Applications of 3D Equilibrium Reconstruction

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Introduction

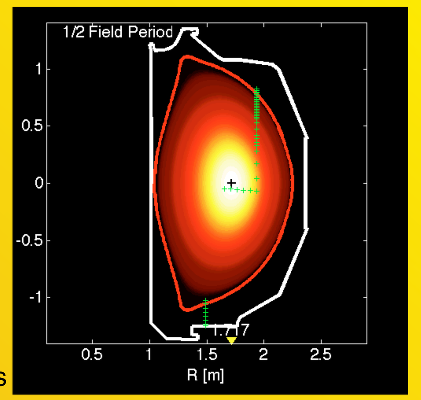
The physics of island divertor control on W7-X, boundary displacements on ITER, and ELM control studies on DIII-D require equilibrium reconstruction and diagnostic prediction capabilities in 3D. The STELLOPT code provides such a tool, allowing researchers to match a 3D equilibrium to 3D measurements of magnetic fields, electron temperature, electron density, ion density, interferometer and polarimetry. In the W7-X device, we are able to forward model diagnostic signals suggesting a set of diagnostics which have the ability to monitor the bootstrap current and control divertor strike points during long-pulse operation. Forward modeling of the 15MA ITER scenario suggests boundary variations on the order of 4 cm with magnetic diagnostic signal variations on the order of 1 Wb accounting for up to 20% the flux loop signal. Full 3D reconstructions of the DIII-D plasma with applied RMP's have been conducted using a full set of magnetic and kinetic diagnostics. In shot 142603, an ELMing double null plasma did not achieve ELM suppression with applied n=3 RMP's and evidence of mode penetration was present in the kinetic profiles. Here minimums in the pressure gradients corresponded to the location of low order rationals. Forward modeling of the equilibria with the Stepped Pressure Equilibrium Code (SPEC) suggests the presence of islands and corroborates the notion of mode penetration.

Computational Tools

VMEC

The VMEC code globally enforces the constraints of ideal MHD; consequently, a continuously nested family of flux surfaces is enforced throughout the plasma domain. Equilibria are defined through the minimization of an MHD energy functional.

$$W = \int \left(\frac{|B|^2}{2\mu_0} + \frac{p}{\gamma - 1} \right) d^3x \qquad \vec{J} \times \vec{B} - \nabla p = 0$$



VMEC DIII-D Equilibria with Thomson data points.

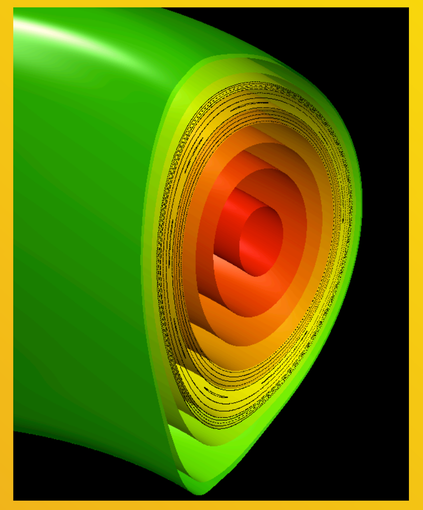
SPEC

The SPEC code relaxes the constraints of ideal MHD locally, and allows for islands and field line chaos at rational surfaces. Number theory tells us that between any two rational numbers an irrational number can be found. This motivates a grid where a finite number of ideal flux surfaces are preserved where the surfaces themselves have rotational transforms which are nobel irrationals (surfaces with the most irrational winding number between two lower order rationals). Between these ideal surfaces a force-free linear field is calculated which obeys the Beltrami equation.

$$W = \sum_l \left[\int \left(\frac{|B_l|^2}{2\mu_0} + \frac{p_l}{\gamma - 1} \right) d^3x - \mu_l \left(\int \left(\vec{A} \cdot \vec{B} \right) d^3x - K_{l0} \right) \right]$$

$$\vec{\nabla} \times \vec{B} = \mu_l \vec{B} \qquad \left[\left[\frac{B^2}{2\mu_0} + \frac{p}{\gamma - 1} \right]\right] = 0$$

In order to guarantee that the rotational transform is continuous, the code iterates on the helicity multiplier (μ). The resulting rotational transform profile is continuous across the interfaces. Despite the stepped pressure approximation, properties such as the Shafranov shift are accurately modeled with as little at 4 surfaces. This is attributed to the preservation of volume integrated quantities such as the pressure and magnetic energy.

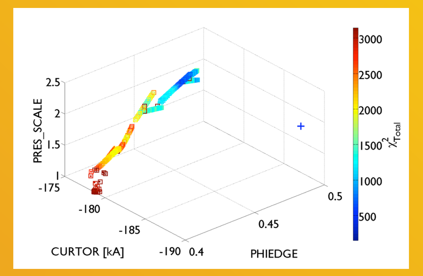


Depiction of SPEC annular regions for ITER.

STELLOPT

The STELLOPT code is a modified Levenberg-Marquardt optimizer which searches parameter space for a good match between the VMEC equilibrium and experimental data. Synthetic magnetic diagnostics are calculated by virtual casing in the DIAGNO code. Synthetic Thomson scattering, charge exchange, interferometry, and MSE polarimetry signals are calculated by the STELLOPT code.

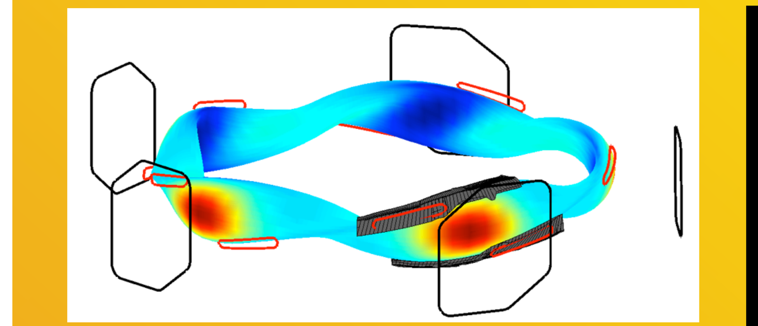
$$\chi^2 = \sum_i \frac{| f_i(TARGET) - f_i(MODEL) |^2}{\sigma_i^2}$$



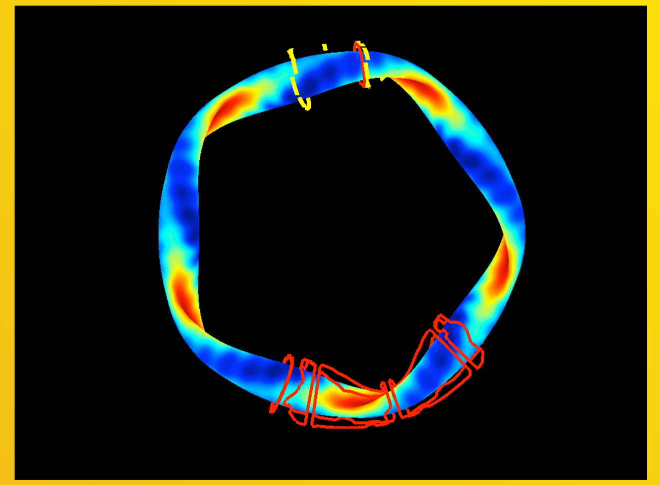
Depiction of STELLOPT parameter space search (LMDIF)

W7-X Forward Diagnostic Modeling

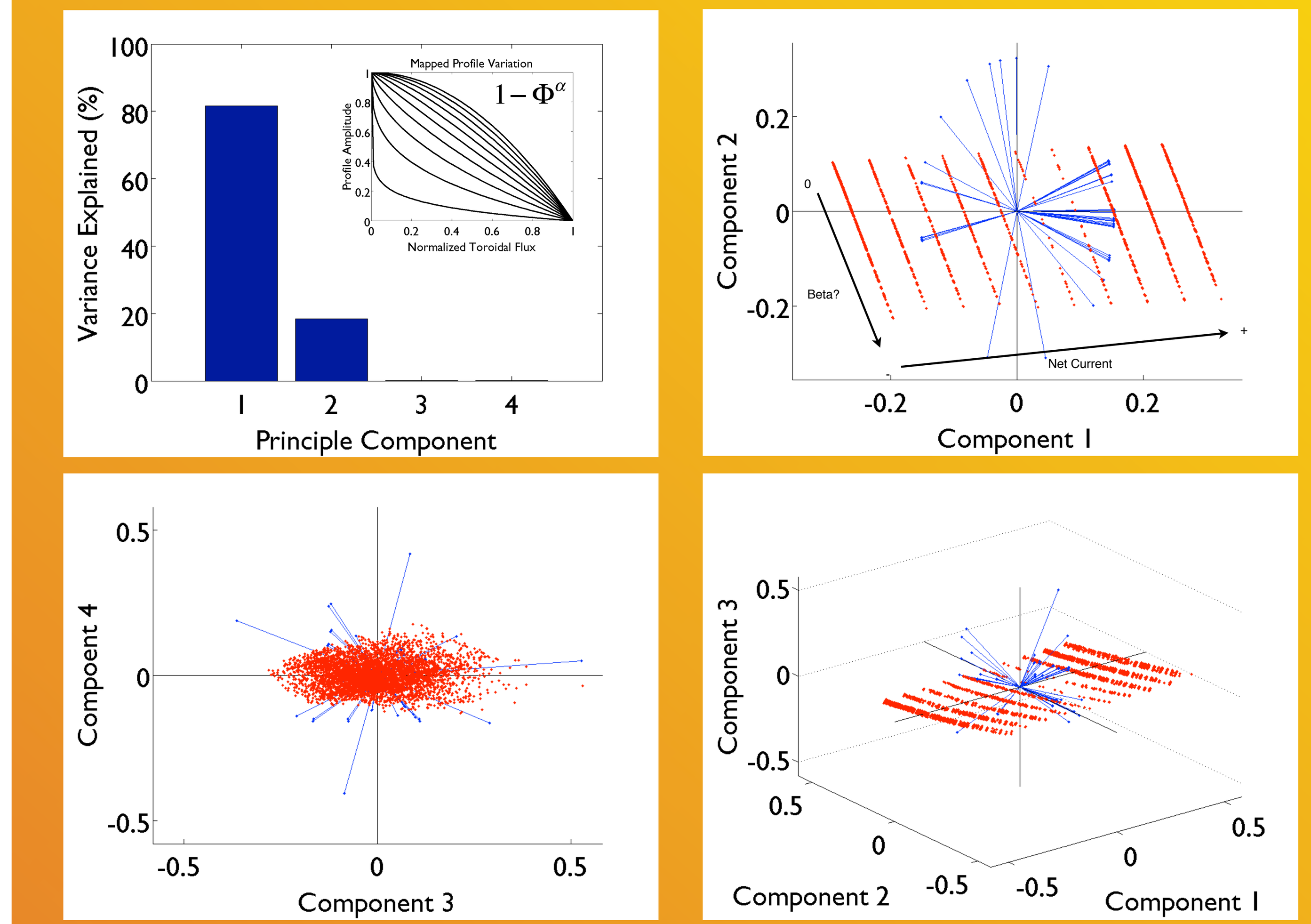
The prediction and control of strike points has been identified as a key topic to the success of the W7-X experiment. The divertor control coils and planar coils have been identified as mechanisms to control the locations of unstable manifold (used as a proxy for the strike points). STELLOPT/DIAGNO2 modeling of the diagnostic response indicates limited ability to detect profile variations.



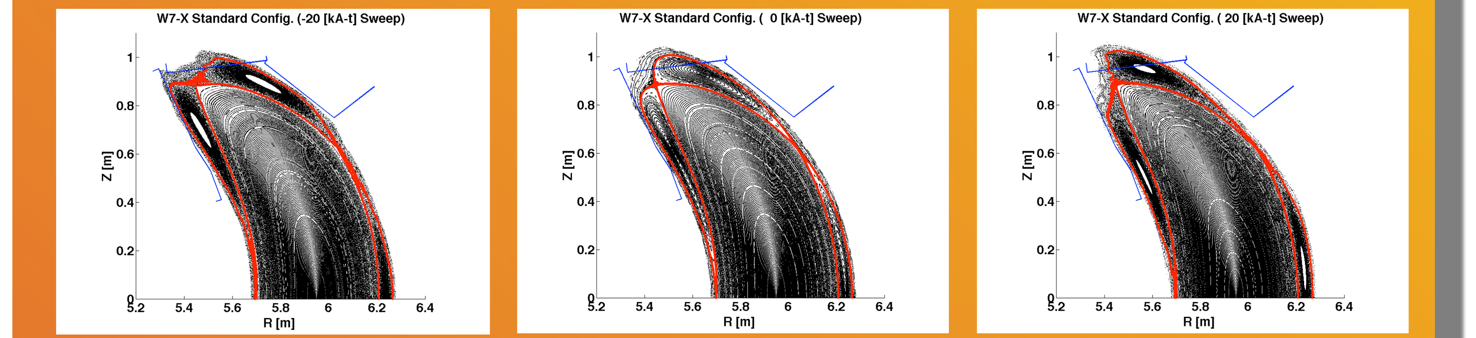
Trim (black) and sweep (red) coils provide the primary actuators for error field correction and divertor control. Both sets of coils drive non-stellarator symmetric fields. A divertor module is plotted in for reference. Color contours indicate B-normal from Trim coils.



Flux loops (red) and segmented Rogowski coils (yellow) provide the primary means for sensing variations of the plasma parameters in real time. Color contours indicate |B| on VMEC boundary.



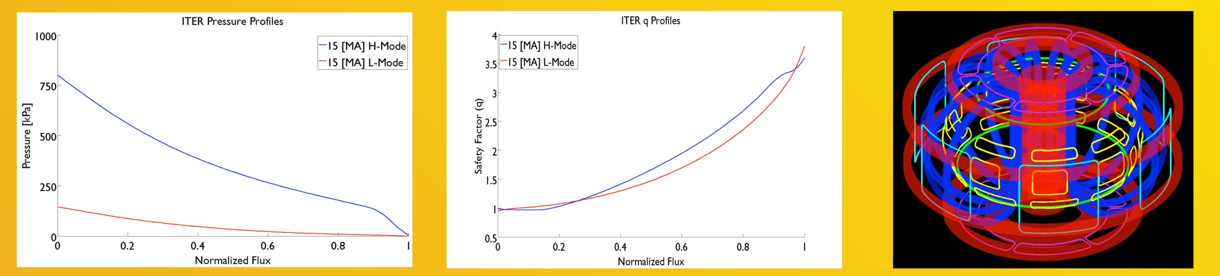
Principle component analysis performed for ~10,000 W7-X Standard Case equilibria (red dots) where net toroidal current (CURTOR), pressure scaling (PRES_SCALE), current density profile (AC[1]) and pressure profile (AM[1]) were varied (alpha coefficient above). The response of the magnetic diagnostics (blue) were evaluated and it was found that net toroidal current and to a lesser extent beta were the only significant components.



Poincare plots of the effect of sweep coil energization on the W7-X standard configuration in vacuum. The unstable manifold (red) provides a proxy for strike point motion on the upper divertor plates (blue). These coils indicate an ability to move strike points almost 10 cm along the divertor plate.

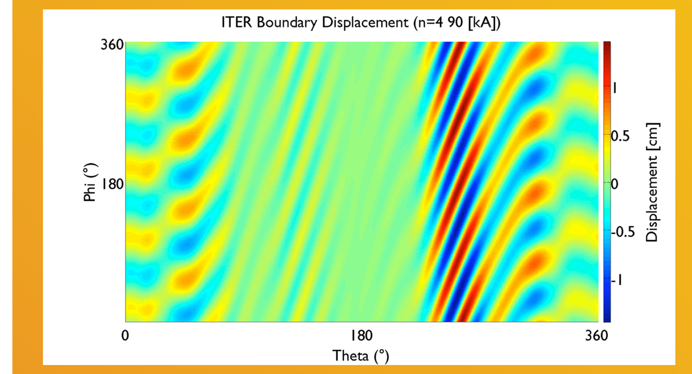
ITER Predictive Modeling

Forward modeling of the ITER 15 MA scenario was performed by the CORSICA code. These simulations provided inputs to the VMEC 3D equilibrium code at the start of flat-top (L-mode) and at full D-T burn (H-mode). Pressure and current profiles were utilized to construct VMEC input files. The in-vessel coils were then energized to examine boundary effects for multiple modes and coil currents. The DIAGNO2 code was then utilized to evaluate the B-Probe in Flux Loop response in ITER. Forward modeling of island structure was performed using the SPEC code.

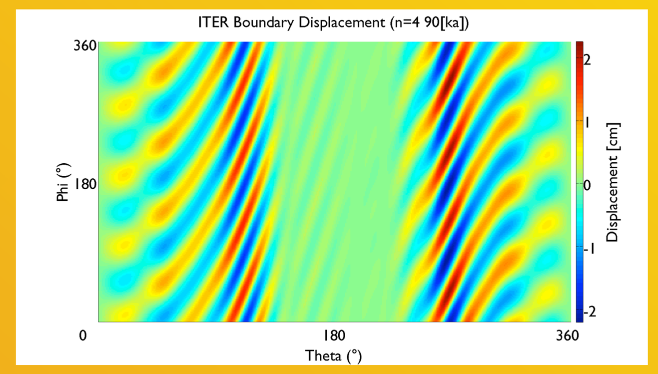


Scenario	Maximum Displacement
n=3 45 [kA]	0.70 [cm]
n=3 90 [kA]	1.88 [cm]
n=4 30 [kA]	0.54 [cm]
n=4 90 [kA]	1.46 [cm]

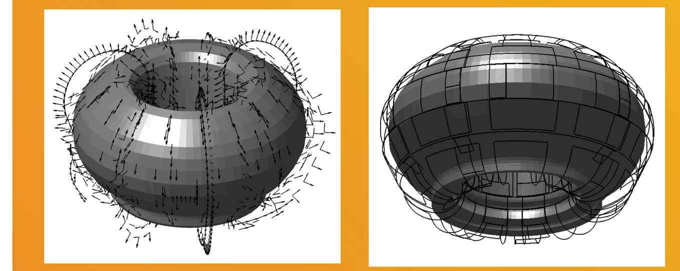
In-Vessel Coil Scenario	Maximum Displacement
n=3 45 [kA]	0.84 [cm]
n=3 90 [kA]	1.84 [cm]
n=4 30 [kA]	1.15 [cm]
n=4 90 [kA]	2.25 [cm]



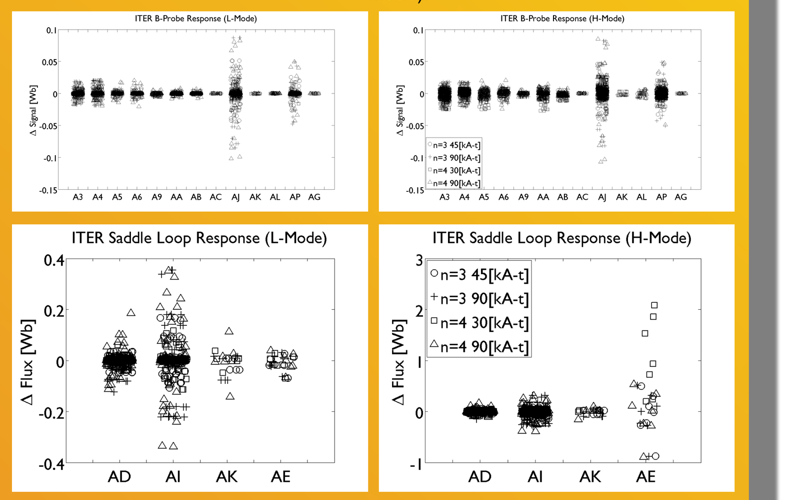
ITER boundary displacements for the 15MA scenario at the start of current flattop (L-Mode).



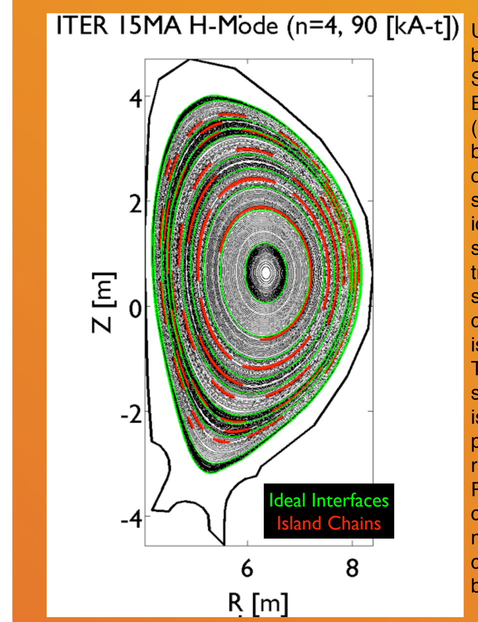
ITER boundary displacements for the 15MA scenario at full burn (H-Mode).



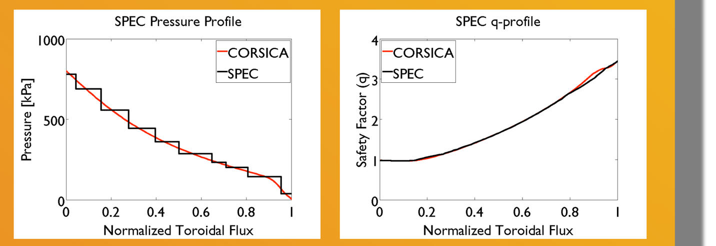
ITER B-Field Probes (left) and flux loops (right). ITER first wall depicted for reference.



ITER magnetic diagnostic response to applied RMP at start of current flat top (left) and full burn (right) for the 15MA scenario. Horizontal scatter has been artificially added for clarity.



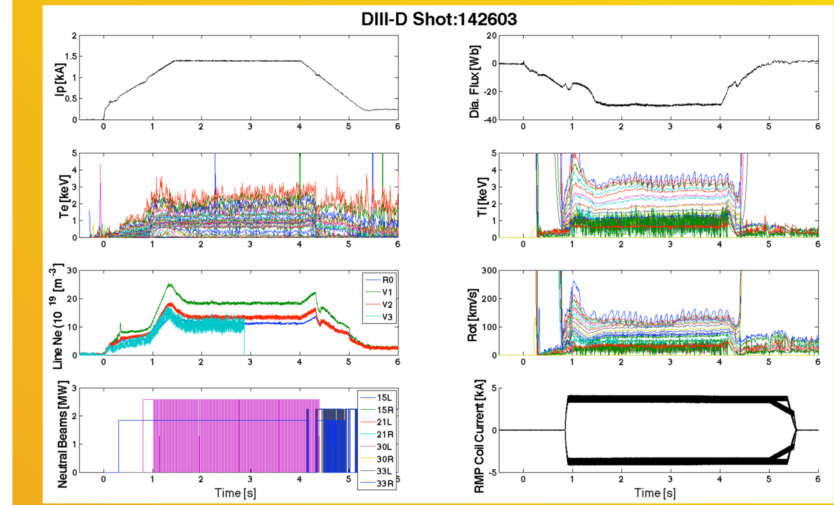
Utilizing the VMEC free boundary solutions, the Stepped Pressure Equilibrium Code (SPEC) was run in fixed boundary allowing for the opening of islands and stochastic regions. Ten (10) ideal interfaces were chosen surfaces with nobel irrational transform (green). These surfaces are mathematically considered the most robust to island formation and breakup. The resulting equilibrium shows the formation of narrow islands (red) throughout the plasma but of very limited radial extent (sub cm). Rotational transform calculations suggest adding more edge ideal interfaces to capture the fine details of the bootstrap current.



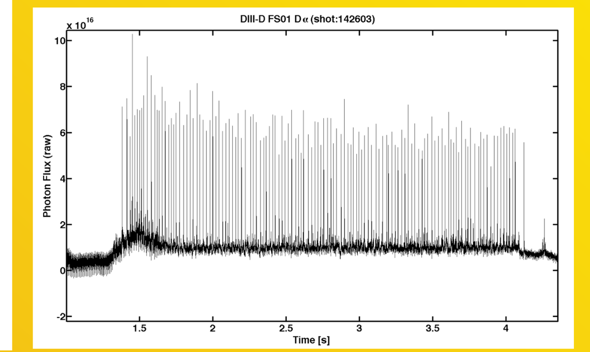
Stepped pressure profile approximation (black) utilized in the SPEC calculation plotted against the profile from CORSICA (red). Calculated q profile from SPEC plotted against that obtained from CORSICA.

DIII-D 3D Equilibrium Reconstruction

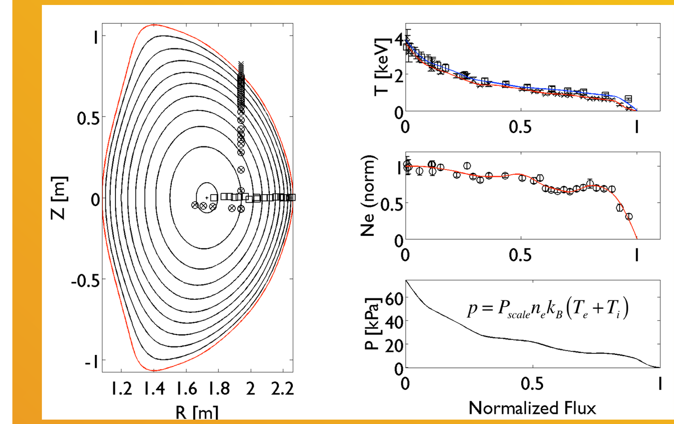
Three dimensional equilibrium reconstructions for DIII-D shot 142603 were performed, matching flux loop, B-probes, Thomson scattering, charge exchange, interferometric, and motional Stark effect diagnostics. The ideal coil set was utilized and the C-coil contribution was ignored. Here chi-squared between VMEC and experimental measures were minimized. Species temperatures and densities were utilized to calculate total pressures.



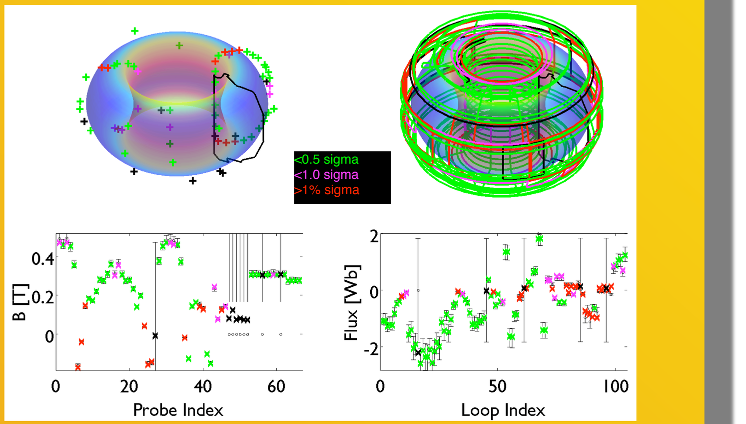
DIII-D shot 142603 time evolution. This was a double null (stellarator symmetric) shot with applied n=3 RMP. Charge exchange profiles indicate a clear sawtooth like behavior in the core.



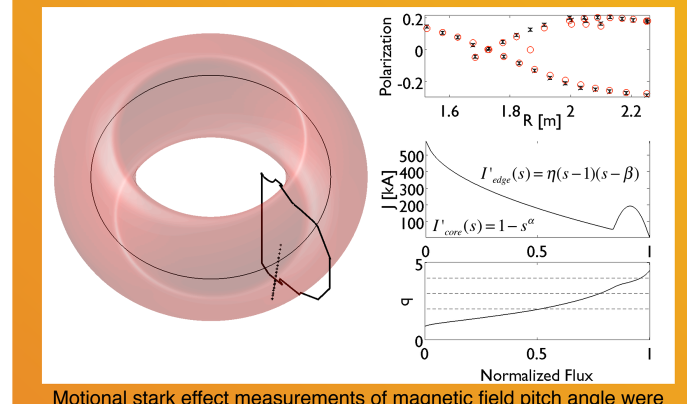
The shot indicated an ELMing characteristic despite applied n=3 RMP.



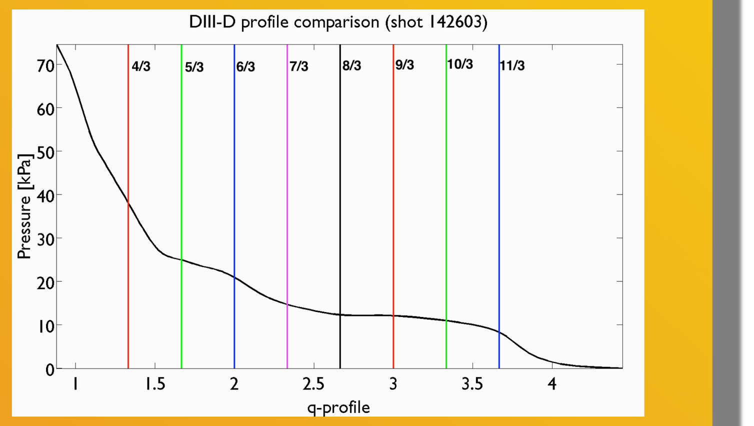
STELLOPT 3D reconstructions were able to match Thomson and Charge Exchange measurements of the electron temperature and density (red) and the ion temperature (blue).



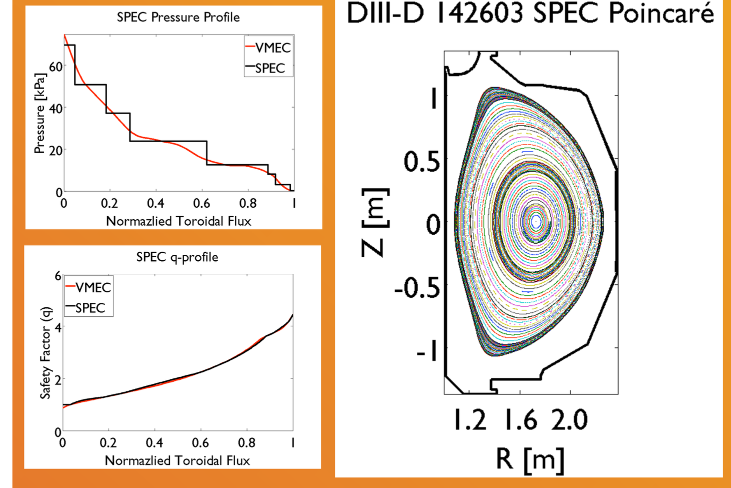
The full set of available magnetic diagnostics were matched in STELLOPT using the DIAGNO2 code. Any mismatch present was small and can be attributed to lack of C-coil and error field effects.



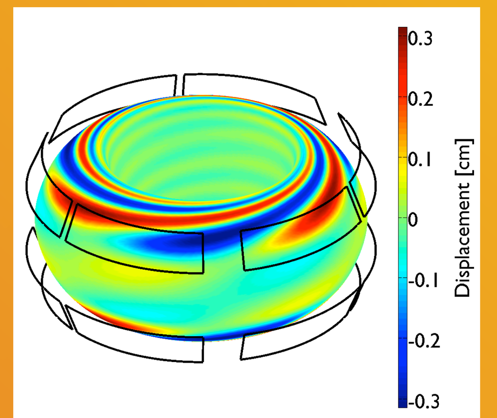
Motional stark effect measurements of magnetic field pitch angle were well matched utilizing a simplified core current model (alpha). Bootstrap current was modeled but MSE data was not available in the edge.



Plotting the reconstructed pressure profile against the reconstructed q-profile, the presence of flat spots near resonant low order rationals (n=3) is clear. This suggests that resonant fields were penetrating the plasma possibly opening small islands.



Forward modeling of the DIII-D 3D reconstructed equilibria with the SPEC code. No large islands are present in the equilibria but a small island is present at the 5/3 surface and flux surface kinking is present near the 8/3 surface.



The reconstructed 3D equilibria indicated sub-cm displacements at the edge of the plasma.