A new class of magnetic confinement device in the shape of a knot

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Abstract

We describe a new class of magnetic confinement device, with the magnetic axis in the shape of a knot. We call such devices “knotatrons”. Examples are given that have a large volume filled with magnetic surfaces, with significant rotational-transform, and with the magnetic field produced entirely by external circular coils.

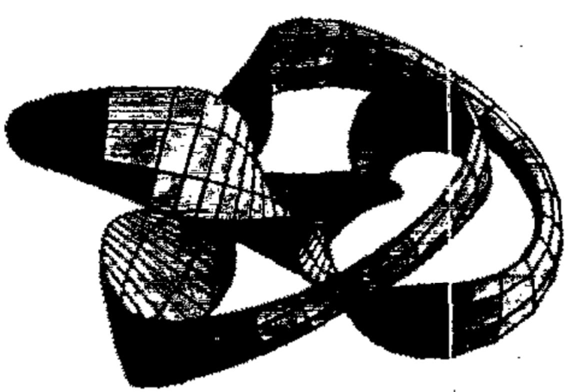
correction:
Knots *have* been considered before!

I have recently (after the PoP article) learnt that knotted configurations were considered in:

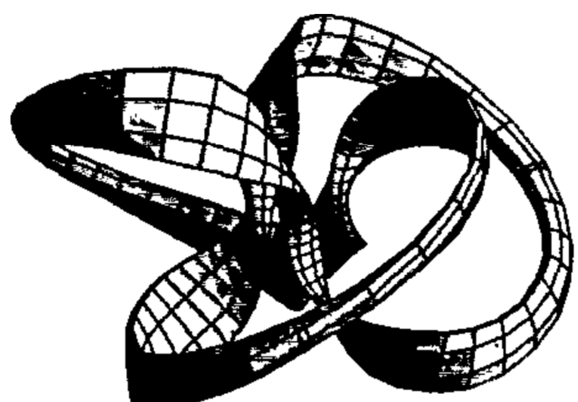
EXISTENCE OF QUASIHELICALLY SYMMETRICAL STELLARATORS
By: GARREN, DA; BOOZER, AH
PHYSICS OF FLUIDS B-PLASMA PHYSICS Volume: 3 Issue: 10 Pages: 2822-2834
Published: OCT 1991

Fourier representations for quasi-helical knots were constructed, as shown below.

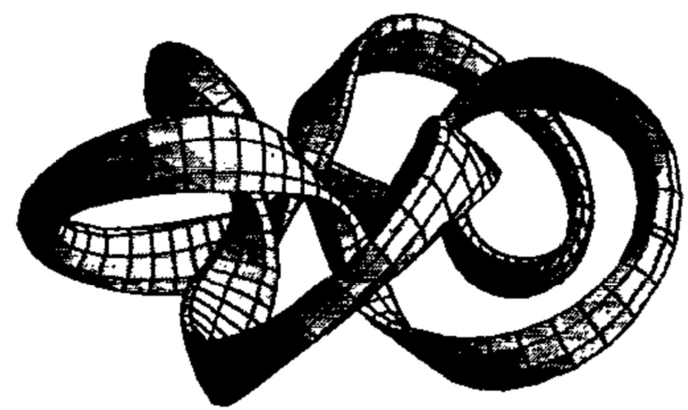
Garren & Boozer Fig.9



Garren & Boozer Fig.11

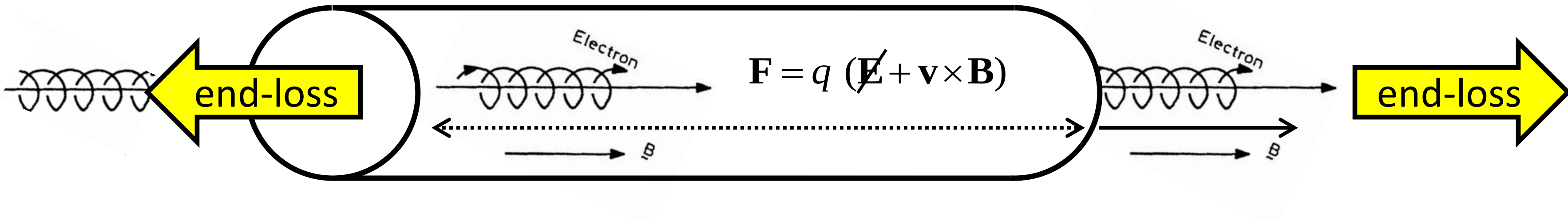


Garren & Boozer Fig.13



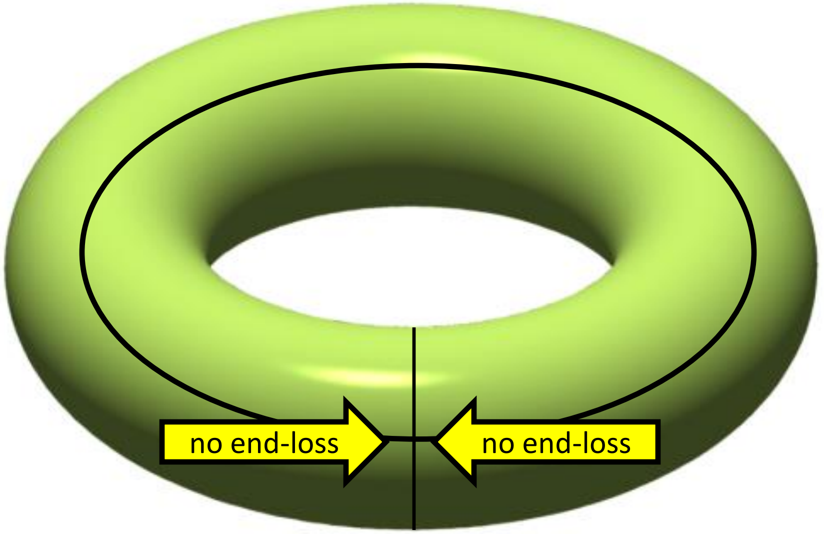
Charged particles are confined *perpendicularly* in a strong magnetic field,
but are “lost” in the *parallel* direction.

An open-ended cylinder has good perpendicular confinement of charged particles,
but particles are lost through the ends.



To eliminate end-losses the magnetic field must "close" upon itself. Joining the ends of a cylinder makes a tokamak.

In an axisymmetric tokamak, the magnetic axis is a circle in real space.



perpendicular particle drifts are caused by inhomogeneous |B|, curvature etc. . .

Because rotational-transform is required to cancel particle drifts,

axisymmetric configurations need toroidal plasma current;

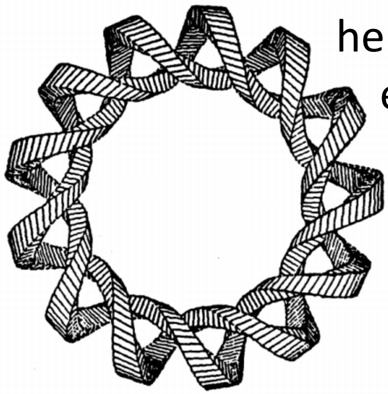
and toroidal plasma current leads to disruptions, . . .

An alternative for producing rotational-transform is by non-axisymmetric shape.

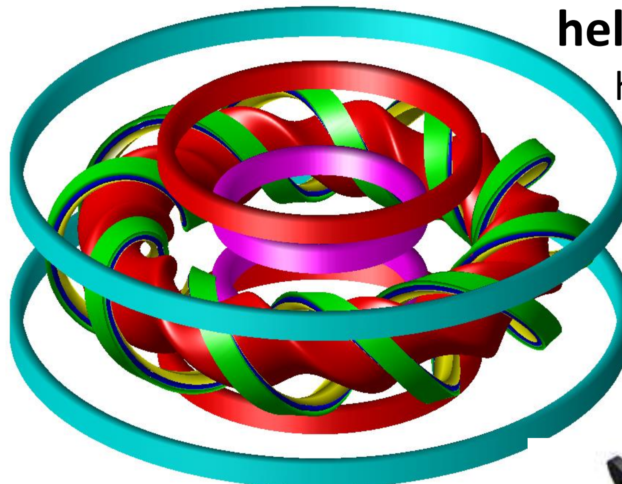
In the non-axisymmetric stellarator the confining magnetic field is produced by external coils, and stellarators are more stable.

In a conventional stellarator, the magnetic axis is smoothly deformable into a circle.

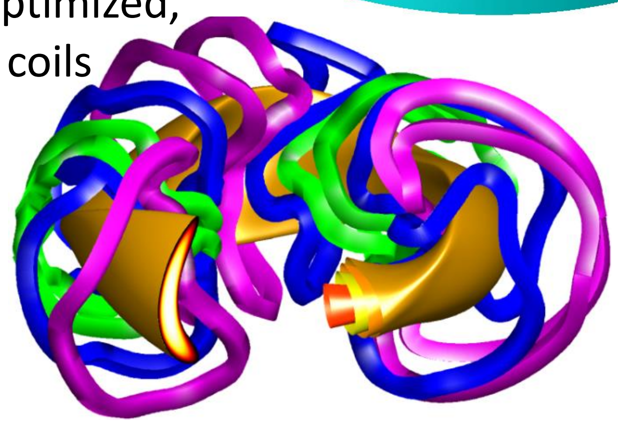
torsatron: continuous, helical coils; e.g. ATF

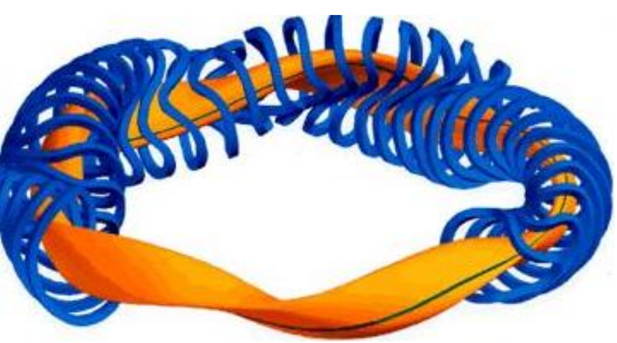


heliotron: helical coils; e.g. LHD



NCSX: optimized, modular coils





helias: helical axis, modular coils; e.g. W7X

heliac: helical axis; e.g. H-1NF, TJ-II.

The magnetic axis of a tokamak is a circle. The magnetic axis of a conventional stellarator is smoothly deformable into a circle.

There is another class of confinement device that:

1) is closed, in that the magnetic axis is topologically a circle (a closed, one-dimensional curve) ;
2) has a large volume of “good flux-surfaces” (as will be shown in following slides) ;
3) has significant rotational-transform without plasma current (because the magnetic axis is non-planar) ;
4) has a magnetic axis that is not smoothly deformable into a circle.
   (without cutting or passing through itself)

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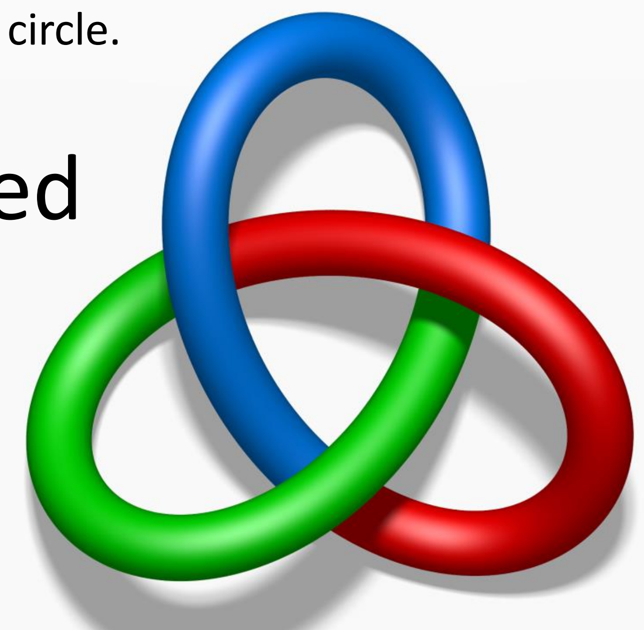
The magnetic field may be closed by forming a knot!



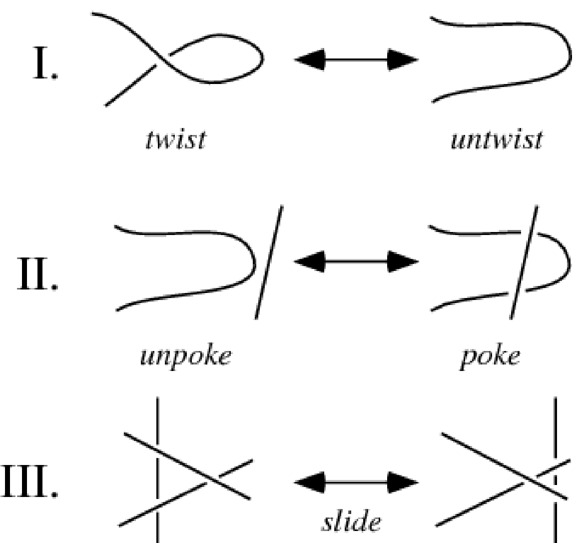
e.g. a colored trefoil knot

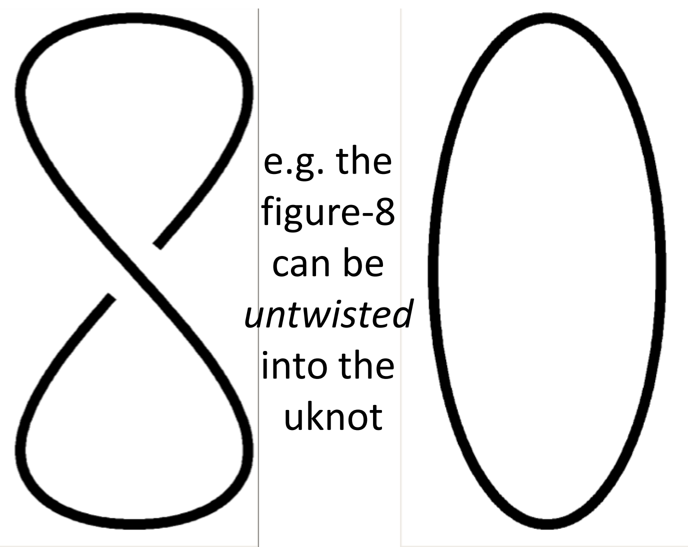
Mathematically, a knot, $K: S^1 \to S^3$, is an *embedding* of the circle, S^1, in real space, $S^3 \equiv R^3$.

Two knots, K_0 and K_1, are *ambient isotopic* if there exists a continuous, one-parameter family of homeomorphisms of S^3, h_t, such that $h_0 \circ K_0 = K_0$ and $h_1 \circ K_0 = K_1$

Equivalently, two knots are ambient isotopic if one knot can be deformed into the other knot without introducing cuts or having the knot passing through itself.

Reidemeister moves





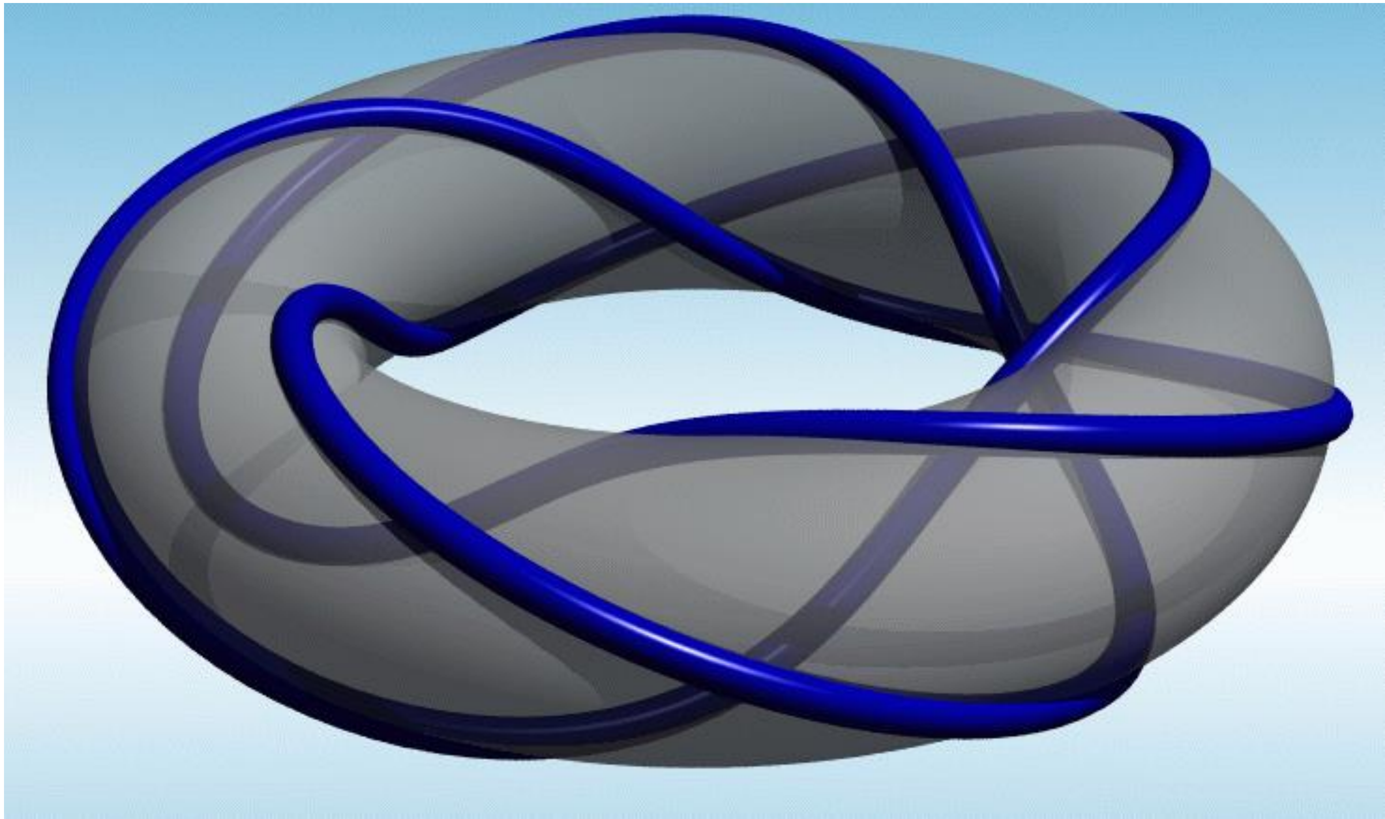
A (p,q) torus knot, with p, q co-prime, wraps p times around poloidally and q times around toroidally on a torus.

$$x(\zeta) = R(\zeta)\ \cos(p\zeta), \quad \text{where } R(\zeta) = R_0 + r\ \cos(q\zeta), \text{ and } R_0, r \text{ are constants}$$

$$y(\zeta) = R(\zeta)\ \sin(p\zeta),$$

$$z(\zeta) = \quad -r\ \sin(q\zeta),$$

is a (p,q) torus-knot.



A suitably placed set of circular current coils can produce a magnetic field with an axis in the shape of a knot.

Let $\mathbf{x}(\zeta) = x(\zeta)\ \mathbf{i}\ +\ y(\zeta)\ \mathbf{j}\ +\ z(\zeta)\ \mathbf{k}$ be a closed curve in real space,

closed: $\mathbf{x}(\zeta + 2\pi) = \mathbf{x}(\zeta)$

tangent $\mathbf{t} \equiv \mathbf{x}' / |\mathbf{x}|$, normal $\mathbf{n} \equiv \mathbf{t}' / \kappa$, bi-normal $\mathbf{b} \equiv \mathbf{t} \times \mathbf{n}$, $\mathbf{b}' \equiv -\tau \mathbf{n}$, $\kappa \equiv$ curvature, $\tau \equiv$ torsion

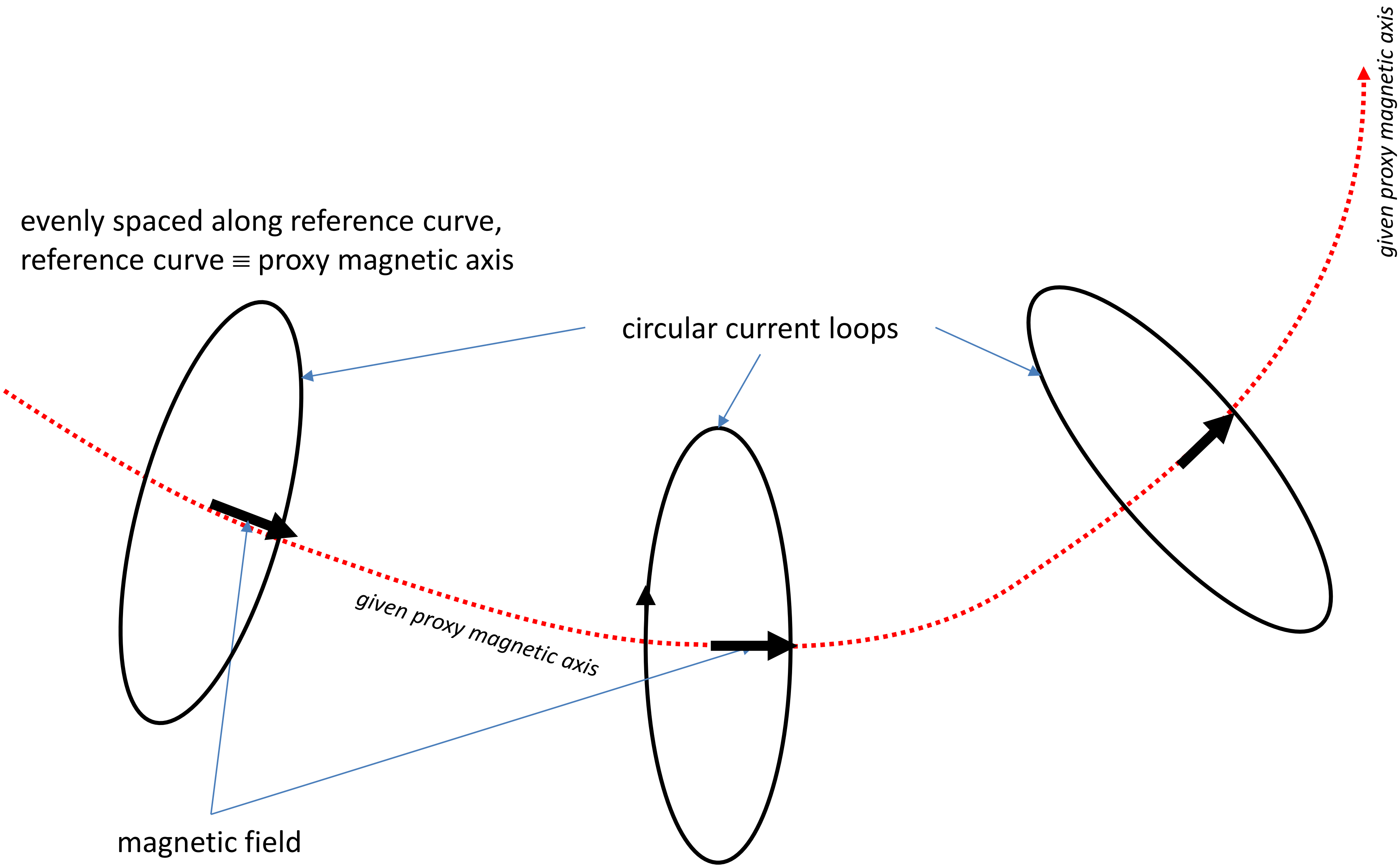
Place a set of $i = 1, ..N$ circular coils, radius a, unit current, equally spaced in ζ,

e.g. $\bar{\mathbf{x}}_i(\vartheta) = \mathbf{x}(\zeta_i) + a \cos(\vartheta)\ \mathbf{n} + a \sin(\vartheta)\ \mathbf{b}$

Adjust the orientation of each coil so that the total magnetic is tangential to $\mathbf{x}(\zeta)$,

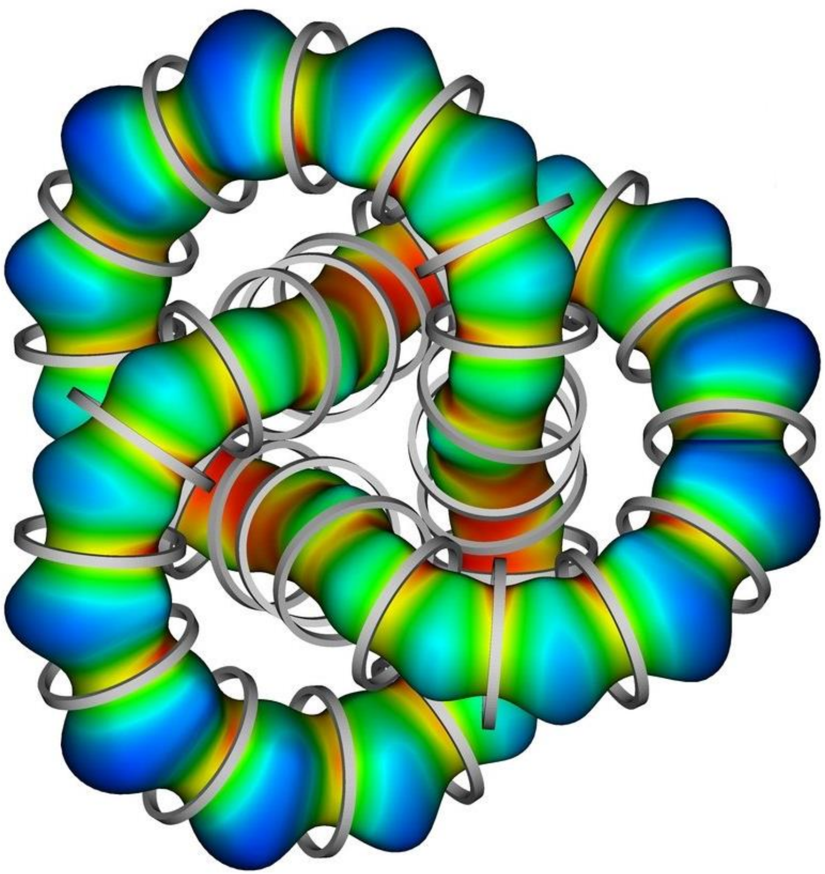
i.e. $\sum_{coils} \mathbf{B}_{coil}(\mathbf{x}_i) \times \mathbf{t}_i = 0$ for all $i = 1, ..N$

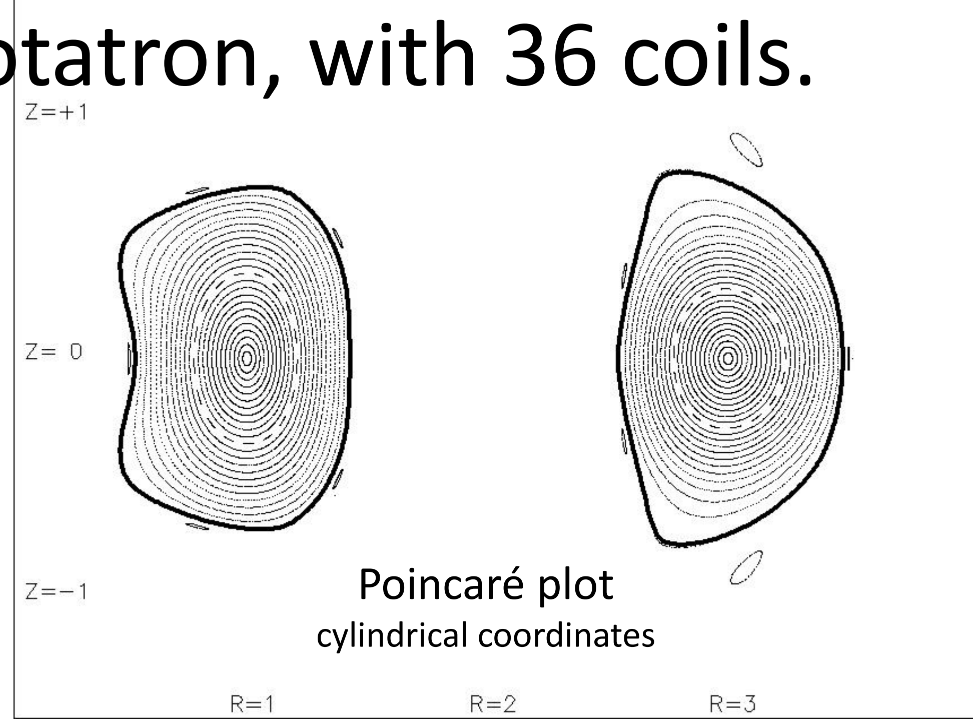
The orientation of a set of circular coils is adjusted to produce the required magnetic axis.

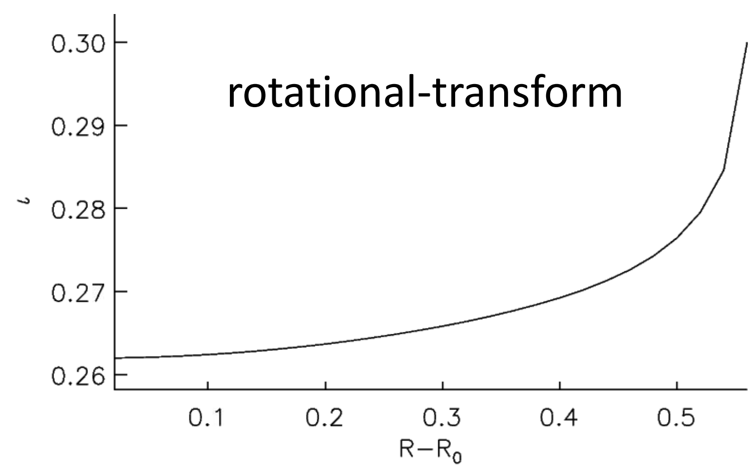


Example: (2,3) torus knotatron, with 36 coils.

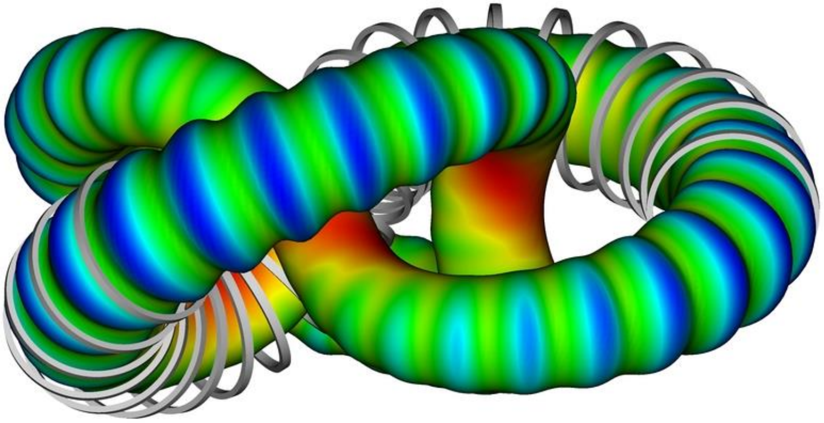
A flux surface in a (2,3) torus-knotatron with 36 circular coils.
The color indicates |B|.

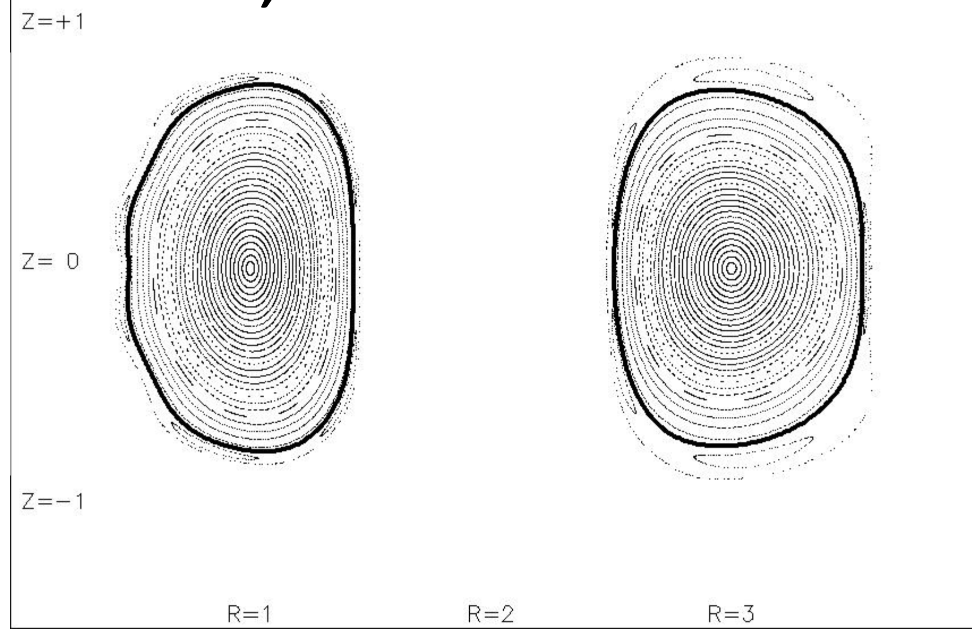


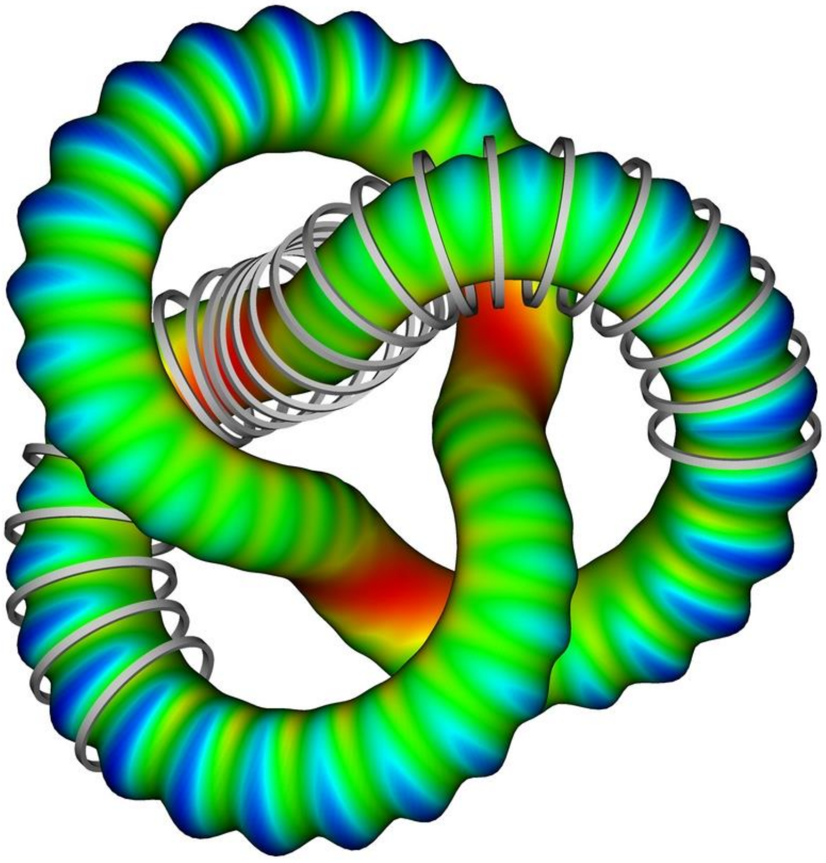


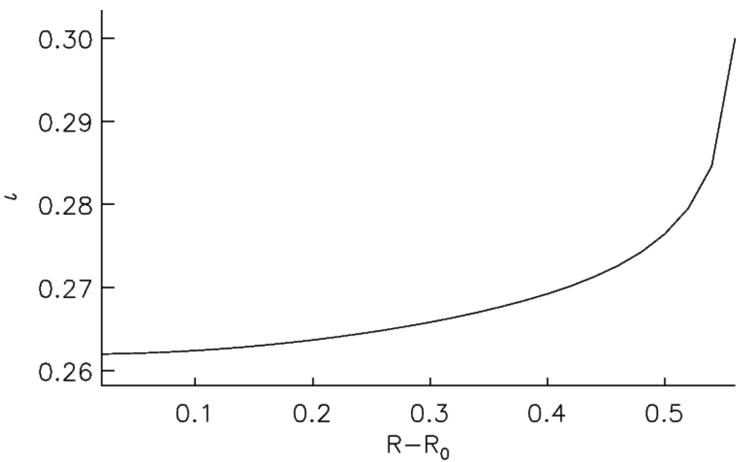


Example: (2,3) torus knotatron, with 72 coils.

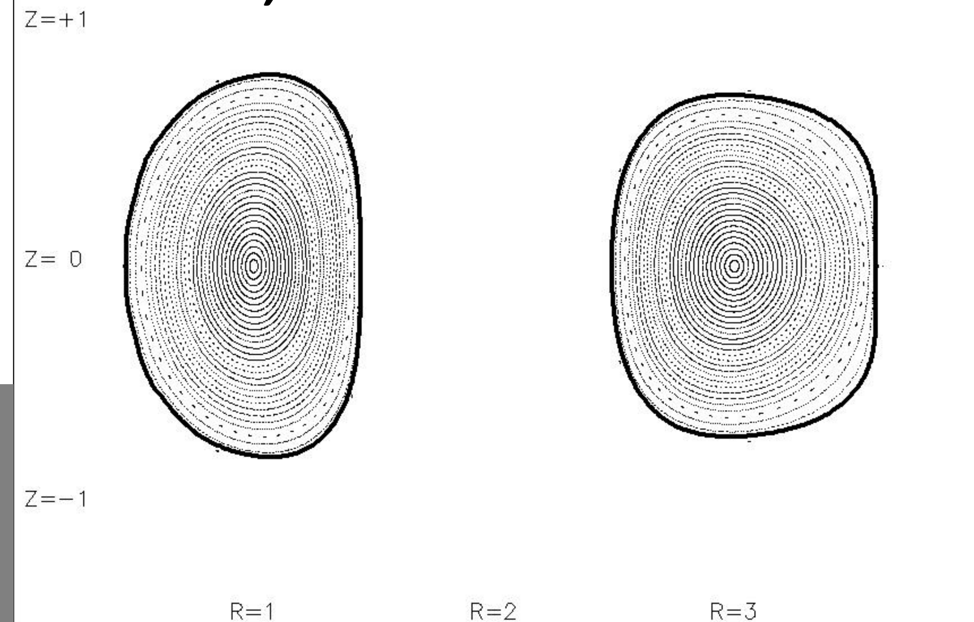


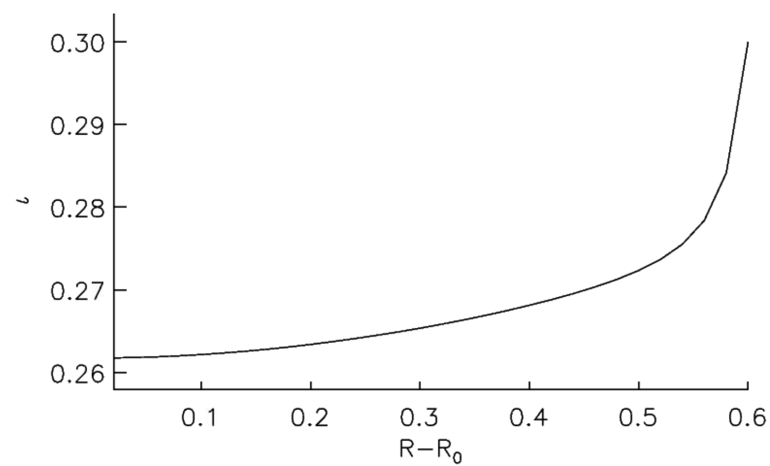




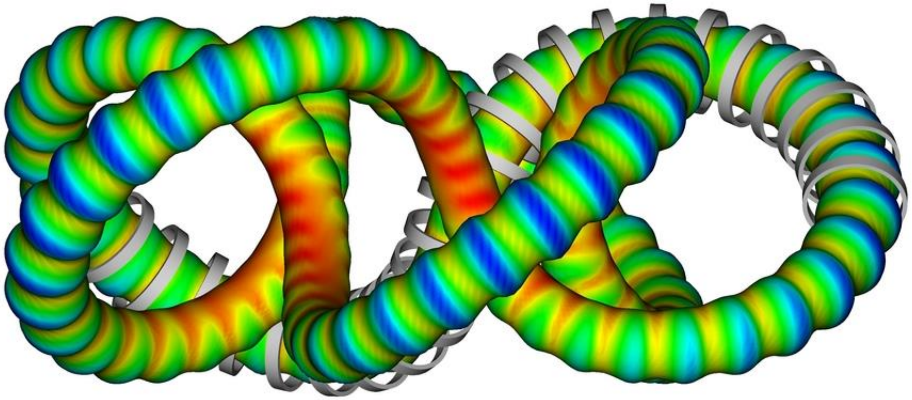


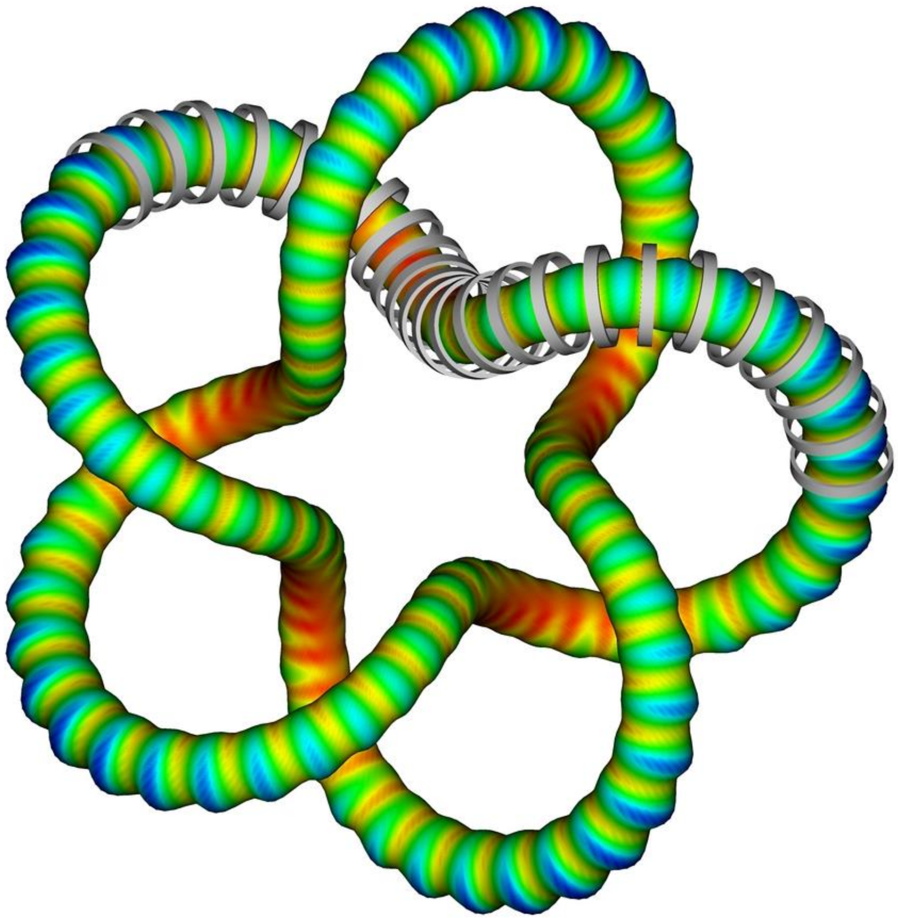
Example: (2,3) torus knotatron, with 108 coils.

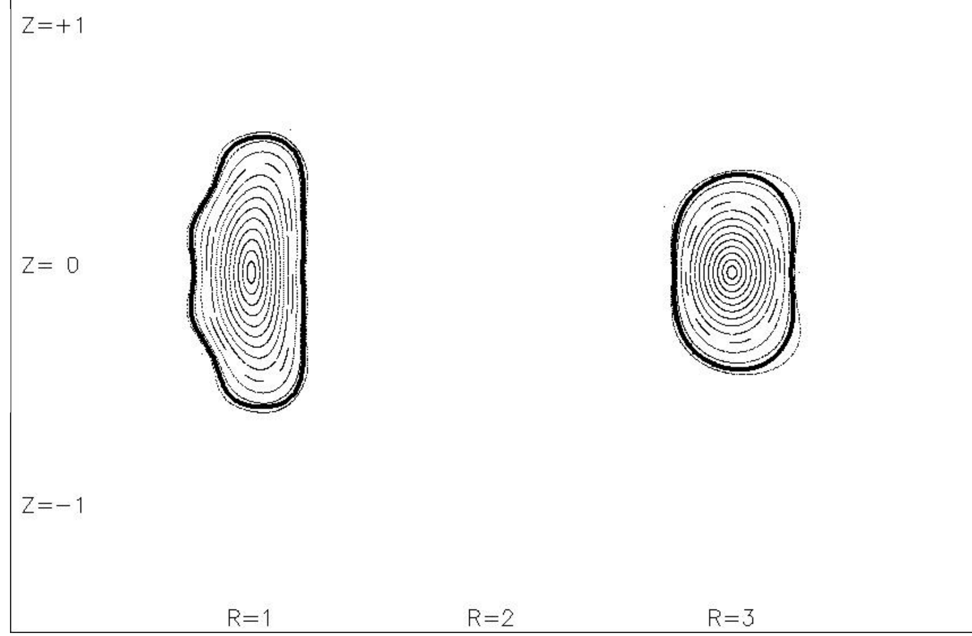


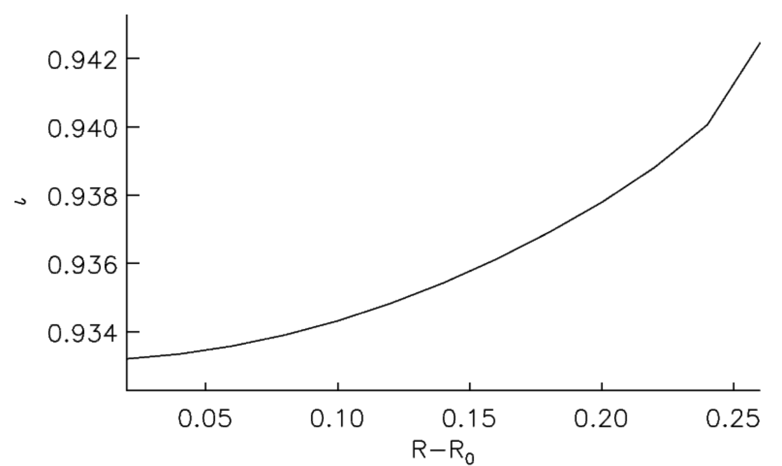


Example: (2,5) torus knotatron

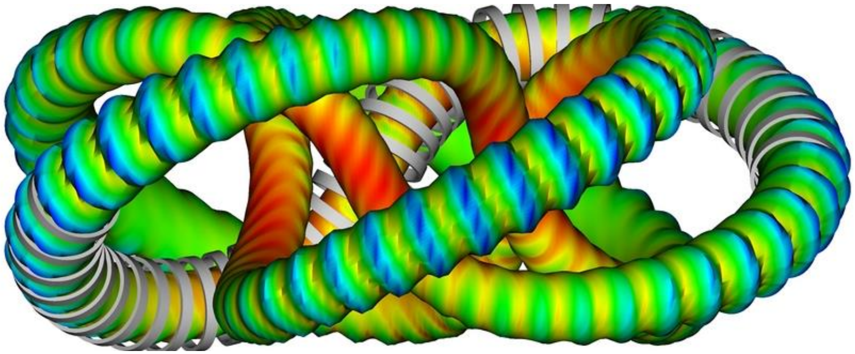


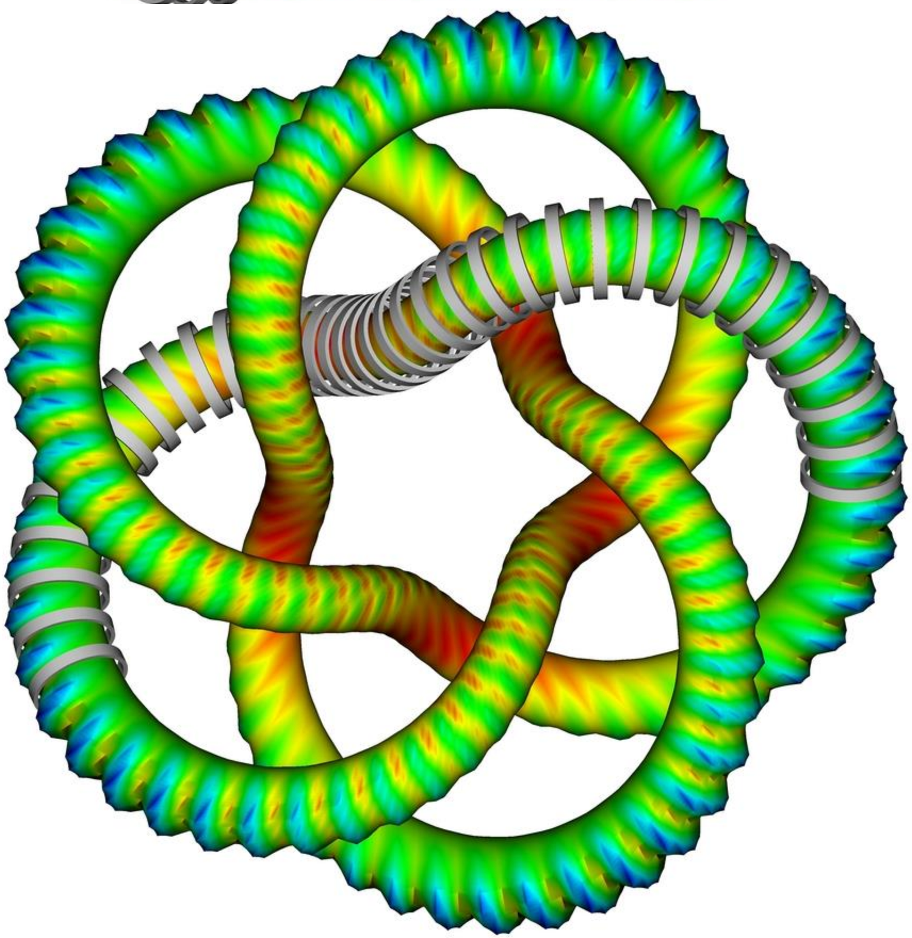


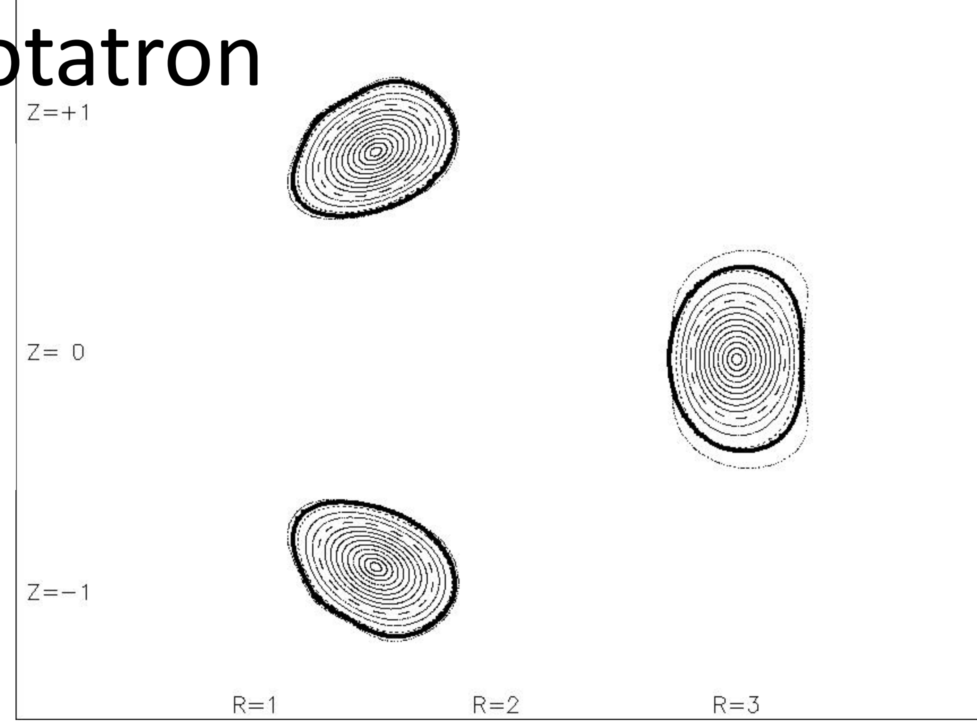


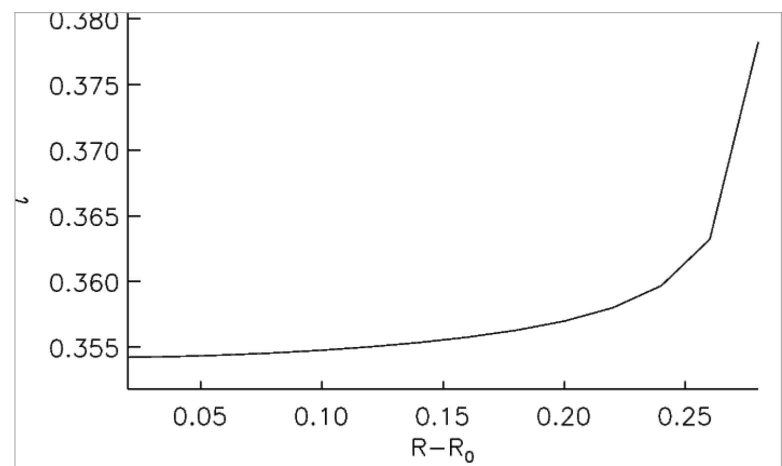


Example: (3,5) torus knotatron

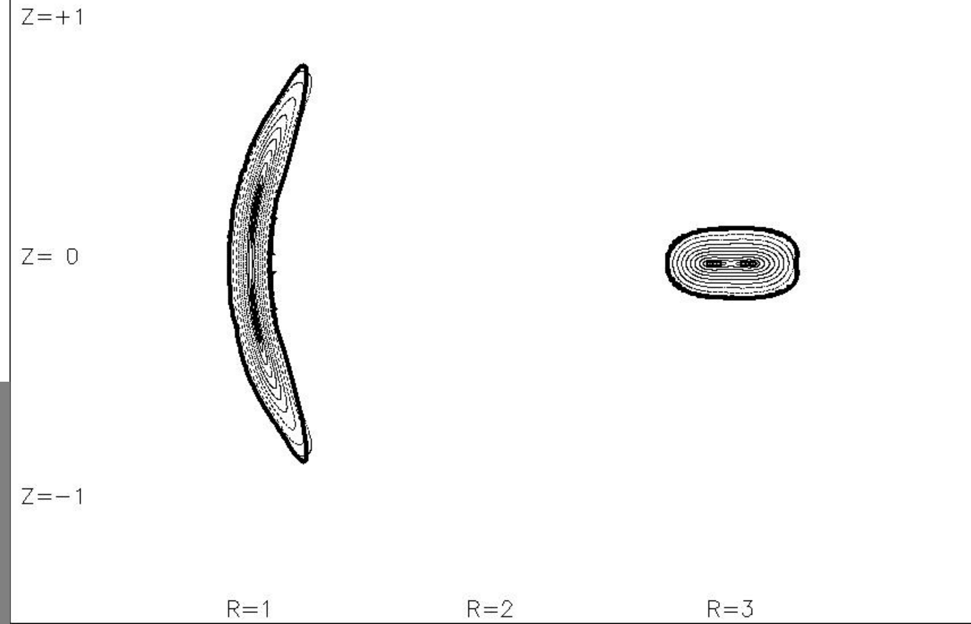


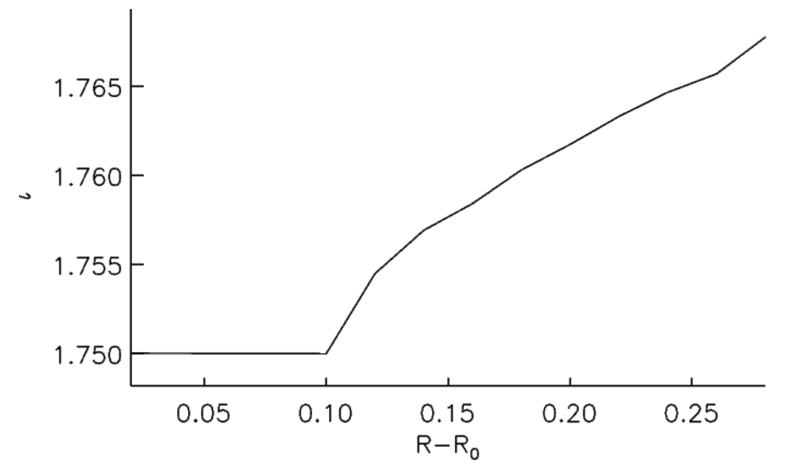




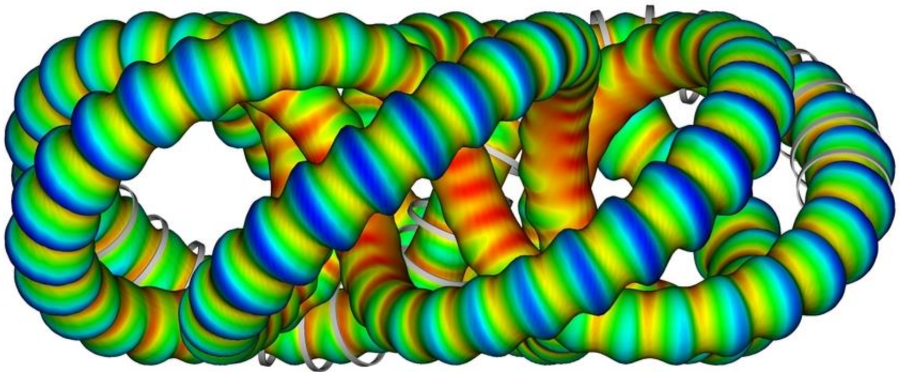


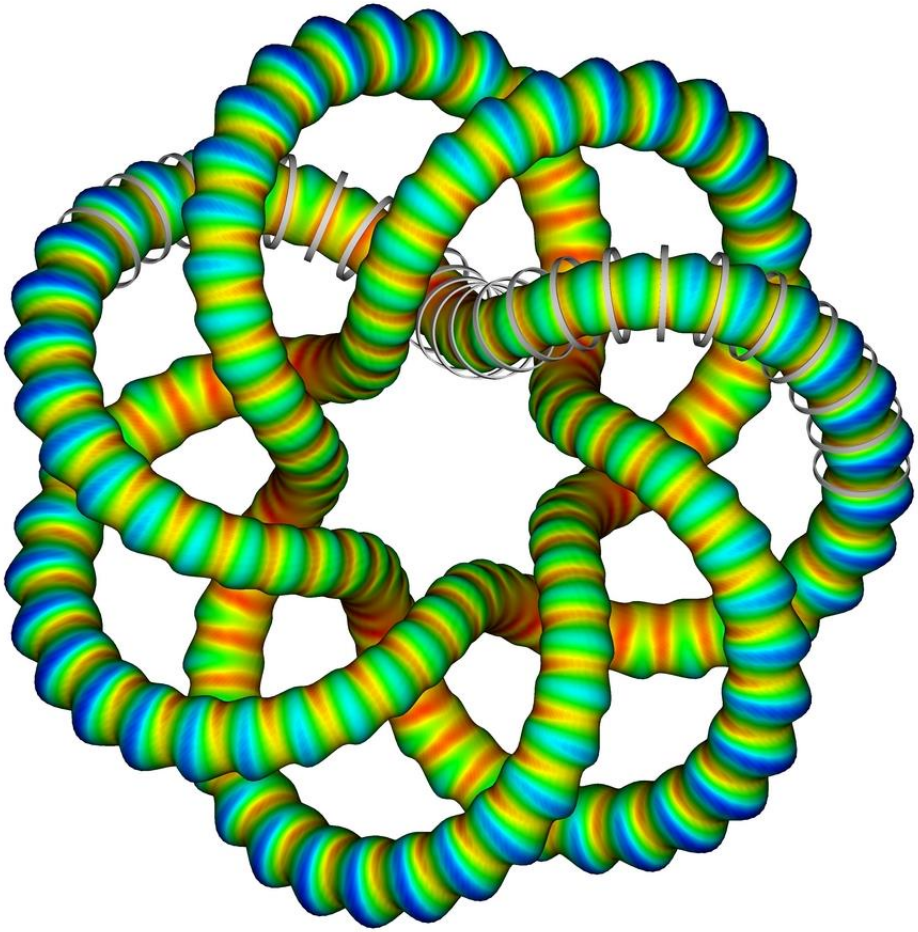
Example: (2,7) torus knotatron

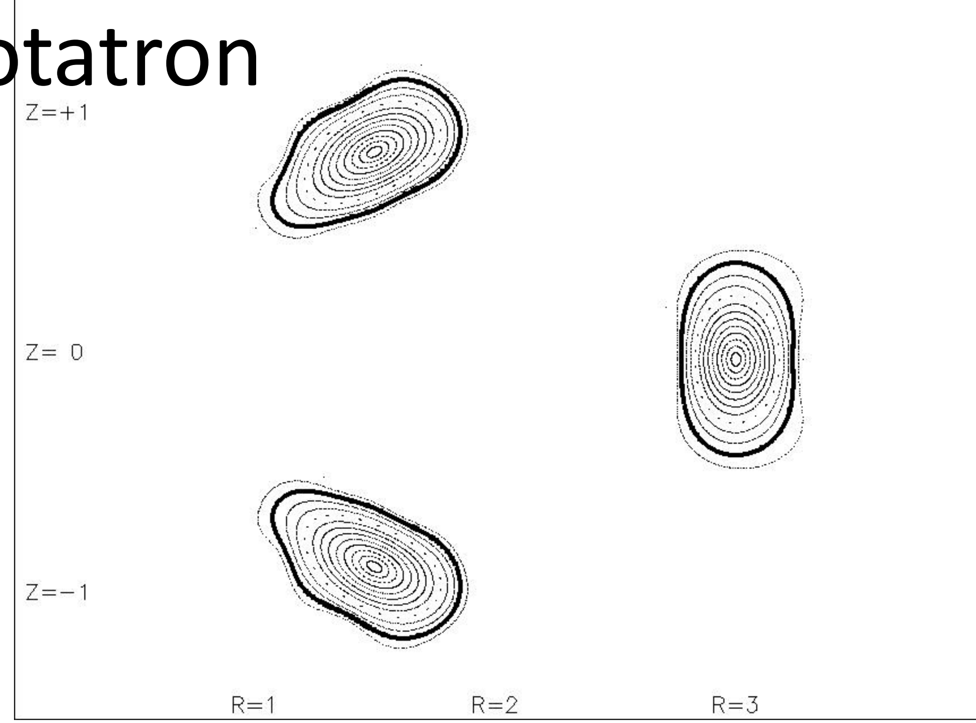


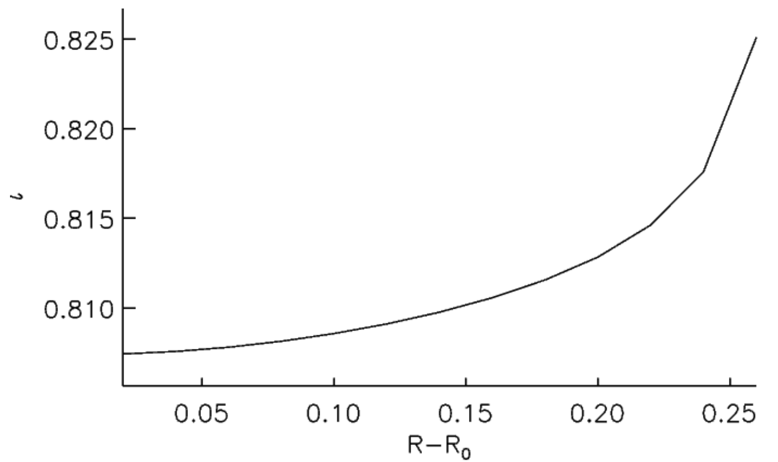


Example: (3,7) torus knotatron

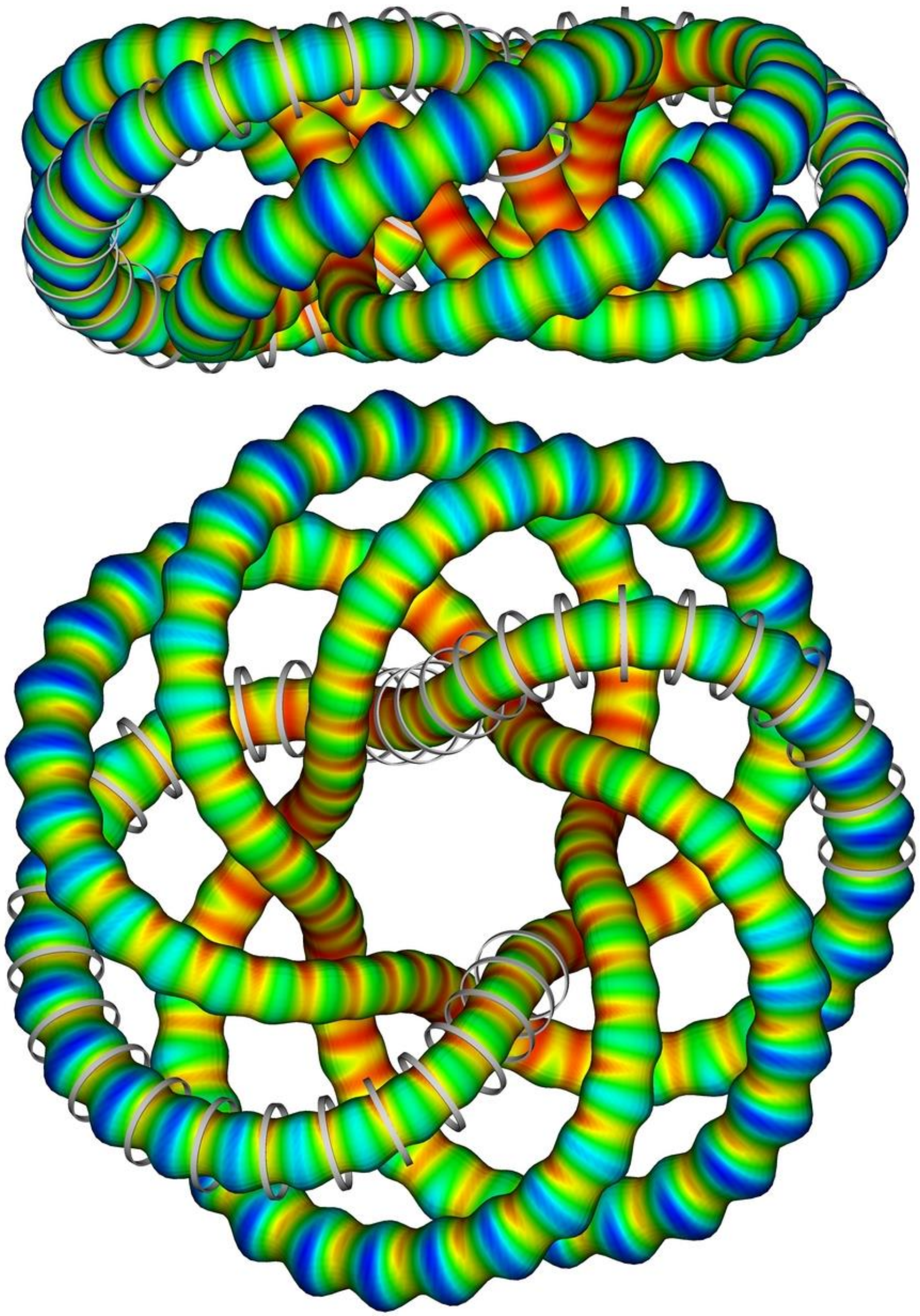


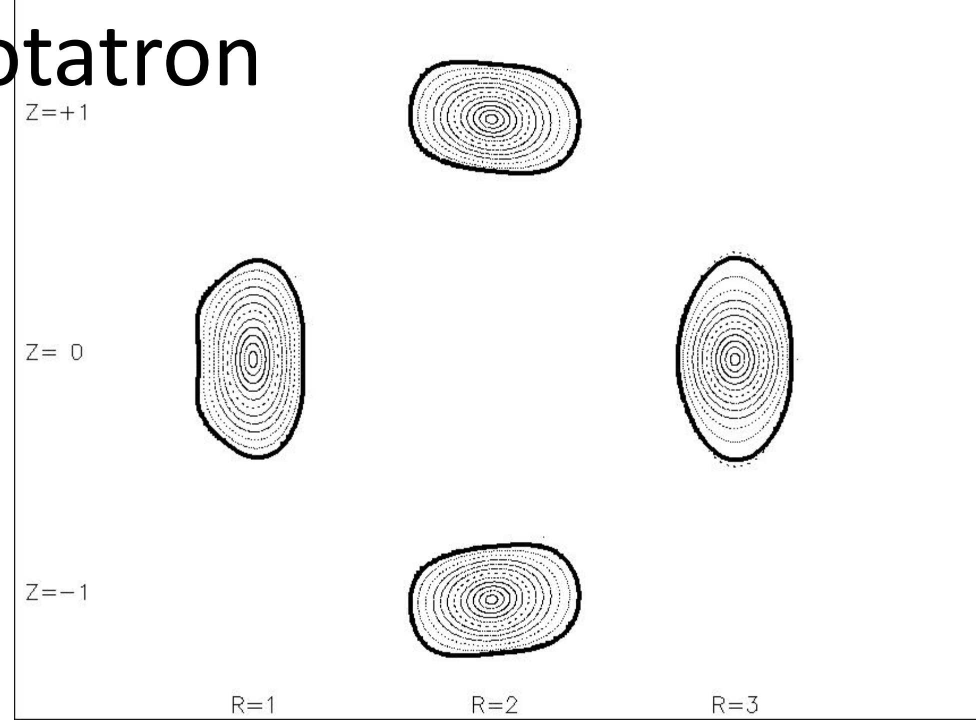


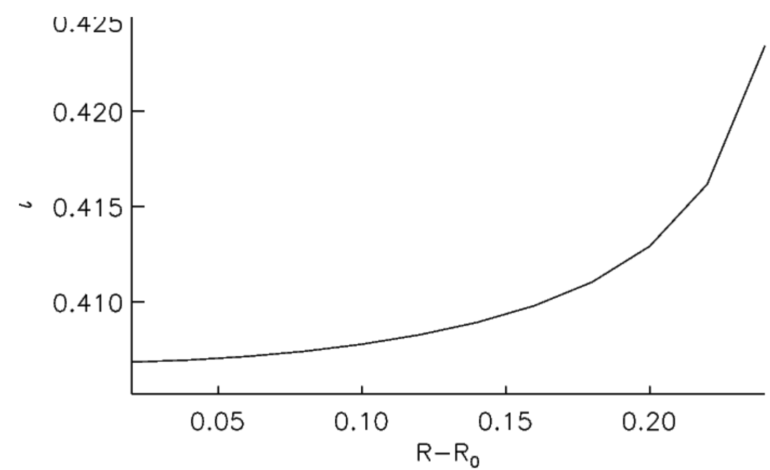




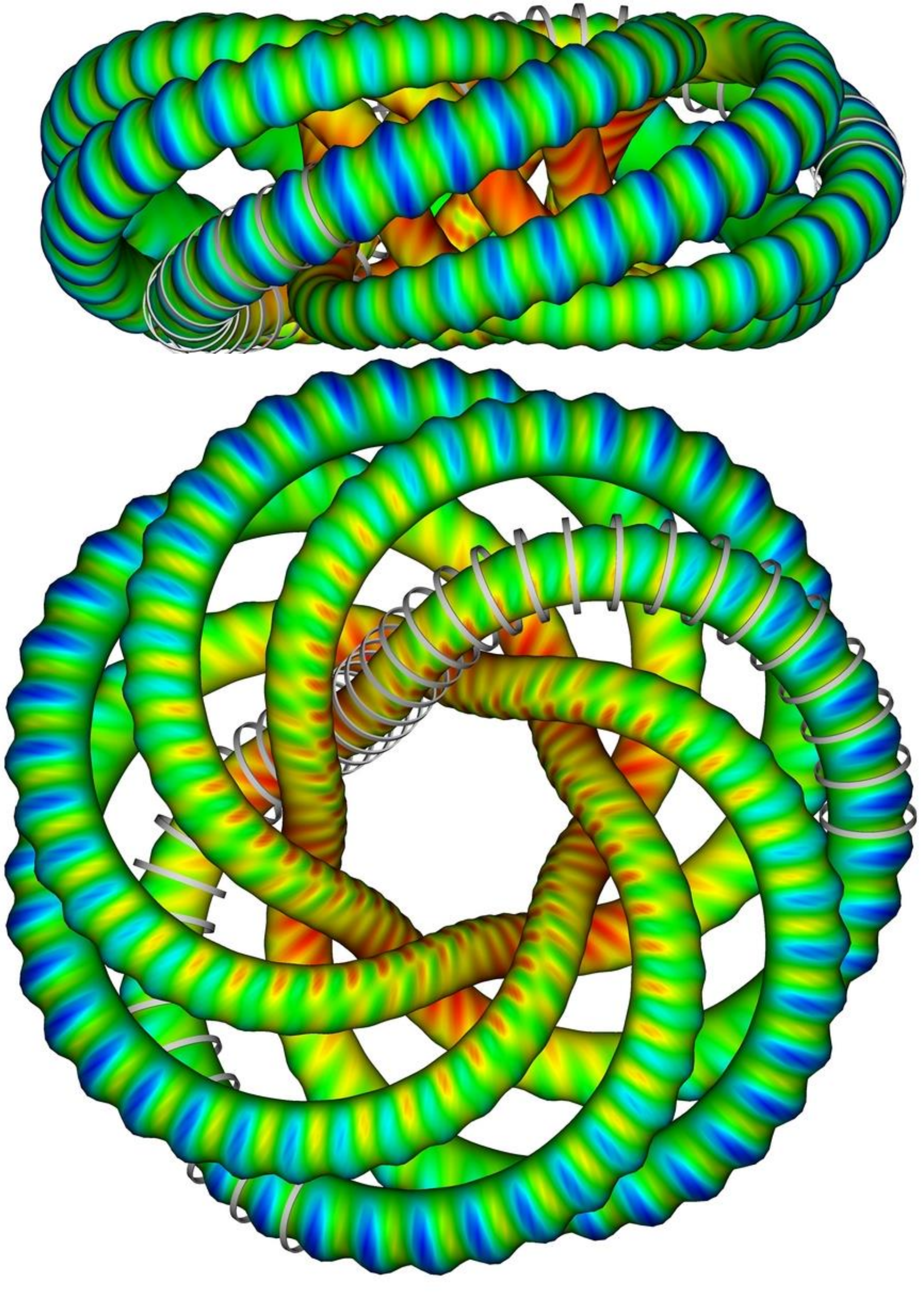
Example: (4,7) torus knotatron

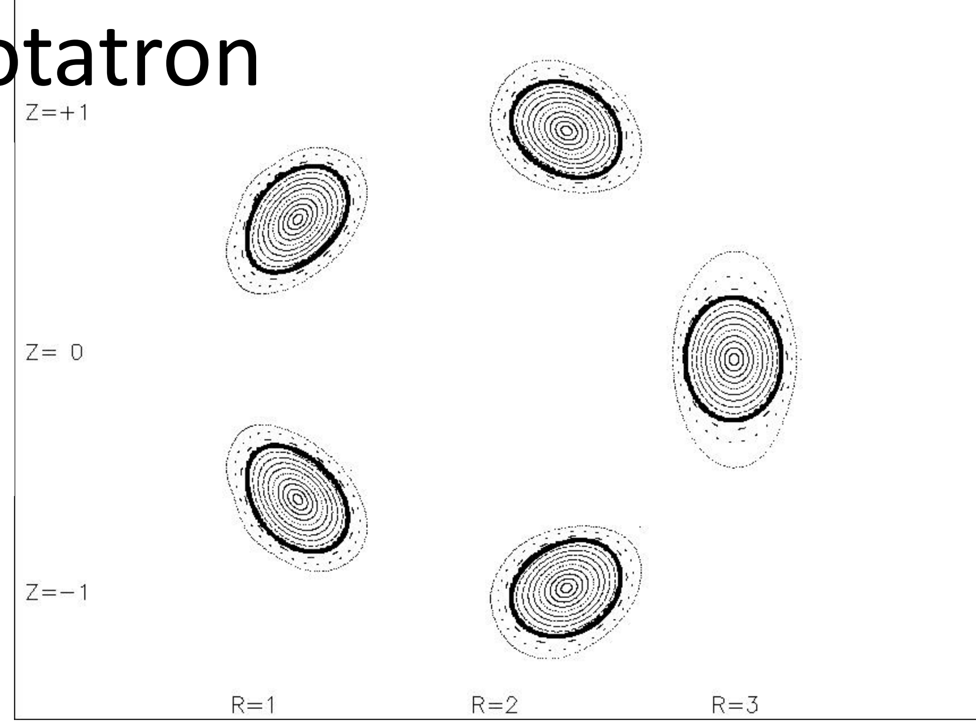


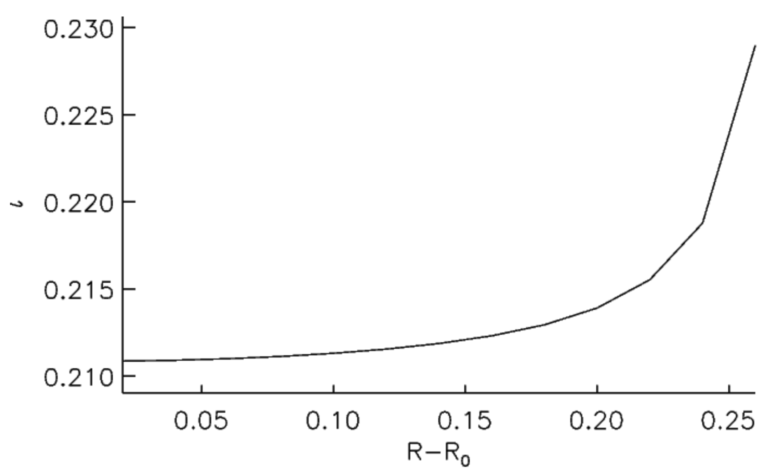




Example: (5,7) torus knotatron

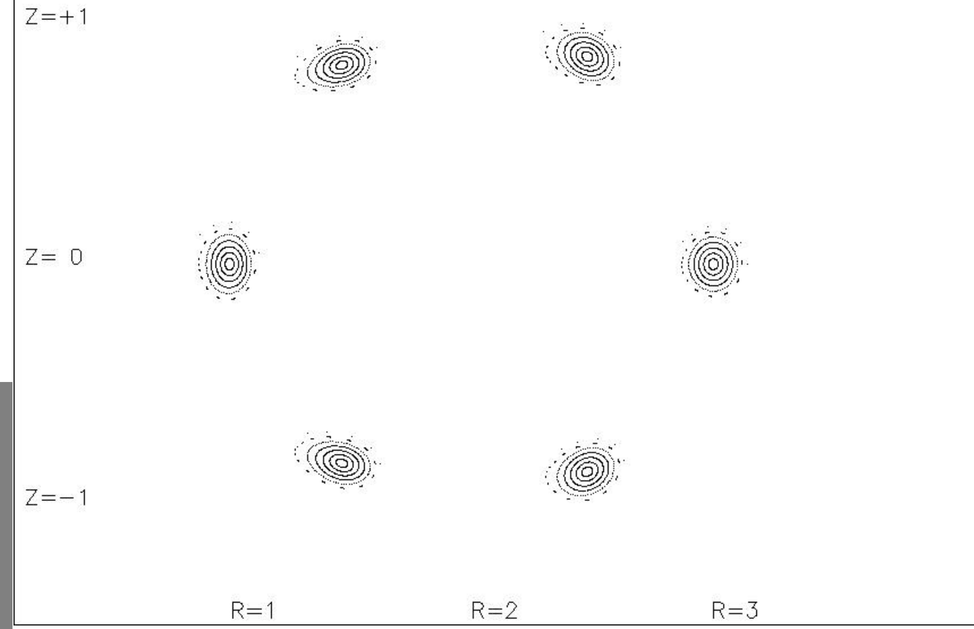


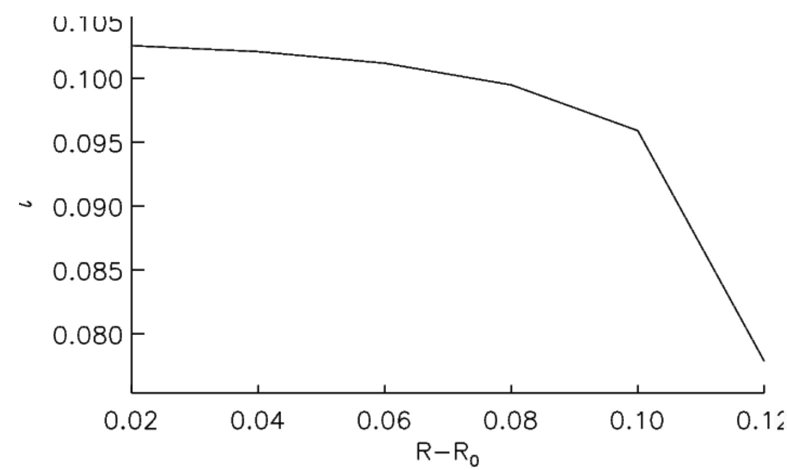




Example: (6,7) torus knotatron

place holder





The knotatron is a new class of stellarator. Both tokamaks & stellarators are *unknotatrons*.

A knotatron is a magnetic confinement device with a magnetic axis that is ambient isotopic to a knot.

The confining magnetic field in a knotatron is produced by currents external to the plasma.

Thus, the knotatron is a new example of a stellarator.

Tokamaks, conventional stellarators have magnetic axes that are ambient isotopic to the circle. The circle is a trivial knot, which is called the *unknot*.

Thus, tokamaks and conventional stellarators are *unknotatrons*.

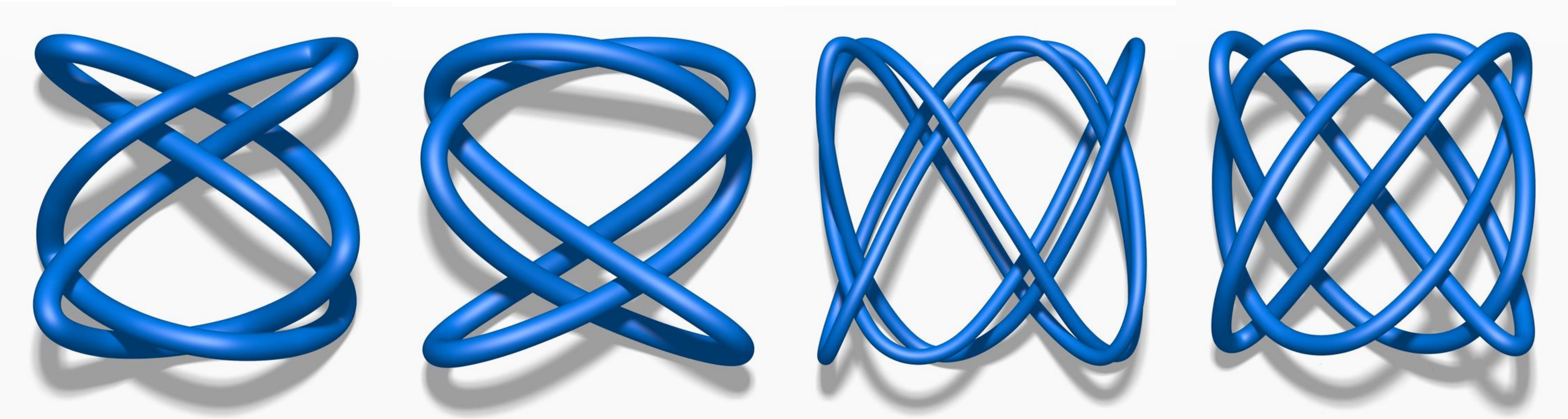
There is also the class of Lissajous knots.

$$x(\zeta) = \cos(n_x\zeta + \varphi_x),$$
$$y(\zeta) = \cos(n_y\zeta + \varphi_y),$$
$$z(\zeta) = \cos(n_z\zeta + \varphi_z),$$

where n_x, n_y and n_z are integers,

and φ_x and φ_y and φ_z are phase shifts.

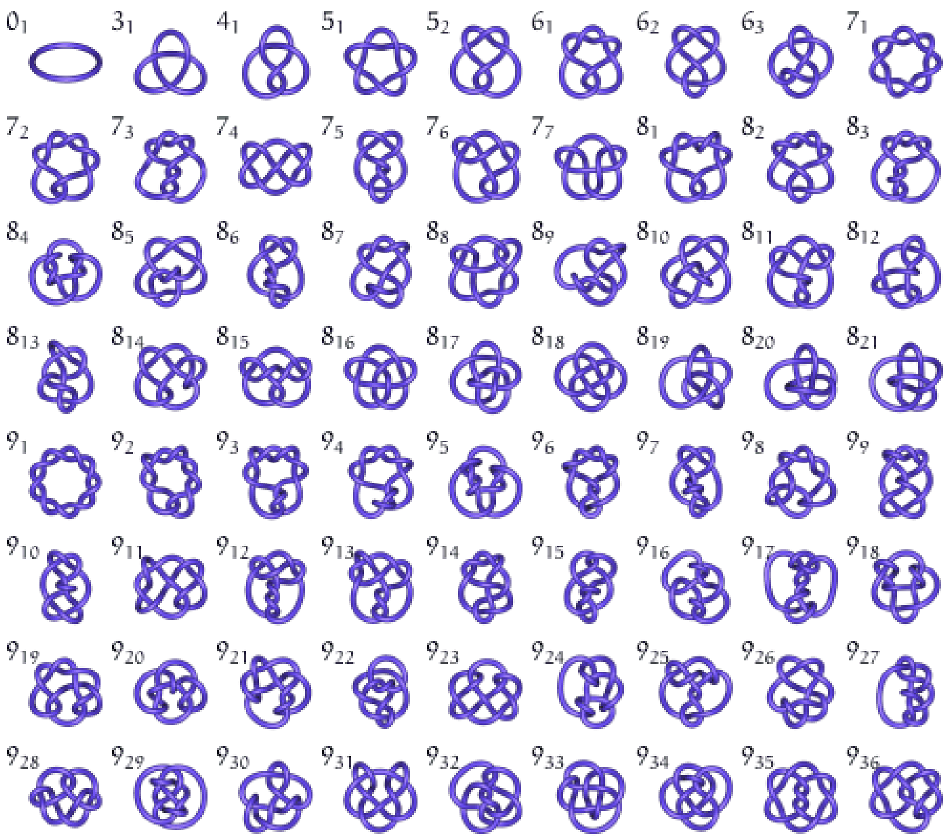
examples . . .



There is an infinite variety of knots.

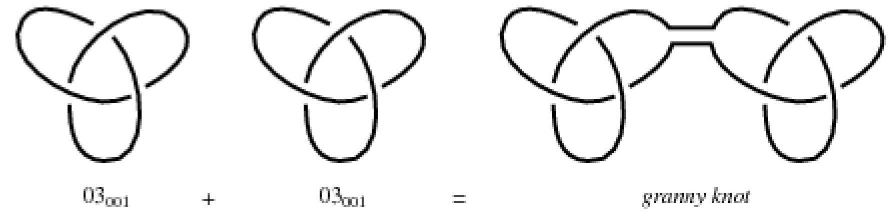
Composite knots are formed from simple knots.

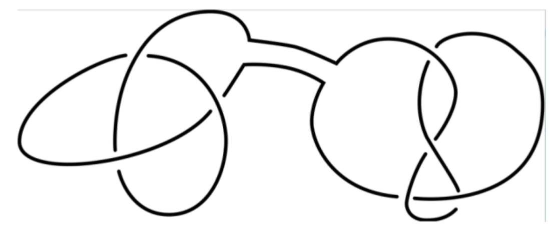
knot tables

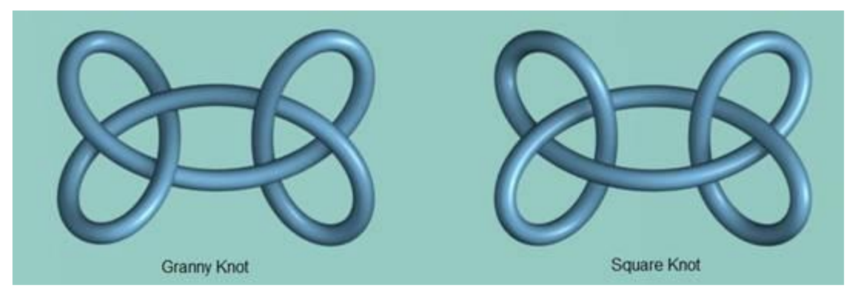


and many more . . . .

composition of knots

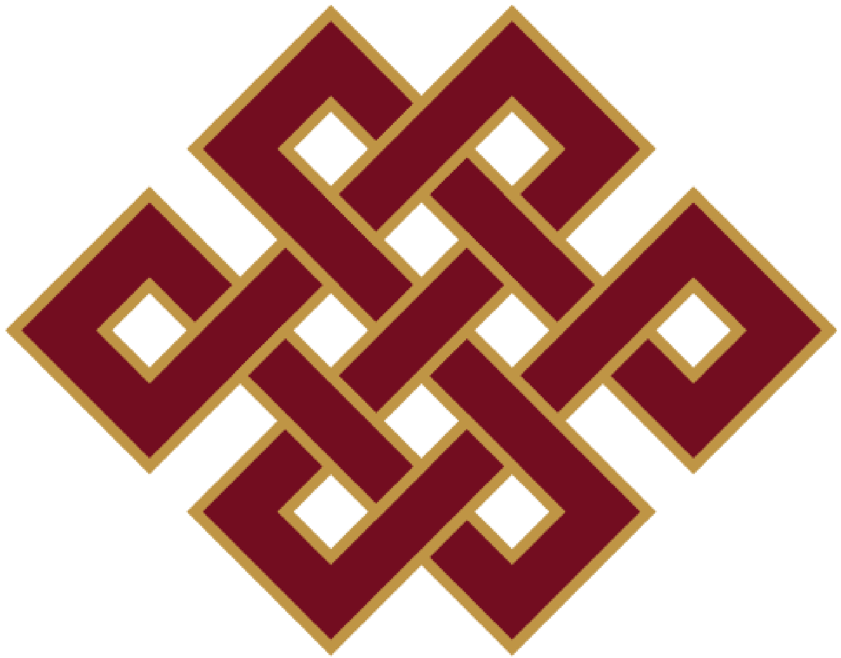






Is there a knot that is optimal for confinement?

the endless knot



Does the knotatron have advantages?

It is not known if knotatrons have advantages over conventional stellarators.

Knotatrons will probably have stability and transport properties similar to stellarators.

Modern stellarators must be carefully designed to have favorable properties.

A greater variety of geometrical shapes are allowed in the knotatron class.

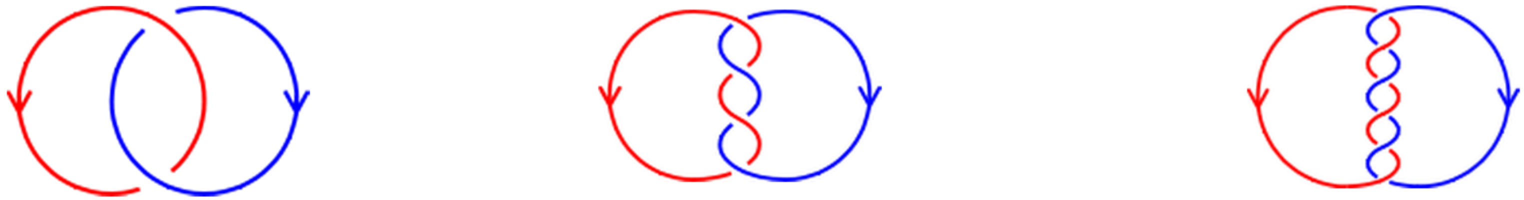
Equilibrium, stability and transport studies will be needed to explore the properties of knotatrons.

Stellarator design algorithms could be used to search for knotatrons with favorable properties.

Is there a quasi-symmetric knotatron?

Yes! [Garren & Boozer, 1991]

The helicity integral measures the “linked-ness” of a magnetic field.



How many times do two closed curves link each other?

$$\text{Gauss linking number} \quad = \quad -\frac{1}{4\pi} \oint \oint \frac{\mathbf{r}}{r^3} \times \, d\mathbf{y} \cdot \, d\mathbf{x}, \qquad \text{where } \; \mathbf{r} \equiv \mathbf{y} - \mathbf{x}$$

How ‘linked’ is a magnetic field?

$$\begin{aligned} \text{Helicity} \quad &= \quad -\frac{1}{4\pi} \int \int \frac{\mathbf{r}}{r^3} \times \mathbf{B}(\mathbf{y}) \cdot \mathbf{B}(\mathbf{x}) \, d^3x \, d^3y \\ &= \int \mathbf{A} \cdot \mathbf{B} \, d^3x \end{aligned}$$

$$\text{Coulomb gauge vector potential} \quad \mathbf{A}(\mathbf{x}) \quad = \quad -\frac{1}{4\pi} \int \frac{\mathbf{r}}{r^3} \times \mathbf{B}(\mathbf{y}) \, d^3y$$

Does the theory of knots and links play a role in plasma confinement?

Taylor Relaxation: weakly resistive plasmas will relax to minimize the energy, but the plasma cannot easily “unlink” itself i.e. constraint of conserved helicity

The figure-eight stellarator

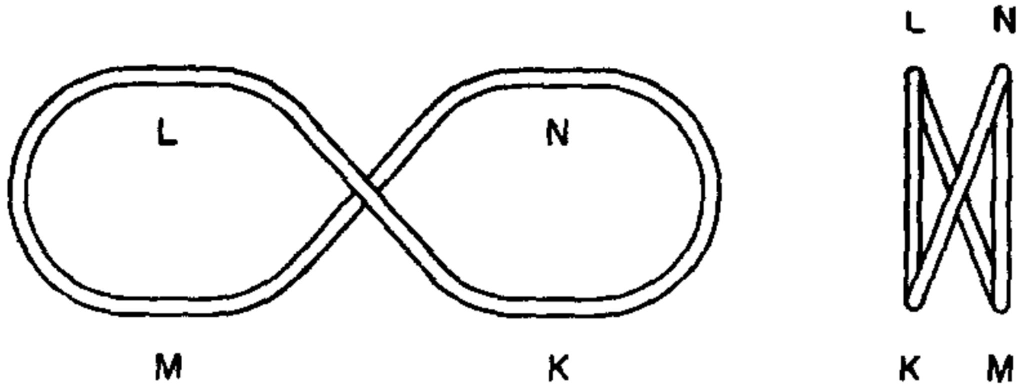


FIG. 2. Top and end views of a figure-eight stellarator.
Spitzer, 1958