} - \frac{\nu_l p^{1/\gamma - 1}}{\gamma} \right)}_{\substack{\nu p^{1/\gamma} = \gamma p/(\gamma-1) = const. \\ \text{in each annulus}}} \delta p \, dv + \underbrace{\int_{V_l} \delta \mathbf{A} \cdot (\nabla \times \mathbf{B} - \mu_l \mathbf{B})\, dv}_{\nabla \times \mathbf{B} = \mu_l \mathbf{B} \text{ in each annulus}} - \int_{\partial V_l} \underbrace{[[p + B^2/2]]}_{\substack{\text{continuity of total pressure} \\ \text{across interfaces}}} \xi . d\mathbf{S} \Big\}$$

Equilibrium solutions when $\nabla \times \mathbf{B} = \mu_l \mathbf{B}$ in annuli, $[[p+B^2/2]]=0$ across interfaces

$\rightarrow$ partial *Taylor relaxation* allowed in each annulus; allows for topological variations/islands/chaos;

$\rightarrow$ global relaxation prevented by ideal constraints; $\rightarrow$ non-trivial *stepped – pressure* solutions;

$\rightarrow$ $\nabla \times \mathbf{B} = \mu_l \mathbf{B}$ is a linear equation for $\mathbf{B}$; depends on interface geometry; solved in parallel in each annulus;

$\rightarrow$ solving force balance $\equiv$ adjusting interface geometry to satisfy $[[p+B^2/2]]=0$;
standard numerical problem finding zero of multi-dimensional function;
call NAG routine: Newton & convex gradient method;

Existence of Three-Dimensional Toroidal MHD Equilibria with Nonconstant Pressure

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We establish an existence result for the three-dimensional MHD equations

$$
\begin{aligned}
(\nabla \times \mathbf{B}) \times \mathbf{B} &= \nabla p \\
\nabla \cdot \mathbf{B} &= 0 \\
\mathbf{B} \cdot n|_{\partial T} &= 0
\end{aligned}
$$

with $p \neq$ const in tori T *without symmetry*. More precisely, our theorems insure the existence of sharp boundary solutions for tori whose departure from axisymmetry is sufficiently small; they allow for solutions to be constructed with an arbitrary number of pressure jumps. © 1996 John Wiley & Sons, Inc.

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→ ***how large the "sufficiently small" departure from axisymmetry can be needs to be explored numerically . . . .***

<u>By *definition*, an equilibrium code must constrain topology;</u>

<u>$\mathbf{B}\cdot\nabla p=0$ means flux surfaces *must* coincide with pressure gradients.</u>

Definition: Equilibrium Code (fixed boundary)

given (1) boundary (2) pressure (3) rotational-transform $\equiv$ inverse q-profile (or current profile);

$\rightarrow$ calculate $\mathbf{B}$ that is consistent with force-balance; pressure profile *is not changed*!

compare with "coupled equilibrium-transport" algorithm:

$\rightarrow$ simultaneously evolve pressure, etc. , while adjusting $\mathbf{B}$;

<u>An equilibrium code must enforce topological constraints;</u>

$\rightarrow$ Parallel transport $\gg$ perpendicular transport; simplest approximation $\mathbf{B}\cdot\nabla p=0$;

$\rightarrow$ The constraint $\mathbf{B}\cdot\nabla p=0$ means the structure of $\mathbf{B}$ and p are intimately connected;

<u>*cannot apriori specify pressure without apriori constraining topology of the field;</u>

$\rightarrow$ pressure gradients must coincide with flux surfaces;

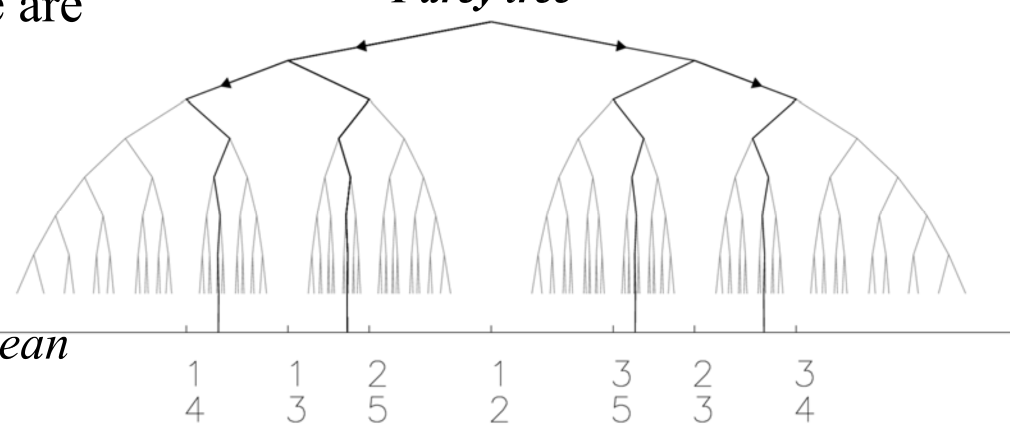
$\rightarrow$ the flux surfaces most likely to survive are strongly irrational $\equiv$ "*noble*";

$\equiv$ limit of alternating path down Farey-tree;

$\equiv$ Fibonacci sequence

$$\frac{p_1}{q_1},\frac{p_2}{q_2},\frac{p_1+p_2}{q_1+q_2},\ldots \rightarrow \frac{p_1+\gamma p_2}{q_1+\gamma q_2},\ \gamma = golden\ mean$$

Farey tree



Extrema of energy functional obtained numerically; introducing the Stepped Pressure Equilibrium Code, SPEC

The vector-potential is discretized

* toroidal coordinates (s,ϑ,ζ), *interface geometry $R_l=\sum_{m,n}R_{l,m,n}\cos(m\vartheta-n\zeta),\, Z_l=\sum_{m,n}Z_{l,m,n}\sin(m\vartheta-n\zeta)$
* exploit gauge freedom $\mathbf{A}=A_\vartheta(s,\vartheta,\zeta)\nabla\vartheta+A_\zeta(s,\vartheta,\zeta)\nabla\zeta$
* Fourier $A_\vartheta=\sum_{m,n}a_\vartheta(s)\cos(m\vartheta-n\zeta)$
* Finite-element $a_\vartheta(s)=\sum_i a_{\vartheta,i}(s)\varphi(s)$ ***piecewise cubic or quintic basis polynomials***

and inserted into constrained-energy functional $F=\sum_{l=1}^{N}\left(W_l-\mu_l H_l/2-\nu_l M_l\right)$

* derivatives w.r.t. vector-potential $\rightarrow$ linear equation for Beltrami field $\nabla\times\mathbf{B}=\mu\mathbf{B}$ ***solved using sparse linear solver***
* field in each annulus computed independently, distributed across multiple cpus
* field in each annulus depends on enclosed toroidal flux (boundary condition) and
  $\rightarrow$ poloidal flux, ψ_P, and helicity-multiplier, μ ***adjusted so interface transform is strongly irrational***
  $\rightarrow$ geometry of interfaces, $\xi\equiv\{R_{m,n},Z_{m,n}\}$

Force balance solved using multi-dimensional Newton method.

* interface geometry is adjusted to satisfy force $\mathbf{F}[\xi]\equiv\{[[p+B^2/2]]_{m,n}\}=0$
* angle freedom constrained by spectral-condensation, adjust angle freedom to minimize $\sum m^2\left(R_{mn}^2+Z_{mn}^2\right)$
* derivative matrix, $\nabla\mathbf{F}[\xi]$, computed in parallel using finite-differences ***minimal spectral width [Hirshman, VMEC]***
* call NAG routine: quadratic-convergence w.r.t. Newton iterations; robust convex-gradient method;

Numerical error in Beltrami field scales as expected

Scaling of numerical error with radial resolution depends on finite-element basis

$\mathbf{A} = A_\vartheta \nabla\vartheta + A_\zeta \nabla\zeta, \quad \mathbf{B} = \nabla\times\mathbf{A}, \quad \mathbf{j} = \nabla\times\mathbf{B},$ need to quantify error $\mathbf{j} - \mu\mathbf{B}$

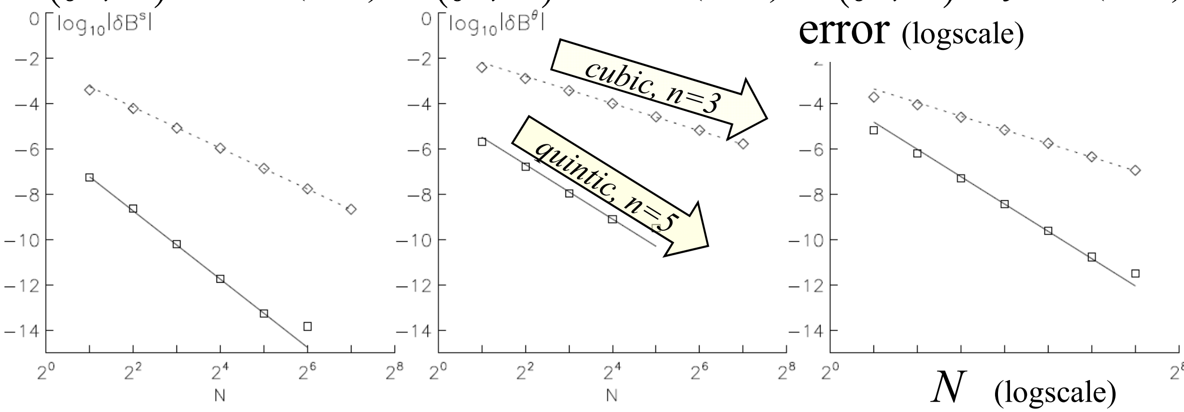
$A_\vartheta, A_\zeta \sim O(h^n)$

h = radial grid size = 1 / N
n = order of polynomial

$$\sqrt{g}B^s = \partial_\vartheta A_\zeta - \partial_\zeta A_\vartheta \sim O(h^n)$$
$$\sqrt{g}B^\vartheta = \qquad - \partial_s A_\zeta \sim O(h^{n-1})$$
$$\sqrt{g}B^\zeta = \partial_s A_\vartheta \qquad \sim O(h^{n-1})$$

$$\sqrt{g}\,j^s \sim O(h^{n-1})$$
$$\sqrt{g}\,j^\vartheta \sim O(h^{n-2})$$
$$\sqrt{g}\,j^\zeta \sim O(h^{n-2})$$

$(\mathbf{j} - \mu\mathbf{B})\bullet\nabla s \sim O(h^{n-1})$ $\quad(\mathbf{j} - \mu\mathbf{B})\bullet\nabla\vartheta \sim O(h^{n-2})$ $\quad(\mathbf{j} - \mu\mathbf{B})\bullet\nabla\zeta \sim O(h^{n-2})$



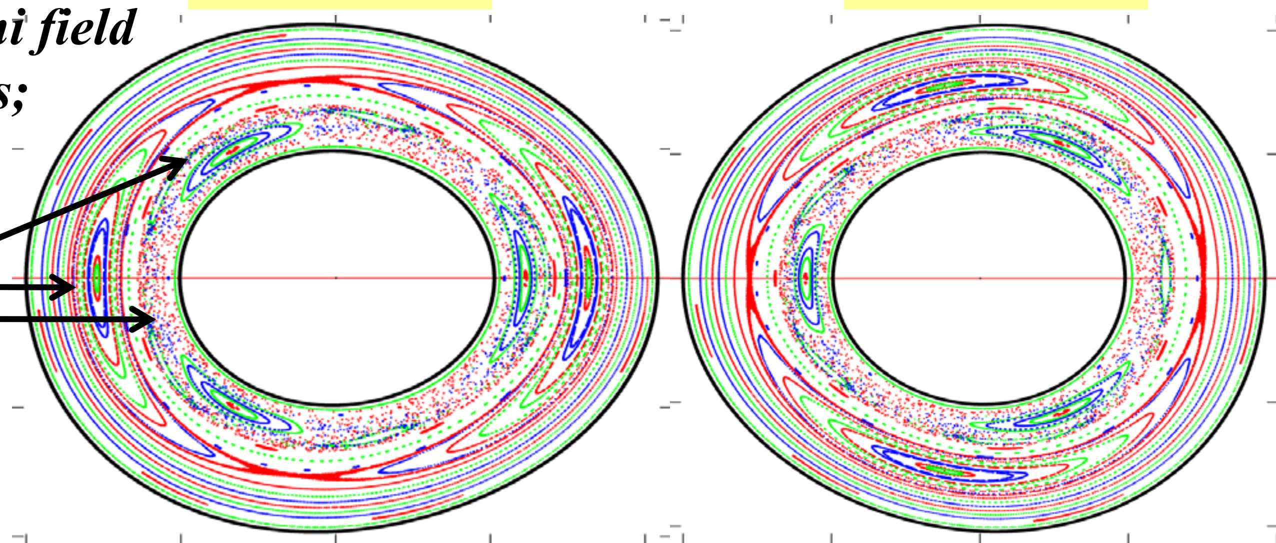
Example of chaotic Beltrami field in single given annulus;

$R = 1.0 + r(\vartheta,\zeta)\cos\vartheta,$
$Z = \qquad r(\vartheta,\zeta)\sin\vartheta,$

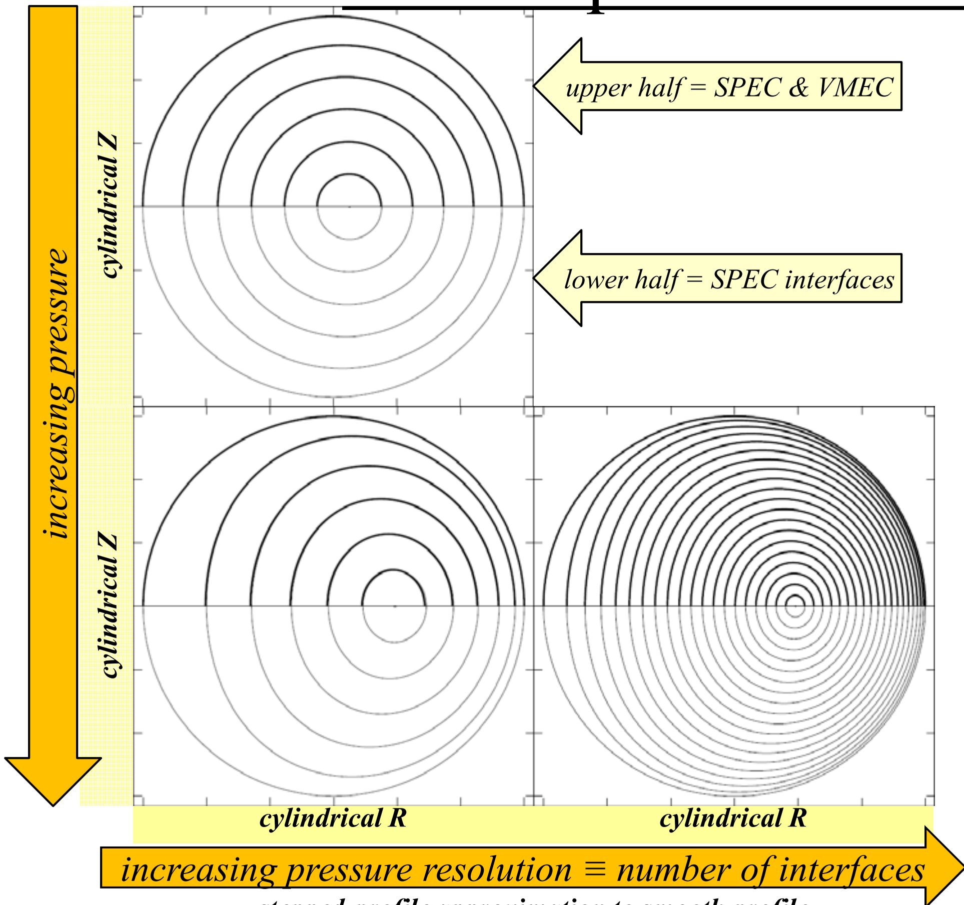
inner surface
$r = 0.1$
outer interface
$r = 0.2 \ + \delta\left[\cos(2\vartheta - \zeta) + \cos(3\vartheta - \zeta)\right]$

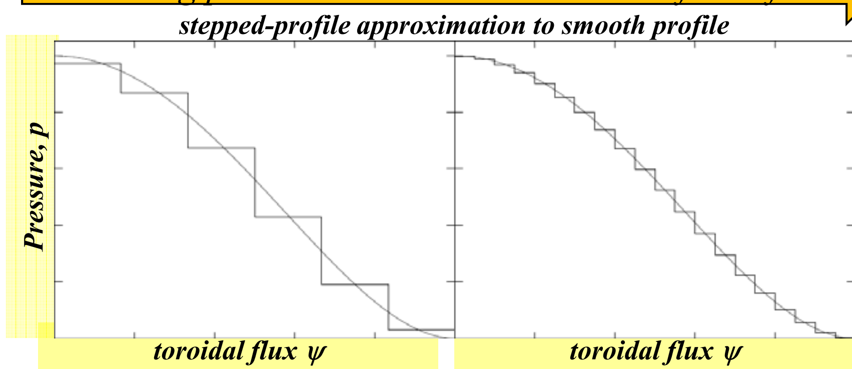
(m,n)=(3,1) island
+ (m,n)=(2,1) island
= chaos

Poincaré plot, ζ=0 ***Poincaré plot, ζ=π***



Stepped-pressure equilibria accurately approximate smooth-pressure *axisymmetric* equilibria





in axisymmetric geometry . . .

→ magnetic fields have family of nested flux surfaces

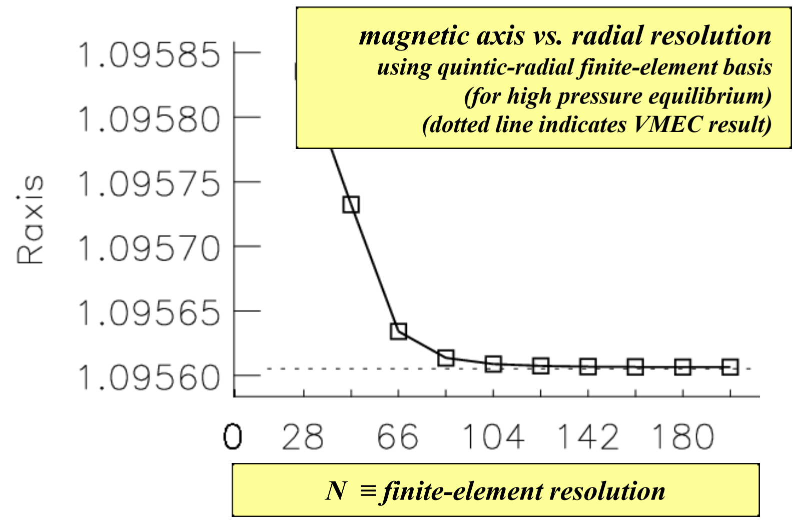
→ equilibria with smooth profiles exist,

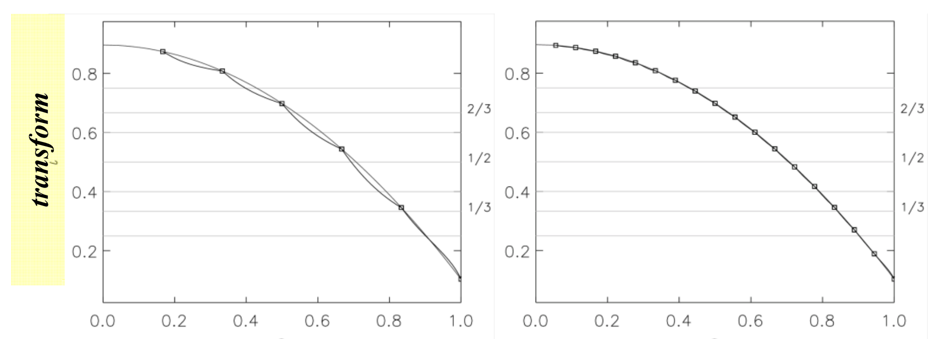
→ may perform benchmarks (e.g. with VMEC)

(arbitrarily approximate smooth-profile with stepped-profile)

→ approximation improves as number of interfaces increases

→ location of magnetic axis converges w.r.t radial resolution

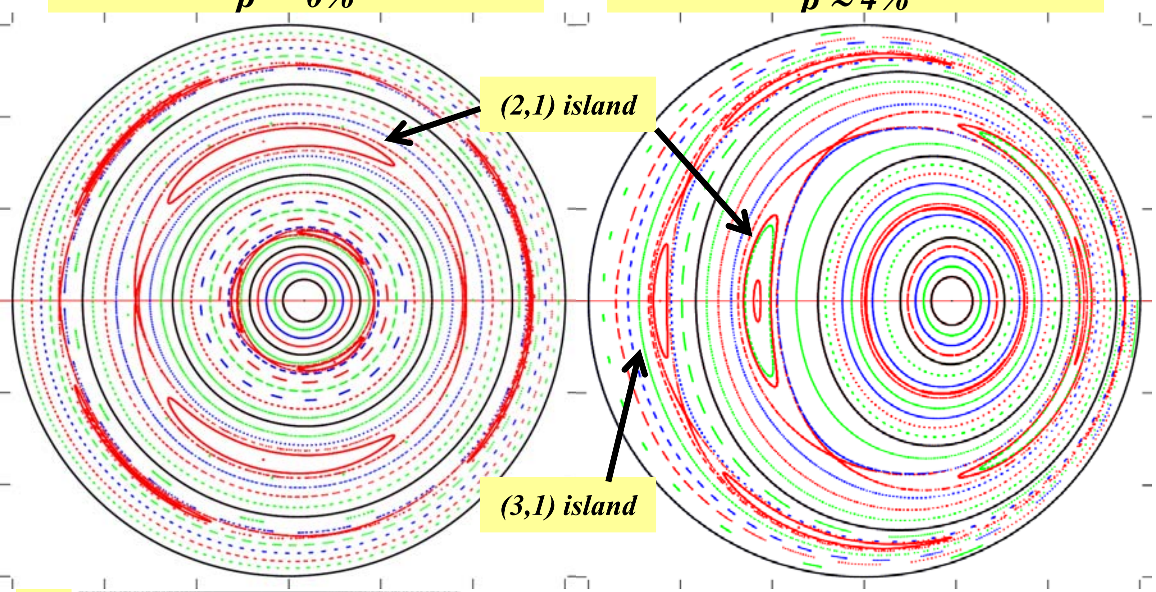




Equilibria with (i) perturbed boundary≡chaotic fields, and (ii) pressure are computed .

zero-pressure equilibrium
$\beta = 0\%$

high-pressure equilibrium
$\beta \approx 4\%$

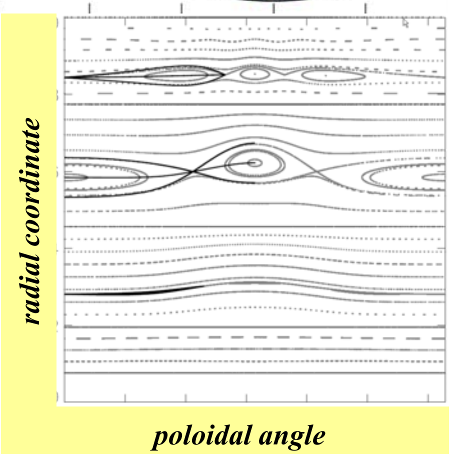


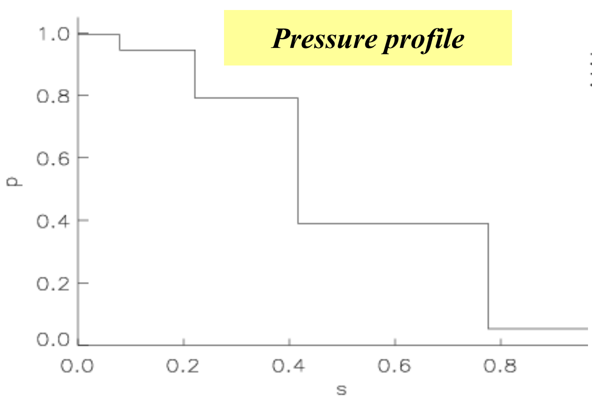
boundary deformation induces islands

$$R = 1.0 + r\cos\vartheta,\ Z = r\sin\vartheta$$

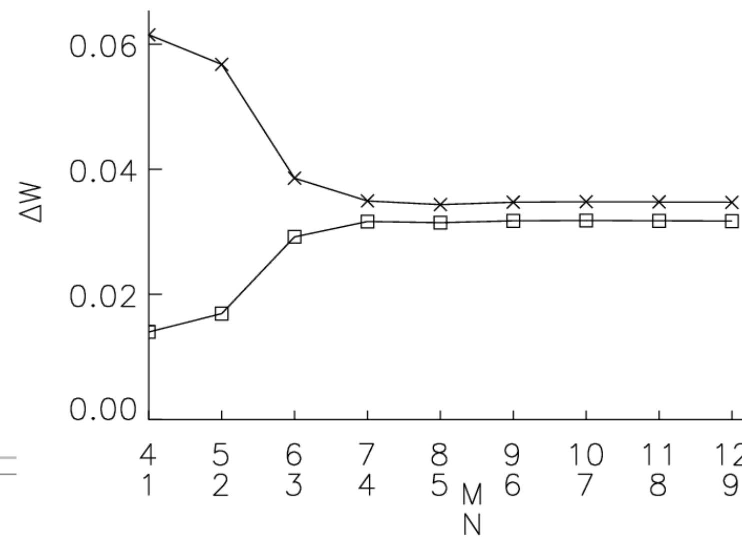
$$r = 0.3 + \delta\cos(2\vartheta - \phi) + \delta\cos(3\vartheta - \phi)$$

Demonstrated convergence with Fourier resolution





Convergence of (2,1) & (3,1) island widths with Fourier resolution for $\beta \approx 4\%$ case

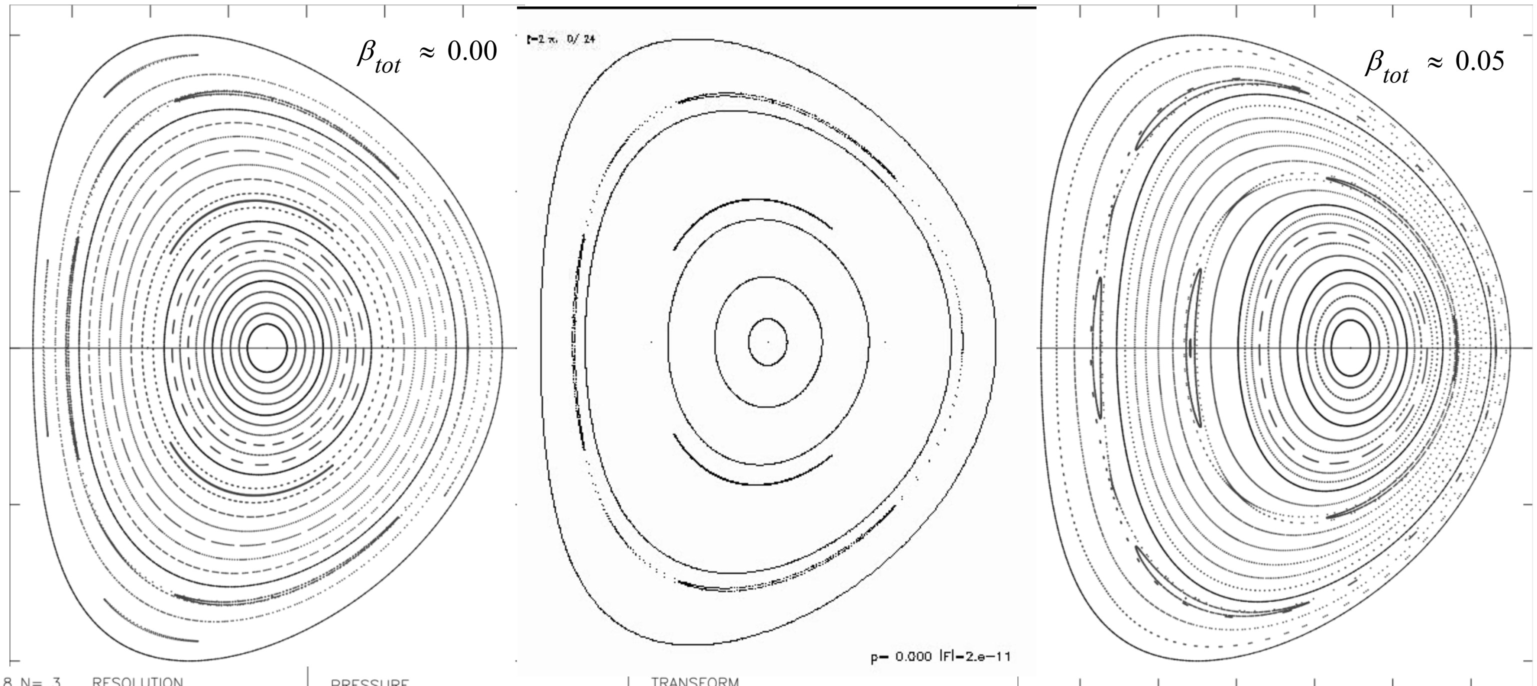


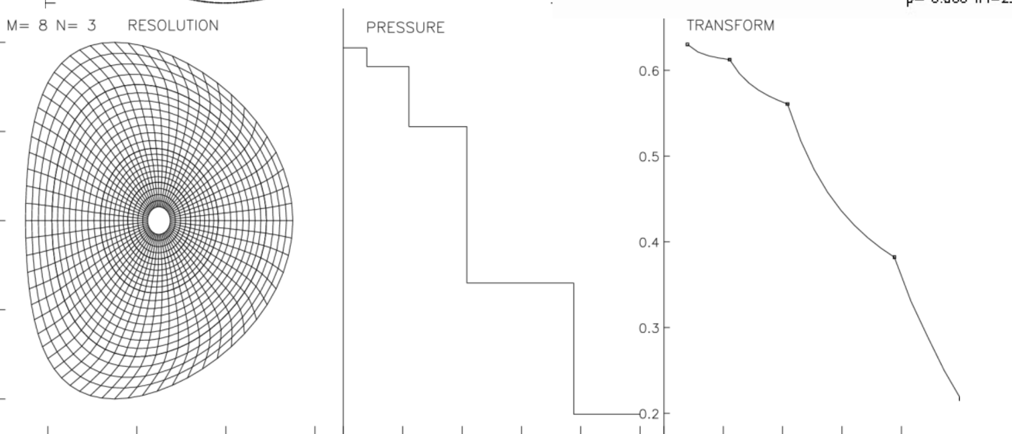
Sequence of equilibria with increasing pressure shows plasma *can* have significant response to external perturbation.

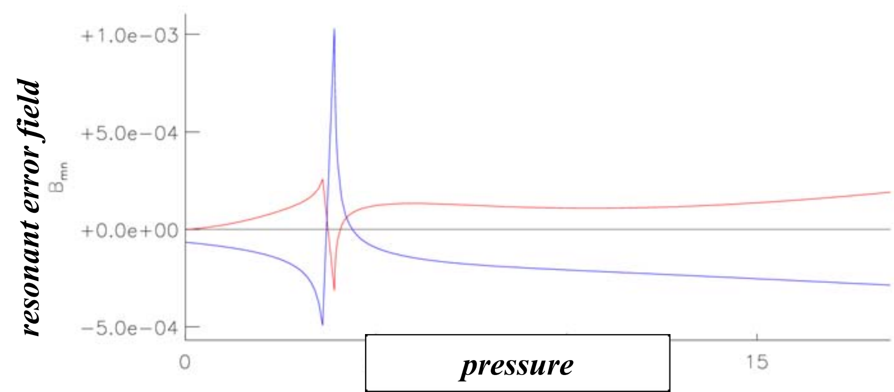
axisymmetric plus small perturbation

$$R = 1.00 + 0.30\cos(\vartheta) + 0.05\cos(2\vartheta) \quad + \quad [\delta_{21}\cos(2\vartheta-\zeta) + \delta_{31}\cos(3\vartheta-\zeta)]\cos(\vartheta)$$

$$Z = 1.00 + 0.40\sin(\vartheta) + \quad [\delta_{21}\cos(2\vartheta-\zeta) + \delta_{31}\cos(3\vartheta-\zeta)]\sin(\vartheta)$$







Summary

→ A partially-relaxed, topologically-constrained energy principle has been presented for MHD equilibria with chaotic fields and non-trivial (i.e. non-constant) pressure

→ The model has been implemented numerically

* using a high-order (piecewise quintic) radial discretization
* an optimal (i.e. spectrally condensed) Fourier representation
* workload distrubuted across multiple cpus,
* extrema located using Newton's method with quadratic-convergence

→ Intuitively, the equilibrium model is an extension of Taylor relaxation to multiple volumes

→ The model has a sound theoretical foundation

* solutions guaranteed to exist (under certain conditions)

→ The numerical method is computationally tractable

* does not invert singular operators
* does not struggle to resolve fractal structure of chaos

→ Convergence studies have been performed

* expected error scaling with radial resolution confirmed
* detailed benchmark with axisymmetric equilibria (with smooth profiles)
* that the island widths converge with Fourier resolution has been confirmed

Toroidal magnetic confinement depends on flux surfaces

Transport in magnetized plasma dominately parallel to **B**

→ if the field lines are not confined (e.g. by flux surfaces), then the plasma is poorly confined

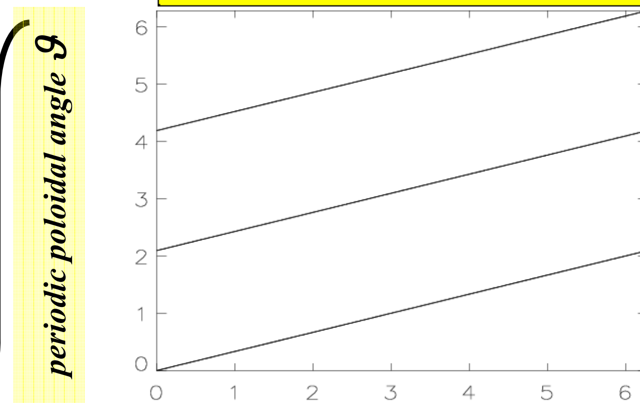
Axisymmetric magnetic fields possess a continuously nested family of flux surfaces

→ nested family of flux surfaces is guaranteed if the system has an ignorable coordinate

magnetic field is called integrable

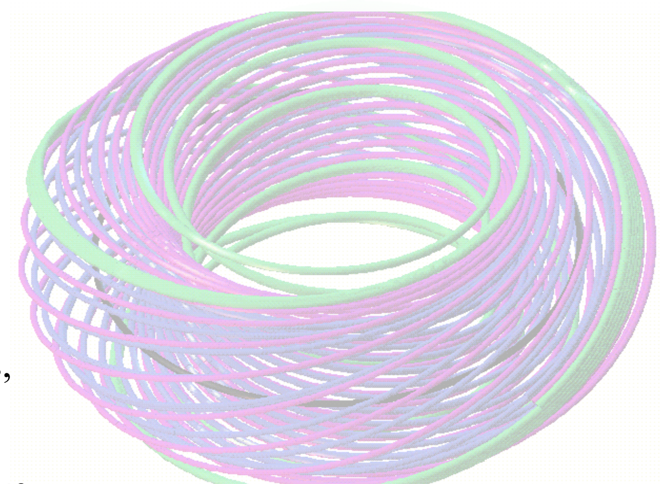
→ rational field-line ≡ periodic trajectory ***family of periodic orbits ≡ rational flux surface***

rational field-line $\vartheta = 0.3333\ldots\,\xi$

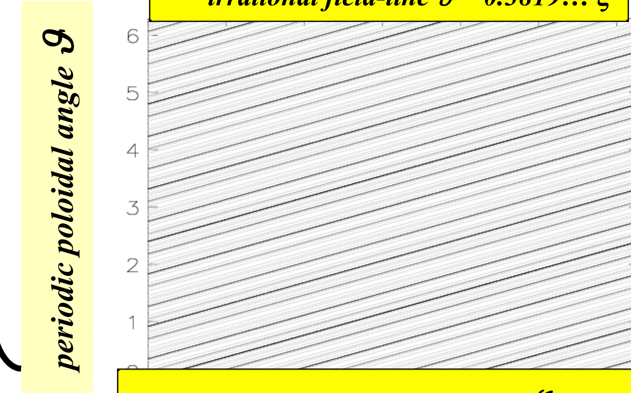


→ irrational field-lines cover *irrational* flux surface

magnetic field lines wrap around toroidal "flux" surfaces



irrational field-line $\vartheta = 0.3819\ldots\,\xi$



periodic toroidal angle ζ

straight-field-line flux coordinates,

$$\mathbf{B}\cdot\nabla\psi = 0$$

$$\mathbf{B} = \nabla\psi\times\nabla\vartheta + \iota(\psi)\nabla\zeta\times\nabla\psi$$

$$\sqrt{g}\,\mathbf{B}\cdot\nabla \equiv \partial_\zeta + \iota\,\partial_\vartheta$$

magnetic differential equation, $\mathbf{B}\cdot\nabla\sigma = s$,

is singular at rational surfaces, $(m\,\iota - n)\,\sigma_{m,n} = i(\sqrt{g}\,s)_{m,n}$

Ideal MHD equilibria are extrema of energy functional

The energy functional is

$$W = \int_V (p + B^2 / 2)\, dv$$

$V \equiv$ global plasma volume

ideal variations

mass conservation $\}$ $\partial_t \rho + \nabla \bullet (\rho \mathbf{v}) = 0$

state equation $\}$ $d_t (p \rho^{-\gamma}) = 0$

Faraday's law, ideal Ohm's law $\}$ $\delta \mathbf{B} = \nabla \times (\delta \xi \times \mathbf{B})$

→ideal variations don't allow field topology to change "frozen-flux"

the first variation in plasma energy is

$$\delta W = \int_V (\nabla p - \mathbf{j} \times \mathbf{B}) \cdot \delta \xi \, dv$$

Euler Lagrange equation for globally ideally-constrained variations
ideal-force-balance $\nabla p = \mathbf{j} \times \mathbf{B}$

→ two surface functions, e.g. the pressure, $p(s)$, and rotational-transform ≡ inverse-safety-factor, $\iota(s)$,
and → a boundary surface (. . for fixed boundary equilibria . . .),
constitute "boundary-conditions" that must be provided to uniquely define an equilibrium solution
. . . . . . The computational task is to compute the magnetic field that is consistent with the given boundary conditions . . .

nested flux surface topology maintained by singular currents at rational surfaces

from $\nabla \bullet (\sigma \mathbf{B} + \mathbf{j}_\perp) = 0$, parallel current must satisfy $\mathbf{B} \bullet \nabla \sigma = -\nabla \bullet \mathbf{j}_\perp$, where $\mathbf{j}_\perp = \mathbf{B} \times \nabla p / B^2$

$$\sigma_{m,n} = \frac{i(\sqrt{g}\ \nabla \bullet \mathbf{j}_\perp)_{m,n}}{(m\iota - n)} + \delta(m\iota - n)$$

→ magnetic differential equations are singular at rational surfaces (periodic orbits)
→ pressure-driven "Pfirsch-Schlüter currents" have 1/ x type singularity
→ δ - function singular currents shield out islands

Topological constraints : pressure gradients coincide with flux surfaces

The ideal interfaces are chosen to coincide with pressure gradients

$\rightarrow$ parallel transport dominates perpendicular transport,

$\rightarrow$ simplest approximation is $\mathbf{B}\cdot\nabla p = 0$

$\rightarrow$ pressure gradients **must** coincide with KAM surfaces $\equiv$ ideal interfaces

$\rightarrow$ structure of B and structure of the pressure are intimately connected;

$\rightarrow$ cannot apriori specify pressure without apriori constraining structure of the field;

[next order of approximation, $\mathbf{B}\cdot\nabla p$ is small, e.g. $\partial_t p = \kappa_\parallel \nabla_\parallel^2 p + \kappa_\perp \nabla_\perp^2 p = 0$, with $\kappa_\parallel \gg \kappa_\perp$, e.g. $\kappa_\perp / \kappa_\parallel \sim 10^{-10}$

*pressure gradients coincide with KAM surfaces, cantori . .

*pressure flattened across islands, chaos with width $> \Delta w_C \sim (\kappa_\perp / \kappa_\parallel)^{1/4}$

* anisotropic diffusion equation solved analytically, p' $\propto 1 / \left(\kappa_\parallel \varphi_2 + \kappa_\perp G\right)$, φ_2 is quadratic-flux across cantori, G is metric term]

$\rightarrow$ where there are significant pressure gradients, there can be no islands or chaotic regions with width $> \Delta wc$

A fixed boundary equilibrium is defined by :

(i) given pressure, $p(\psi)$, and rotational-transform profile, $\iota(\psi)$

(ii) geometry of boundary;

(a) only stepped pressure profiles are consistent (numerically tractable) with chaos and $\mathbf{B}\cdot\nabla p = 0$

(b) the computed equilibrium magnetic field must be consistent with the input profiles

(a) + (b) = where the pressure has gradients, the magnetic field must have flux surfaces.

$\rightarrow$ non-trivial stepped pressure equilibrium solutions are *guaranteed* to exist

Taylor relaxation: a weakly resistive plasma will relax, *subject to single constraint* of conserved helicity

Taylor relaxation, [Taylor, 1974]

$$\underbrace{W = \int_V \left(p + B^2/2\right) dv}_{\text{plasma energy}}, \qquad \underbrace{H = \int_V \left(\mathbf{A}\cdot\mathbf{B}\right) dv}_{\text{helicity, } \mathbf{B}=\nabla\times\mathbf{A}}$$

Constrained energy functional $F = W - \mu H / 2$, $\mu \equiv$ Lagrange multiplier

Euler-Lagrange equation, for *unconstrained* variations in magnetic field, $\nabla \times \mathbf{B} = \mu \mathbf{B}$

linear force-free field ≡ Beltrami field

But, . . . Taylor relaxed fields have no pressure gradients

Ideal MHD equilibria and Taylor-relaxed equilibria are at opposite extremes . . . .

Ideal-MHD → imposition of *infinity* of ideal MHD constraints

non-trivial pressure profiles, but structure of field is ***over-constrained***

Taylor relaxation → imposition of *single* constraint of conserved global helicity

structure of field is not-constrained, but pressure profile is trivial, i.e. ***under-constrained***

We need something in between . . .

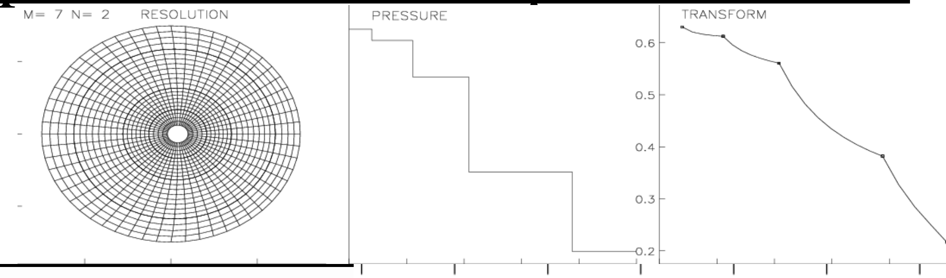
. . . perhaps an equilibrium model with *finitely* many ideal constraints, and *partial* Taylor relaxation?

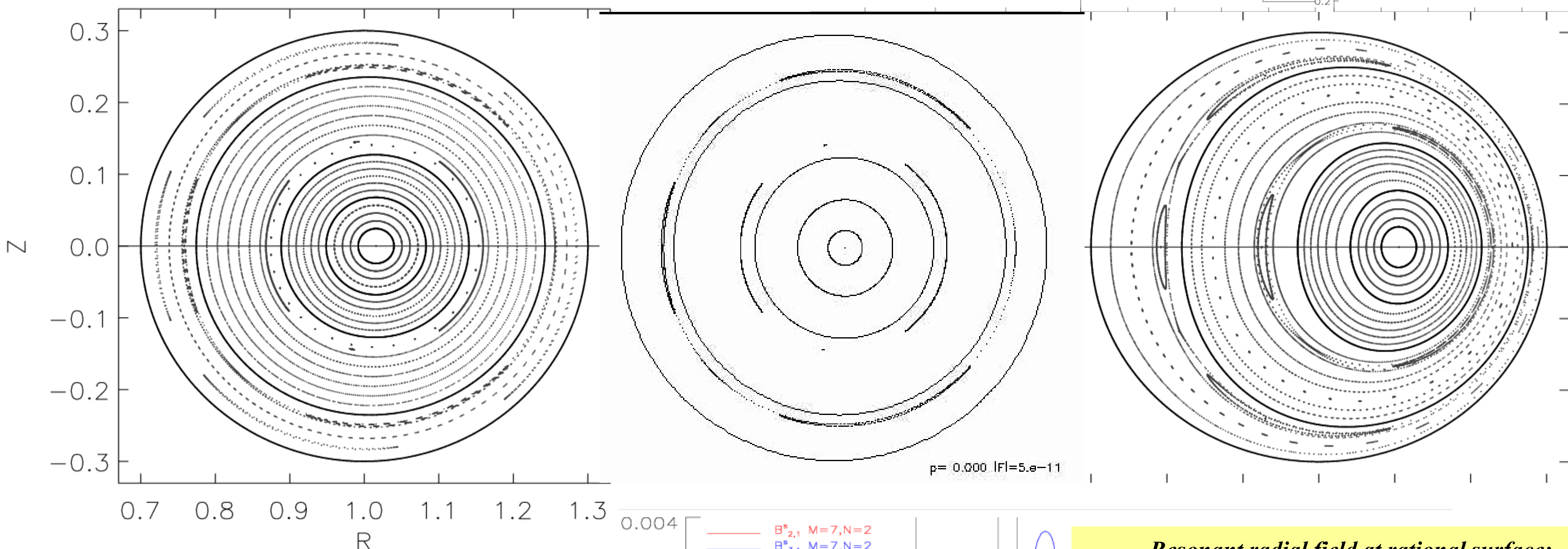
Sequence of equilibria with increasing pressure shows plasma *can* have significant response to external perturbation.

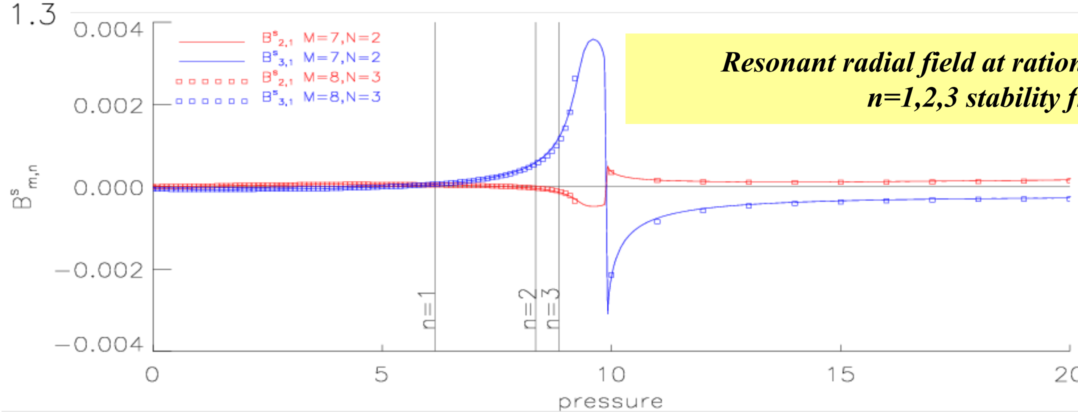
axisymmetric plus perturbation $\delta_{21}=\delta_{31}=10^{-4}$

$$R = 1.00 + [0.30 + \delta_{21}\cos(2\vartheta - \zeta) + \delta_{31}\cos(3\vartheta - \zeta)]\cos(\vartheta)$$

$$Z = 1.00 + [0.30 + \delta_{21}\cos(2\vartheta - \zeta) + \delta_{31}\cos(3\vartheta - \zeta)]\sin(\vartheta)$$





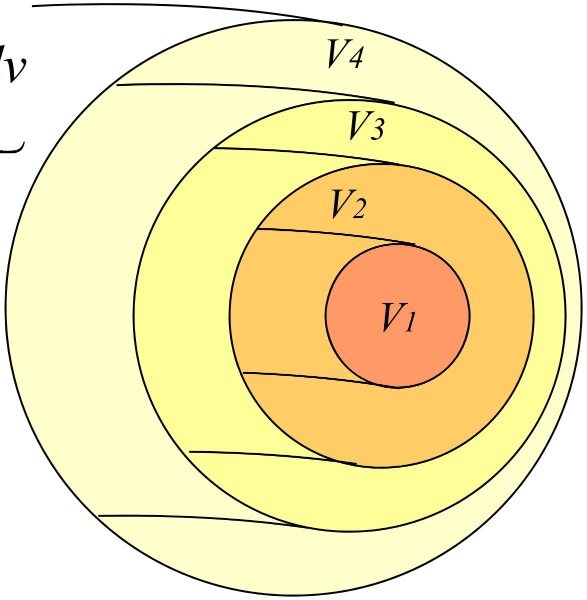


Resonant radial field at rational surface;
n=1,2,3 stability from PEST;

Introducing the multi-volume, partially-relaxed model of MHD equilibria with topological constraints

Energy, helicity and mass integrals

$$\underbrace{W_l = \int_{V_l} \left(\frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv}_{\text{plasma energy}}, \quad \underbrace{H_l = \int_{V_l} (\mathbf{A} \cdot \mathbf{B})\, dv}_{\text{helicity}}, \quad \underbrace{M_l = \int_{V_l} p^{1/\gamma}\, dv}_{\text{mass}}$$



Multi-volume, partially-relaxed energy principle

* A set of N nested toroidal surfaces enclose N annulur volumes

$\rightarrow$ the interfaces are assumed to be ideal, $\delta \mathbf{B} = \nabla \times (\delta \boldsymbol{\xi} \times \mathbf{B})$

* The multi-volume energy functional is

$$\boxed{F = \sum_{l=1}^{N} \left(W_l - \mu_l H_l / 2 - \nu_l M_l \right)}$$

→ field remains tangential to interfaces,
→ a finite number of ideal constraints, imposed topologically!

Euler-Lagrange equation for *unconstrained* variations in A

In each annulus, the magnetic field satisfies $\nabla \times \mathbf{B}_l = \mu_l \mathbf{B}_l$

Euler-Lagrange equation for variations in interface geometry

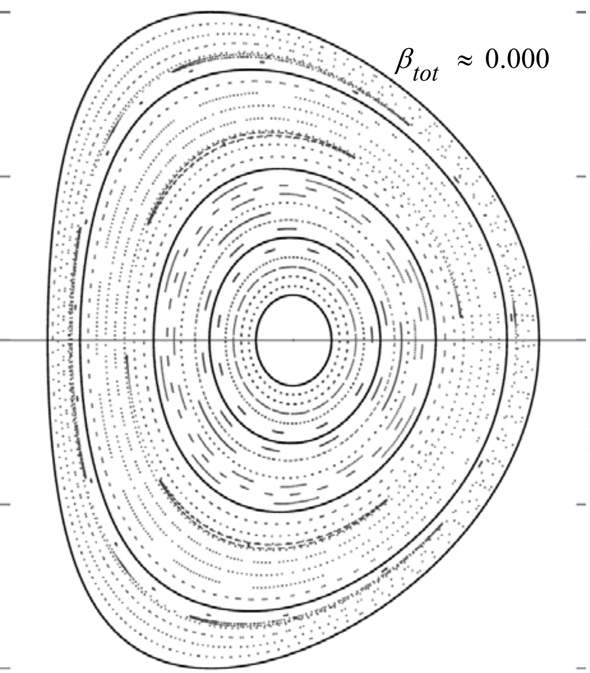
Across each interface, pressure jumps allowed, but total pressure is continuous [[p+ $B^2/2$]]=0

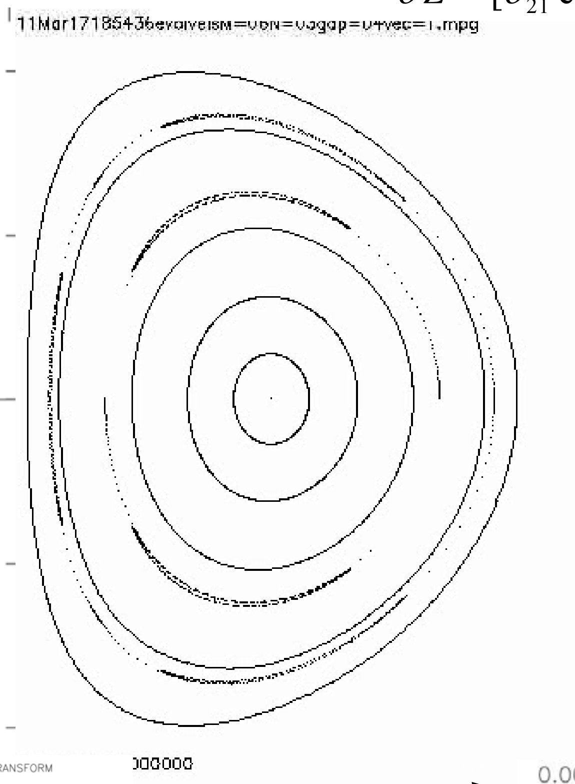
$\rightarrow$ an analysis of the force-balance condition is that the interfaces must have strongly irrational transform

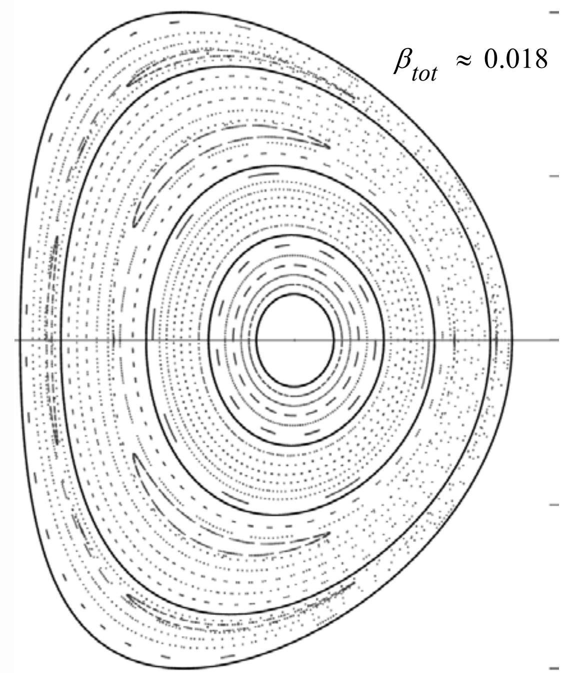
ideal interfaces coincide with KAM surfaces

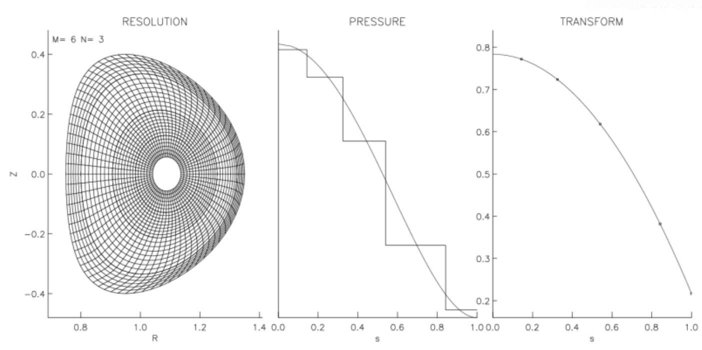
Sequence of equilibria with slowly increasing pressure

$$\begin{aligned} &axisymmetric: && R = 1.00 + 0.30\cos(\vartheta) + 0.05\cos(2\vartheta) \\ &plus && Z = 1.00 + 0.40\sin(\vartheta) \\ &perturbation && :\delta R = [\delta_{21}\cos(2\vartheta - \zeta) + \delta_{31}\cos(3\vartheta - \zeta)]\cos(\vartheta) \\ & && \delta Z = [\delta_{21}\cos(2\vartheta - \zeta) + \delta_{31}\cos(3\vartheta - \zeta)]\sin(\vartheta) \end{aligned}$$









$T_{\mathbf{F}} \approx 20\,\mathrm{s}$

$T_{\nabla\mathbf{F}} \approx 60\,\mathrm{m}$