CCFE CULHAM CENTRE FOR FUSION ENERGY

PPPL PRINCETON PLASMA PHYSICS LABORATORY

CONSORZIO RFX

Australian National University

Advanced MHD models of anisotropy, flow and chaotic fields

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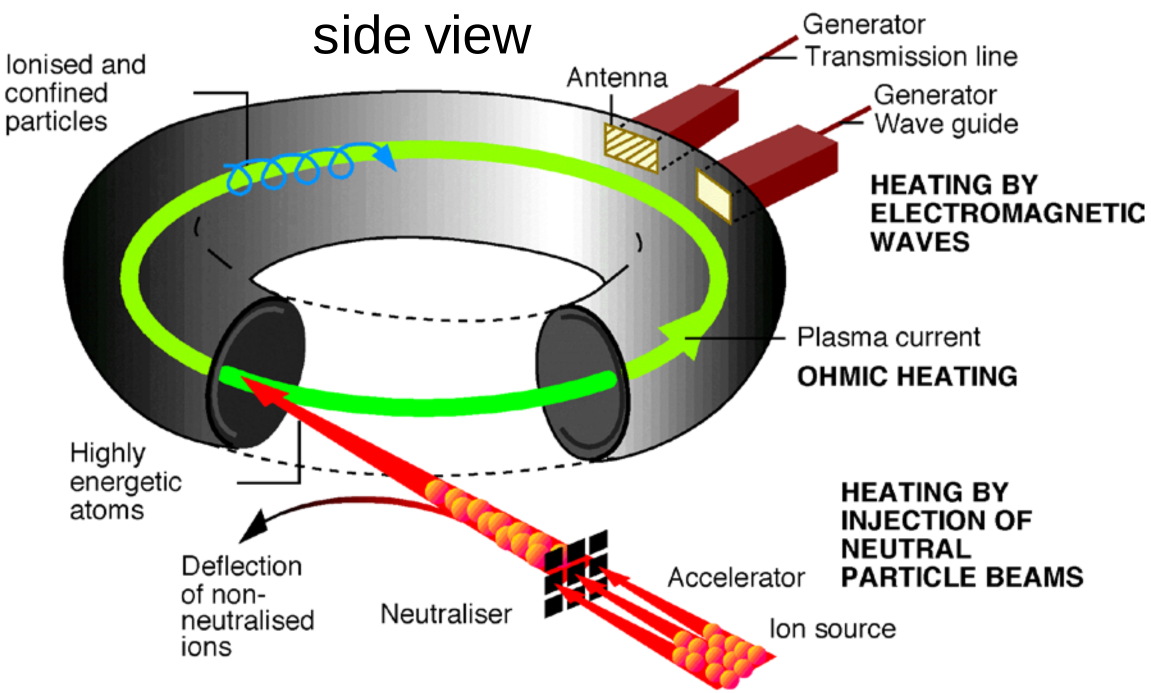
Outline

- Anisotropy: equilibrium and stability
  - Expected impact of anisotropy
  - Development of anisotropy into EFIT++ , HELENA
  - Demonstrate impacts of anisotropy on J_{ϕ}, plasma parameters
  - Development of MHD single adiabatic stability model
  - Compute impact on MAST equilibrium, stability
  - Future directions
- Multiple Relaxed Region MHD model
  - resolves chaotic field regions, islands, flux surfaces in fully 3D plasmas
  - Stepped Pressure Equilibrium Code.
  - Demonstrate two interface model to describe helical plasmas in reverse field pinches
  - Highlight some recent progress
  - Future directions
- Conclusions

"MHD with anisotropy in velocity, pressure"

- Pressure different parallel and perpendicular to field due mainly to *directed* neutral beam injection



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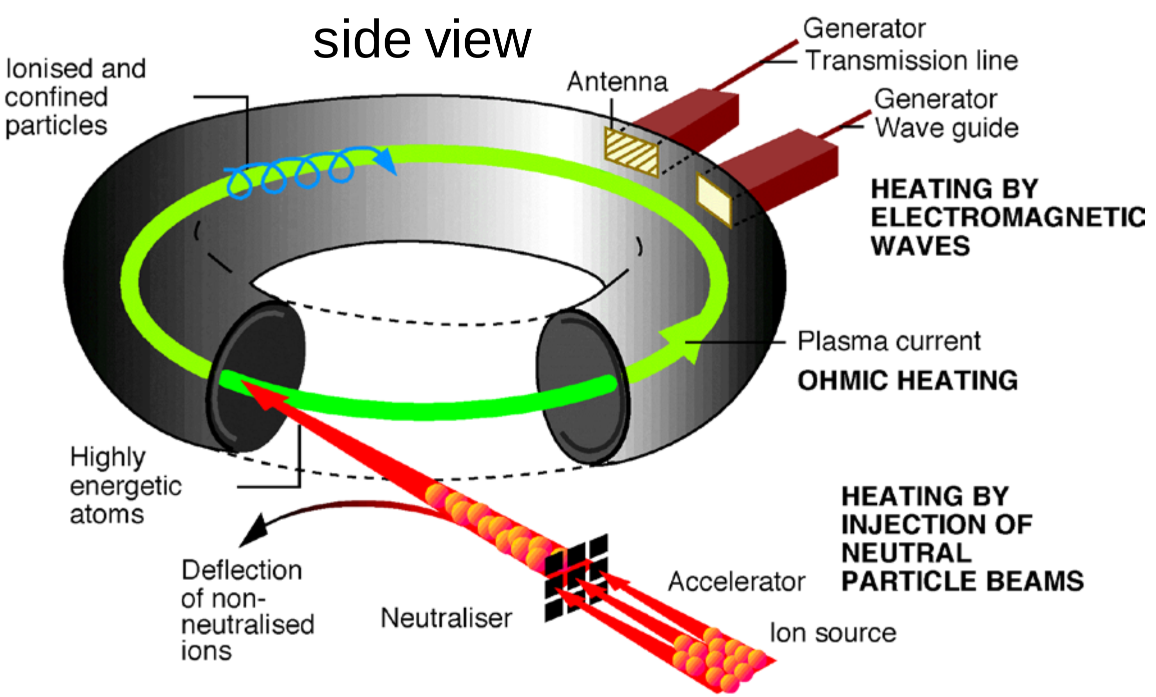

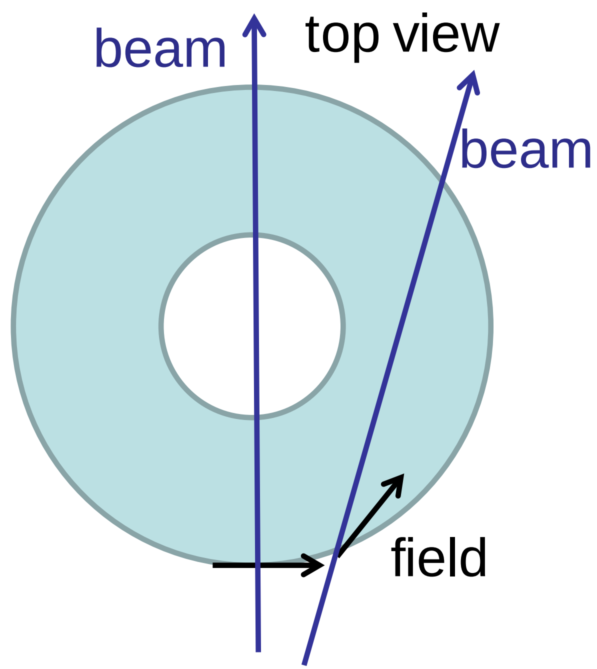

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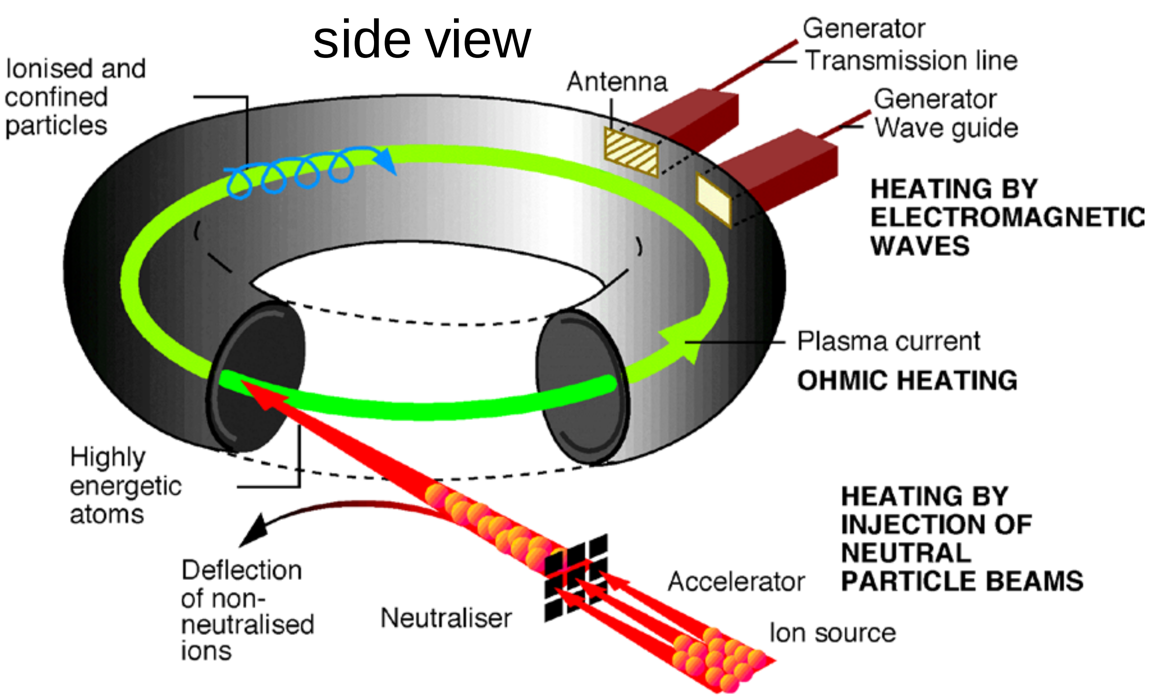

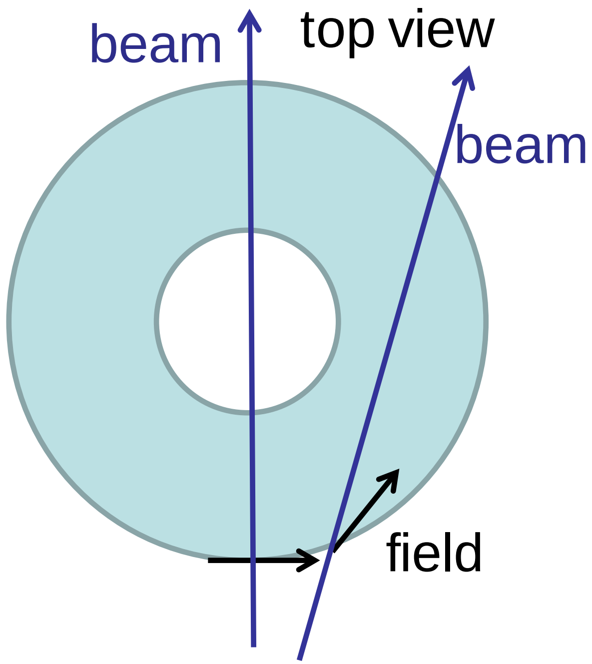

$\Rightarrow$ Pressure is a tensor $\overline{\mathbf{P}} = p_\perp \overline{\mathbf{I}} + \Delta\mathbf{BB}/\mu_0$, $\Delta = \dfrac{\mu_0\left(p_{||} - p_\perp\right)}{B^2}$

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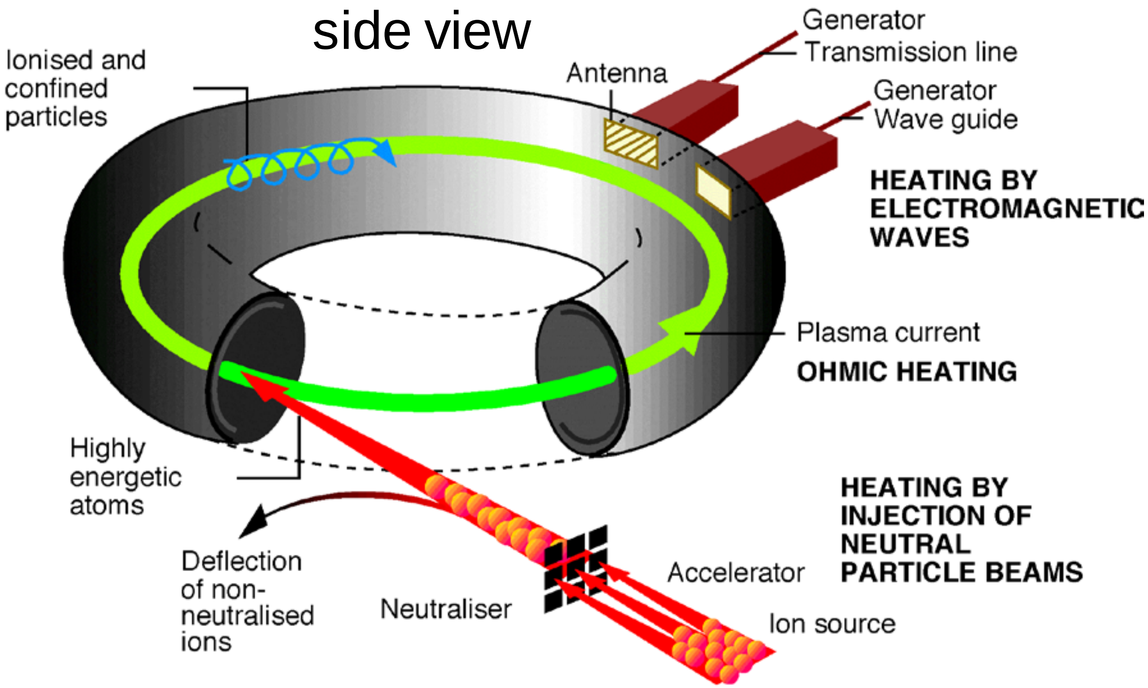

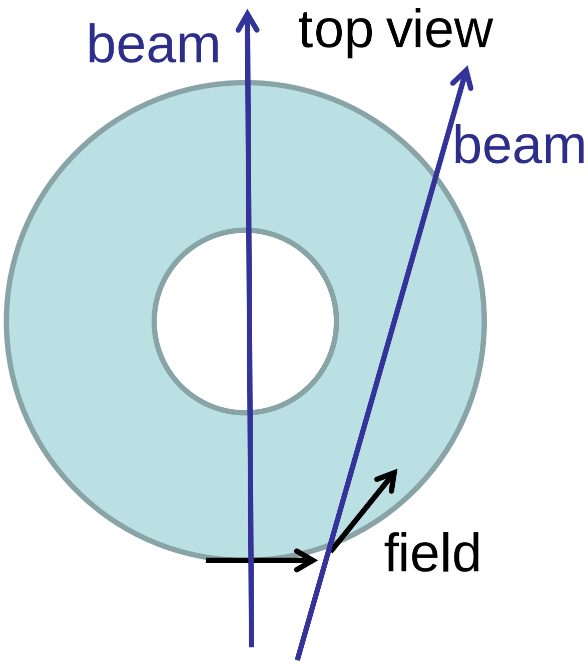

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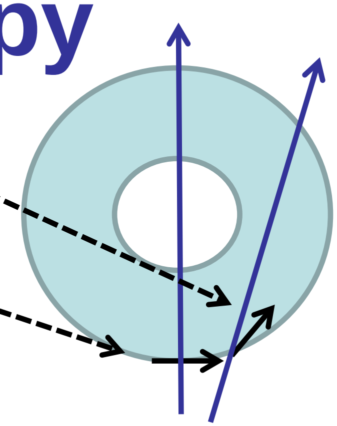
- Momentum

$$\boxed{\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \left(\mathbf{v}\cdot\nabla\right)\cdot\mathbf{v}\right) = -\nabla\cdot\overline{\mathbf{P}} + \mathbf{J}\times\mathbf{B}}$$

Expected impact of anisotropy

- Small angle θ_b between beam, field $\Rightarrow p_{||} > p_{\perp}$
- Beam orthogonal to field, $\theta_b = \pi/2 \Rightarrow p_{\perp} > p_{||}$

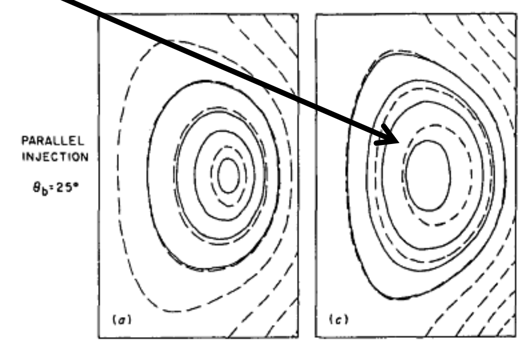


- If $p_{||}$ sig. enhanced by beam, $p_{||}$ surfaces distorted and displaced inward relative to flux surfaces

  [Cooper et al, Nuc. Fus. 20(8), 1980]

Peaked pressure profile

Broad pressure profile



Parallel pressure contours(solid)

Flux surfaces (dashed)

- If $p_{\perp} > p_{||}$, an increase will occur in centrifugal shift :

  [R. Iacono, A. Bondeson, F. Troyon, and R. Gruber, Phys. Fluids B 2 (8). August 1990]

- Compute $p_{\perp}$ and $p_{||}$ from moments of distribution function, computed by TRANSP

[M J Hole, G von Nessi, M Fitzgerald, K G McClements, J Svensson, PPCF 53 (2011) 074021]

- Infer $p_{\perp}$ from diamagnetic current $\mathbf{J}_{\perp}$

[see V. Pustovitov, PPCF 52 065001, 2010 and references therein]

MHD with rotation & anisotropy

- Inclusion of anisotropy and flow in equilibrium MHD equations

[R. Iacono, et al Phys. Fluids B 2 (8). 1990]

$$\nabla \cdot (\rho \mathbf{v}) = 0, \qquad \rho \mathbf{v} \cdot \nabla \mathbf{v} = \mathbf{J} \times \mathbf{B} - \nabla \cdot \overline{\mathbf{P}}, \qquad \nabla \cdot \mathbf{B} = 0$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \qquad \nabla \times (\mathbf{v} \times \mathbf{B}) = 0,$$

$$\overline{\mathbf{P}} = p_\perp \overline{\mathbf{I}} + \Delta \mathbf{B}\mathbf{B} / \mu_0 \,, \qquad \Delta = \frac{\mu_0 (p_{||} - p_\perp)}{B^2}$$

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$$\overline{\mathbf{P}}=p_\perp\overline{\mathbf{I}}+\Delta\mathbf{BB}/\mu_0\,,\qquad \Delta=\frac{\mu_0\left(p_{||}-p_\perp\right)}{B^2}$$

- Frozen flux gives velocity plus axis-symmetry

$$\mathbf{v}=\frac{\psi_M'(\psi)}{\rho}\mathbf{B}-R\phi_E'(\psi)\mathbf{e}_\varphi.$$

Equilibrium eqn becomes:

$$\boxed{\nabla\cdot\left[\tau\left(\frac{\nabla\psi}{R^2}\right)\right]=-\frac{\partial p_{||}}{\partial\psi}-\rho H_M'(\psi)+\rho\frac{\partial W}{\partial\psi}-I_M'(\psi)\frac{I}{R^2}-\psi_M''(\psi)\mathbf{v}\cdot\mathbf{B}+R\rho v_\phi\phi_E''(\psi)}$$

$$I=RB_\varphi$$

$$I_M(\psi)=\tau\,I-\mu_0R^2\psi_M'(\psi)\phi_E'(\psi)$$

$$H_M(\psi)=W_M(\rho,B,\psi)-\frac{1}{2}\left[R\phi_E'(\psi)\right]^2+\frac{1}{2}\left[\frac{\psi_M'(\psi)B}{\rho}\right]^2,$$

$$\left\{I_M(\psi),\psi_M(\psi),\phi_E(\psi),H_M(\psi),\frac{\partial p_{||}}{\partial\psi},\frac{\partial W}{\partial\psi}\right\}$$

Set of 6 profile constraints

$$\tau=1-\Delta-\mu_0\left(\psi_M'\right)^2/\rho,$$

Neglect poloidal flow

- Suppose $\mathbf{v} = -R\phi_E'(\psi)\mathbf{e}_\varphi = R\Omega(\psi)\mathbf{e}_\varphi \quad \Rightarrow \quad F(\psi) = I_M(\psi)/\tau$

  and equilibrium eqn becomes:

$$\boxed{\nabla\cdot\left[(1-\Delta)\left(\frac{\nabla\psi}{R^2}\right)\right] = -\frac{\partial p_\parallel}{\partial\psi} - \rho H'(\psi) + \rho\frac{\partial W}{\partial\psi} - \frac{F'(\psi)F'(\psi)}{R^2(1-\Delta)} + R^2\rho\Omega(\psi)\Omega'(\psi)}$$

Set of 5 profile constraints $\left\{F(\psi),\Omega(\psi),H(\psi),\frac{\partial p_\parallel}{\partial\psi},\frac{\partial W}{\partial\psi}\right\}$

- $\partial W/\partial\psi$: different for MHD/ double-adiabatic/ guiding centre

- If two temperature Bi-Maxwellian model chosen

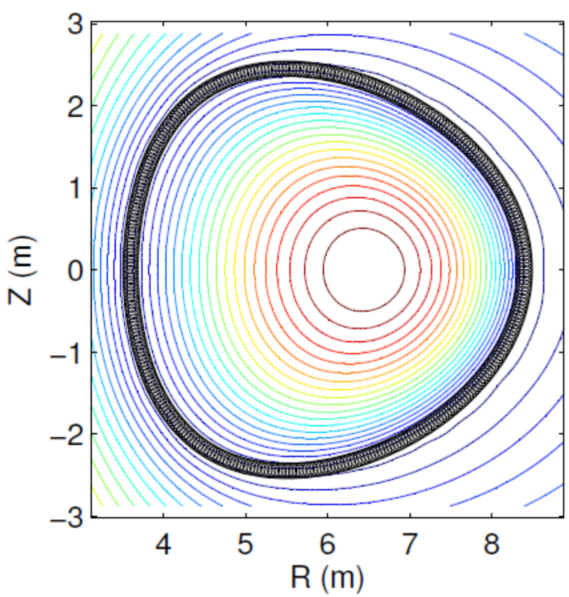
$$p_\parallel(\rho, B\psi) = \frac{k_B}{m}\rho T_\parallel(\psi) \qquad p_\perp(\rho, B\psi) = \frac{k_B}{m}\rho T_\perp(\psi) = \frac{k_B}{m}\rho T_\parallel(\psi)\frac{B}{B-\theta(\psi)T_\parallel}$$

$$\{F(\psi),\Omega(\psi),H(\psi),T_\parallel(\psi),\theta(\psi)\}$$

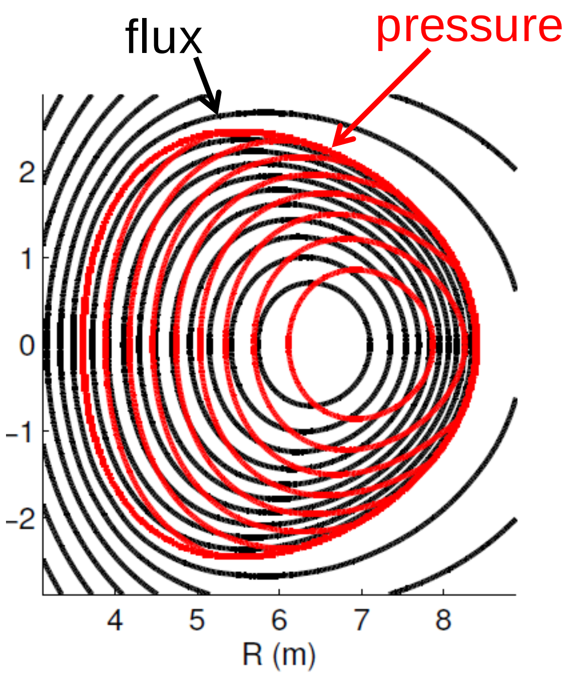
EFIT TENSOR: reconstruction code

- Adds kinetic constraints to magnetic-only constraints of EFIT
- Reveals J_ϕ sensitive to heat transport constraints
- Soloviev benchmarks computed for isotropic, anisotropic and flow cases.
- Used for MAST #13050, #18696
- Installed for both MAST and JET

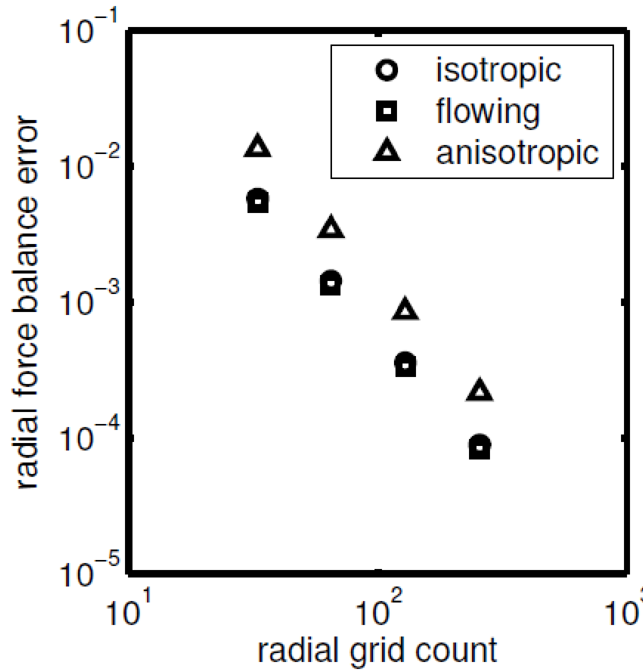
[Fitzgerald, Appel, Hole, Nucl. Fusion **53** (2013) 113040]



Soloviev:
β_t=0.07



Extended Soloviev:
β_t=0.07, M_ϕ=0.8, Δ=0.004,



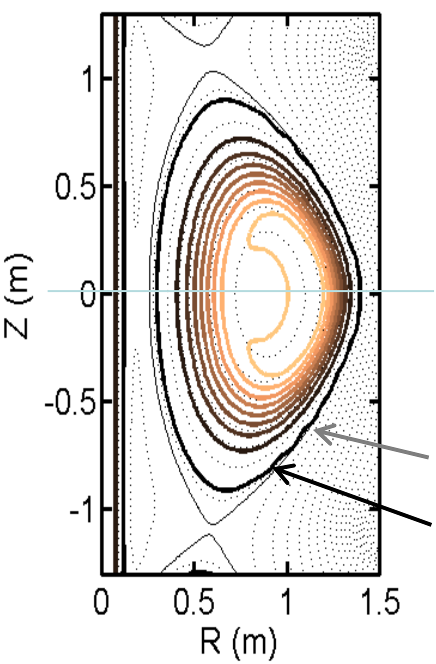
Solution Convergence

HELENA+ATF: parametric scans, stability

- Companion code written to enable stability studies.
- Can be used to study how equilibrium changes with anisotropy

$$J_\varphi = \underbrace{R\frac{B_p^2}{B^2}\left(\frac{\partial p_\parallel}{\partial\Psi}\right)_B}_{p_\parallel} + \underbrace{R\frac{B^2-B_p^2}{B^2}\left(\frac{\partial p_\perp}{\partial\Psi}\right)_B}_{p_\perp} + \underbrace{\frac{1-\Delta}{2R}\left(\frac{\partial(RB_\varphi)^2}{\partial\Psi}\right)_B}_{toroidal\,field} - \underbrace{R\nabla\cdot\frac{\Delta\nabla\Psi}{R^2}}_{nonlinear}$$

MAST-like equilibrium

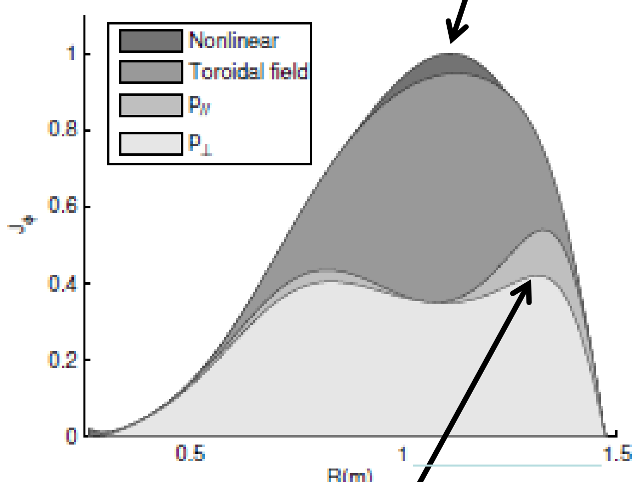


flux

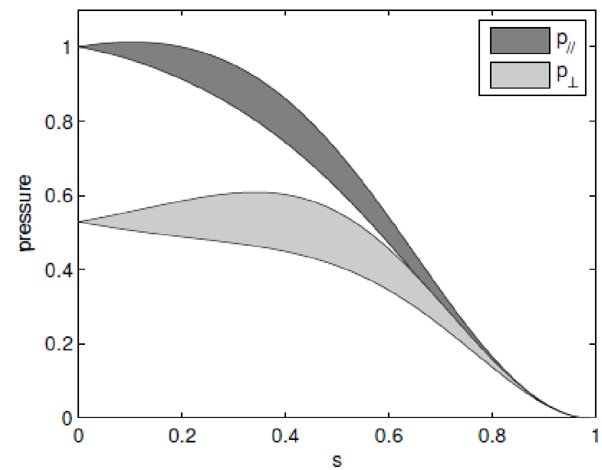
surfaces of constant $p_\parallel$.

J_ϕ components

$J_{\phi nl}$ core localised



$J_{\phi\parallel}$ dominant outboard

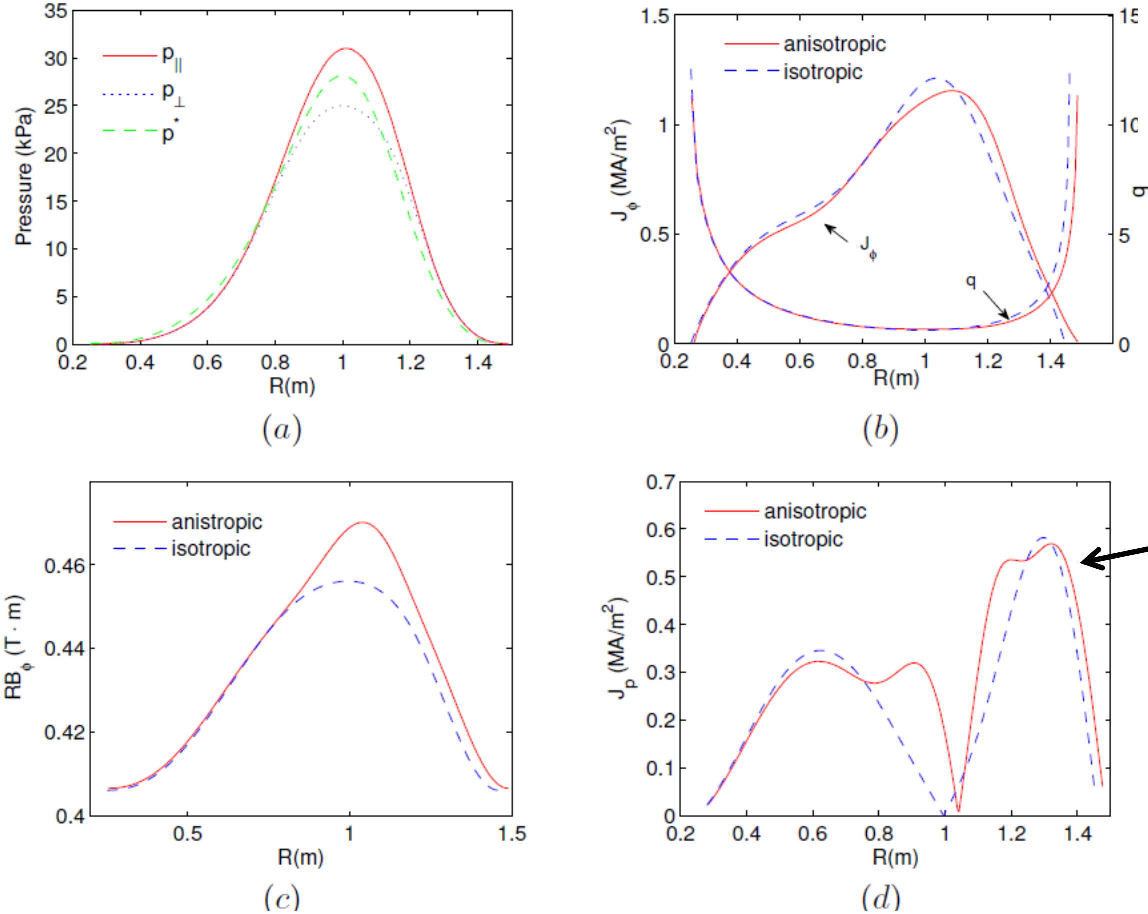


$s \propto \sqrt{r}$

[Qu, Fitzgerald, Hole, Plasma Phys. Control. Fusion **56** (2014) 075007]

HELENA+ATF: parametric scans, stability

$$J_\varphi = \underbrace{R\frac{B_p^2}{B^2}\left(\frac{\partial p_\parallel}{\partial \Psi}\right)_B}_{p_\parallel} + \underbrace{R\frac{B^2-B_p^2}{B^2}\left(\frac{\partial p_\perp}{\partial \Psi}\right)_B}_{p_\perp} + \underbrace{\frac{1-\Delta}{2R}\left(\frac{\partial (RB_\varphi)^2}{\partial \Psi}\right)_B}_{toroidal\,field} - \underbrace{R\nabla\cdot\frac{\Delta\nabla\Psi}{R^2}}_{nonlinear}$$



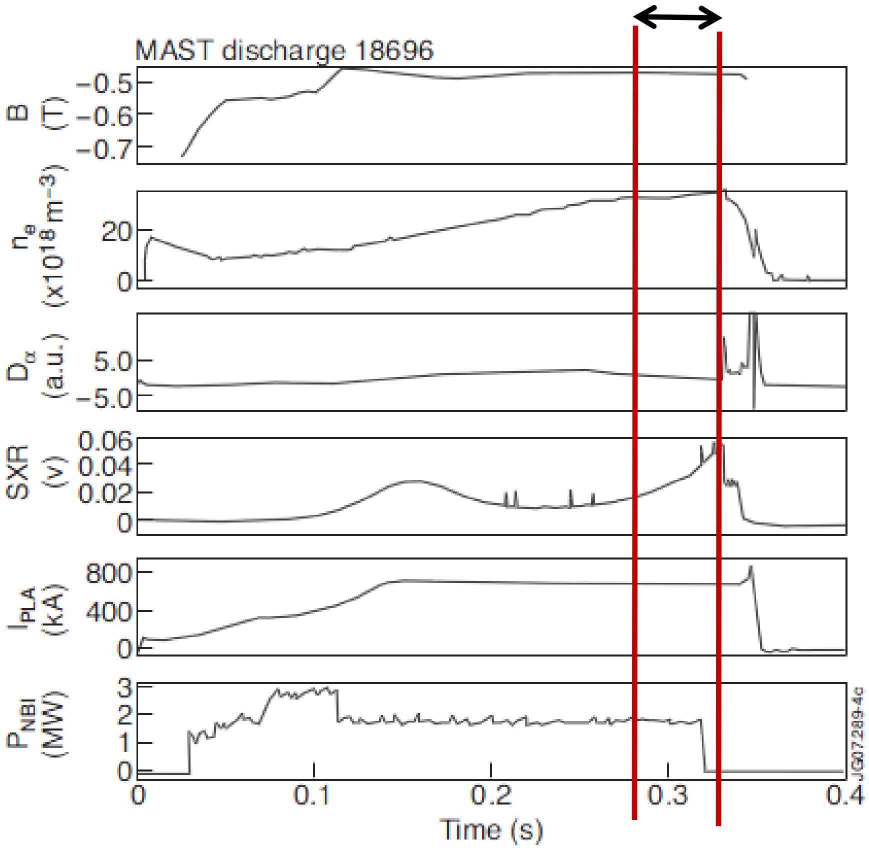
$$\mu_0 \boldsymbol{J_p} = \nabla(RB_\varphi)\times\nabla\varphi$$

$p_{||}/p_\perp \approx 1.25$

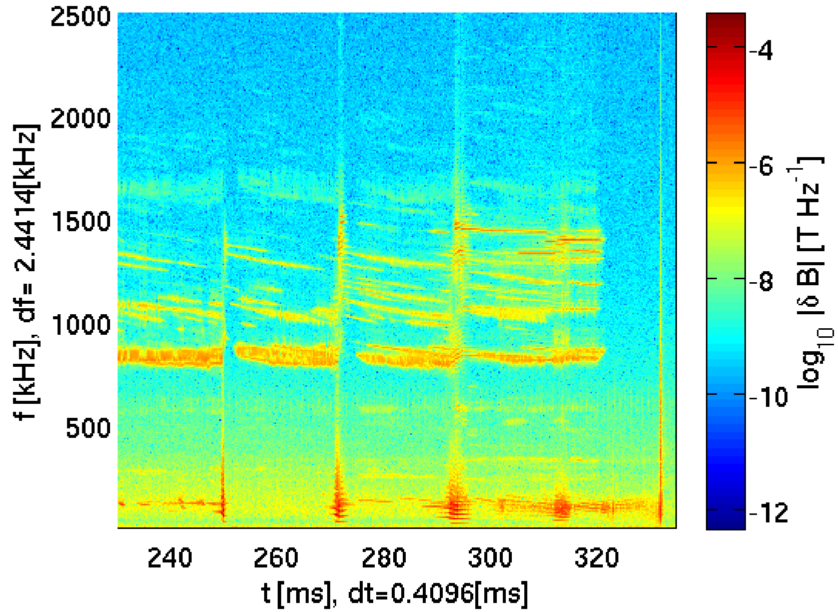
Most significant difference in $\mathbf{J_p}$ which can effect change in stability

Anisotropy on MAST: #18696

- MAST #18696
- 1.9MW NB heating
- I_p = 0.7MA, β_n=2.5
- TRANSP simulation available
- Magnetics shows CAEs



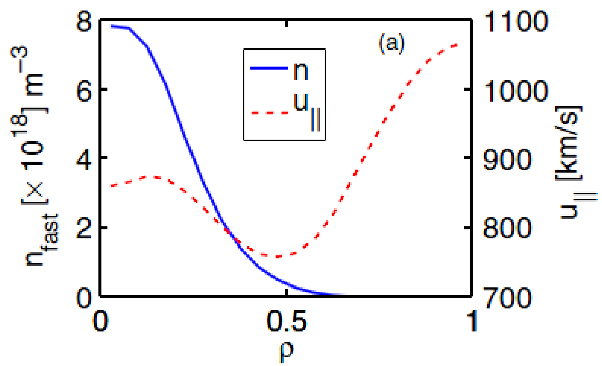
Magnetics

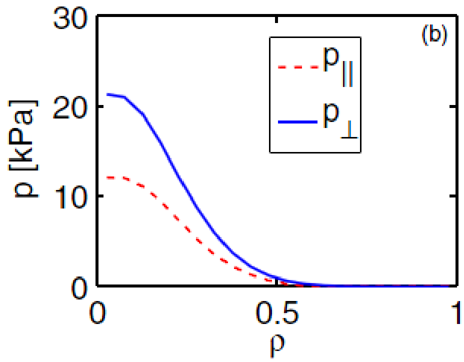


[M.P. Gryaznevich et al, Nuc. Fus. 48, 084003, 2008.; Lilley *et al* 35th EPS Conf. Plas.Phys. 9 - 13 June 2008 ECA Vol.32D, P-1.057]

- What is the impact on q profile due to presence of anisotropy and flow?

Beam population $p_\perp/p_\parallel \approx 1.7$





$p_\perp/p_\parallel \approx 1.7$

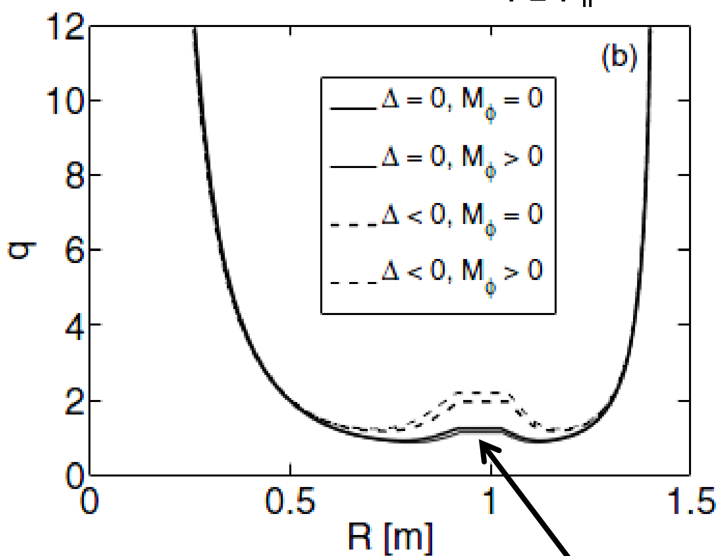
$$\rho = \sqrt{\Phi / \Phi_0}$$

Φ = toroidal flux

Impact on plasma computed using FLOW, EFIT TENSOR

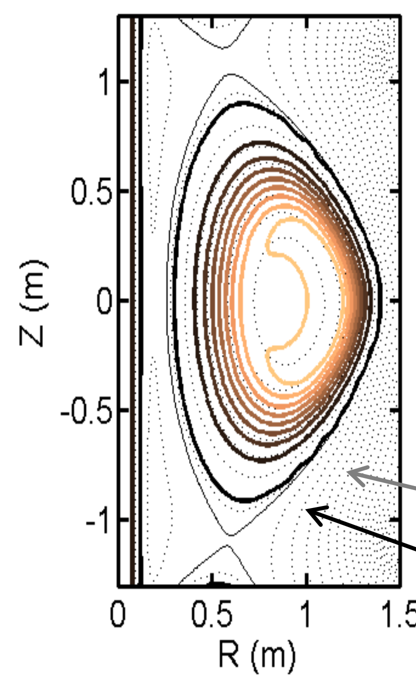
FLOW scans $\Delta < 0$: $p_\perp/p_\parallel \approx 1.7$

$\Delta = 0$: $p_\perp/p_\parallel = 1$



Low grid resolution of FLOW at core

EFIT++ (TENSOR)



Calculation of MAST #18696 at 290ms.

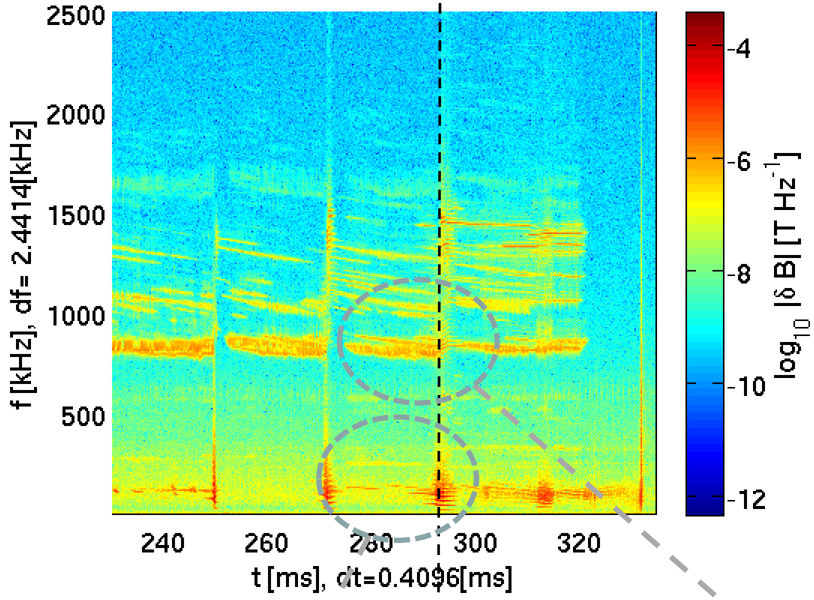
$p_\perp / p_\parallel \sim 1.7$

(slowing down beam particles)

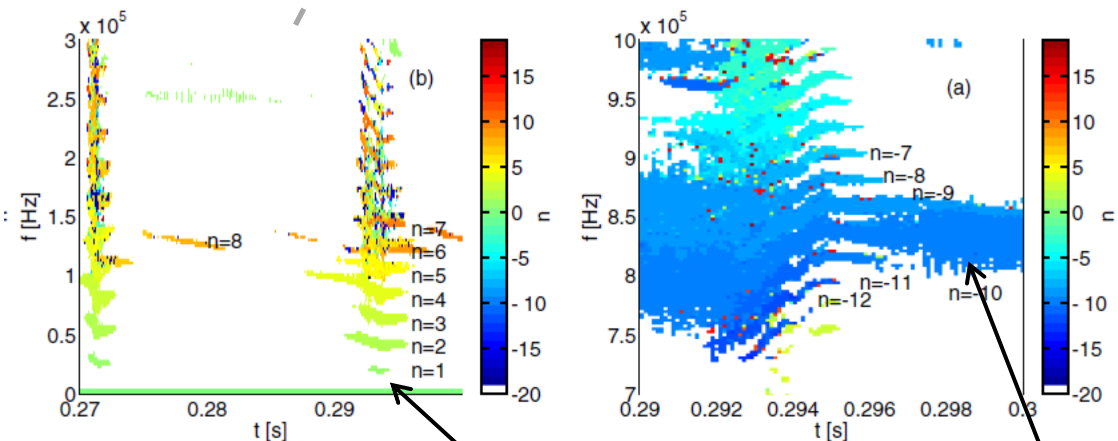
poloidal flux

surfaces of constant $p_\parallel$.

Anisotropy on MAST



- How do predicted mode frequencies change due to changes in *q* produced by anisotropy and flow?

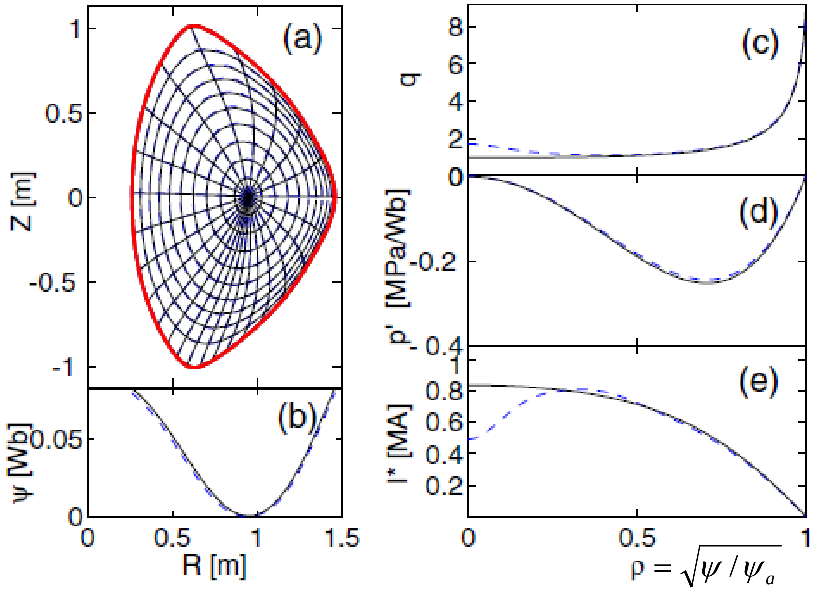


n=1 mode

n=-10 mode

- Appetiser: What is the change in ideal MHD stability of n=1 TAE?

Increased shear gives multiple TAEs:
changes radial structure

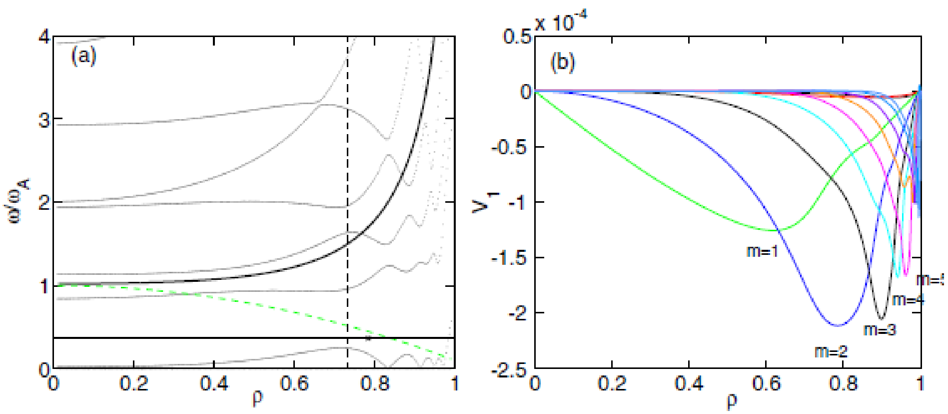


- Reshape plasma to have larger reverse shear

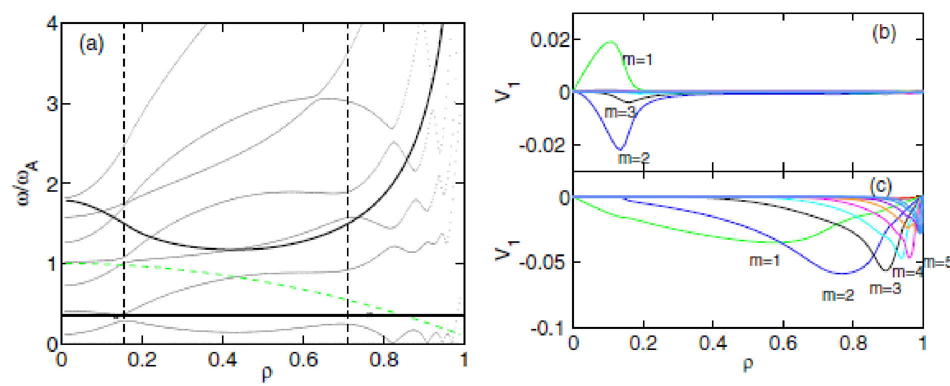
$$I^*(s) \rightarrow I^*(s) + \underbrace{I_0 \exp\left[-\frac{(s-s_0)^2}{2\sigma_0^2}\right]}_{\text{core}} + \underbrace{I_1 \exp\left[-\frac{(s-s_1)^2}{2\sigma_1^2}\right]}_{\text{reverse shear}}$$

I_0, I_1 varied to match q_0=1.7, q_{min}=1.24

[M J Hole, G von Nessi, M Fitzgerald and the MAST team, PPCF, 55 014007, 2013]



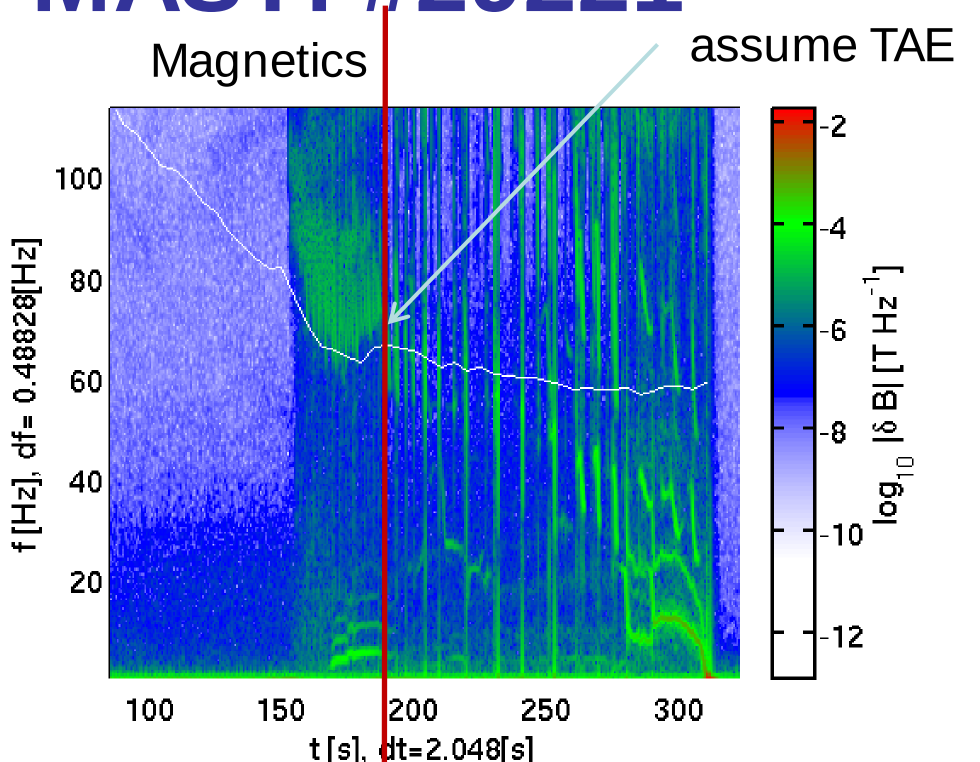
Single global TAE at (m,n) = (1,1)

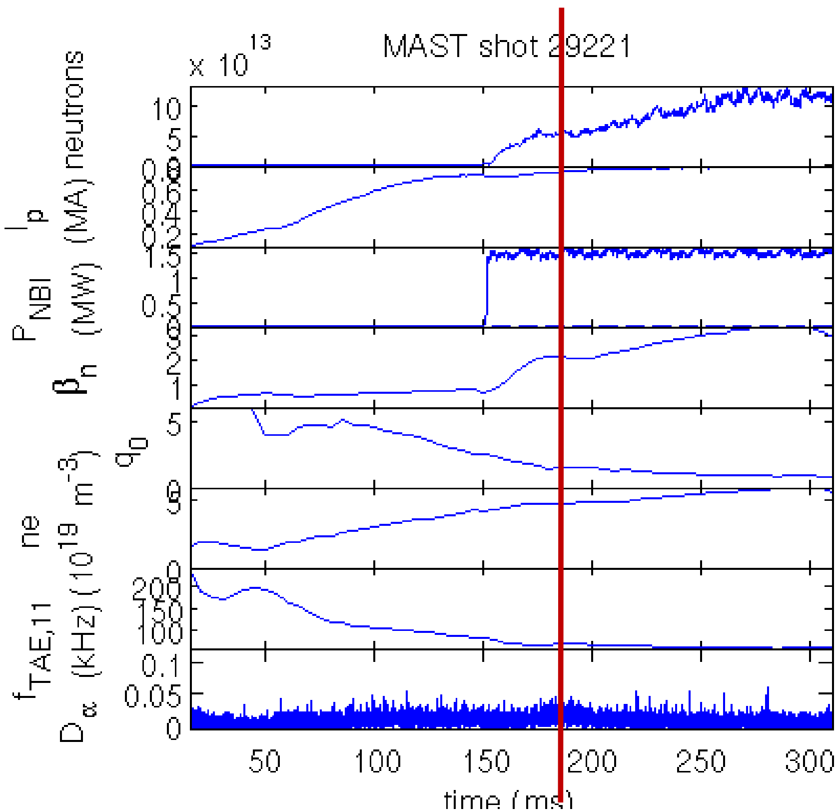


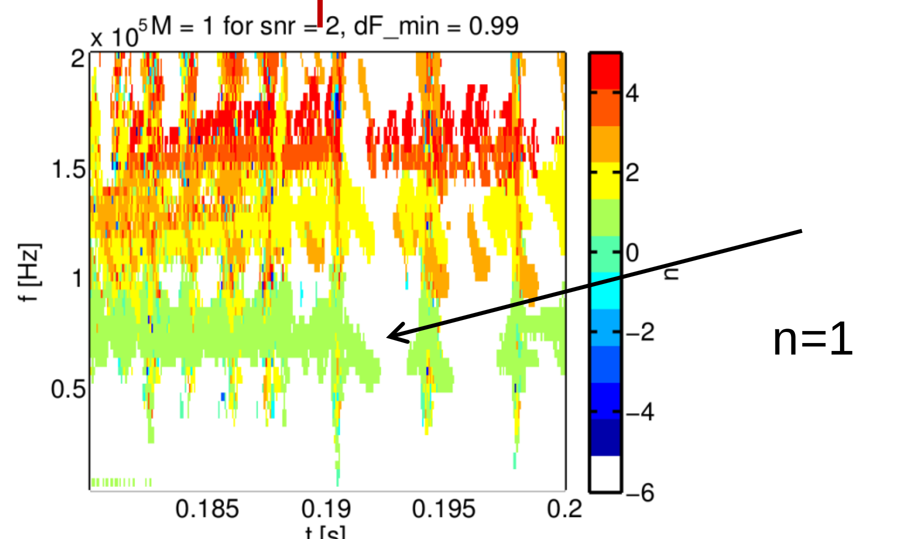
Reverse shear produces second (m,n)) = (1,1) odd TAE resonance in the core

Anisotropy on MAST: #29221

- MAST #29221
- 1.6MW NB heating
- I_p = 0.9MA, β_n~3
- Magnetics shows TAEs, tearing modes fishbones, long-lived modes

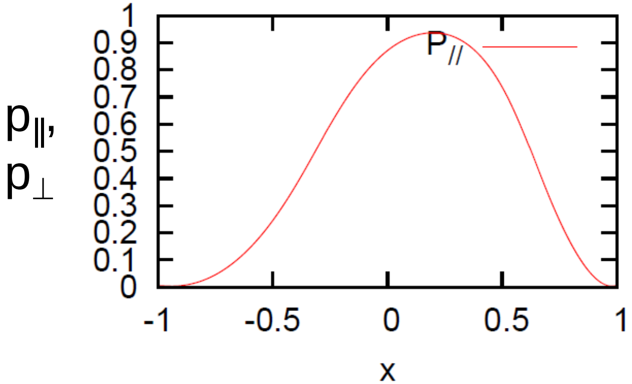




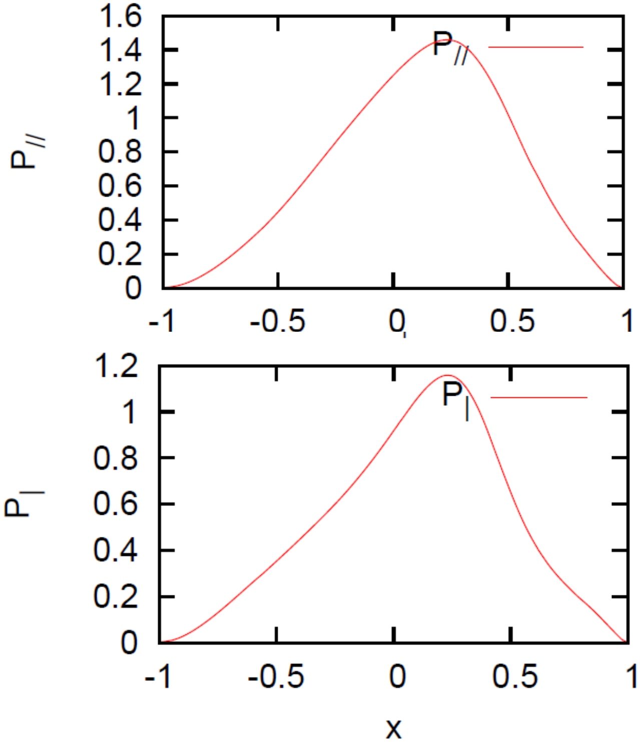


Beam + thermal population: $p_{\|} / p_{\perp} \approx 1.4$

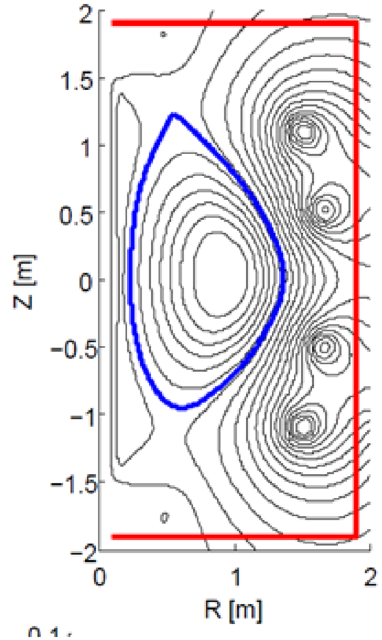
HELENA+ATF / EFIT TENSOR: no flow + p*



HELENA+ATF / EFIT TENSOR: no flow, anisotropy



$p_{\|}/p_{\perp}$ = 1.4 at r/R=0.5



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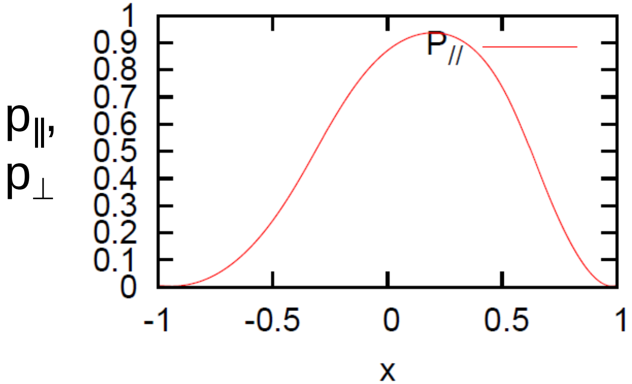

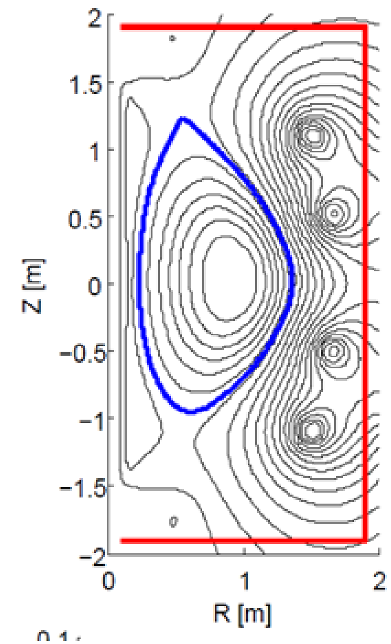

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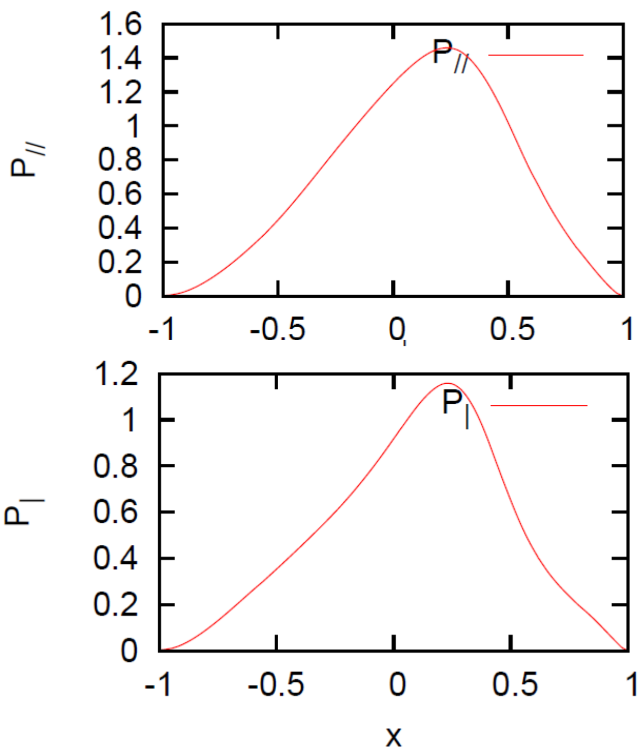

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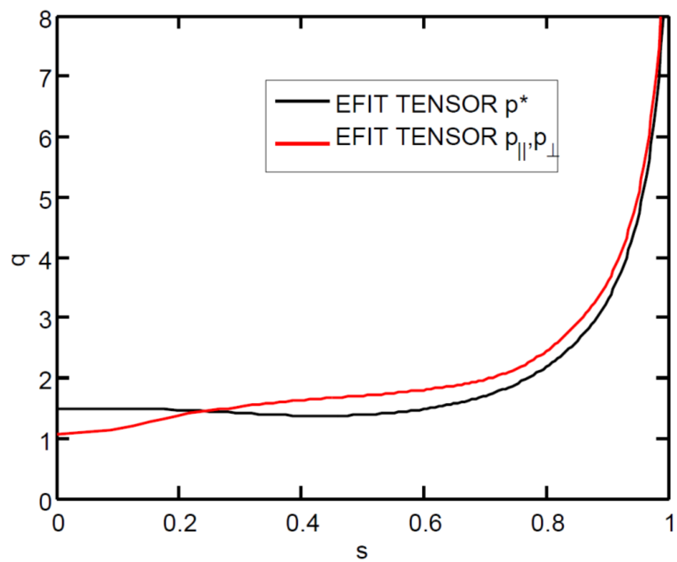

- What is the impact on stability due to this q profile?

Stability: incompressional

- Normal mode treatment: Linearise around time dependent oscillations of form exp[i (ω t - m θ- n ϕ)]
- Without compressibility, Mikhailovski (*) show perturbed Lagrangian distribution function is zero, meaning that the Euler perturbed distribution function is

$$\tilde{f} = -\boldsymbol{\xi}_\perp \cdot \nabla F$$

The fluid closure equations are*

$$p_{\parallel 1} = -\xi_n \left[\frac{\partial p_\parallel}{\partial n} - (p_\parallel - p_\perp) \frac{\partial \ln B}{\partial n} \right]$$

$$p_{\perp 1} = -\xi_n \left[\frac{\partial p_\perp}{\partial n} - (2p_\perp + \hat{c}) \frac{\partial \ln B}{\partial n} \right]$$

$$\hat{c} \equiv \sum_s M_s \int \frac{B}{v_\parallel} (\mu B)^2 \frac{\partial F_s}{\partial \varepsilon} d\mu d\varepsilon.$$

*A B Mikhailovskii, Instabilities in a confined plasma, IOP publishing (1998)

Stability: compressional

- Using existing model
  - *Double-adiabatic (CGL)*
    - Collisionless, $p_\parallel$ and $p_\perp$ do **independent** work
    - No streaming particle heat flow
    - Does not reduce to MHD in the isotropic limit

$$\frac{d}{dt}\left(\frac{p_\perp}{\rho B}\right) = 0,$$

$$\frac{d}{dt}\left(\frac{p_\parallel B^2}{\rho^3}\right) = 0.$$

Stability: compressional

- Using existing model
  - *Double-adiabatic (CGL)*
    - Collisionless, $p_{\parallel}$ and $p_{\perp}$ do **independent** work
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    - Does not reduce to MHD in the isotropic limit
  - *Double-polytropic law*
    - Generalisation of CGL but unclear rationale for choices of adiabatic index $\gamma_{\perp}$, $\gamma_{||}$

$$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho B}\right) = 0,$$

$$\frac{d}{dt}\left(\frac{p_{\parallel} B^2}{\rho^3}\right) = 0.$$

$$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho B^{\gamma_{\perp}-1}}\right) = 0$$

$$\frac{d}{dt}\left(\frac{p_{\parallel} B^{\gamma_{\parallel}-1}}{\rho^{\gamma_{\parallel}}}\right) = 0$$

Stability: compressional

- Using existing model
  - *Double-adiabatic (CGL)*
    - Collisionless, $p_{\|}$ and $p_{\perp}$ do **independent** work
    - No streaming particle heat flow
    - Does not reduce to MHD in the isotropic limit

$$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho B}\right) = 0,$$

$$\frac{d}{dt}\left(\frac{p_{\|} B^2}{\rho^3}\right) = 0.$$

  - *Double-polytropic law*
    - Generalisation of CGL but unclear rationale for choices of adiabatic index $\gamma_{\perp}$, $\gamma_{||}$

$$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho B^{\gamma_{\perp}-1}}\right)=0$$

$$\frac{d}{dt}\left(\frac{p_{\|} B^{\gamma_{\|}-1}}{\rho^{\gamma_{\|}}}\right)=0$$

- New extension to MHD
  - *Single adiabatic (SA) model*
    - $p_{\|}$ and $p_{\perp}$ doing **joint** work
    - Accounting for the isotropic part of the perturbation $\widetilde{\mathrm{P}} \rightarrow \tilde{p}\mathrm{I}$
    - Can reduce to MHD in isotropic limit

[Fitzgerald, Hole, Qu, submitted PPCF 08/09/2014]

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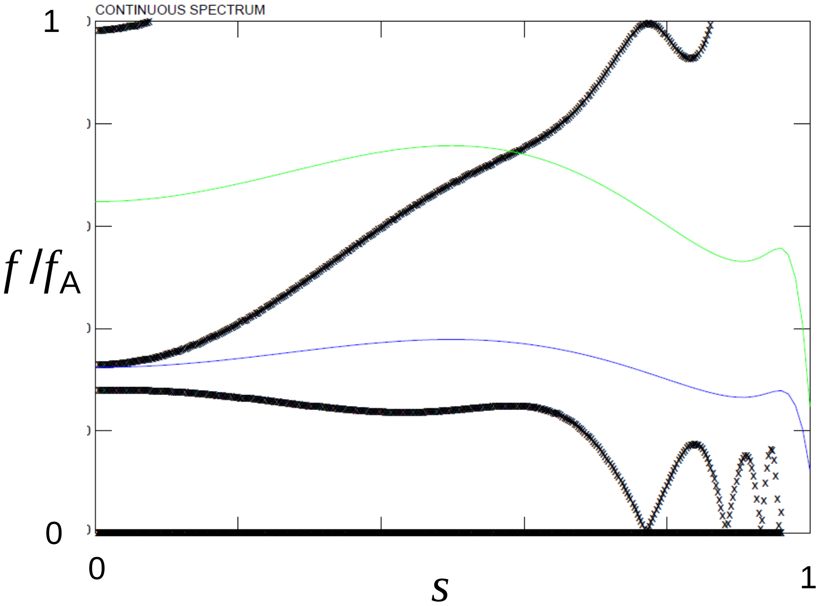
[Fitzgerald, Hole, Qu, submitted PPCF 08/09/2014]

- Implemented both SA and CGL in CSCAS (CSMIS-A) and MISHKA (MISHKA-A)

Incompressible continuum for MAST

isotropic

n=1, γ=0



R_{mag} =

f_A at magnetic axis = 280kHz

Incompressible continuum for MAST

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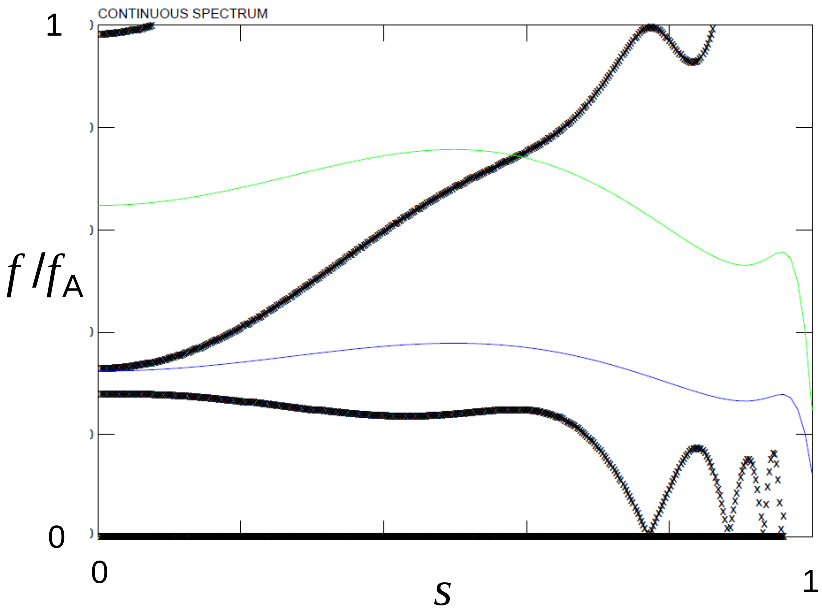
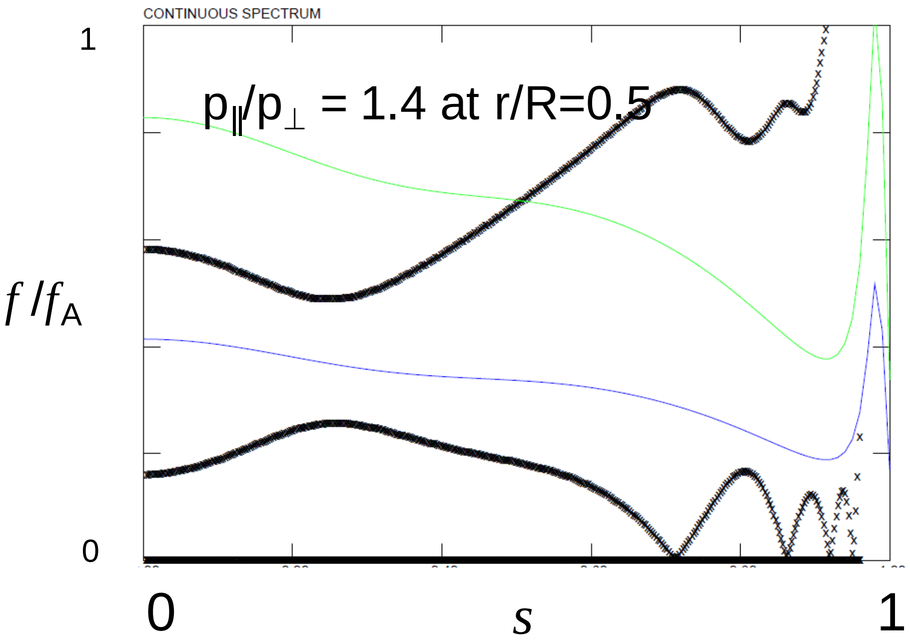
n=1, γ=0



anisotropic

n=1, γ=0



R_{mag} =
f_A at magnetic axis = 280kHz

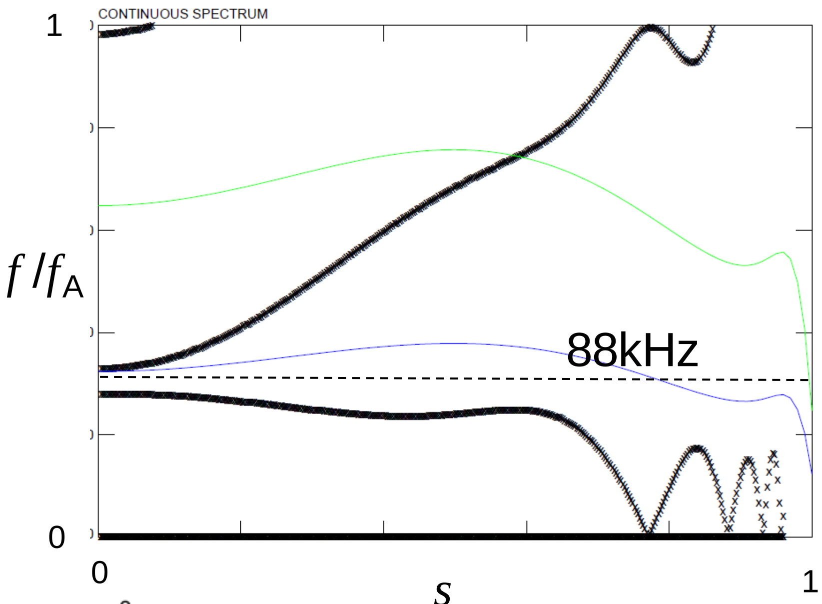
R_{mag} =
f_A at magnetic axis = 260kHz

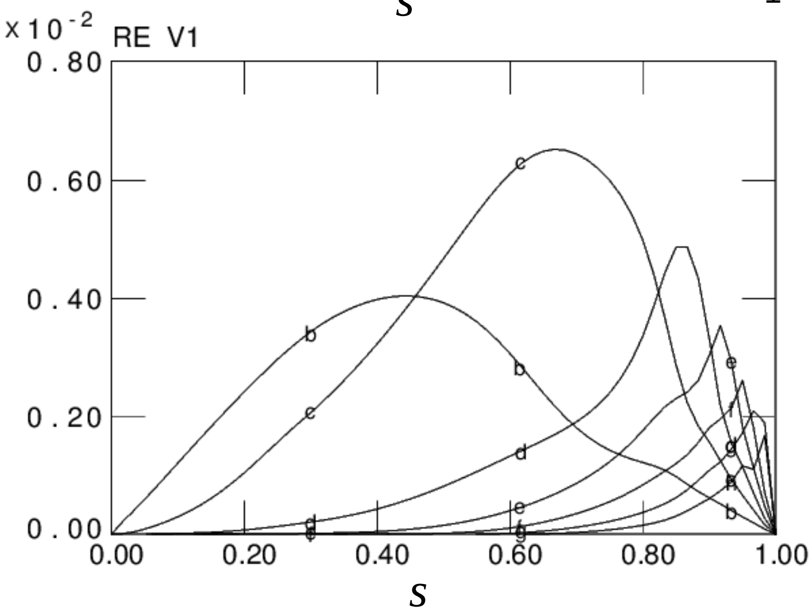
isotropic Δf_{TAE} < anisotropic Δf_{TAE}
$\Rightarrow$anisotropic modes likely to have less continuum damping

Anisotropic mode profile broader

isotropic

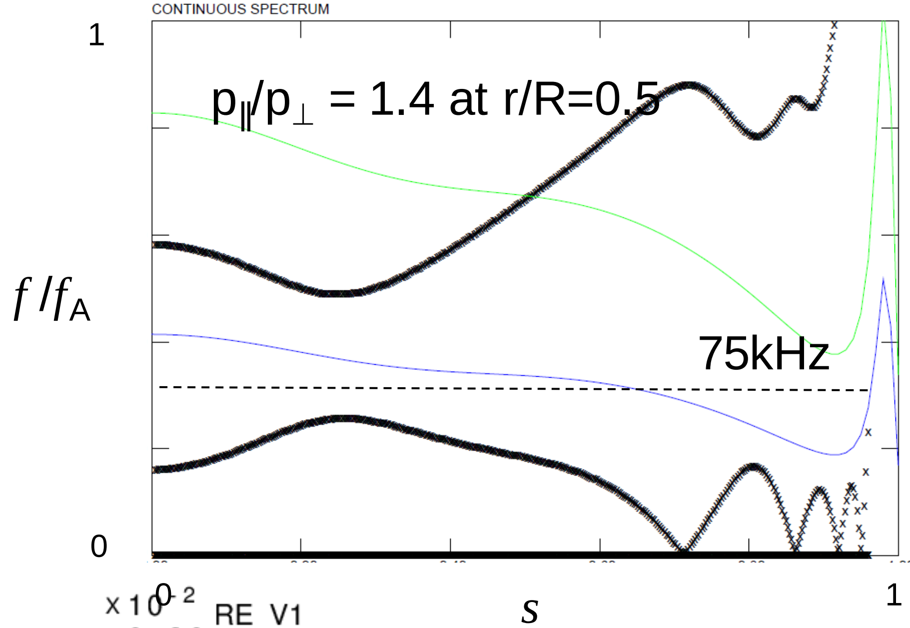
n=1, γ=0

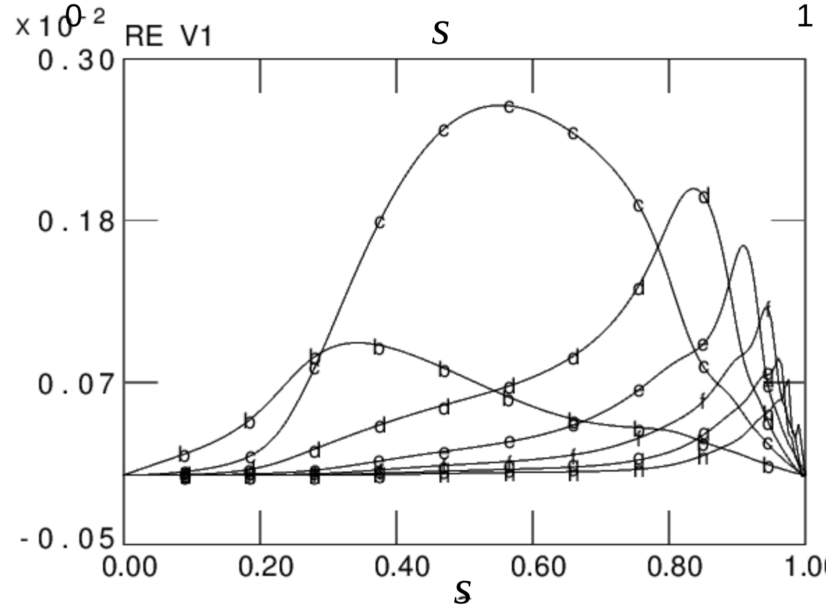




anisotropic

n=1, γ=0





Ongoing work in Anisotropy and Flow

- Demonstrated significant anisotropy in MAST, $0.6<p_{||}/p_{\perp}<1.4$
  - ➢ Can produce significant change in equilibrium
    - change central safety factor (helicity) by up to 15%
      [M J Hole *et al* PPCF **53**, 074021, 2011]
    - Can produce significant poloidal current.
      [Qu, Fitzgerald, Hole, Plasma Phys. Control. Fusion **56** (2014) 075007]
  - ➢ Can change stability:
    - Through change in q in ideal MHD – introduce multiple gap modes
      [M J Hole *et al*, PPCF, **55** 014007, 2013]
    - In incompressible plasmas, lead to: wider gaps, reduced continuum damping, broader radial structure.
    - Developed new Single Adiabatic model. Implemented in MISHKA, CSCAS. [Fitzgerald, Hole, Qu, submitted PPCF 08/09/2014]

To do…

- Couple EFIT TENSOR, MISHKA to wave-particle interaction code HAGIS for self-consistent evolution
- Explore wave-particle interaction new physics

Outline

- Anisotropy: equilibrium and stability
  - Expected impact of anisotropy
  - Development of anisotropy into EFIT++ , HELENA
  - Demonstrate impacts of anisotropy on J_{ϕ}, plasma parameters
  - Development of MHD single adiabatic stability model
  - Compute impact on MAST equilibrium, stability
  - Future directions
- Multiple Relaxed Region MHD model
  - resolves chaotic field regions, islands, flux surfaces in fully 3D plasmas
  - Stepped Pressure Equilibrium Code.
  - Demonstrate two interface model to describe helical plasmas in reverse field pinches
  - Highlight some recent progress
  - Future directions
- Conclusions

3D equilibria in toroidal plasmas

- Simplest model to approximate global, macroscopic force-balance is magnetohydrodynamics (MHD).

$$\nabla p = \mathbf{J} \times \mathbf{B}, \qquad \nabla \times \mathbf{B} = \mathbf{J}, \qquad \nabla \cdot \mathbf{B} = 0$$

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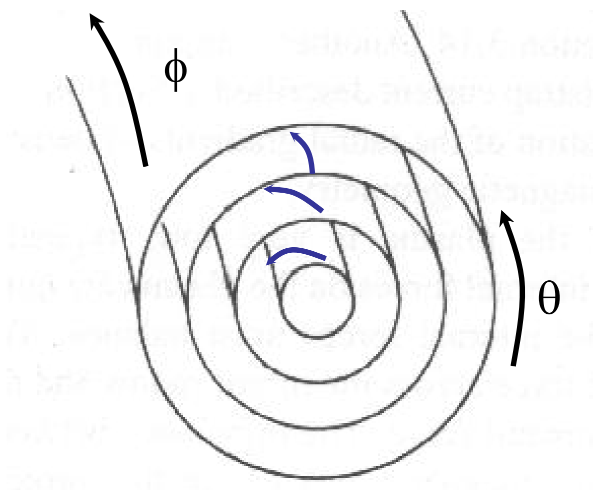
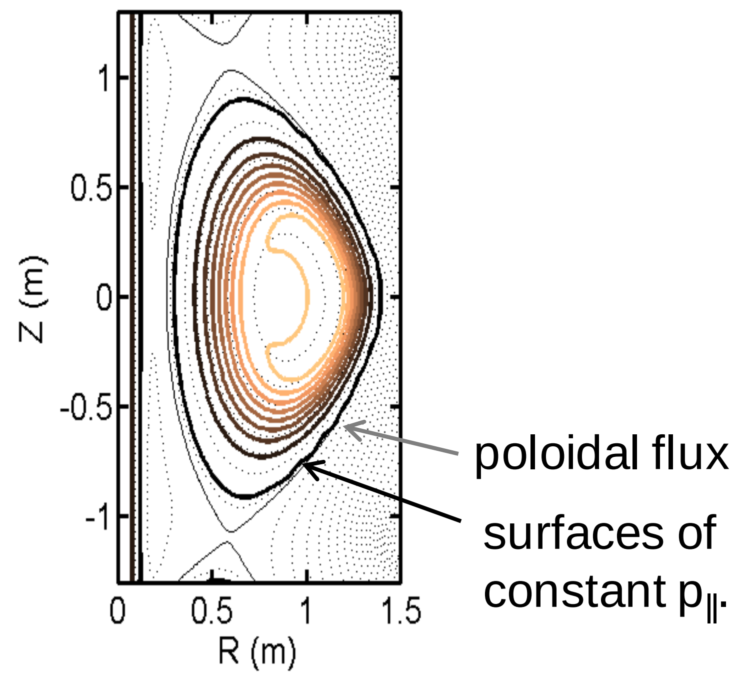
- Toroidal symmetry $\Rightarrow$ field lies in nested flux surfaces





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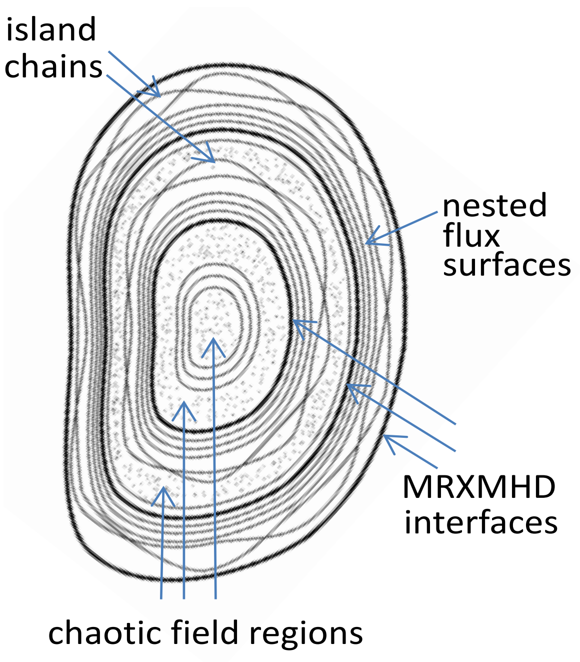
- Non-axisymmetric $\Rightarrow$ field does **not** lie in nested flux surfaces **unless** surface currents allowed.



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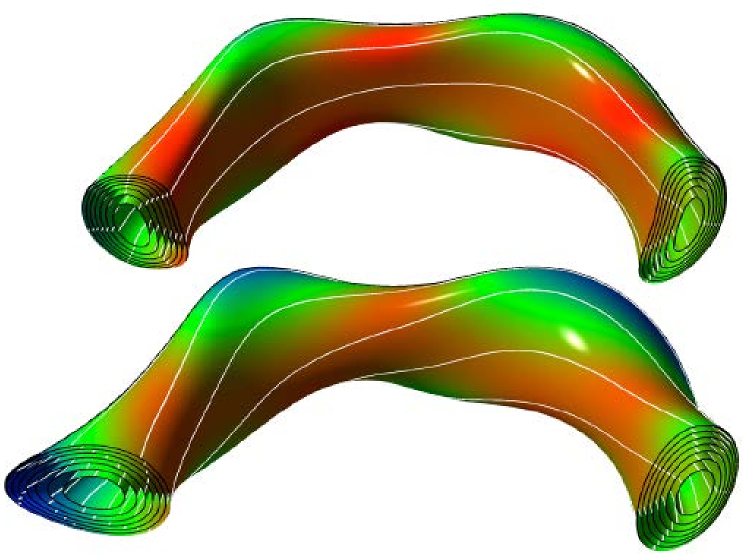
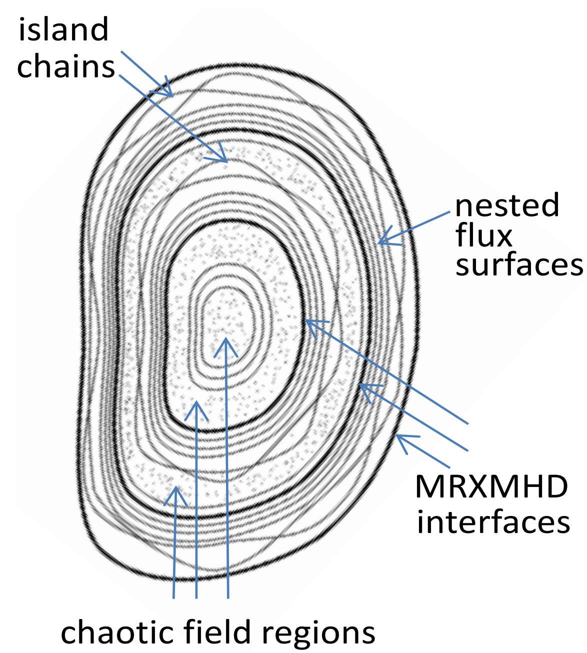


[CTH stellarator, Hanson et al, IAEA 2012]



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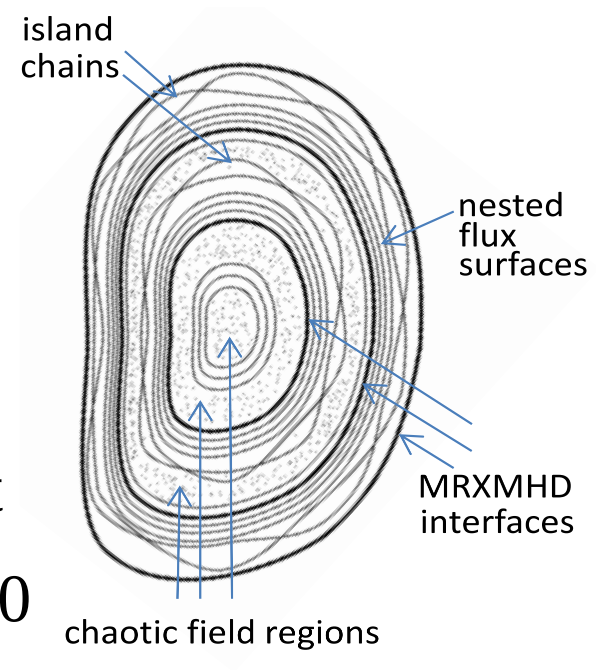
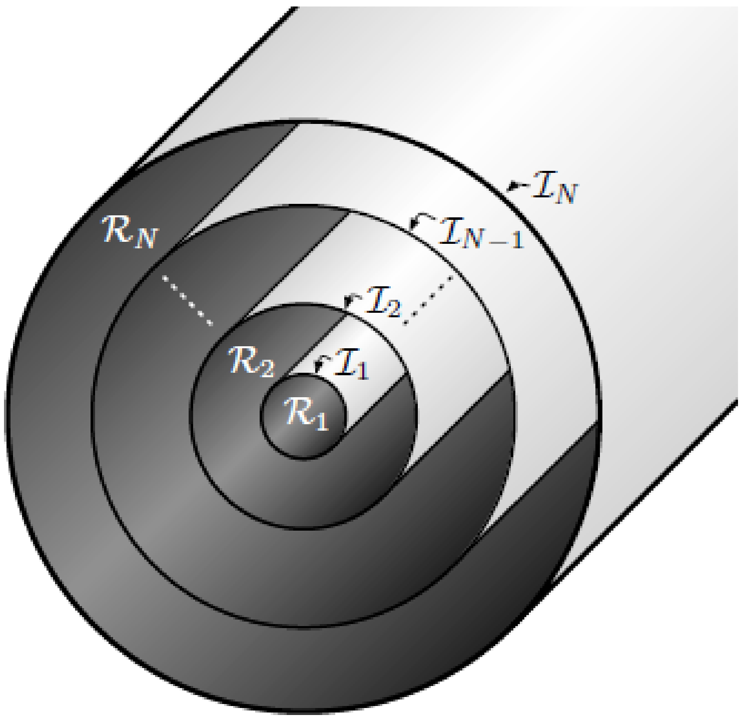
- Existing 3D solvers (e.g. VMEC) assume nested flux surfaces.
- Generalised Taylor relaxation model: Multiple Relaxed Region MHD (MRXMHD) supports full complexity of field: nested flux surfaces, magnetic islands, chaotic regions.

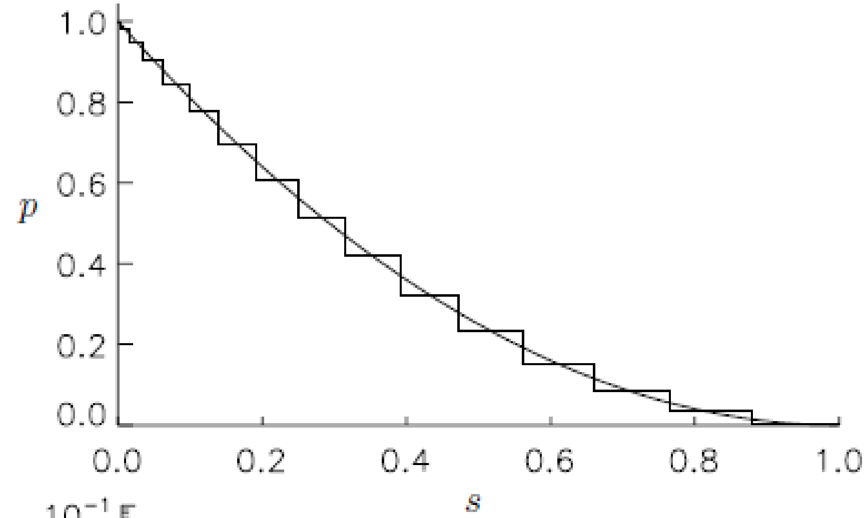


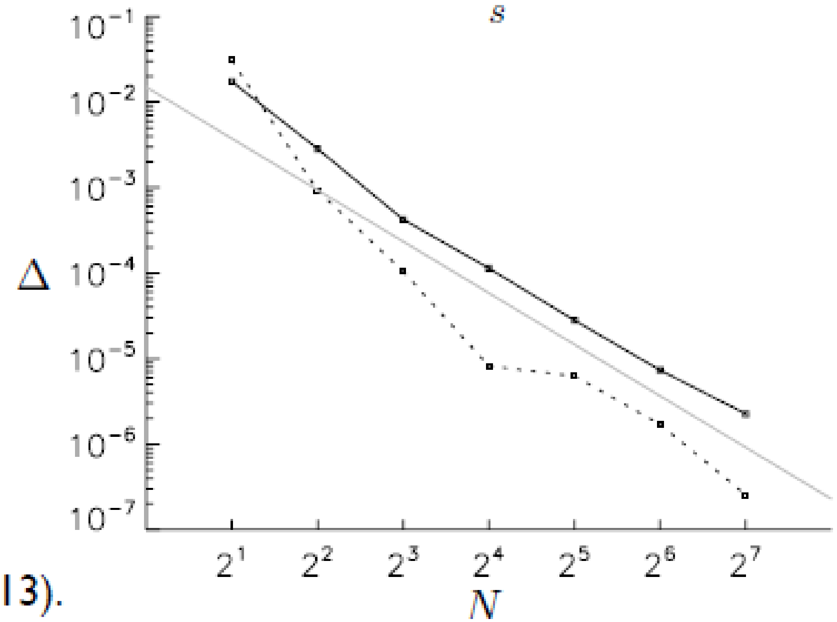
Volume: $\nabla \times \mathbf{B} = \mu_l \mathbf{B} \qquad P_l = \text{constant}$

Interfaces: $[[P_l + B^2/(2\mu_0)]] = 0 \qquad \mathbf{B} \cdot \mathbf{n} = 0$

MRXMHD approaches ideal MHD as N→∞







[1] G. Dennis et al., *Phys. Plasmas* **20**, 032509 (2013).

Stepped Pressure Equilibrium Code, SPEC

[Hudson et al Phys. Plasmas 19, 112502 (2012)]

Hudson

Vector potential is discretised using mixed Fourier & finite elements

- Coordinates (s,φ, ζ)
- Interface geometry $R_i = \sum_{l,m,n} R_{lmn} \cos(m\vartheta - n\zeta),\ Z_i = \sum_{l,m,n} Z_{lmn} \sin(m\vartheta - n\zeta)$
- Exploit gauge freedom $\mathbf{A} = A_\vartheta(s,\vartheta,\zeta)\nabla\vartheta + A_\zeta(s,\vartheta,\zeta)\nabla\zeta$
- Fourier $A_\vartheta = \sum_{m,n} \alpha(s)\cos(m\vartheta - n\varsigma)$
- Finite-element $a_\vartheta(s) = \sum_i a_{\vartheta,i}(s)\varphi(s)$

& inserted into constrained-energy functional $F = \sum_{l=1}^{N} (W_l - \mu_l H_l / 2)$

- Derivatives wrt **A** give Beltrami field $\nabla \times \mathbf{B} = \mu\mathbf{B}$
- Field in each annulus computed independently, distributed across multiple cpu's
- Field in each annulus depends on enclosed toroidal flux, poloidal flux, interfaces ξ

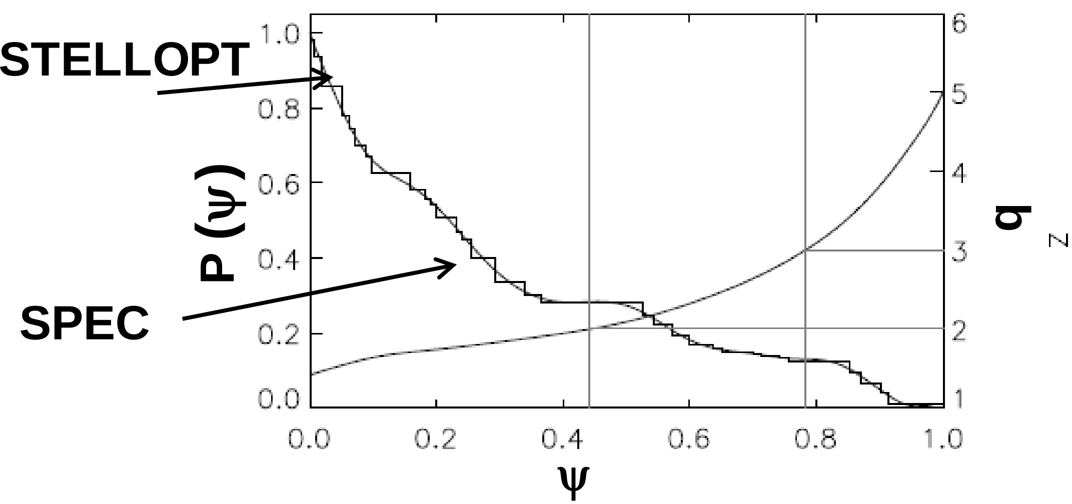
Force balance solved using multi-dimensional Newton method

- Interface geometry adjusted to satisfy force balance $\mathbf{F}[\xi] = \{[[p + B^2/2]]_{m,n}\} = 0$
- Angle freedom constrained by spectral condensation,
- Dertivative matrix ∇F[ξ] computed in parallel using finite difference

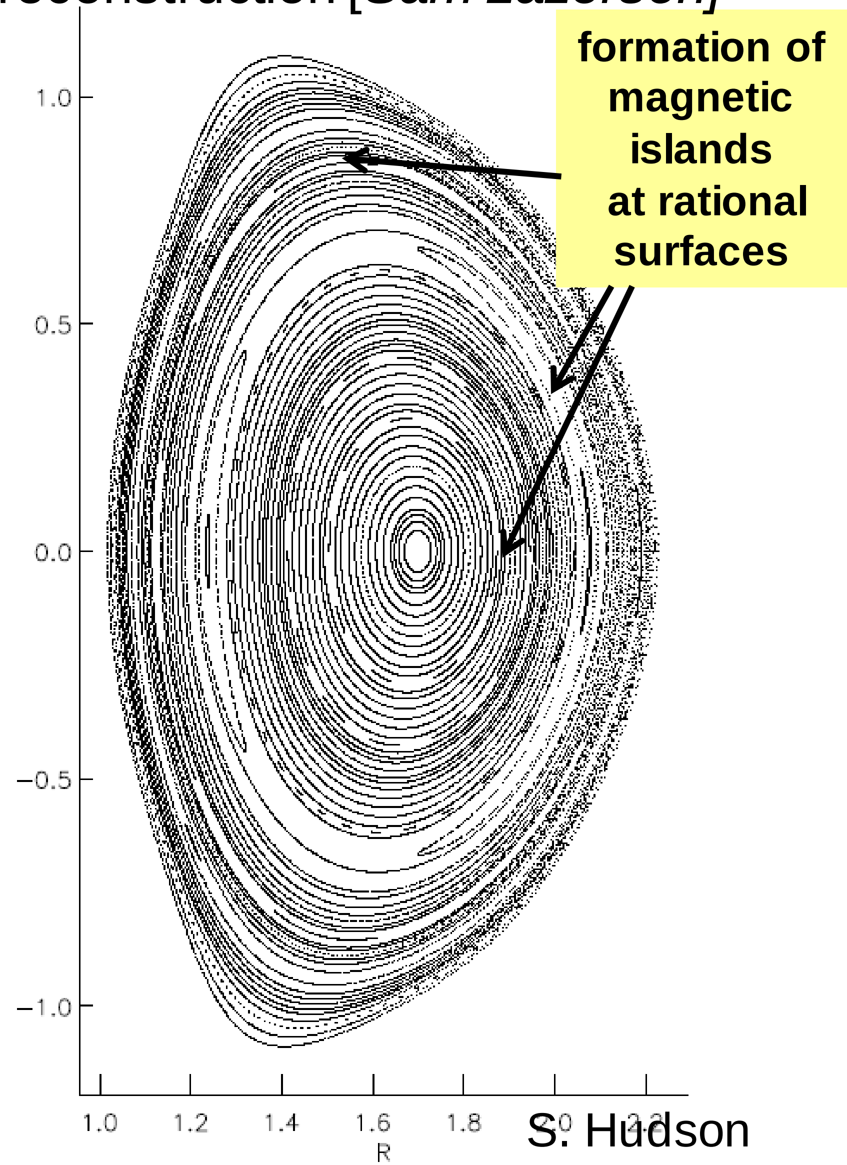
Example: DIIID with n=3 applied error field

[Hudson et al Phys. Plasmas 19, 112502 (2012)]

- 3D boundary, p, q-profile from STELLOPT reconstruction [*Sam Lazerson]*
- Irrational interfaces chosen to coincide with pressure gradients.



- Island formation is permitted
- No rational "shielding currents" included in calculation.

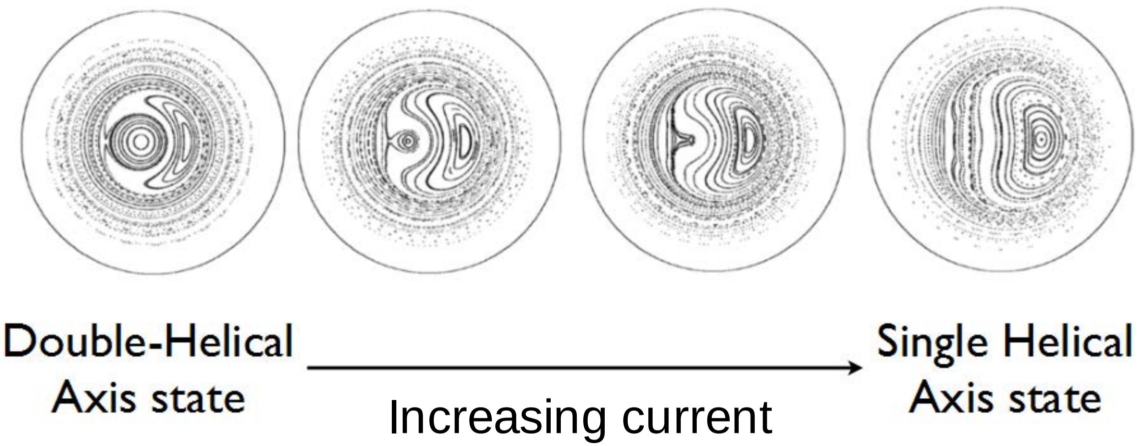


S. Hudson

Spontaneously formed helical states

Dennis, Hudson, Terranova, Dewar, Hole

- The quasi-single helicity state is a stable helical state in RFP: becomes purer as current is increase



"Experimental" Poincaré plot

[Fig. 6 of P. Martin et al., Nuclear Fusion 49, 104019 (2009)]

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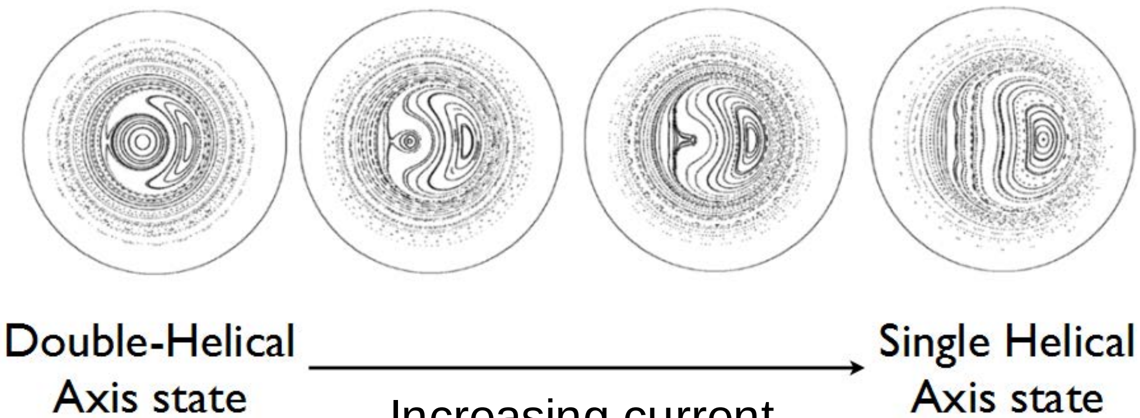
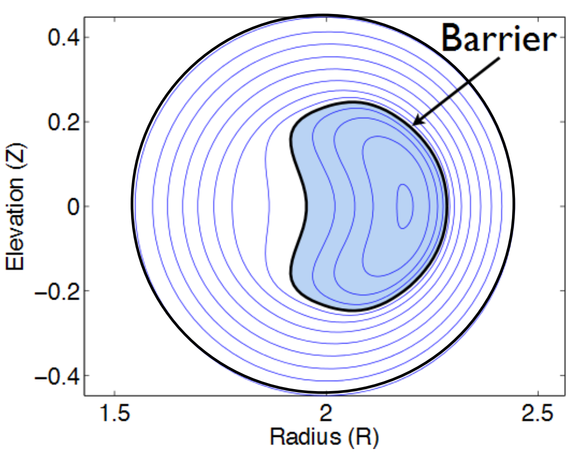


Increasing current

“Experimental” Poincaré plot

[Fig. 6 of P. Martin et al., Nuclear Fusion 49, 104019 (2009)]

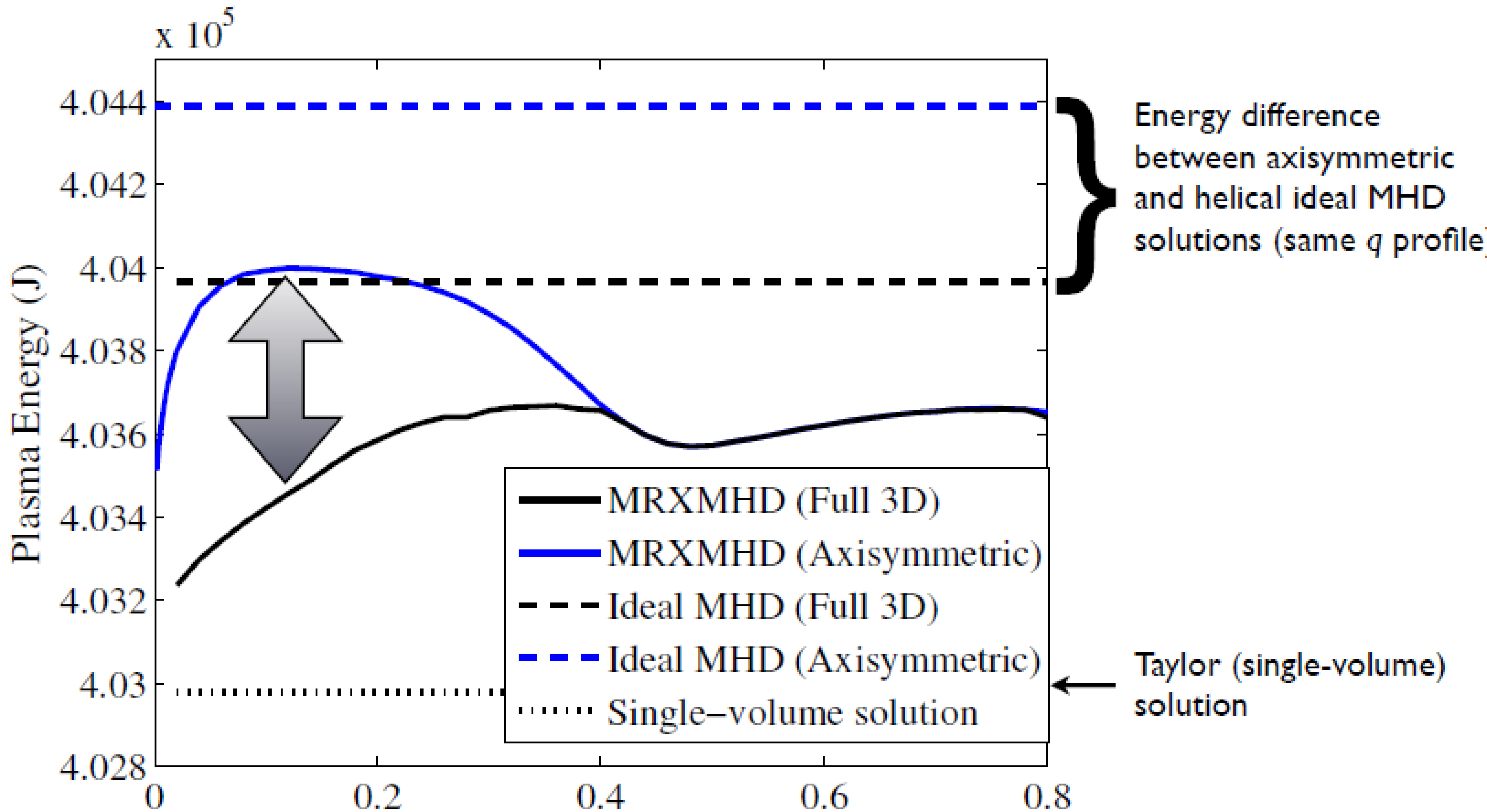


- Ideal MHD with assumed nested flux surfaces can ***not*** model the DAX state
- Might MRXMHD with 2 barriers offer a minimal description to describe DAX and SHAX states in the RFP?
- Model RFX-mod QSH state by a 2-interface minimum energy MRXMHD state.

[G. R. Dennis *et al* , Phys. Rev. Lett. **111**, 055003, 2013]

Plasma is a minimum energy state

- RFP bifurcated state has lower energy (preferred) than comparable axis-symmetric state

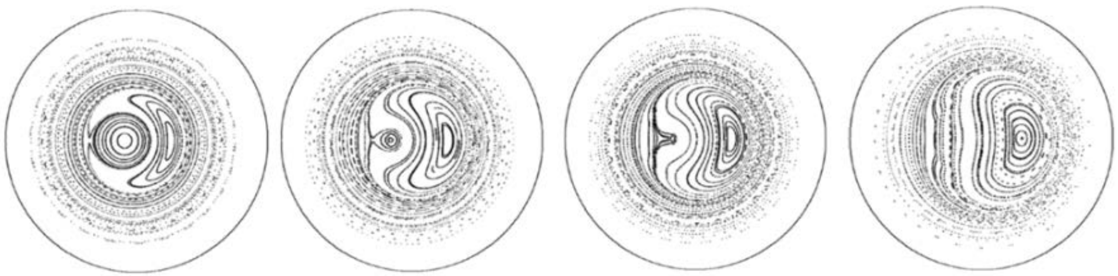


Ideal MHD flux surface chosen as ideal barrier

Spontaneously formed helical states

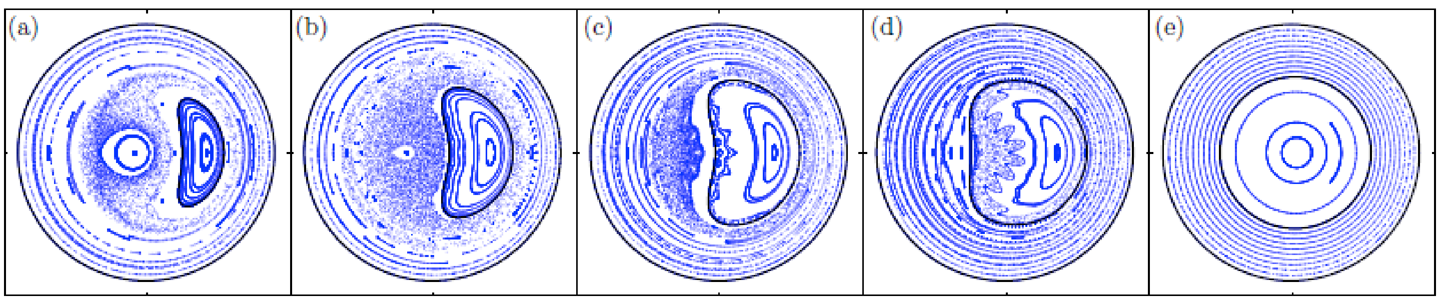
“Experimental” Poincaré plot

[Fig. 6 of P. Martin et al., Nuclear Fusion 49, 104019 (2009)]



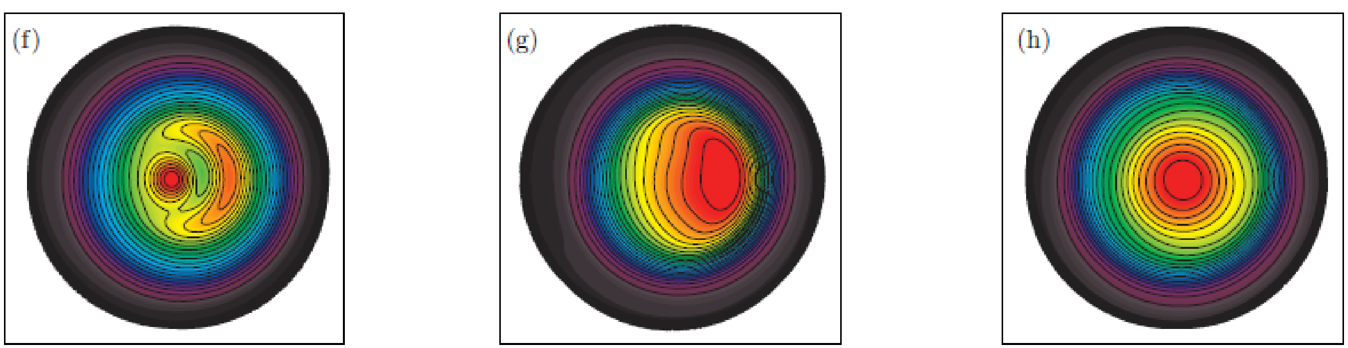
Double-Helical Axis state → Single Helical Axis state

MRXMHD Poincaré plot G. R. Dennis PRL



$\lambda = 0.1$ $\lambda = 0.2$ $\lambda = 0.3$ $\lambda = 0.4$ $\lambda = 0.5$

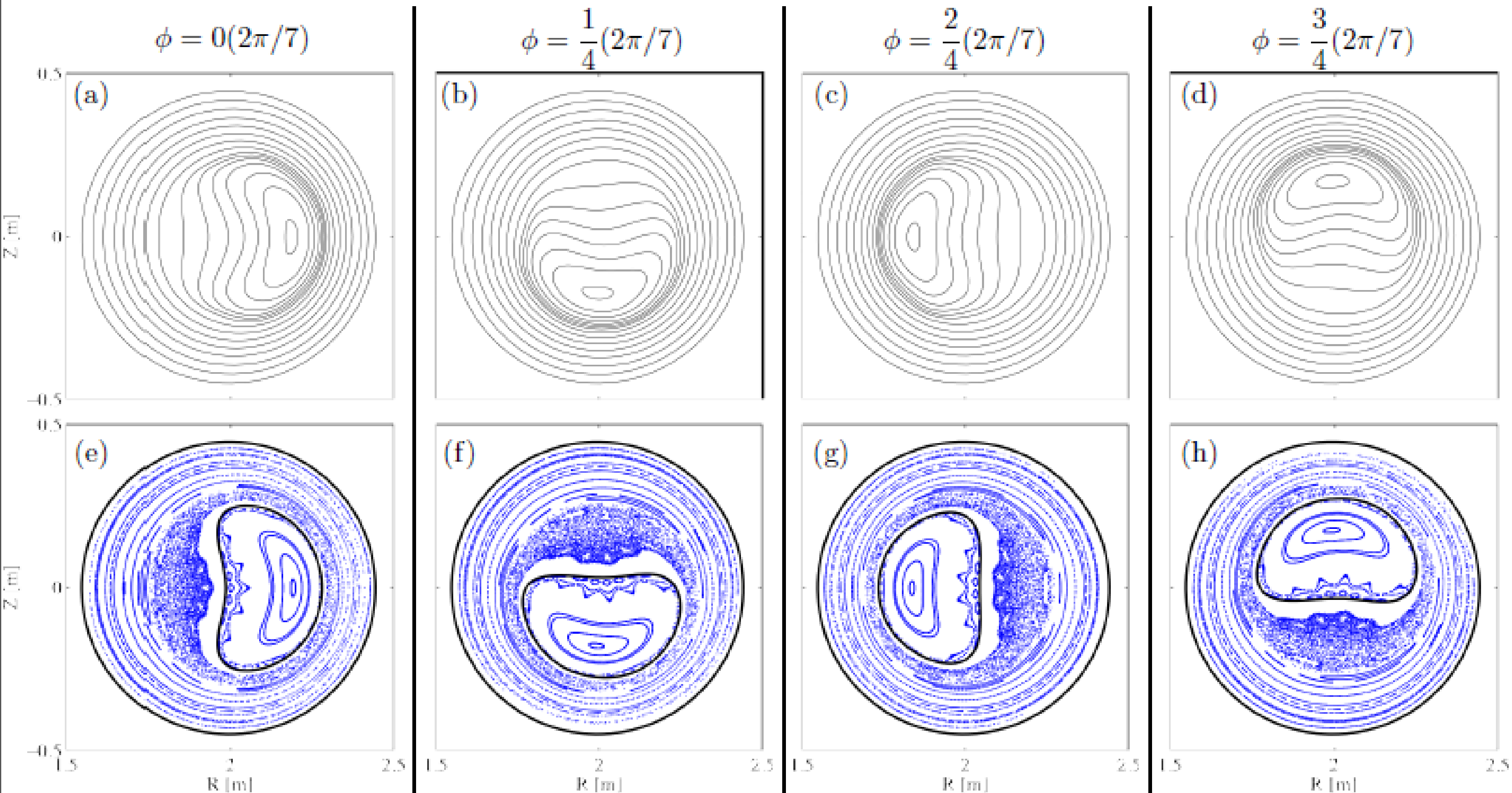
Soft X-ray data



DAx state SHAx state Axisymmetric multiple-helicity state

VMEC / SPEC comparison reveals chaos

Different toroidal cross-sections at $\lambda = 0.4$



Recent progress in MRxMHD

- Extended MRxMHD to include non-zero plasma flow

  [G.R. Dennis, S.R. Hudson, R.L. Dewar, M.J. Hole, sub. Phys Plas. 15/01/2014]

- Generalized straight field line coordinates concept to fully 3D plasmas

  [R. L. Dewar, S. R. Hudson, A. Gibson, *Plasma Phys. Control. Fusion,* **55,** 014004, 2013]

- Related helical bifurcation of a Taylor relaxed state to a tearing mode

  [Z. Yoshida and R. L. Dewar , *J. Phys. A: Math. Theor.* **45**, 365502, 2012]

- Related ghost surfaces and isotherms in chaotic fields

  [S. R. Hudson and J. Breslau**,** Phys. Rev. Let., **100**, 095001, 2008]

- Developed techniques to establish pressure jump a surface can support.

  [M. McGann, ANU PhD thesis, 2013]

Recent progress in MRxMHD

- Computed the high-*n* stability of a pressure discontinuity in a 3D plasma.
  [D. Barmaz, ANU Masters Thesis 2011]

- Developed “plasmoids”, representing partial magnetic island chains
  [R. L. Dewar *et al*, Phys. Plas. **20**, 0832901, 2013.]

Conclusions

Anisotropy

- Extended EFIT++ to include anisotropy and flow
- Code benchmarked to Extended Soloviev
- Demonstrated strong dependence of J_ϕ with anisotropy
- Extended HELENA to include anisotropy: examined components of J_ϕ and variation of p with flux surfaces
- Developed new single adiabatic stability model, incompressible stability treatment and stability code.

MRxMHD

- Introduced/ motivated multi-region relaxed MHD, and SPEC 3D MHD code
- Described helical axis RFP with 2-interface MRXMHD model
- Summarised recent developments and directions

Constraining the flux functions to transport codes or experiment

$$\left\{F(\psi), \Omega(\psi), H(\psi), T_{\|}(\psi), \theta(\psi)\right\}$$

- TRANSP computes *f(E,λ)*: Moments give $p_{\perp}$, $p_{||}$, $u_{\|}$,
- Dependency of flux functions on (R,Z) mesh

$$T_{\|}(R_i, Z_i) = \frac{p_{\|}(R_i,Z_i)}{\left(\frac{k}{m}\right)\rho(R_i,Z_i)}$$

$$F(R_i, Z_i) = R_i B_\phi(R_i, Z_i)[1 - \Delta(R_i, Z_i)]$$

$$\Omega(R_i, Z_i) = \frac{v_\phi(R_i,Z_i)}{R_i}$$

$$H(R_i, Z_i) = \frac{p_{\|}(R_i,Z_i)}{\rho(R_i,Z_i)} \ln\left(\frac{\rho(R_i,Z_i)p_{\|}(R_i,Z_i)}{\rho_0 p_{\perp}(R_i,Z_i)}\right) - \frac{v_\phi^2(R_i,Z_i)}{2}$$

$$\theta(R_i, Z_i) = \frac{\left(\frac{k}{m}\right)\rho(R_i,Z_i)B(R_i,Z_i)}{p_{\|}(R_i,Z_i)} - \frac{\left(\frac{k}{m}\right)\rho(R_i,Z_i)B(R_i,Z_i)}{p_{\perp}(R_i,Z_i)}$$

Analytic extension to Soloviev

A. G-S Soloviev solution

$$\bar{\psi} = \left[x - \frac{1}{2}\epsilon\left(1 - x^2\right)\right]^2 +$$

$$\left(1 - \frac{1}{4}\epsilon^2\right)\left[1 + \epsilon\tau x\left(2 + \epsilon x\right)\right]\left(\frac{y}{\sigma}\right)^2$$

$$\psi = \left(\frac{a^2 B_0}{\alpha}\right)\bar{\psi}$$

$$R = ax + R_0$$

$$Z = ay$$

$$p_S(\bar{\psi}) = p'[1 - \bar{\psi}]$$

$$F_S^2(\bar{\psi}) = F'^2[1 - \bar{\psi}] + R_0^2 B_0^2$$

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B. G-S with flow, anisotropy

$$p_\perp = p_\perp(R, B, \psi)$$
$$\nabla \cdot \left[\left(\frac{\nabla\psi}{R^2}\right)\right] - \frac{F^2}{(1-\Delta)^2}\nabla_\psi \log(1-\Delta) =$$
$$-\frac{1}{(1-\Delta)}\left(\frac{\partial p_\perp}{\partial\psi}\right)_{B,R} - \frac{F(\psi)F'(\psi)}{R^2(1-\Delta)^2}$$
$$\left.\begin{aligned}\left(\frac{\partial p_\perp}{\partial R}\right)_{\psi,B} &= \rho R\Omega(\psi)^2 \\ \left(\frac{\partial p_\perp}{\partial B}\right)_{\psi,R} &= -\Delta B\end{aligned}\right\} \quad (\#)$$

To maintain same ψ geometry as A keep $p_\perp'(\psi)$ and $F'(\psi)$ same, while satisfying (#). Choose

$$p_\perp(R, B, \psi) = \frac{1}{2}\rho_0\Omega_0^2 R^2 - \frac{\Delta_0}{2}B^2 + \sigma_0 p_S(\psi)$$
$$p_\parallel(R, B, \psi) = \frac{1}{2}\rho_0\Omega_0^2 R^2 + \frac{\Delta_0}{2}B^2 + \sigma_0 p_S(\psi)$$
$$F^2(\psi) = \sigma_0^2 F_S^2(\psi)$$

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$$F_S^2(\bar{\psi}) = F'^2[1 - \bar{\psi}] + R_0^2 B_0^2$$

- Solution exhibits de-coupling of magnetic and pressure surfaces, *but* functional dependence of pressure in analytical solution unrealistic because of lack of transport physics i.e. p⊥(ρ,B, ψ)/ ρ ≠ T⊥(B,ψ), as in EFIT TENSOR.

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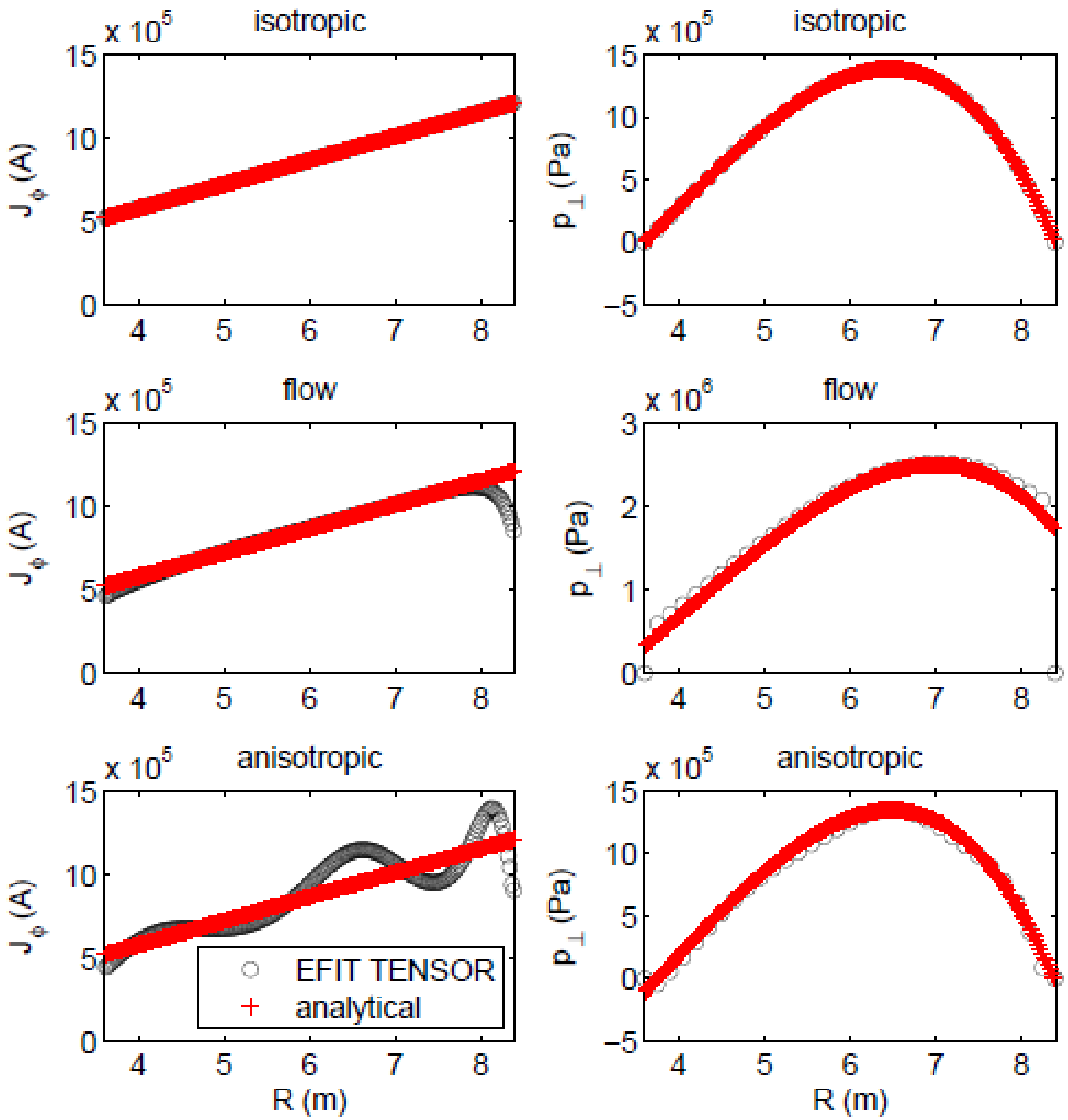
To maintain same ψ geometry as A keep $p_\perp'(\psi)$ and F'(ψ) same, while satisfying (#). Choose

$$p_\perp(R, B, \psi) = \frac{1}{2}\rho_0\Omega_0^2 R^2 - \frac{\Delta_0}{2}B^2 + \sigma_0 p_S(\psi)$$
$$p_\parallel(R, B, \psi) = \frac{1}{2}\rho_0\Omega_0^2 R^2 + \frac{\Delta_0}{2}B^2 + \sigma_0 p_S(\psi)$$
$$F^2(\psi) = \sigma_0^2 F_S^2(\psi)$$

J_ϕ a strong function of anisotropy



e.g. ITER-like plasma

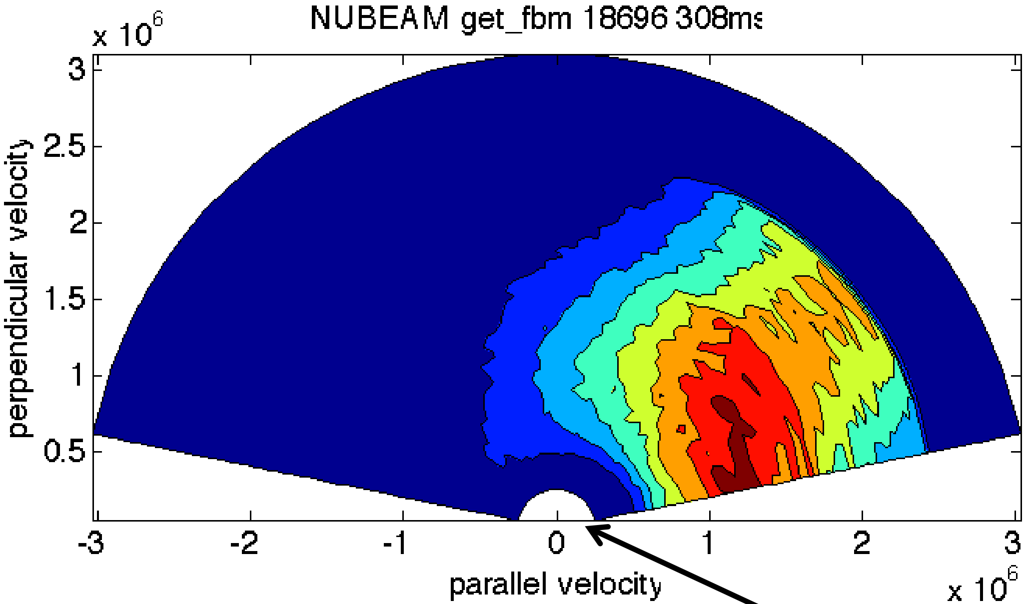
ϵ	0.4
σ	1
τ	1
R_0	6 m
B_0	5 T
α	-3
ρ_0	1×10^{-7}
Ω_0	0 or $7\times 10^5 \mathrm{rad\ s^{-1}}$
Δ_0	0 or 4×10^{-3}
I_p	16 MA
q^*	1.6
β_p	1.0
β_T	0.07

$p_\perp/p_\parallel \sim 1.06$

$P_\perp$ a good match between EFIT TENSOR and analytic working,
however
J_ϕ very different – inferred magnetic topology can be radically different

$p_{||}$, $p_{\perp}$, flow from $f(E,\lambda)$ moments

r/a=0.25



$$E = 0.5mv^2, \quad v_{||} = v\cos\lambda$$

$$n = \int_0^\infty \int_{-1}^1 \hat{f}(E,\lambda)\,\mathrm{d}\lambda\,\mathrm{d}E$$

$$nu_{\|} = \int_0^\infty \int_{-1}^1 v_{\|}\hat{f}(E,\lambda)\,\mathrm{d}\lambda\,\mathrm{d}E$$

$$p_{\|} = m\int_0^\infty \int_{-1}^1 (v_{\|}-u_{\|})^2\hat{f}(E,\lambda)\,\mathrm{d}\lambda\,\mathrm{d}E$$

$$p_{\perp} = \frac{m}{2}\int_0^\infty \int_{-1}^1 v_{\perp}^2\hat{f}(E,\lambda)\,\mathrm{d}\lambda\,\mathrm{d}E.$$

[35th EPS 2008; M.K.Lilley et al]

Thermal population

$v_{||} > v_{\perp}$ in disitribution function, *however...*

$p_{||}$ computed with subtracted $u_{||} \Rightarrow p_{||} < p_{\perp}$

In single fluid limit, need to add thermal species and recompute moments to get complete anisotropy.

[M J Hole, G von Nessi, M Fitzgerald, K G McClements, J Svensson, PPCF 53 (2011) 074021]

Generalised Taylor Relaxation:

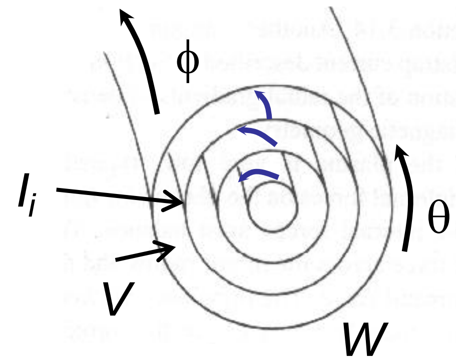
Multiple Relaxed Region MHD (MRXMHD)

R. L. Dewar

- Assume each invariant tori I_i act as ideal MHD barriers to relaxation, so that Taylor constraints are localized to subregions.

New system comprises:

- N plasma regions P_i in relaxed states.
- Regions separated by ideal MHD barrier I_i.
- Enclosed by a vacuum V,
- Encased in a perfectly conducting wall W



$$W_l = \int_{R_i} \left(\frac{B_l^{\,2}}{2\mu_0} + \frac{P_l}{\gamma - 1} \right) d\tau^3$$

$$H_l = \int_V (\mathbf{A}_l \cdot \mathbf{B}_l) d\tau^3$$

Seek minimum energy state:

$$F = \sum_{l=1}^{N} \left(W_l - \mu_l H_l / 2 \right)$$

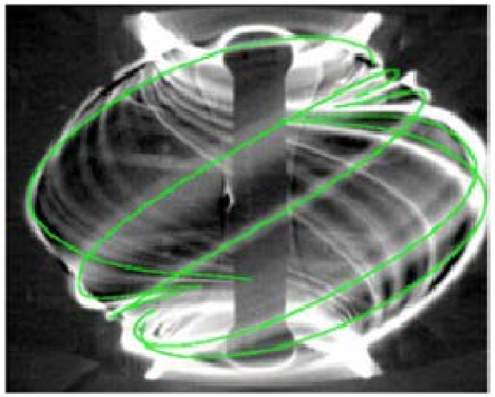
$$\begin{aligned}
P_l :& \quad \nabla \times \mathbf{B} = \mu_l \mathbf{B} \\
& \quad P_l = \text{constant} \\
I_l :& \quad \mathbf{B} \cdot \mathbf{n} = 0 \\
& \quad [[P_l + B^2/(2\mu_0)]] = 0 \\
V :& \quad \nabla \times \mathbf{B} = 0 \\
& \quad \nabla \cdot \mathbf{B} = 0 \\
W :& \quad \mathbf{B} \cdot \mathbf{n} = 0
\end{aligned}$$

Ongoing work/ plans for MRxMHD (1)

- Free-boundary extension: including vacuum region and external conductors

Enables calculation of stability to external modes and response due to Resonant Magnetic Perturbation (RMP) coils – designed to kill Edge Localised Modes (ELMs)

Filamentary structures during an ELM in MAST with the magnetic field lines overlaid.



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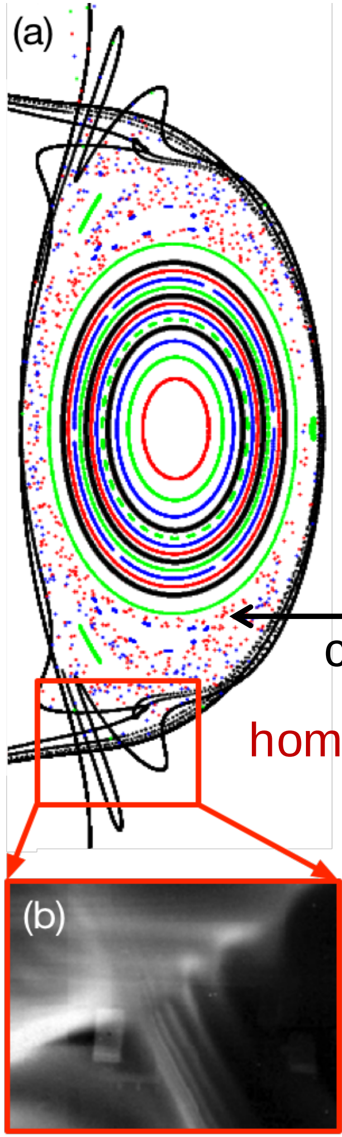
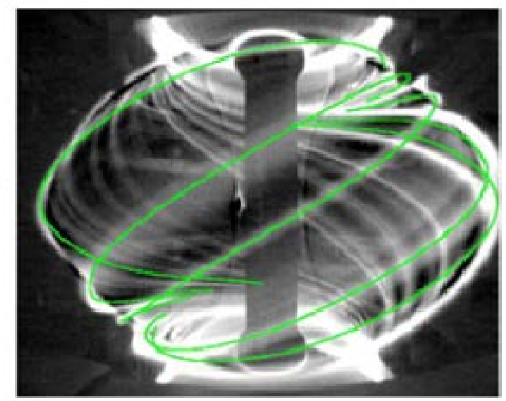


chaotic edge field

homoclinc tangle

Filamentary structures during an ELM in MAST with the magnetic field lines overlaid.



(a) prototype calculation by S. Hudson performed for an illustrative cross-section with a large perturbation

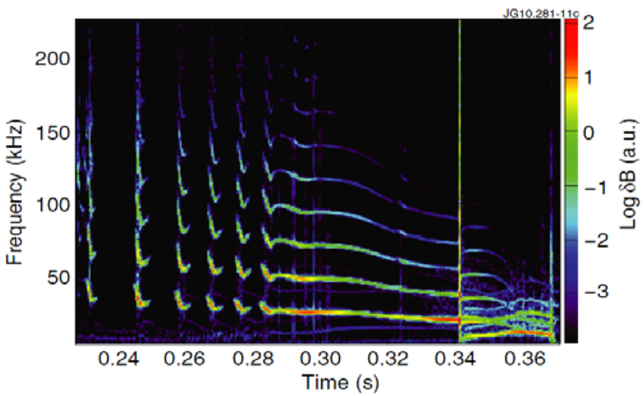
(b) lobe structure observed in MAST in divertor target region during RMP.

[A. Kirk *et al*, Plasma Phys. Control. Fusion **55**, 124003, 2013]

Ongoing work/ plans for MRxMHD (2)

- Each region is relaxed, but boundary interfaces can be unstable to island formation, and support pressure jumps, currents:
  - Determine interface disruption-limit by island formation
  - Global stability of MRxMHD equilibria – determine stability to formation of islands and chaotic fields.
  - Impact of flow-shear on ELMs stability in RMP modified plasmas.

- Explanation of helical states exist in tokamaks form an energy principle: islands, “long-lived” modes, sawteeth



Fishbone oscillations (bursty, up to 0.28s) that initiate a long-living n = 1 kink mode (at frequency 30 kHz) in MAST discharge #16038.

[B N Breizman, S E Sharapov, PPCF, **53**, 054001, 2011]

Single-adiabatic model

- Extending the MHD model to anisotropy
  - isotropic perturbed pressure ~p and assuming zero net heat flow. The perturbation p~ is isotropic

$$\overleftrightarrow{P}_0 + \overleftrightarrow{P}_1 = p_\perp \boldsymbol{I} + (p_\parallel - p_\perp)\boldsymbol{bb} + \tilde{p}\boldsymbol{I} =$$

$$\begin{pmatrix} p_\perp + \tilde{p} & 0 & 0 \\ 0 & p_\perp + \tilde{p} & 0 \\ 0 & 0 & p_\parallel + \end{pmatrix}$$

$$\widetilde{\mathrm{P}} \rightarrow \tilde{p}\mathrm{I} + \widetilde{\pi}$$

$$\mathrm{Tr}\ \nabla \cdot \widetilde{Q} \rightarrow 0$$

$$\mathrm{Tr}\ \widetilde{\pi} \rightarrow 0$$

Assumes zero net heat flow

Generalizing to Anisotropic plasma (non-compressional)

- The Lagrangian perturbed distribution function is zero

$$f_{bounce} \ll f_{mode} \rightarrow CGL$$

$$\tilde{f} = -\boldsymbol{\xi}_\perp \cdot \nabla F - \cancel{\frac{\partial F}{\partial \varepsilon}\langle \frac{1}{i\omega}\frac{M}{e}E_\parallel + v_\parallel^2 \boldsymbol{\xi}_\perp \cdot \boldsymbol{\kappa} - \mu B \boldsymbol{\xi}_\perp \cdot \boldsymbol{\kappa} - \mu B \nabla \cdot \boldsymbol{\xi}_\perp \rangle_{bounce}}$$

- The fluid closure equations are*

$$p_{\parallel 1} = -\xi_n \left[\frac{\partial p_\parallel}{\partial n} - (p_\parallel - p_\perp)\frac{\partial \ln B}{\partial n} \right]$$

$$p_{\perp 1} = -\xi_n \left[\frac{\partial p_\perp}{\partial n} - (2p_\perp + \hat{c})\frac{\partial \ln B}{\partial n} \right]$$

Note for Matthew: this equation ha
1.magnetic drift effect (only EXB dr
2. non-local resonance and Laudau
3. FOW and FLR effects
otherwise you need more complica

$$\hat{c} \equiv \sum_s M_s \int \frac{B}{v_\parallel}(\mu B)^2 \frac{\partial F_s}{\partial \varepsilon} d\mu d\varepsilon.$$

*A B Mikhailovskii, Instabilities in a confined plasma, IOP publishing (1998)

Impact of anisotropy & flow on stability

- MHD cannot deal with anisotropy
  – MHD pressure is isotropic: , $p_{\|}$ and $p_{\perp}$ are combined and indisitnighuisbal. P =
  $$\frac{d}{dt}\left(\frac{p}{\rho^{5/3}}\right) = 0$$
  × Parallel heat flow is extreme due to streaming particles
  × Kinetic effects are significant (Landau damping)
- Use a fluid model as a first approximation:
  – **New** Single adiabatic (SA) model extension to MHD
  – Double-adiabatic (CGL)
    - Collisionless, $p_{\|}$ and $p_{\perp}$ doing **independent** work
    - No streaming particle heat flow
    - Does not reduce to MHD in the isotropic limit

  $$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho B}\right) = 0,$$
  $$\frac{d}{dt}\left(\frac{p_{\|} B^2}{\rho^3}\right) = 0.$$

  – Double-polytropic law
    - Extension of CGL but not physically solid

  $$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho B^{\gamma_{\perp}-1}}\right) = 0$$
  $$\frac{d}{dt}\left(\frac{p_{\|} B^{\gamma_{\|}-1}}{\rho^{\gamma_{\|}}}\right) = 0$$

  – Going to higher order moments and truncate at arbitrary order