Theory and numerics of the Stepped Pressure Equilibrium Code.

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Toroidal magnetic plasma confinement devices are inherently 'three-dimensional'. Devices in which the magnetic field is produced primarily by external coils, such as LHD, are necessarily non-axisymmetric; and even the nominally axisymmetric tokamaks unavoidably have small nonaxisymmetric imperfections. Thus, the task of computing 3D magnetohydrodynamic equilibria is of fundamental importance for stable plasma confinement. Generally, 3D magnetic fields are non-integrable, and a weakly resistive plasma will evolve so that magnetic islands and chaotic field-lines will develop. The existence of magnetic islands and chaotic field-lines greatly complicates the computation of equilibrium solutions. In this poster, we shall describe the theory and numerics of the Stepped Pressure Equilibrium Code, SPEC, which computes equilibrium solutions allowing for arbitrary magnetic field topology.

Our model, which we call multi-region, relaxed MHD, is based on a constrained energy principle that combines ideal MHD and Taylor relaxation. The plasma region is comprised of a set of N nested annular regions which are separated by a discrete set of toroidal interfaces, $\mathcal{I}_l$. In each volume, $\mathcal{V}_l$, bounded by the $\mathcal{I}_{l-1}$ and $\mathcal{I}_l$ interfaces, the plasma energy, W_l, and the global-helicity, H_l, are given by the integrals

$$W_l = \int_{\mathcal{V}_l} \left(\frac{p_l}{\gamma - 1} + \frac{B^2}{2} \right) dv, \qquad H_l = \int_{\mathcal{V}_l} \mathbf{A} \cdot \mathbf{B} \, dv, \tag{1}$$

where $\mathbf{A}$ is the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$, and p_l is the pressure. We seek to minimize the total plasma energy subject to the constraints of conserved helicity and $p_l V_l^\gamma = const.$ in each volume, where V_l is the volume of $\mathcal{V}_l$. Arbitrary variations in both the magnetic field in each annulus and the geometry of the interfaces are allowed, except that we assume the magnetic field remains tangential to the interfaces which act as 'ideal-barriers' and coincide with pressure gradients. The Euler-Lagrange equations show that in each annulus the magnetic field satisfies $\nabla \times \mathbf{B} = \mu_l \mathbf{B}$, and across each interface the total pressure is continuous, $[[p + B^2/2]] = 0$.

We have implemented this model in a code, the Stepped Pressure Equilibrium Code, SPEC, which uses a mixed Fourier, finite-element representation for the vector potential. Quintic polynomial basis functions give rapid convergence in the radial discretization, and the freedom in the poloidal angle is exploited to minimize a "spectral-width". For given interface geometries the Beltrami fields in each annulus are constructed in parallel, and a Newton method (with quadratic-convergence) is implemented to adjust the interface geometry to satisfy force-balance.

Details regarding the numerical algorithms employed will be presented. Convergence studies and three-dimensional equilibrium solutions with non-trivial pressure and islands and chaotic fields will be presented.