Taylor Relaxation and Reversed Field Pinches

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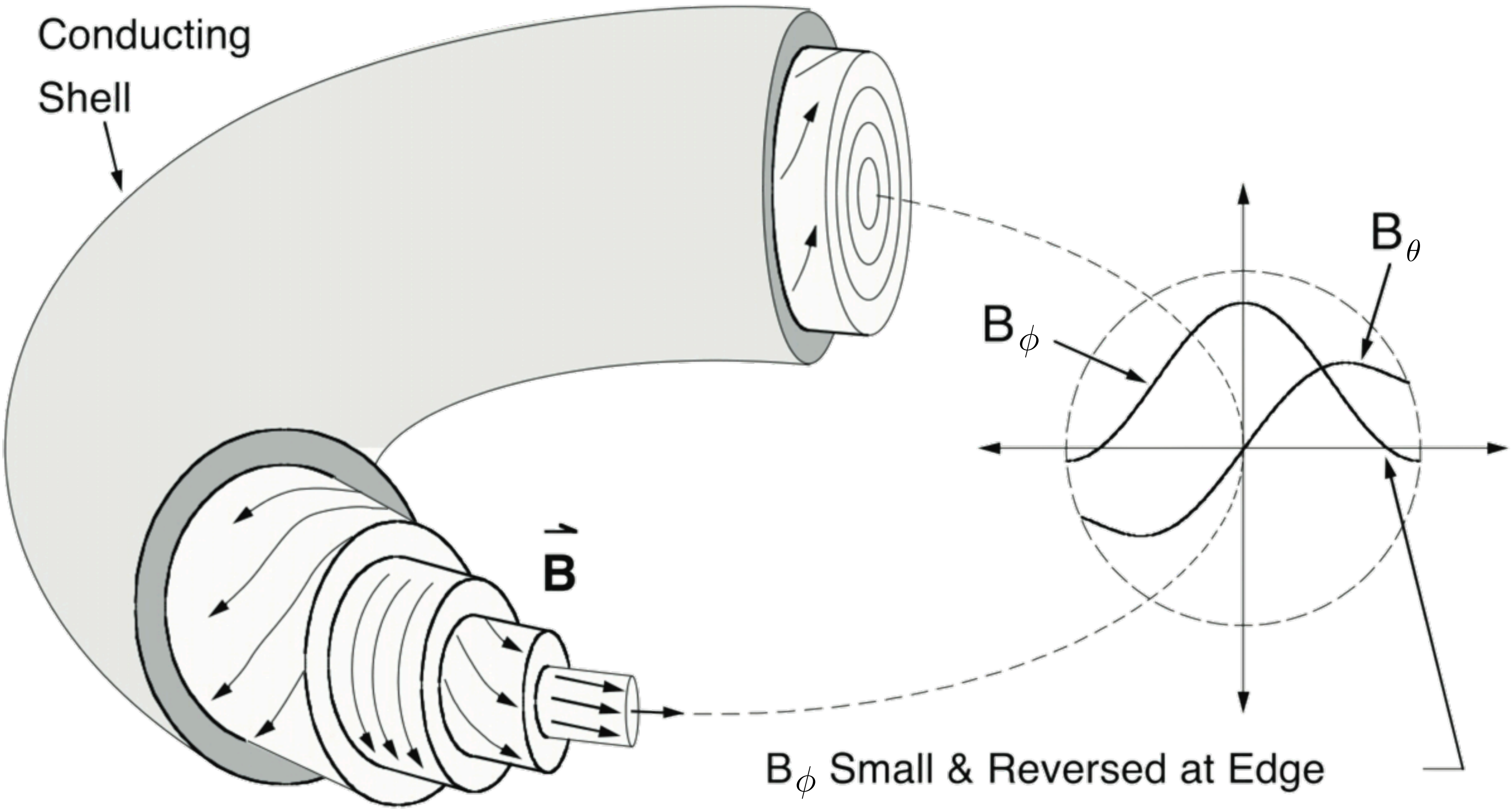
The Reversed Field Pinch is a toroidal plasma confinement device (like a tokamak)



Magnetic Field Structure of the RFP

Figure source: Burning Plasma Assessment Committee, *Burning Plasma: Bringing a Star to Earth,* The National Academies Press (2004).

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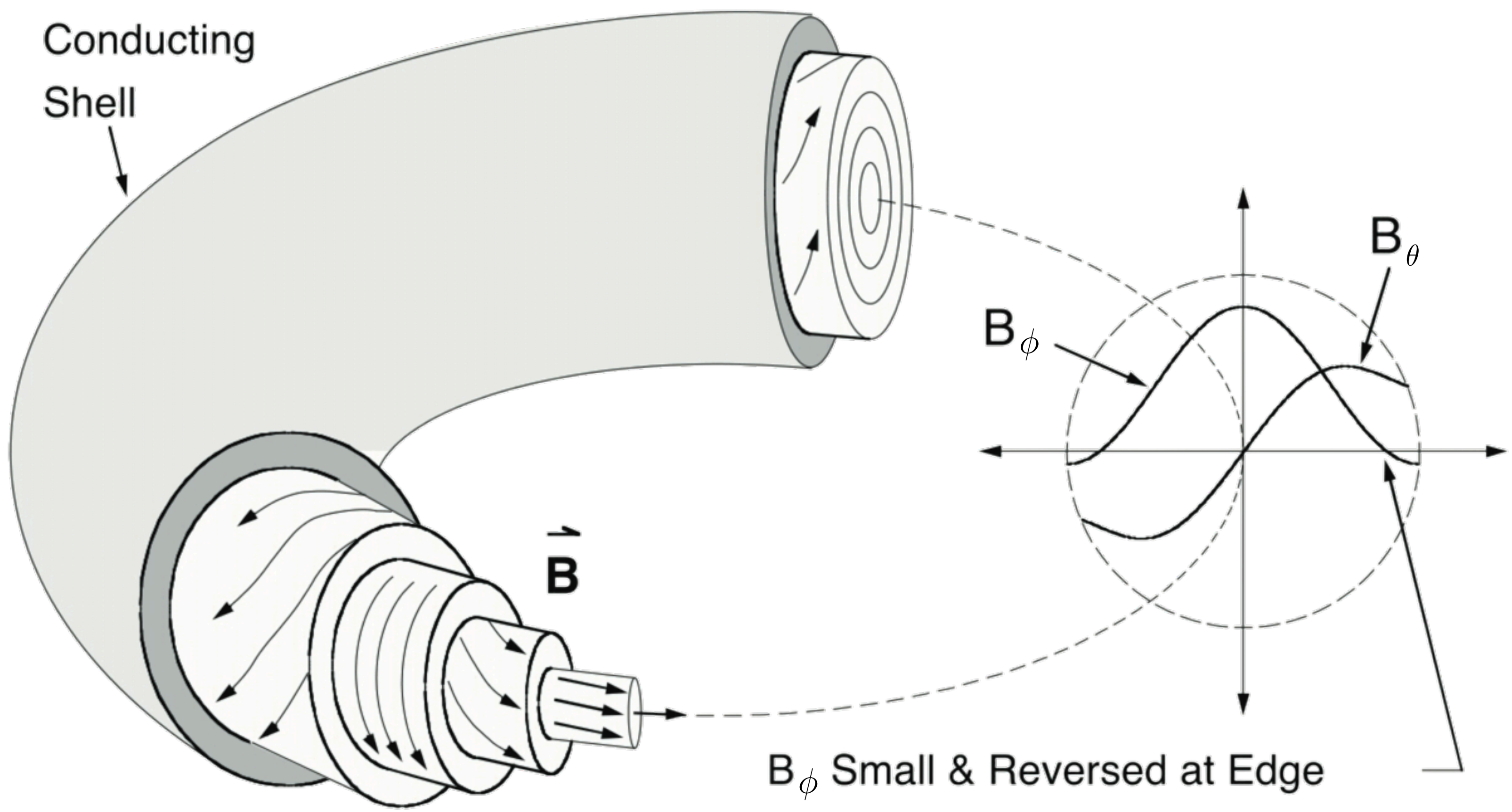

Magnetic Field Structure of the RFP

Typically unstable ⇒ low confinement

A more stable state has recently been observed

It's helical, self-organised (i.e. formed spontaneously)

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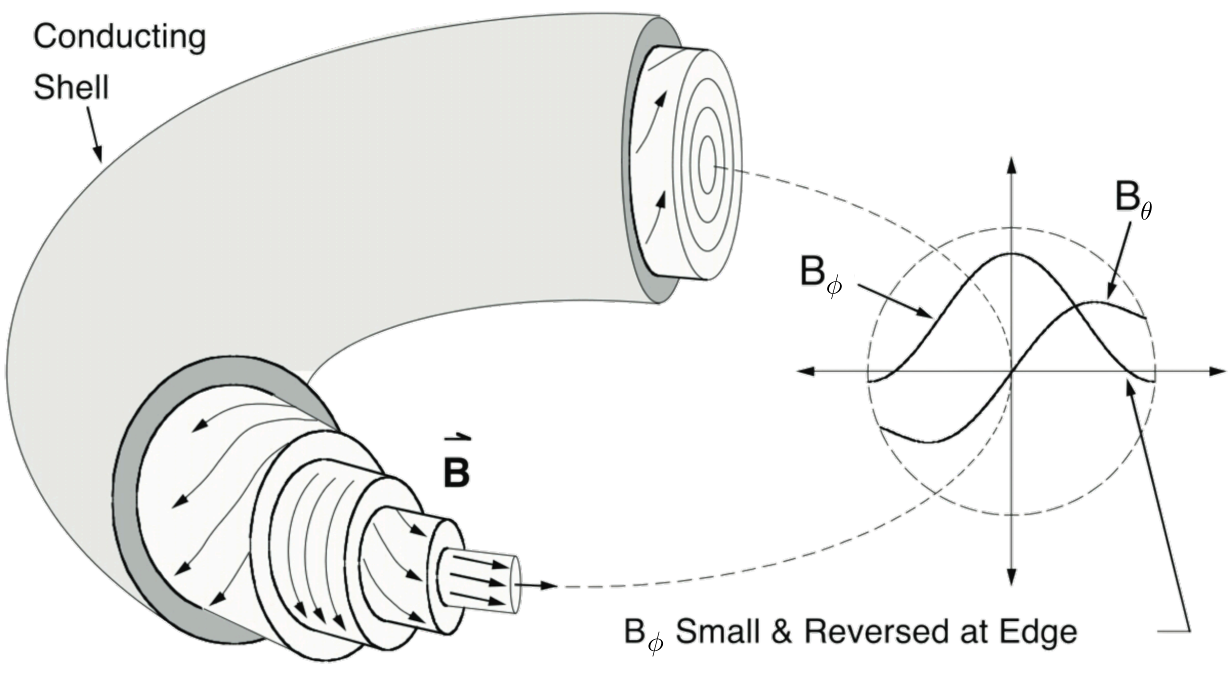

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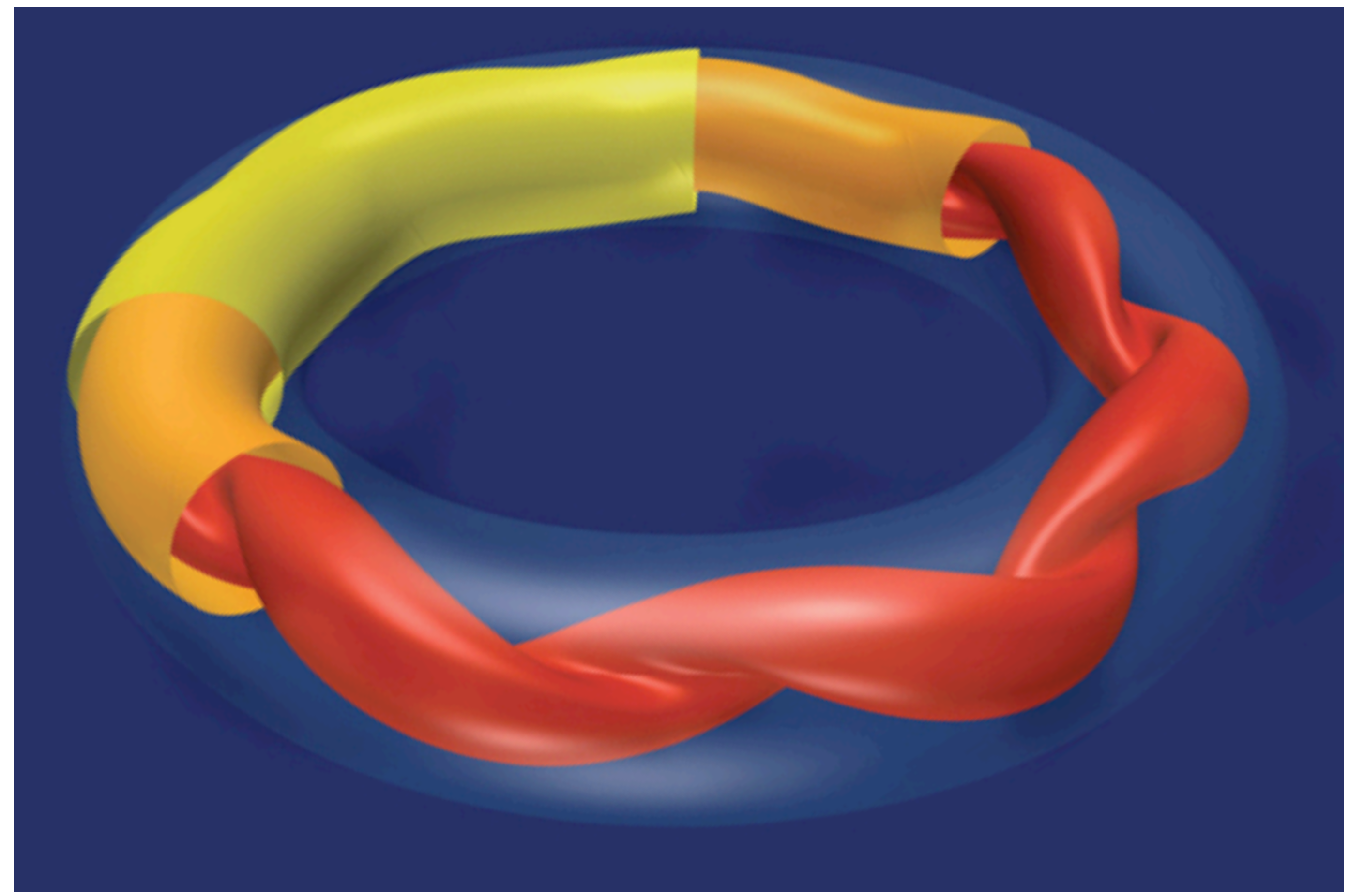

Poor confinement

Better confinement

Right figure source: David Shand, *Nature Physics Cover* **5**:8, August 2009

Taylor's theory $(\mathbf{J} = \mu \mathbf{B})$ is a good description of 'typical' Reversed Field Pinches

Taylor's theory: Plasma quantities are only conserved *globally*

Ideal MHD: Plasma quantities conserved on *every flux surface*

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Goal: *minimal* description of helical states in RFP

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$$\mathbf{J} = \mu \mathbf{B}$$

$$\mathbf{J} = \mu(\mathbf{x}) \mathbf{B}$$

Taylor's theory (Linear force-free fields)

Ideal MHD (Nonlinear force-free fields)

Fewer constraints ⟷ More constraints

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$$\mathbf{J} = \mu \mathbf{B}$$

Taylor's theory (Linear force-free fields) ✗

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$\mathbf{J} = \mu \mathbf{B}$	$\mathbf{J} = \mu_i \mathbf{B}$	$\mathbf{J} = \mu(\mathbf{x}) \mathbf{B}$
Taylor's theory (Linear force-free fields) ✗	??? (Piecewise linear force-free fields)	Ideal MHD (Nonlinear force-free fields) ✓

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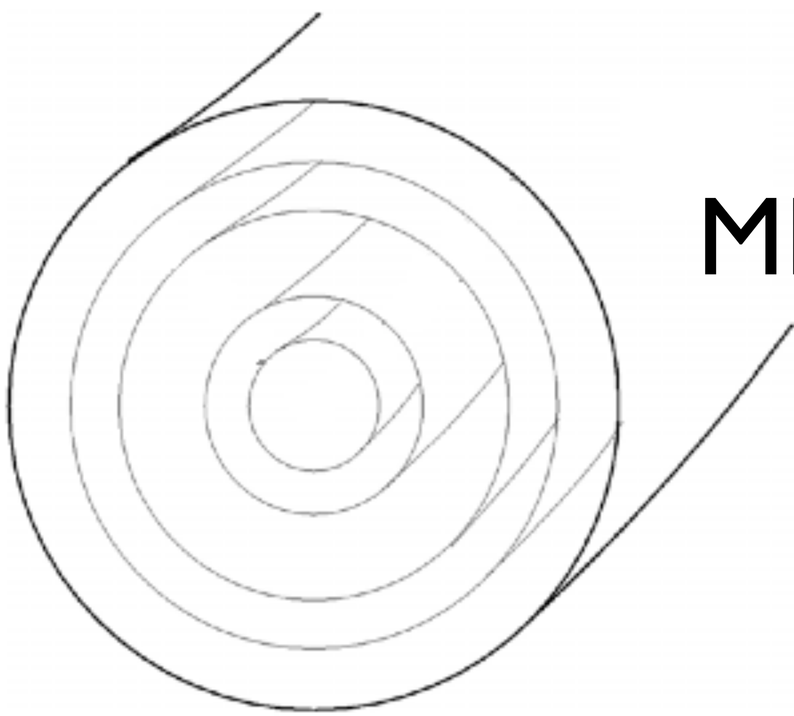
MRXMHD conserves plasma quantities in a finite number of plasma volumes

Linear force-free field: $$\mathbf{J} = \nabla \times \mathbf{B} = \mu \mathbf{B}$$



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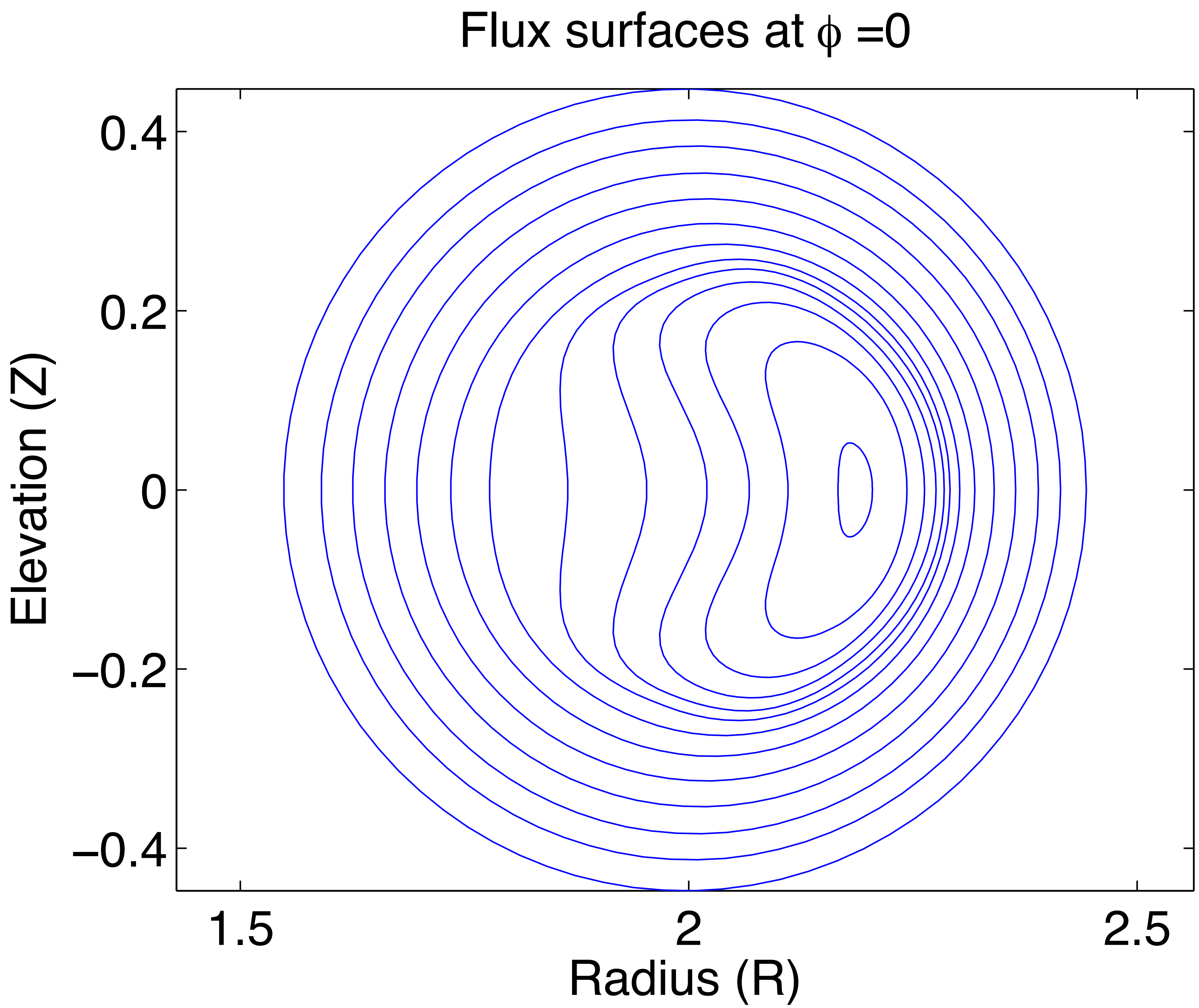
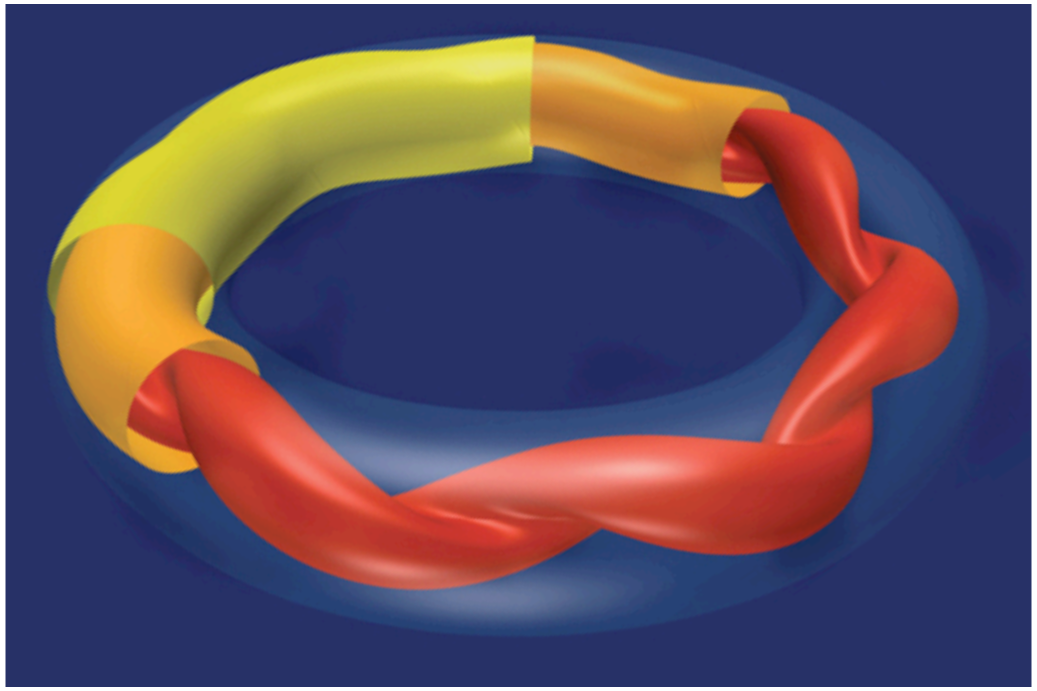
Nonlinear force-free field: $$\mathbf{J} = \nabla \times \mathbf{B} = \mu(\mathbf{x}) \mathbf{B}$$

Figure source: M.J. Hole, S.R. Hudson and R.L. Dewar, *Nuclear Fusion* **47**, 746 (2007)

We take an Ideal MHD solution and reduce the number of constraints





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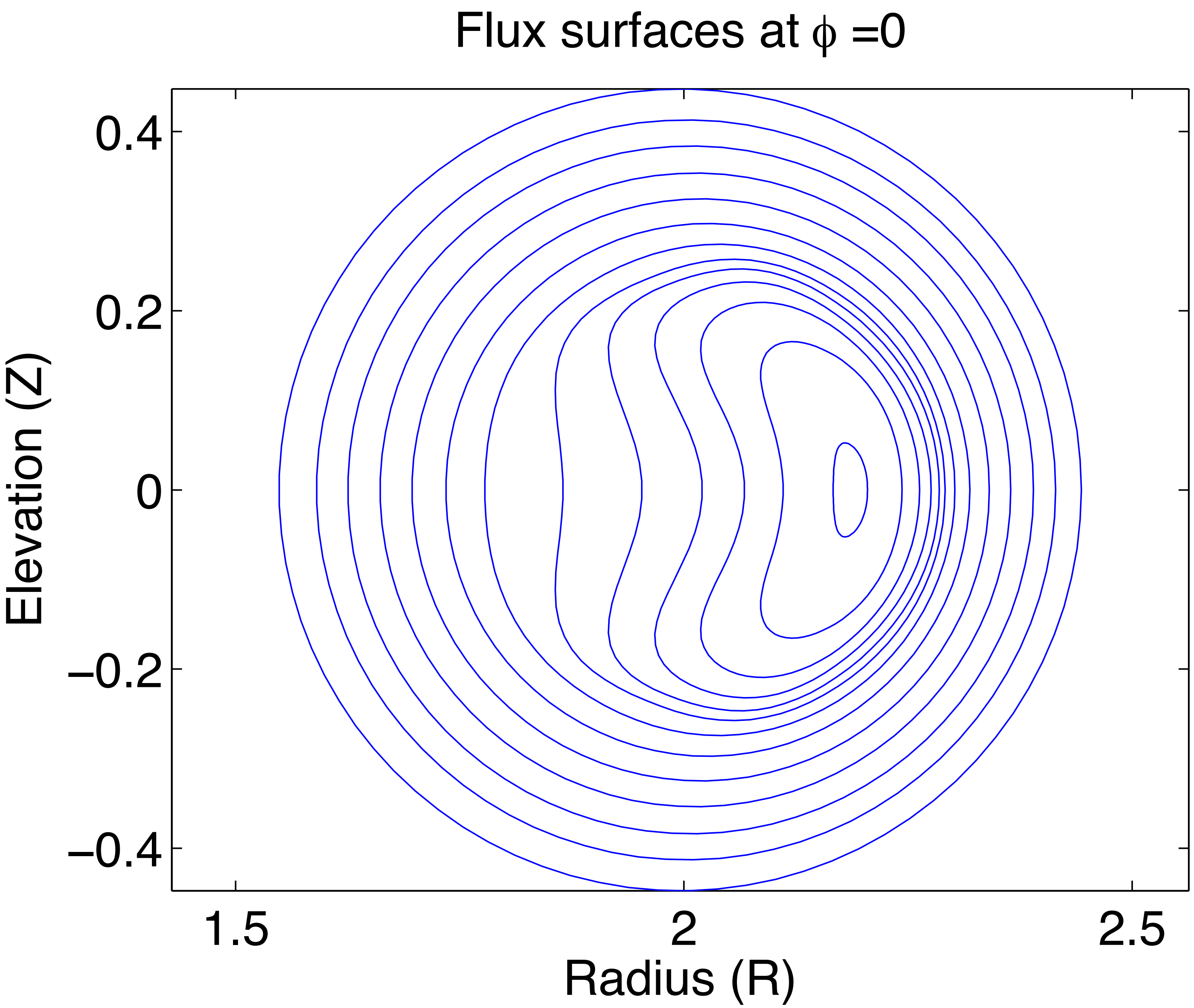

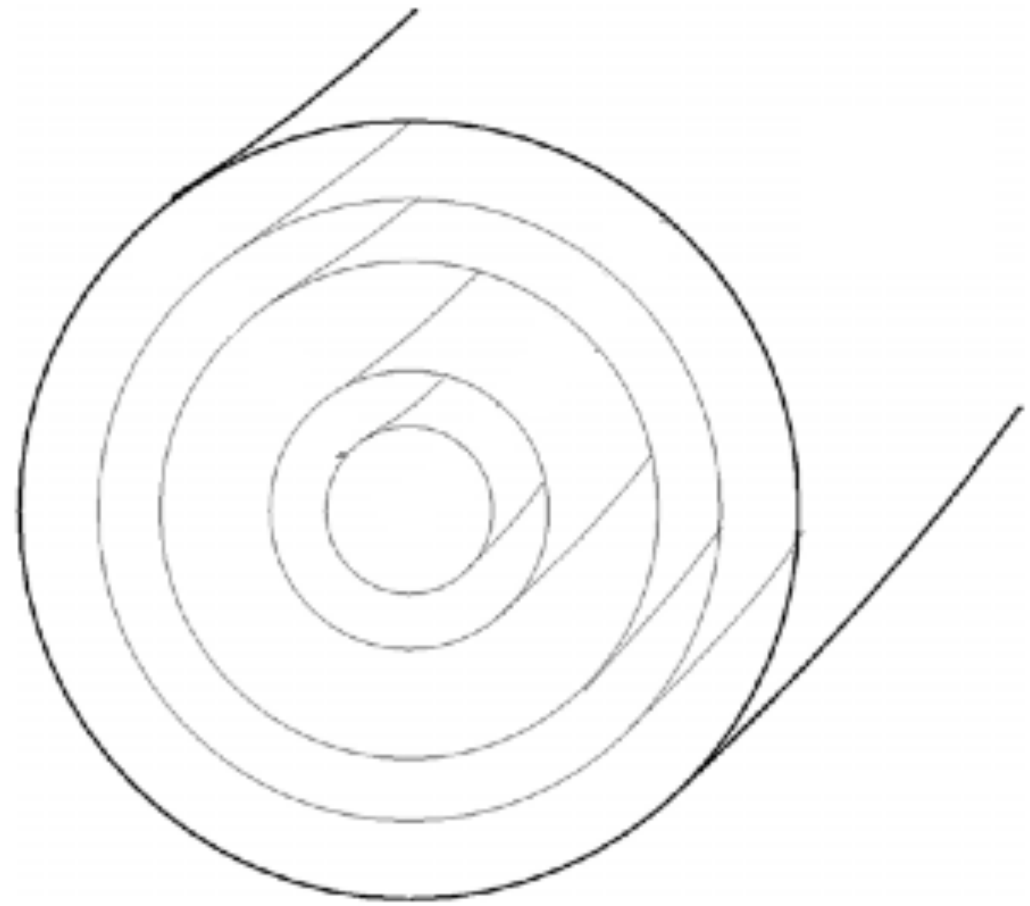

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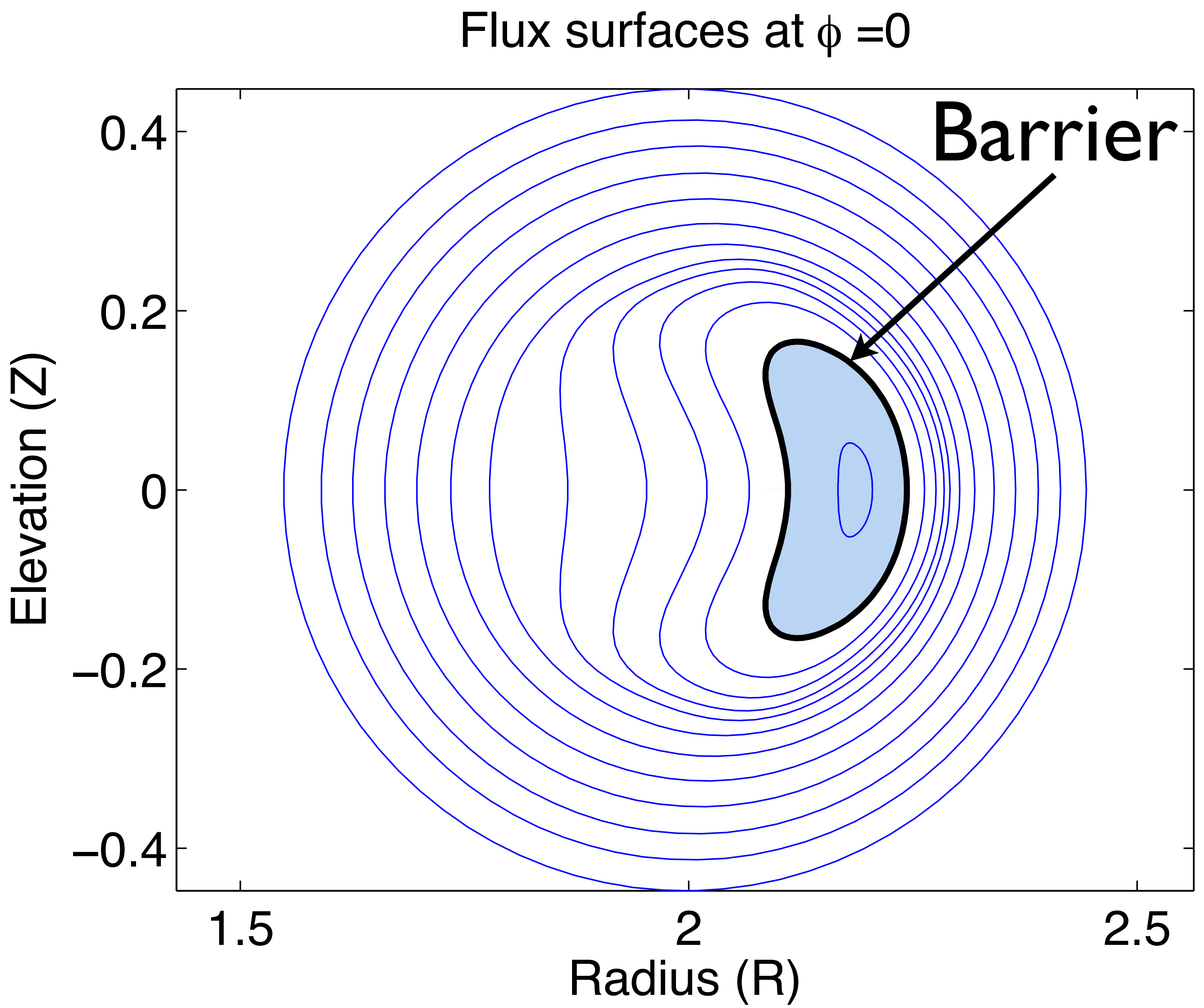

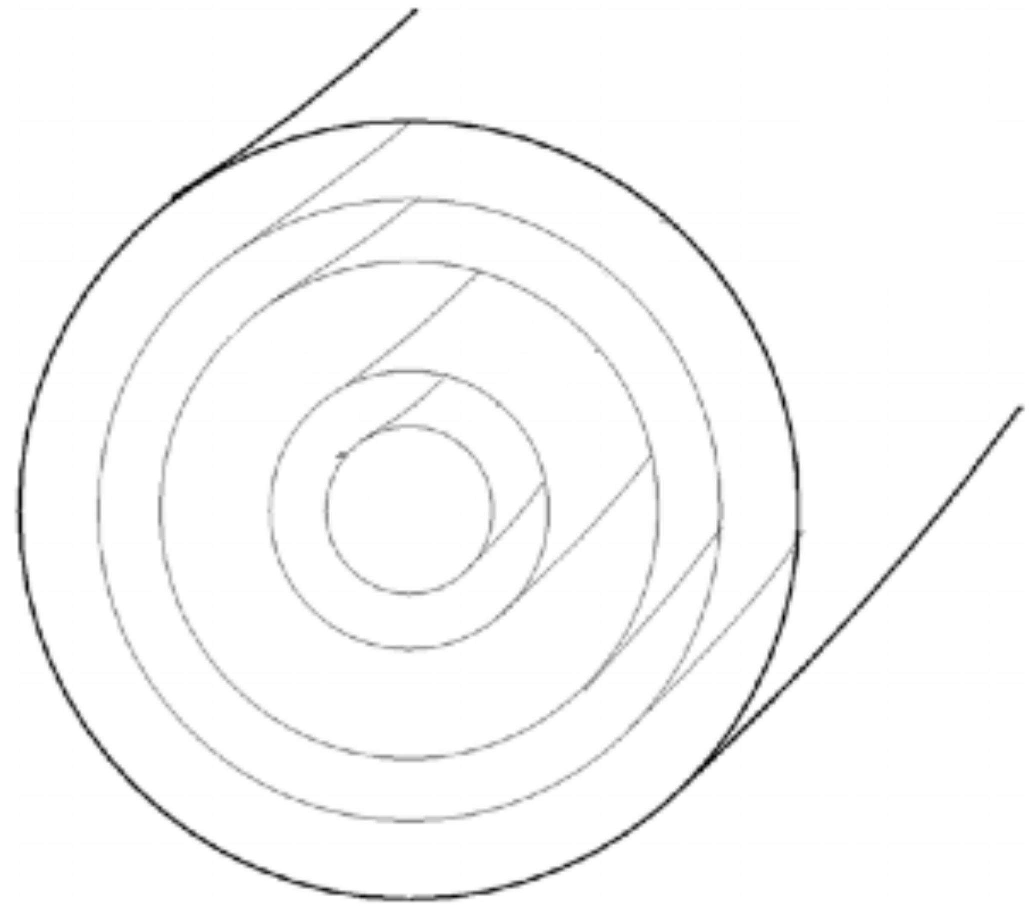

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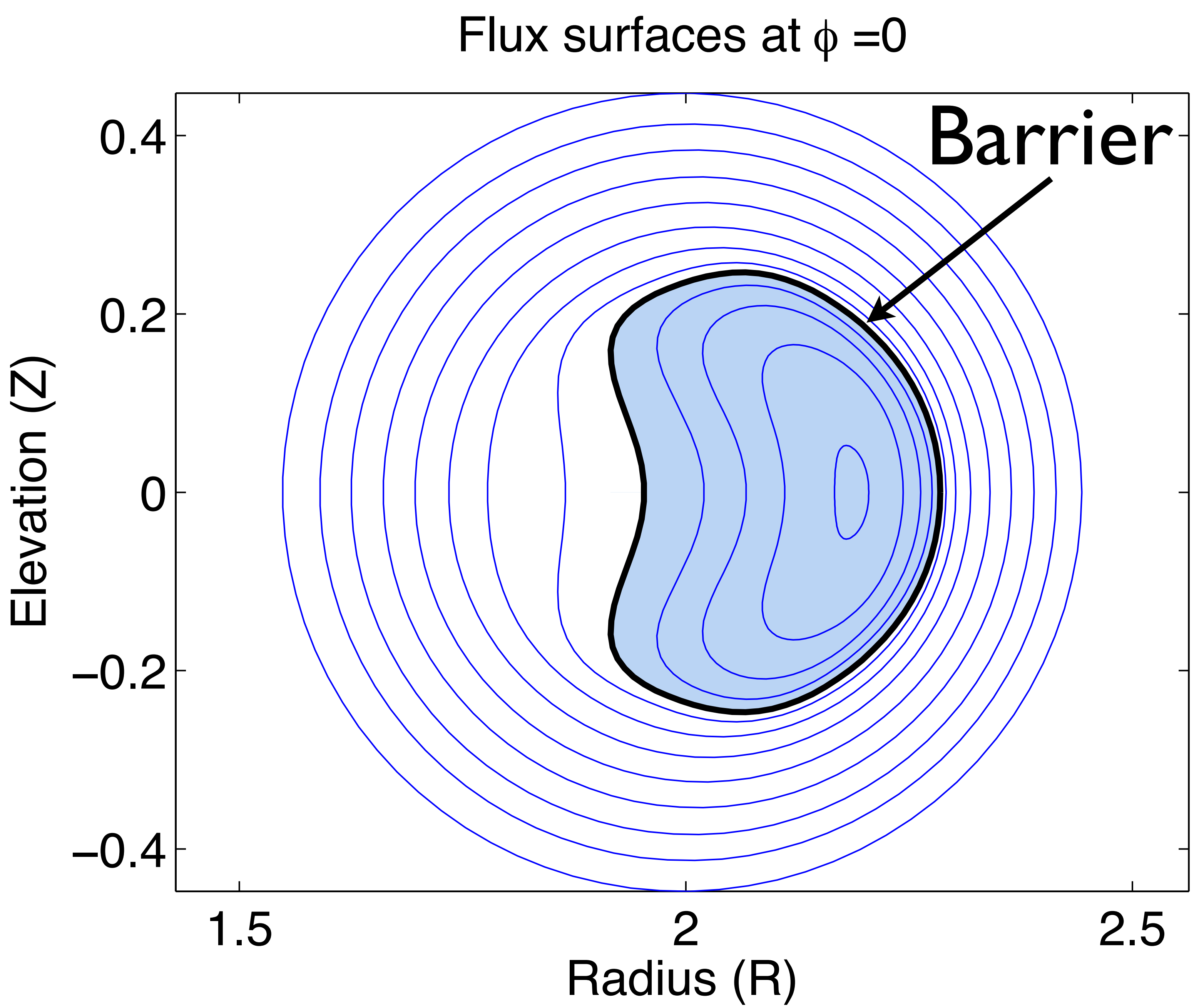

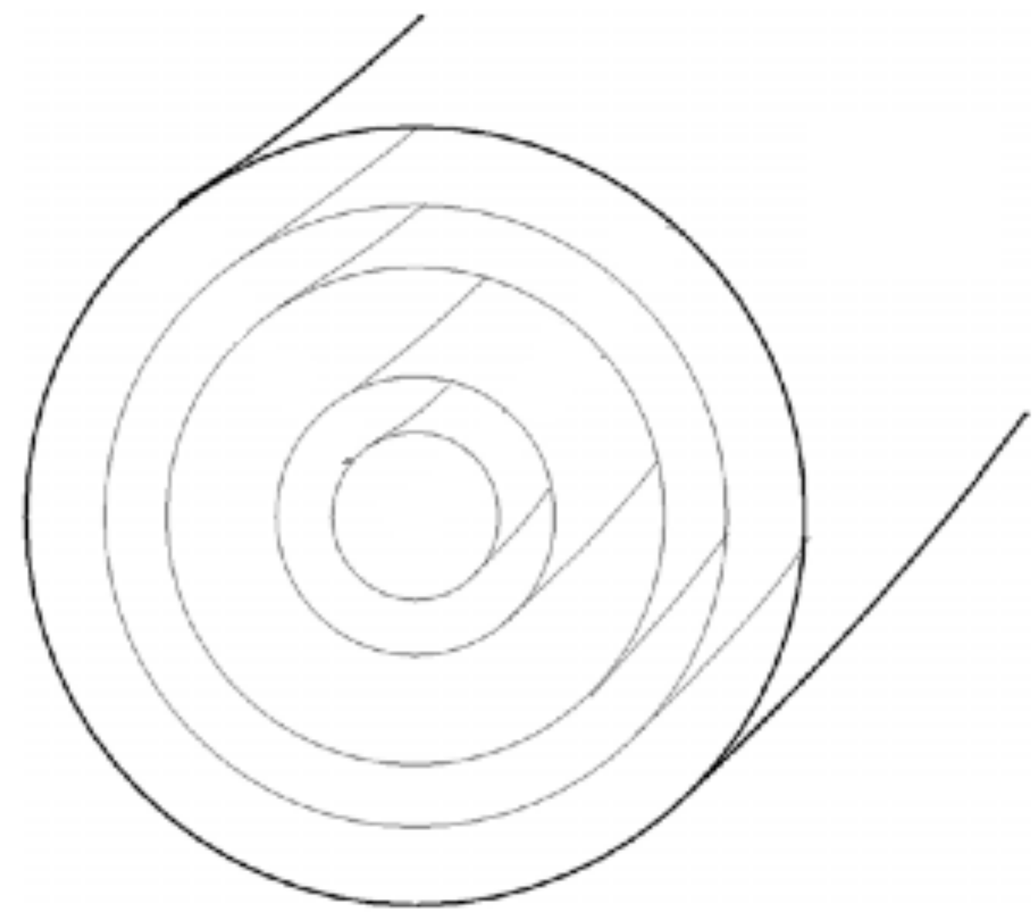

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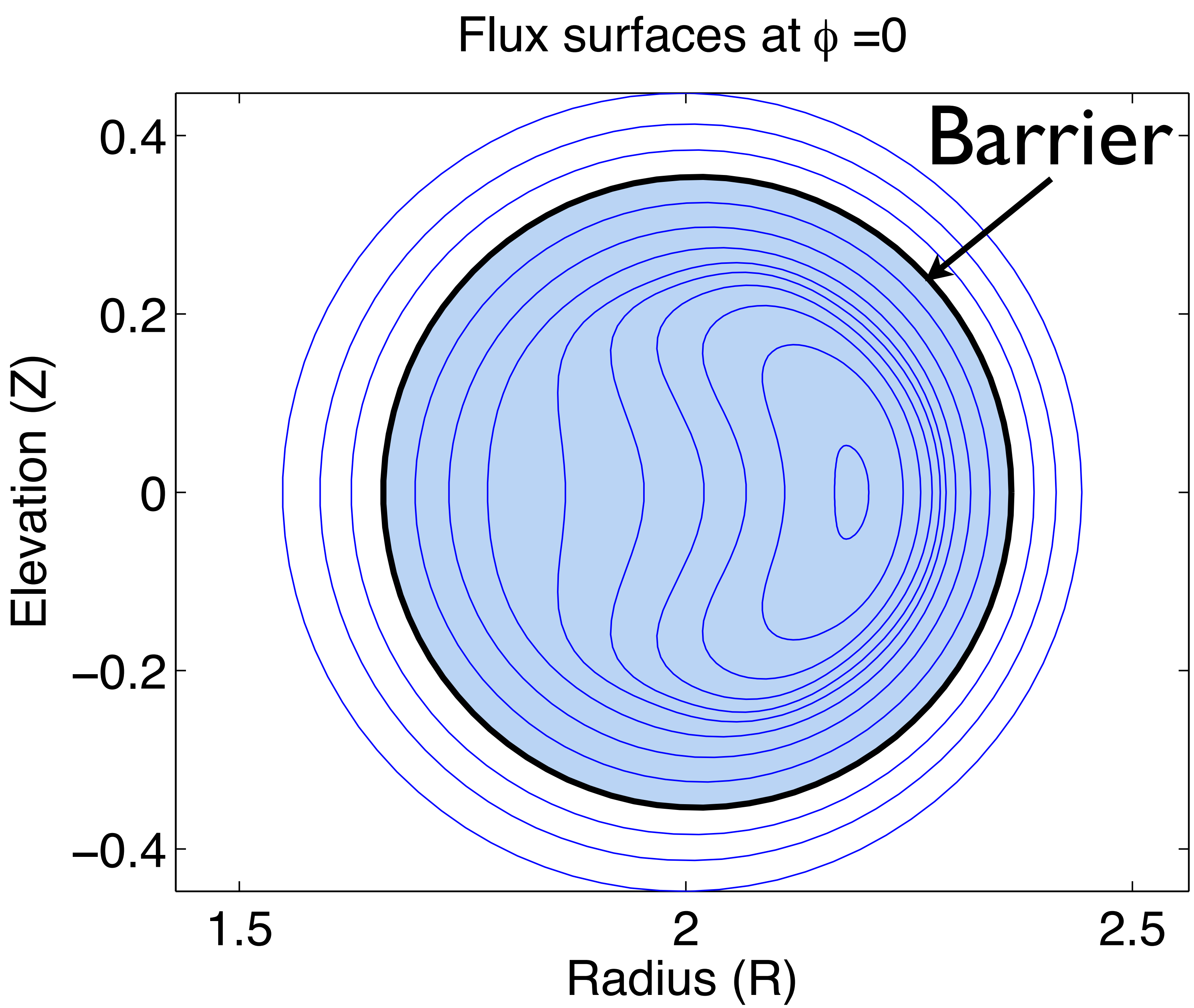
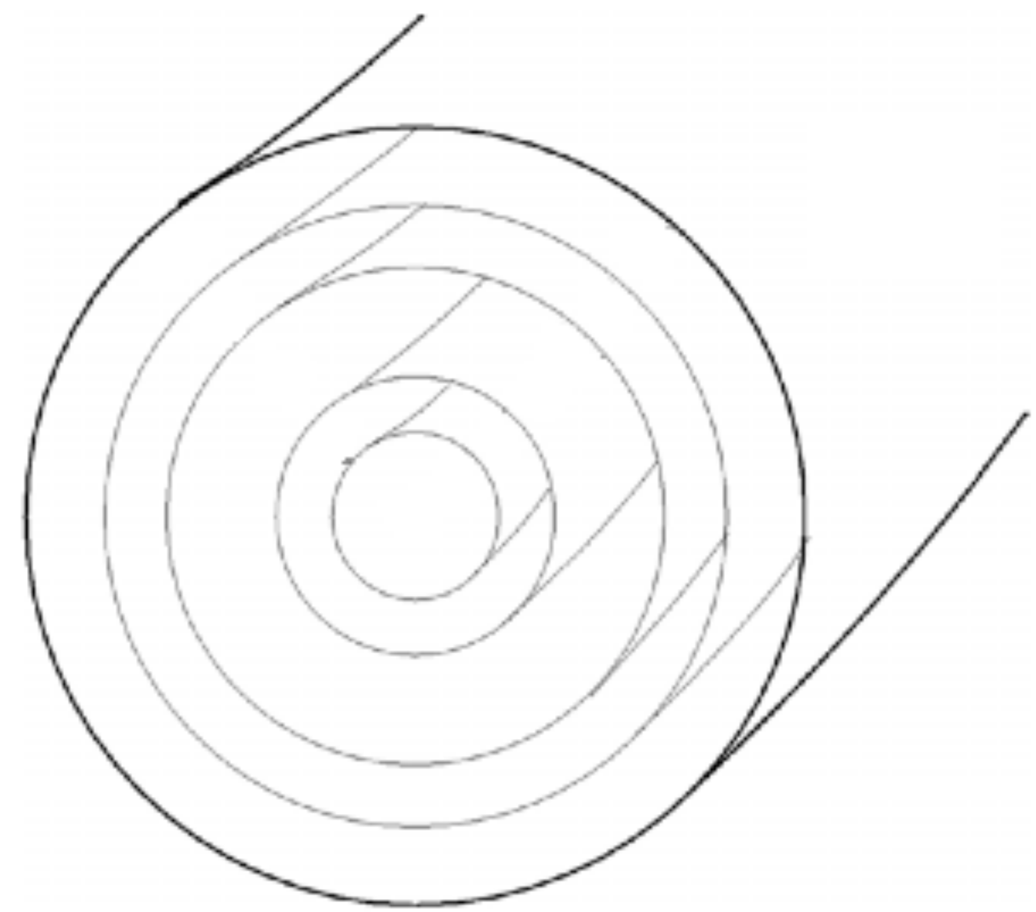
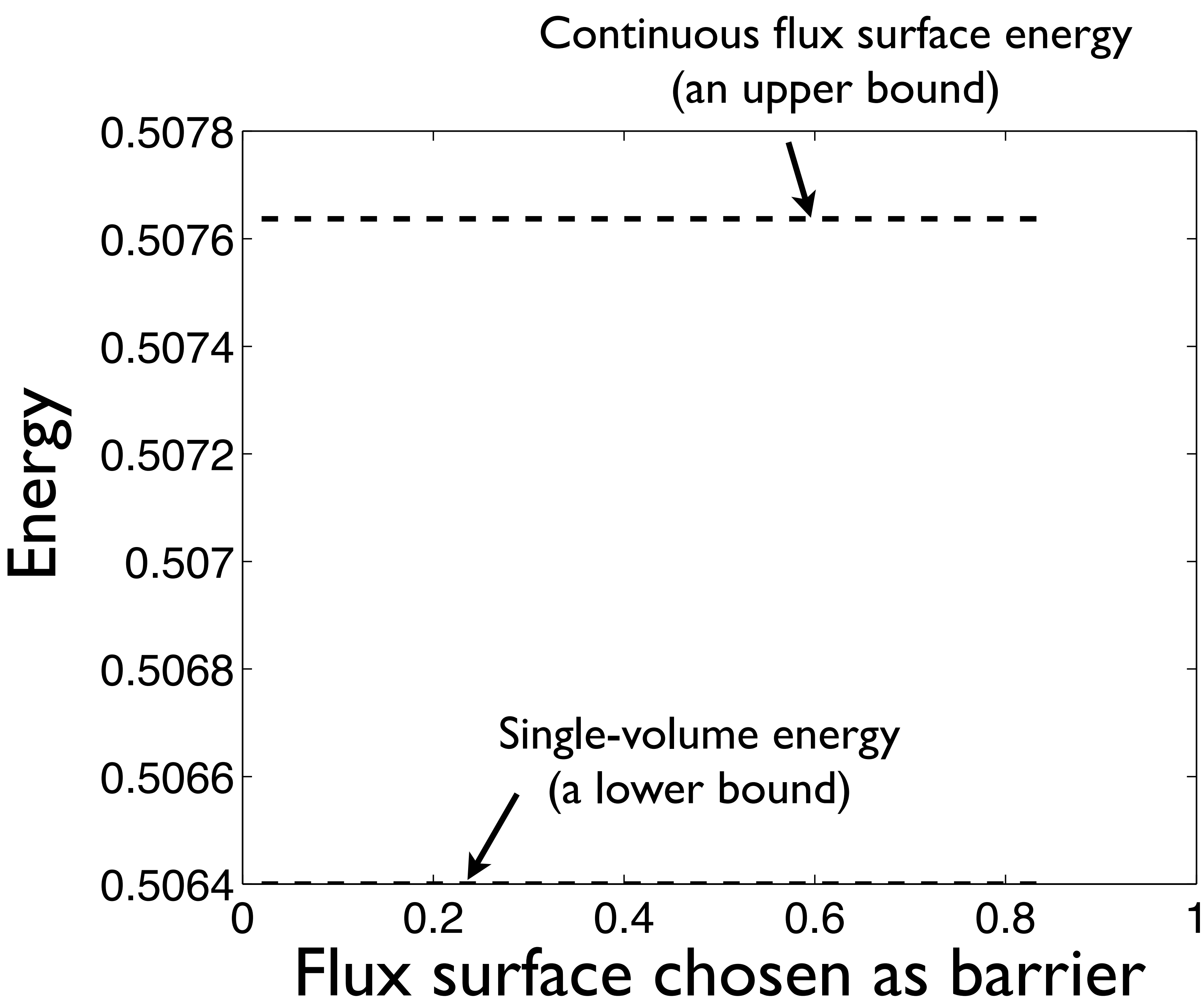
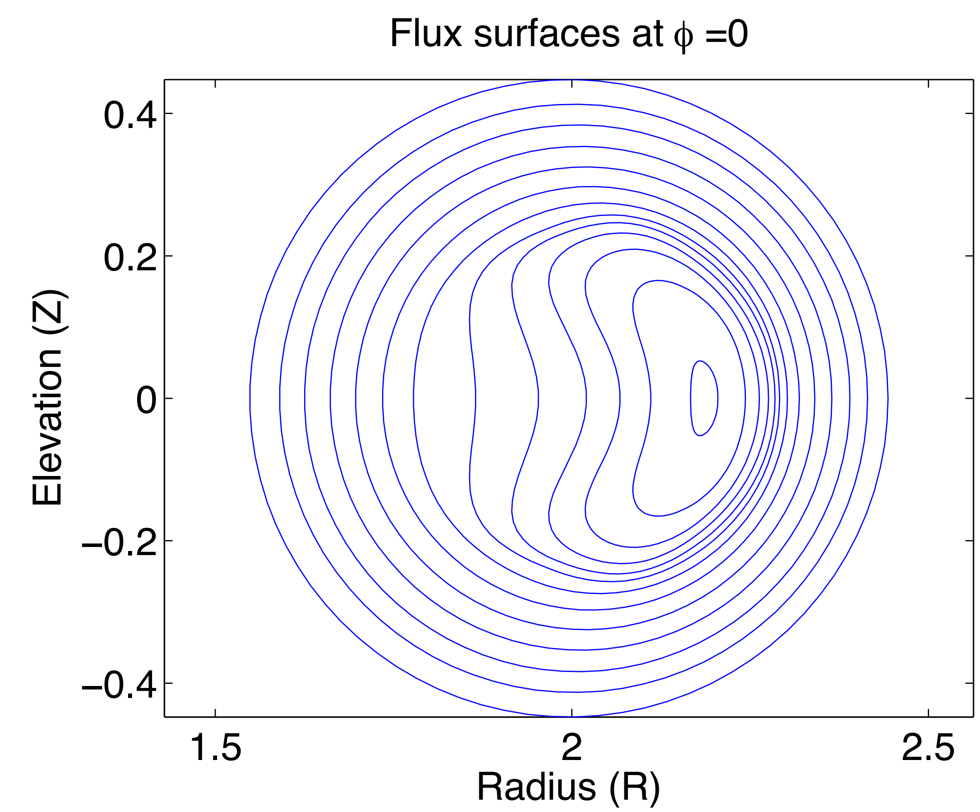
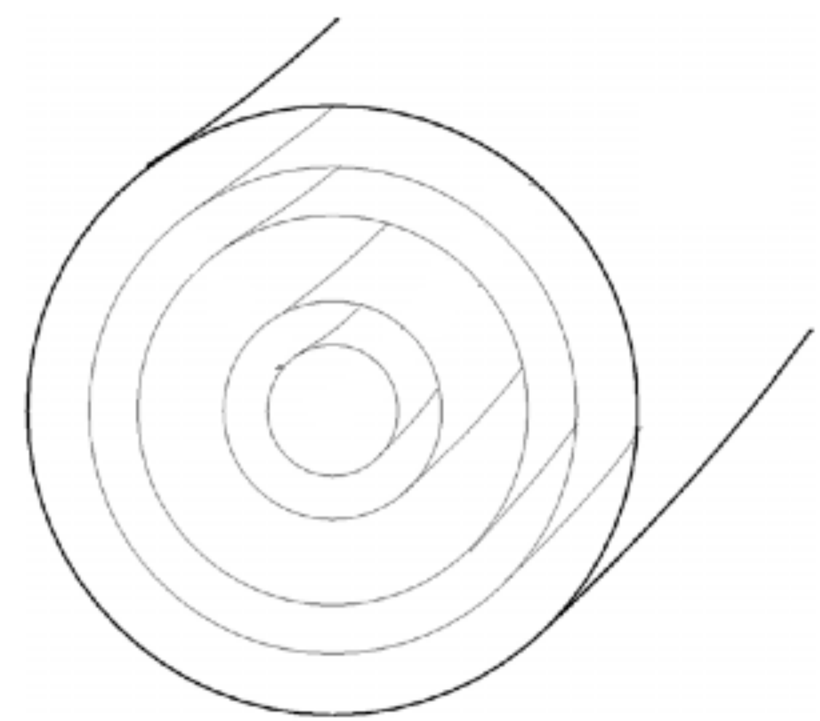




The plasma equilibrium is a minimum energy state







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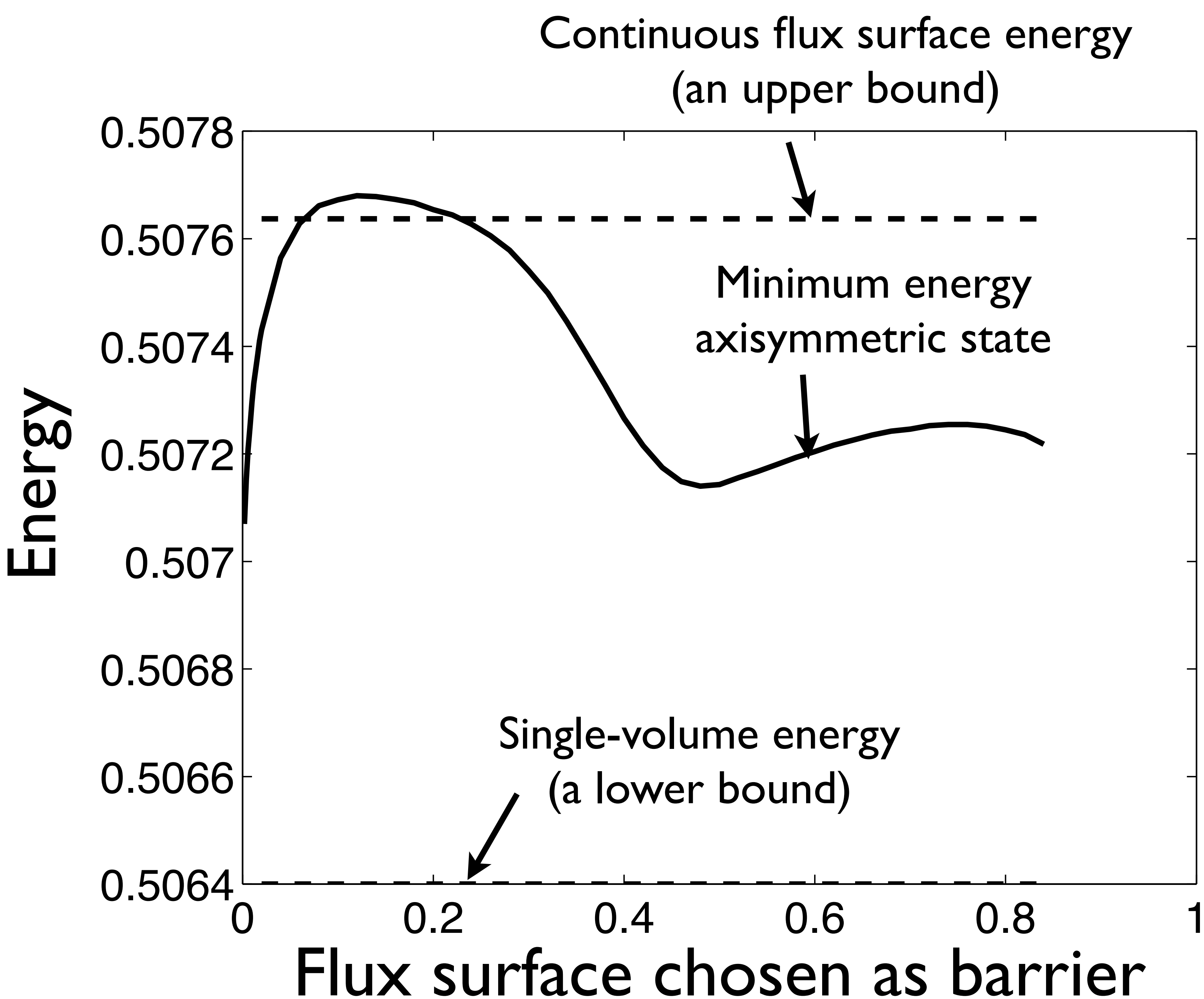

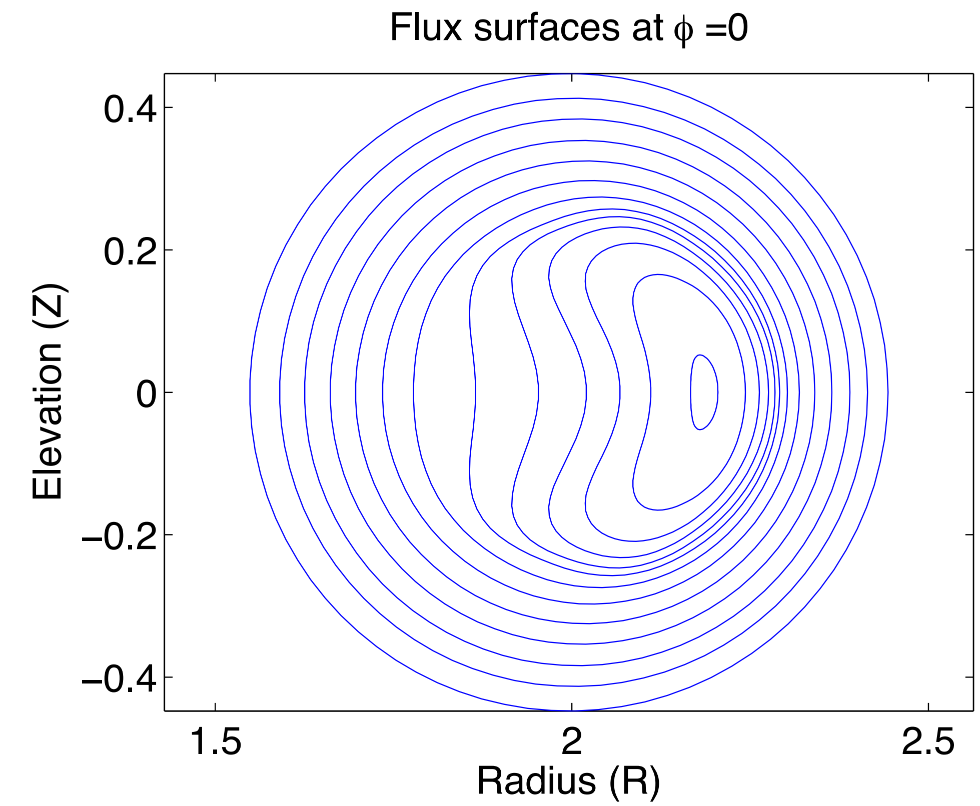

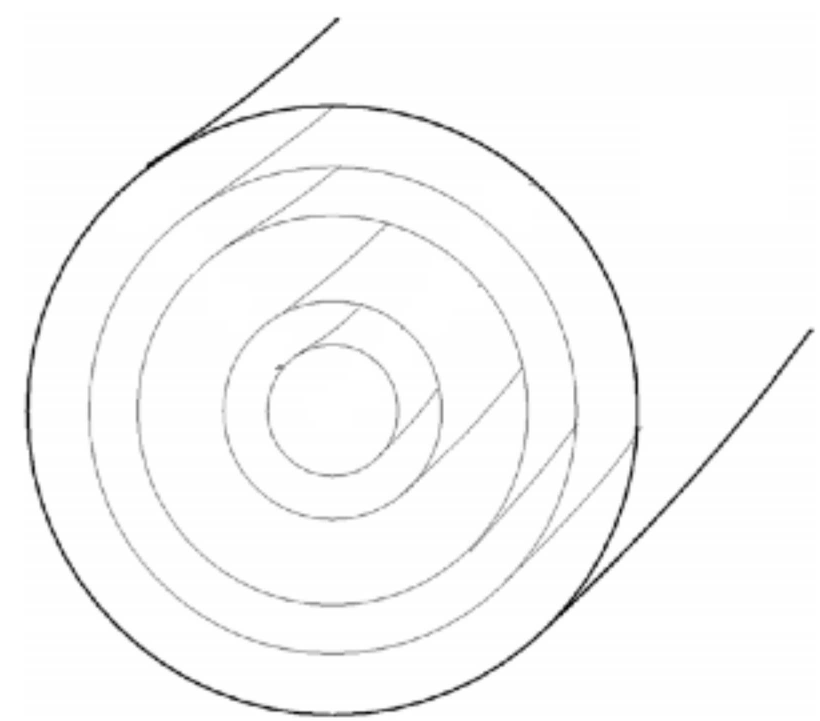

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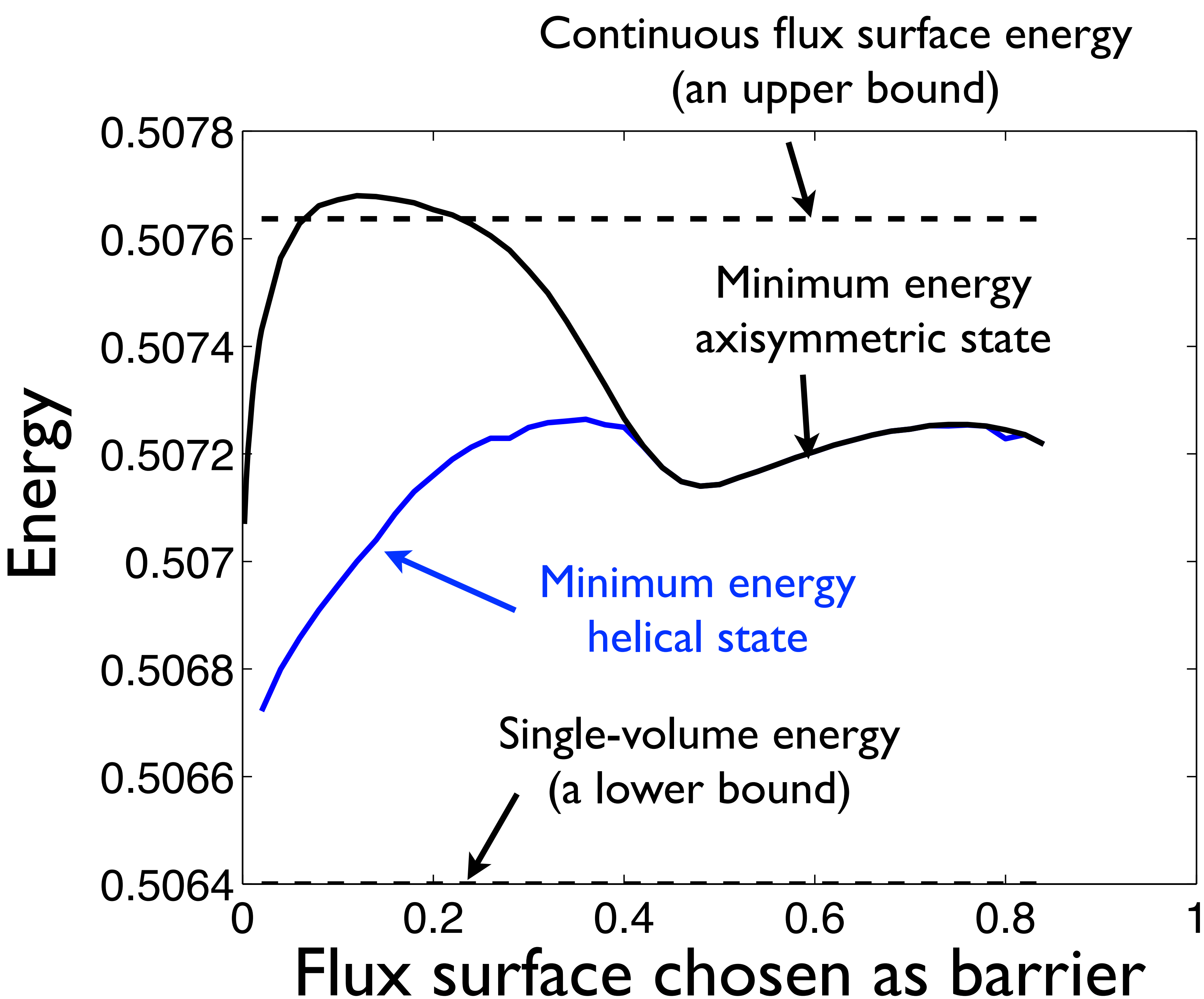

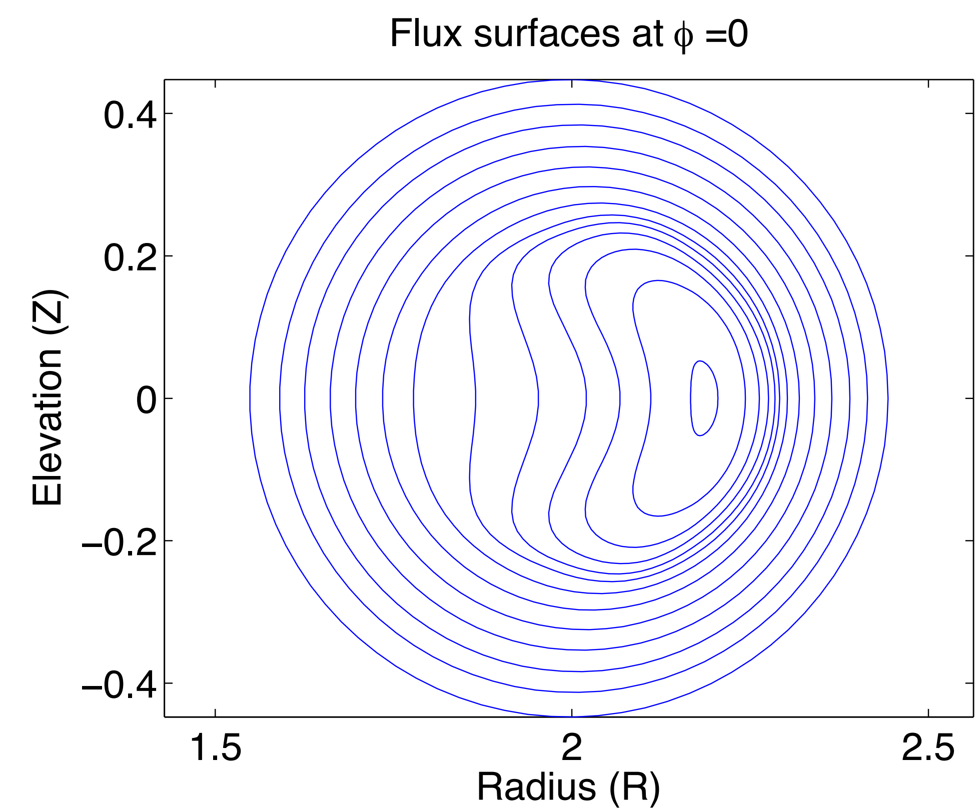

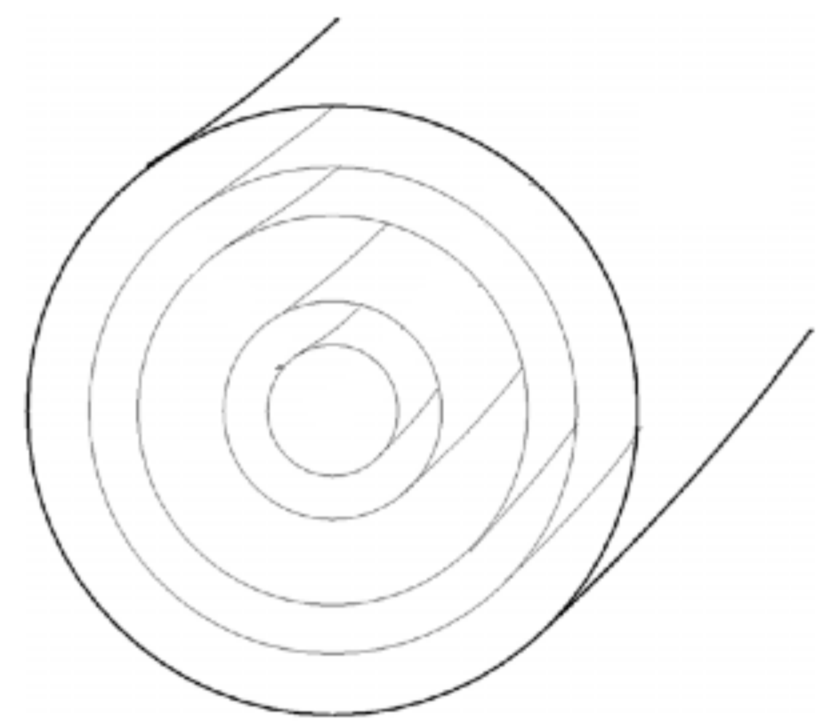

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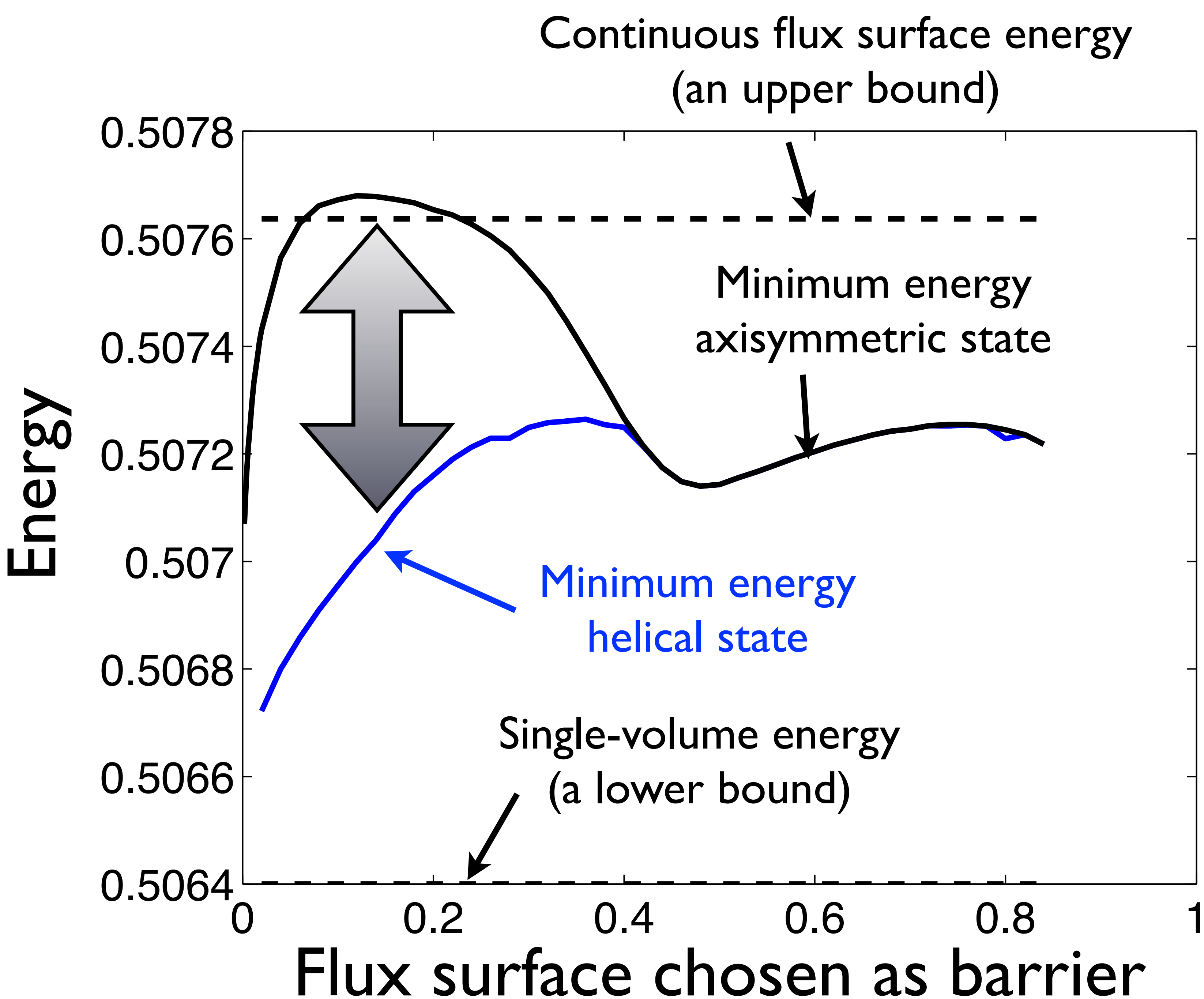
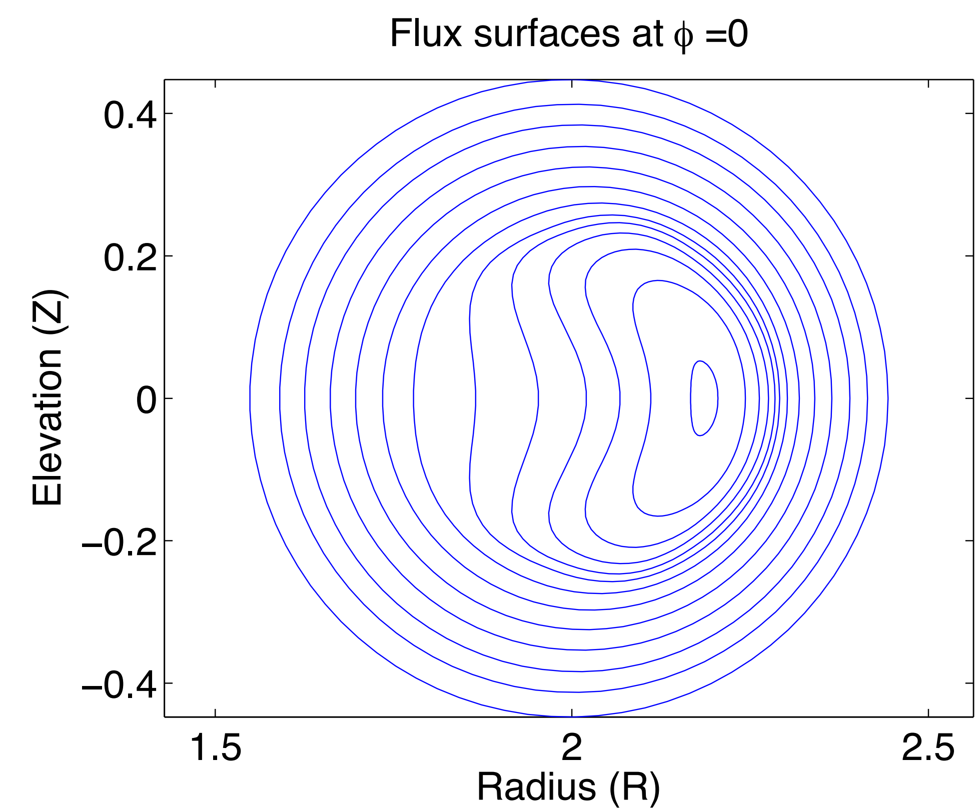
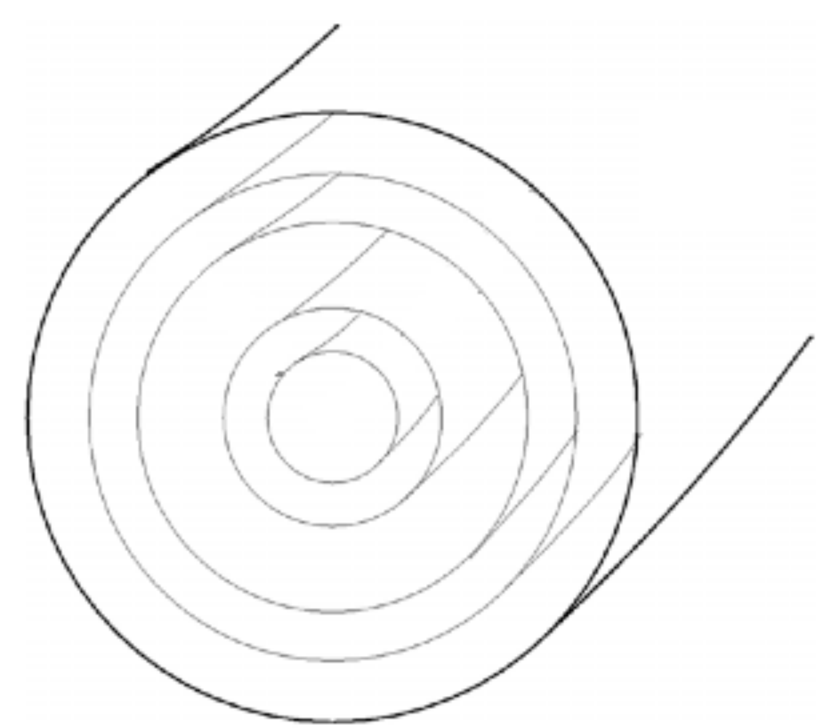
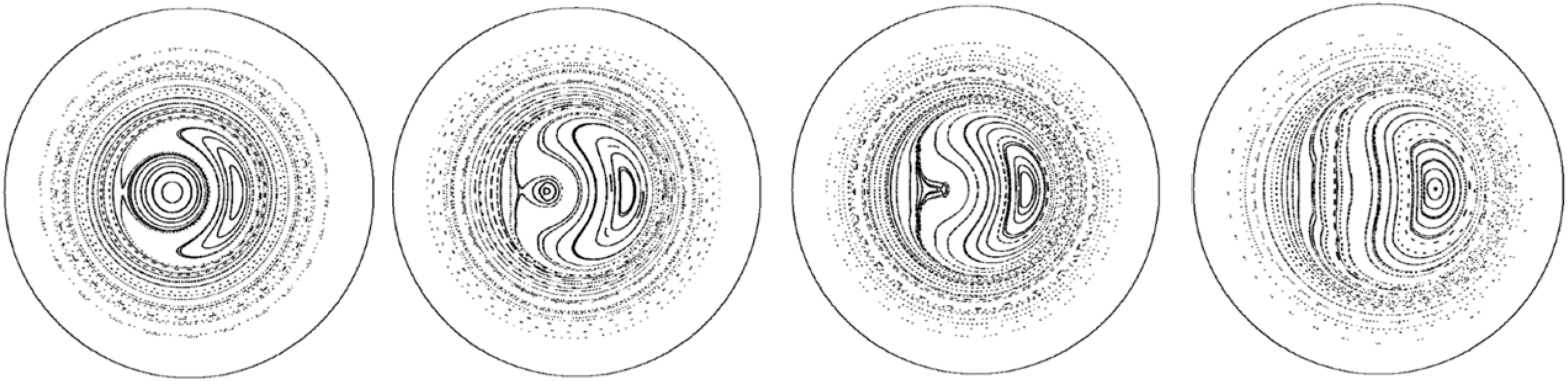






Experimental Poincaré plots



Quasi-single helicity → Single Helical Axis

Top figure source: P. Martin *et al.*, *Nuclear Fusion* **49**, 104019 (2009).

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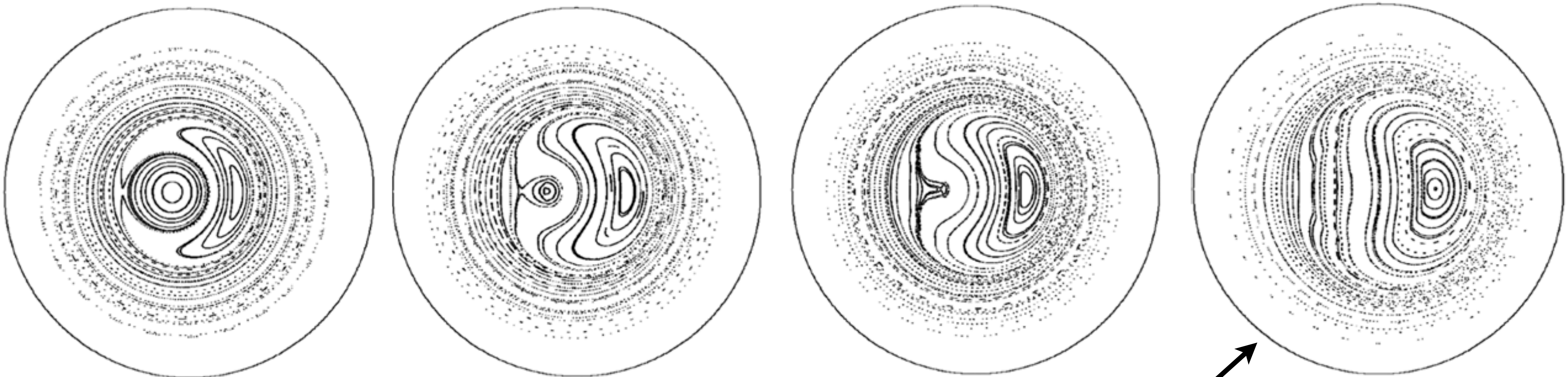

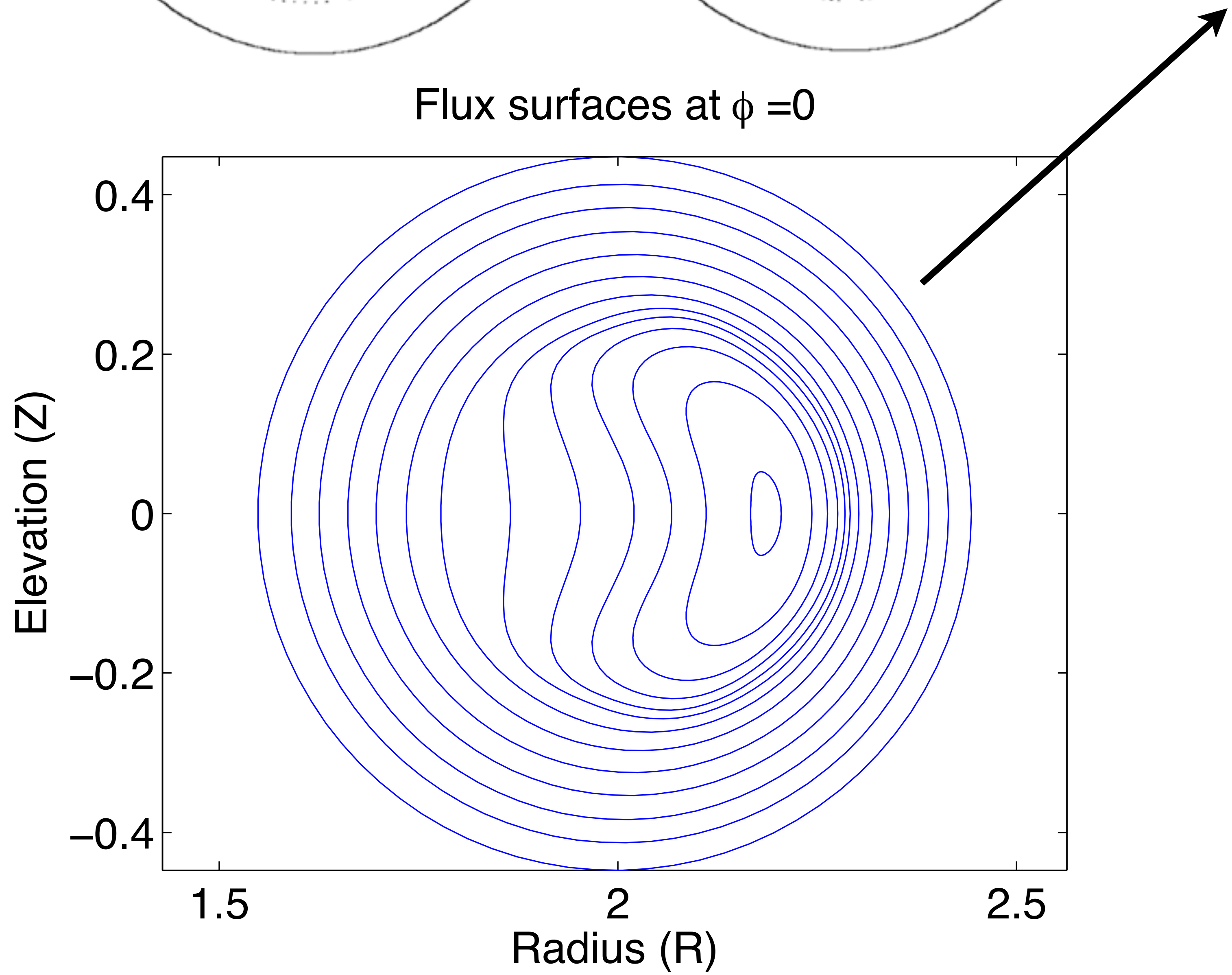

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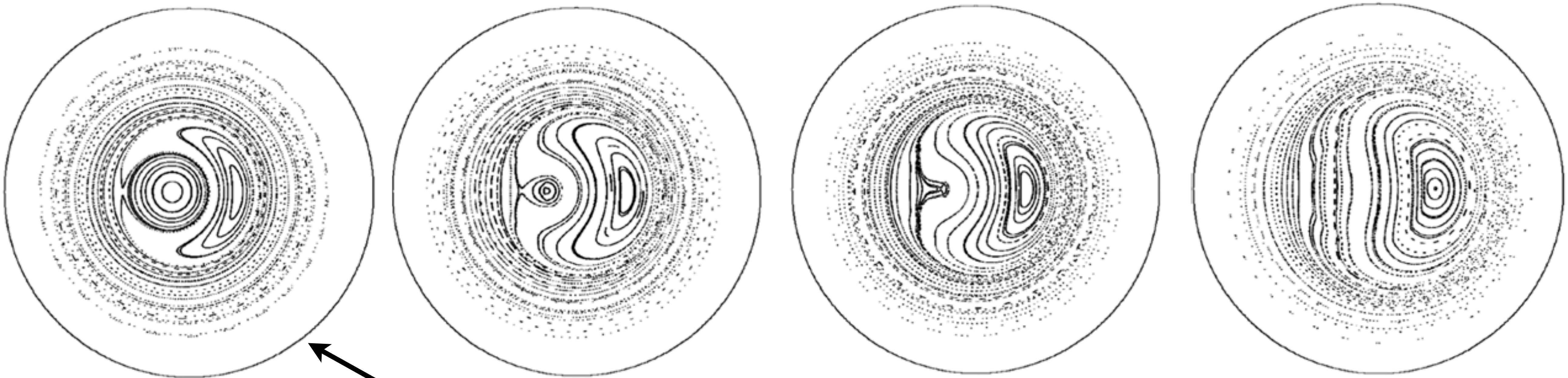

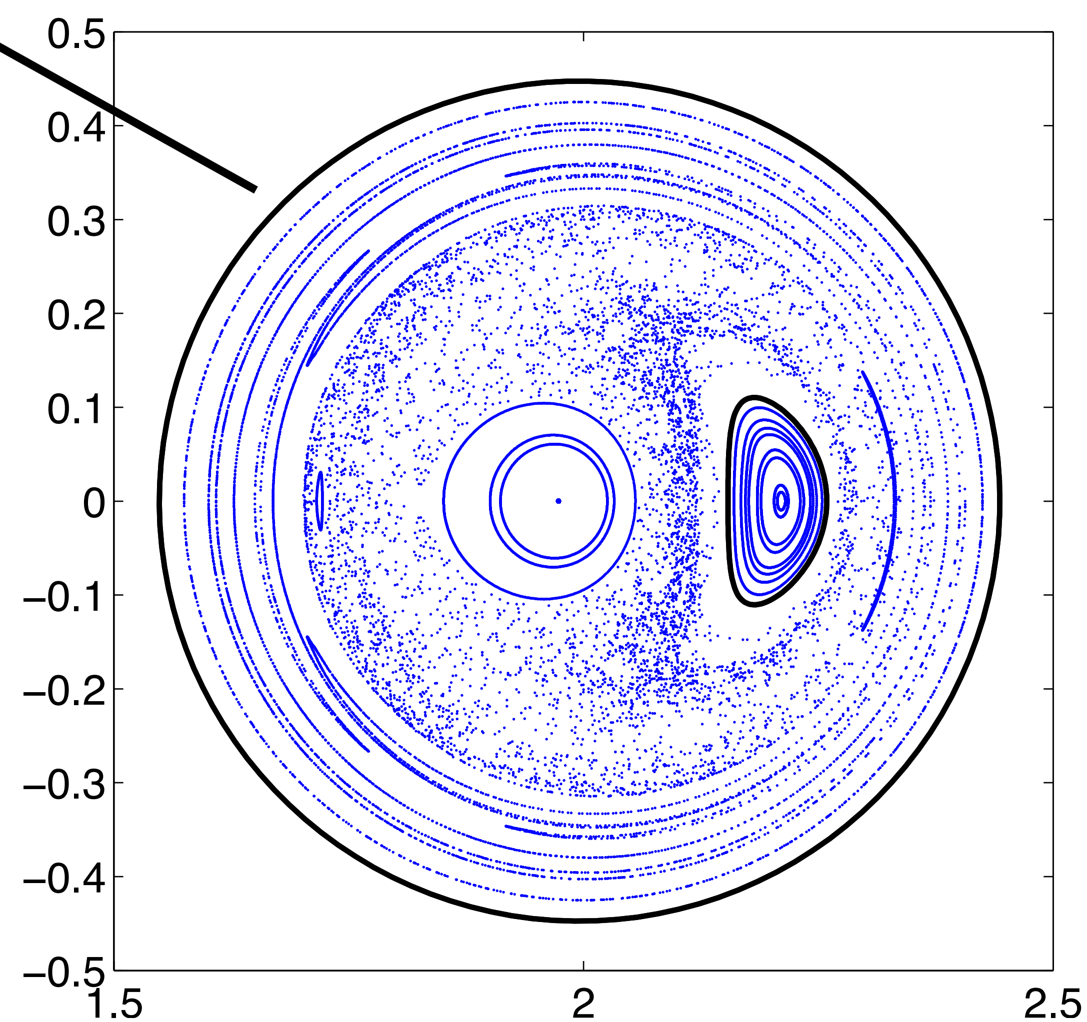

Theoretical Poincaré plots

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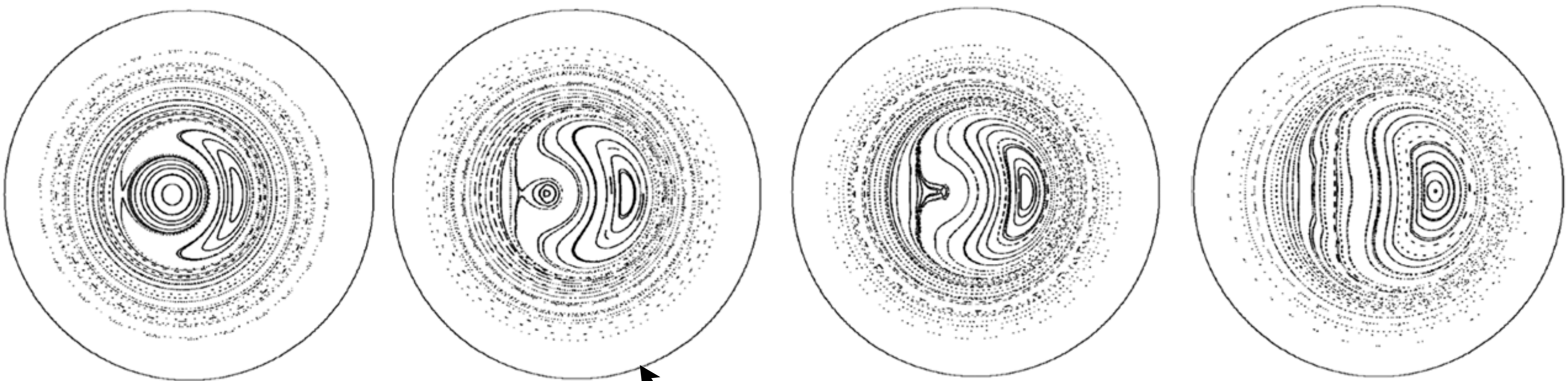

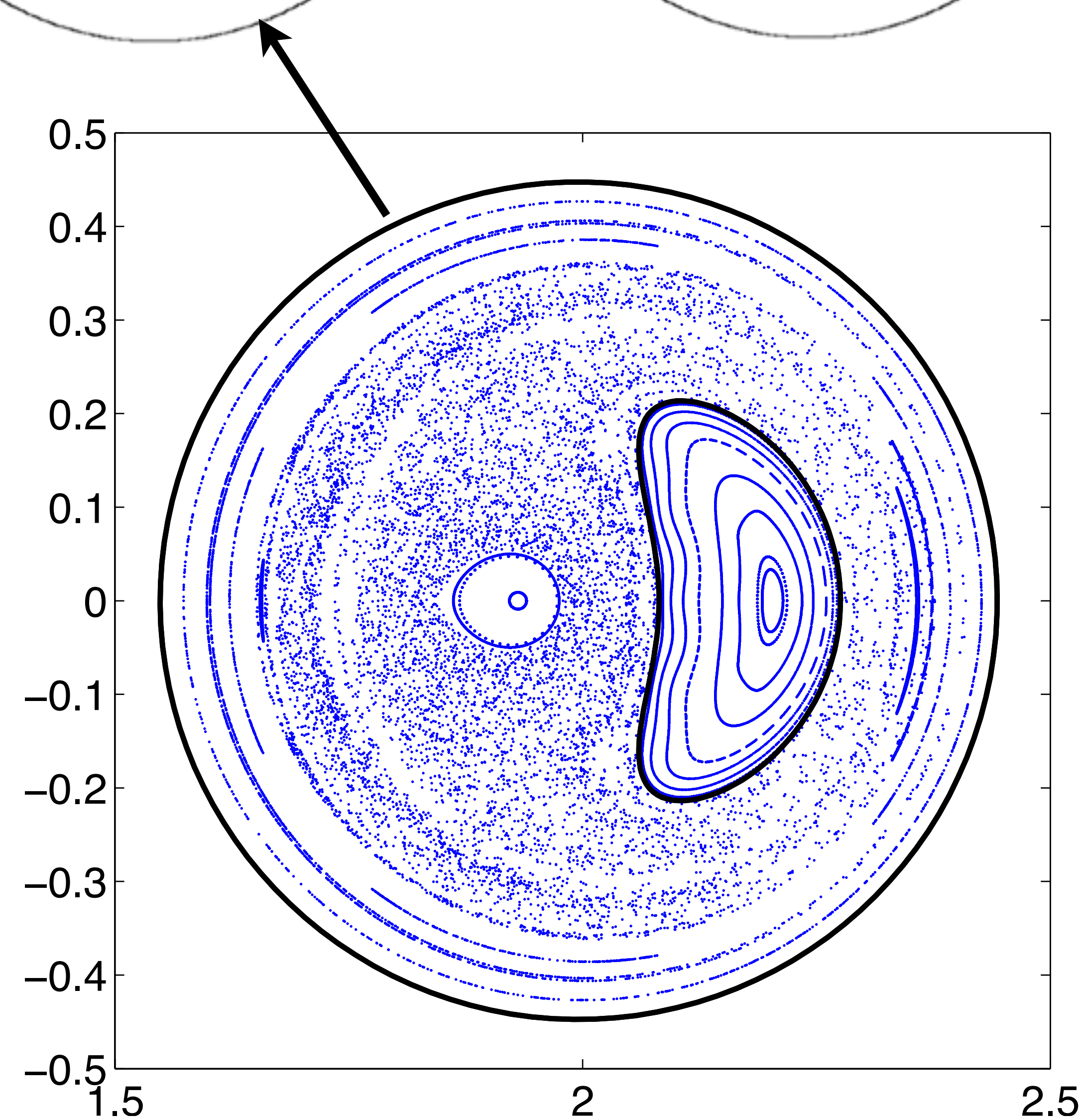

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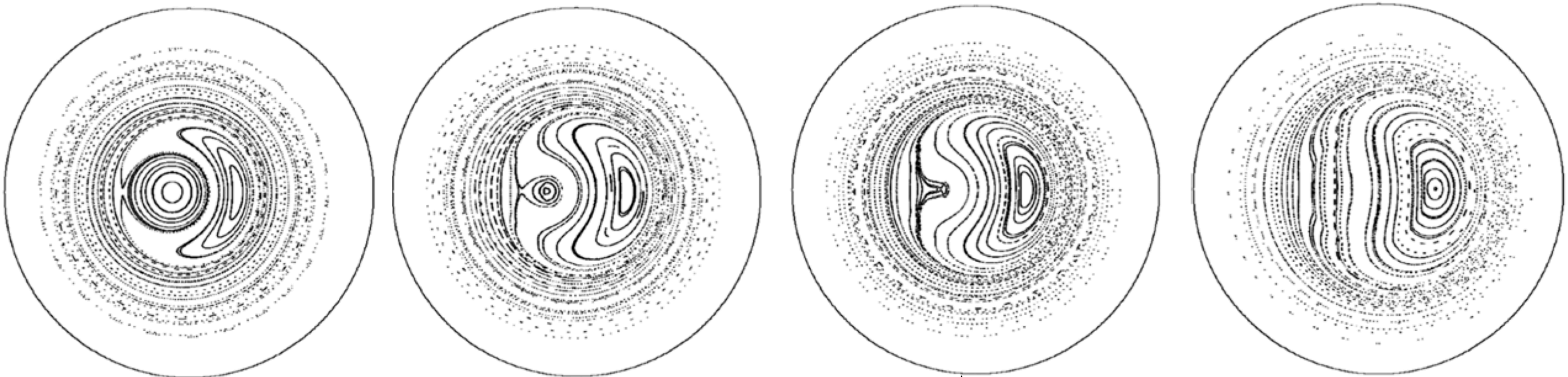

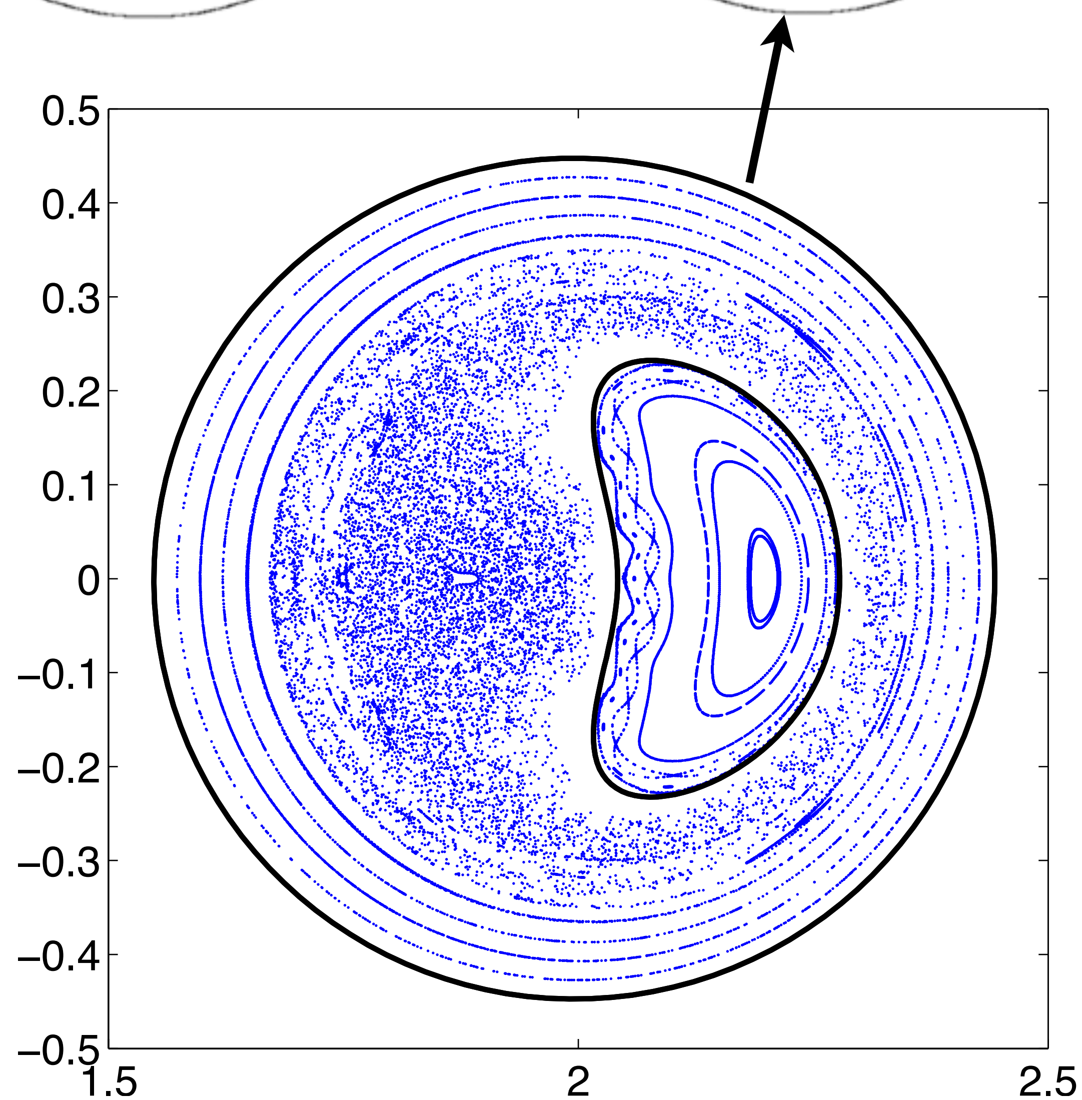

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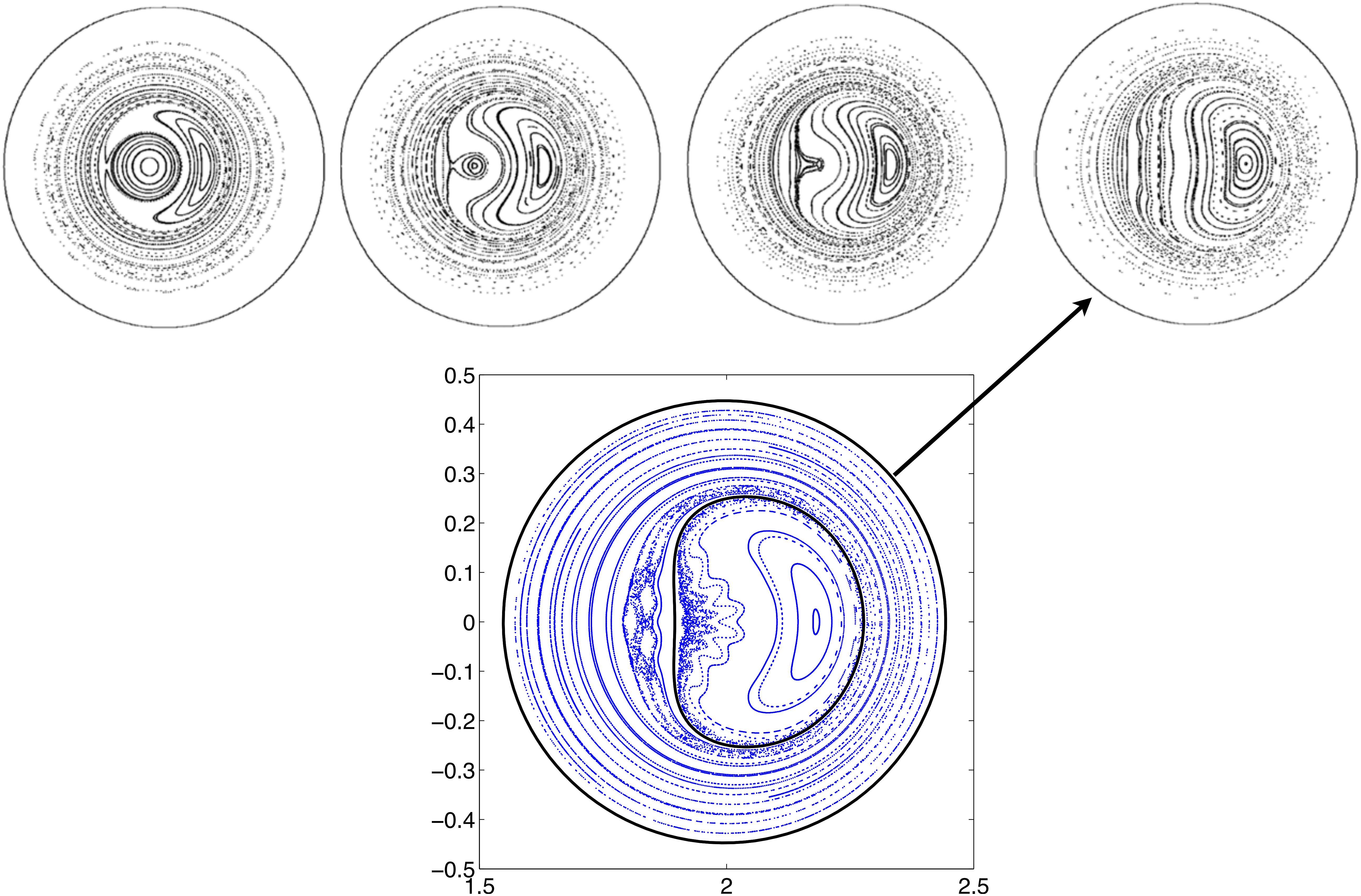

Theoretical Poincaré plots

MRXMHD gives a good qualitative explanation of the high-confinement state in Reversed Field Pinches

With a *minimal* model we reproduced the helical pitch and structure of the Quasi-Single Helicity state in RFP

With MRXMHD we reproduced the second magnetic axis. This is the *first* equilibrium model to be able to reproduce such structures.

MRXMHD is a well-formulated model that interpolates between linear force-free fields and nonlinear force-free fields

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