

# A new class of magnetic confinement device in the shape of a knot

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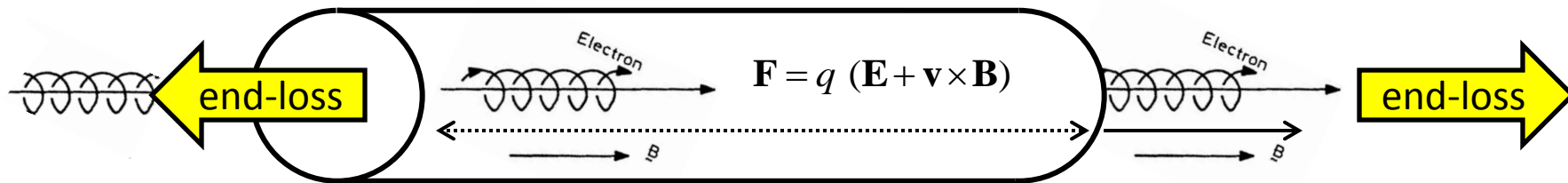
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## *Abstract*

We describe a new class of magnetic confinement device, with the magnetic axis in the shape of a knot. We call such devices “knotatrons”. Examples are given that have a large volume filled with magnetic surfaces, with significant rotational-transform, and with the magnetic field produced entirely by external circular coils.

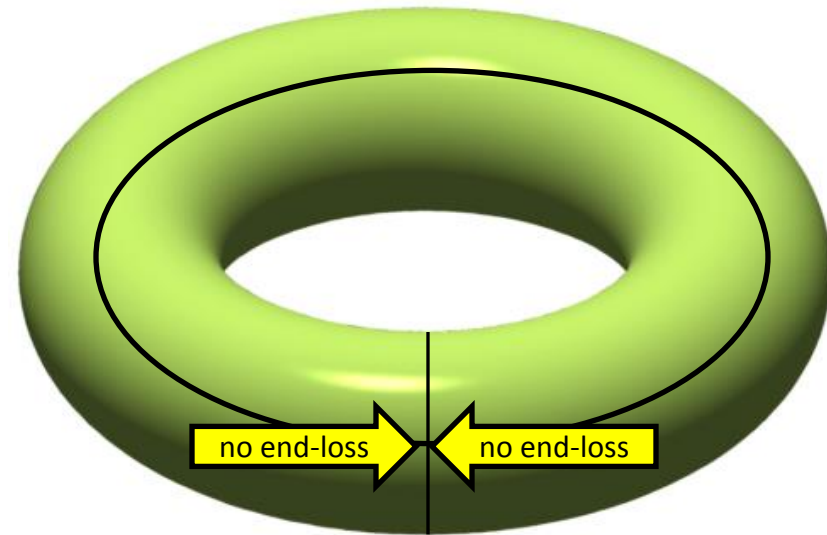
Charged particles are confined *perpendicularly* in a strong magnetic field, but are “lost” in the *parallel* direction.

An open-ended cylinder has good perpendicular confinement of charged particles, but particles are lost through the ends.



To eliminate end-losses  
the magnetic field must “close” upon itself.  
Joining the ends of a cylinder makes a tokamak.

In an axisymmetric tokamak, the  
magnetic axis is a **circle** in real space



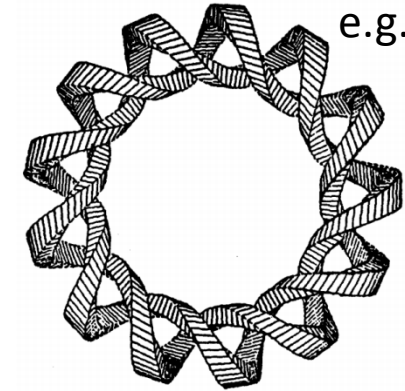
Because rotational-transform is required to cancel particle drifts,  
axisymmetric configurations need toroidal plasma current;  
and toroidal plasma current leads to disruptions, . . .

An alternative for producing rotational-transform is by non-axisymmetric shape.

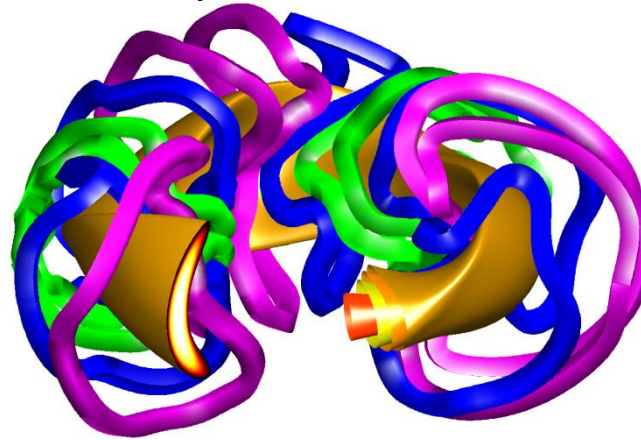
In the non-axisymmetric stellarator the confining magnetic field is produced by external coils, and stellarators are more stable.

**torsatron:** continuous, helical coils;

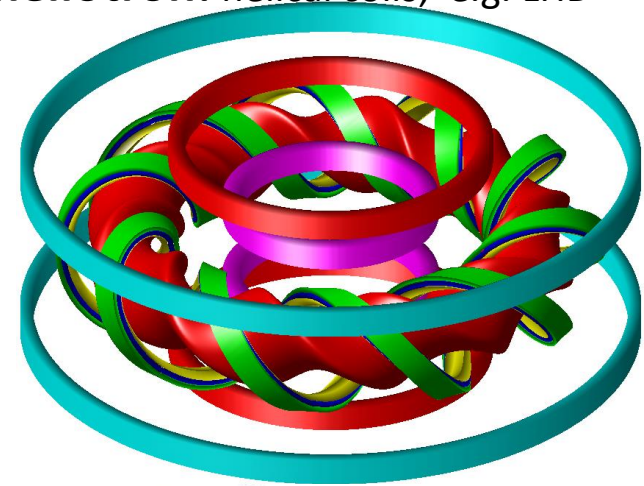
e.g. ATF



**NCSX:** optimized, modular coils

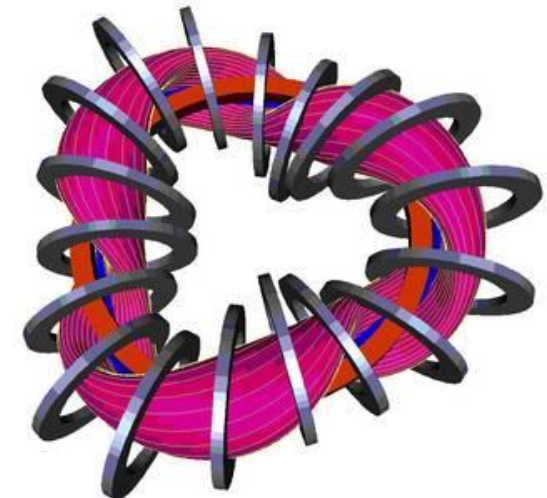
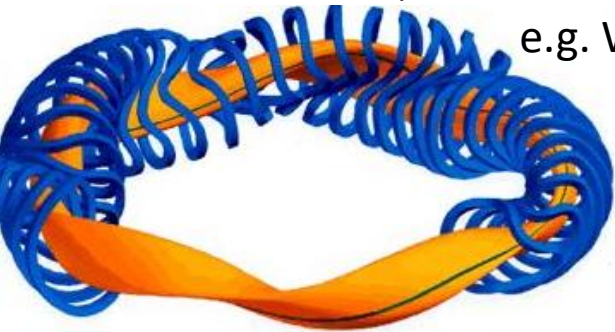


**heliotron:** helical coils; e.g. LHD



**helias:** helical axis, modular coils;

e.g. W7X



**heliac:** helical axis; e.g. H-1NF, TJ-II.

The magnetic axis of a tokamak is a circle.

The magnetic axis of a conventional stellarator is smoothly deformable into a circle.

There is another class of confinement device that:

- 1) is closed, in that the magnetic axis is topologically a circle ( a closed, one-dimensional curve ) ;
- 2) has a large volume of “good flux-surfaces” ( as will be shown in following slides ) ;
- 3) has significant rotational-transform without plasma current ( because the magnetic axis is non-planar ) ;
- 4) has a magnetic axis that is not smoothly deformable into a circle.  
( without cutting or passing through itself )

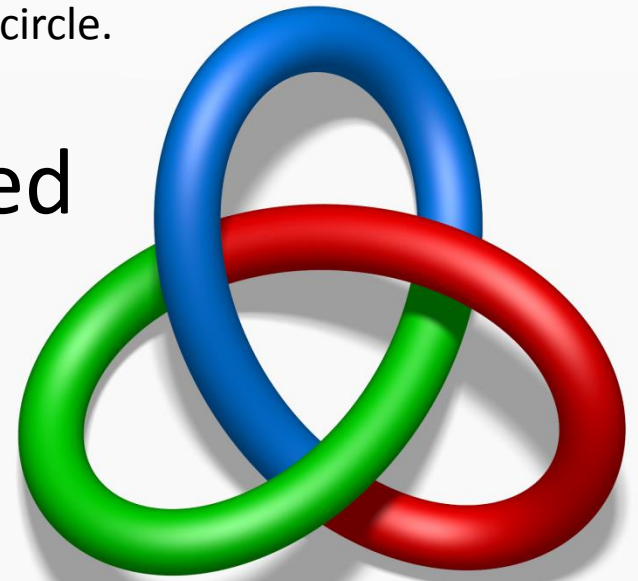
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The magnetic field may be closed by forming a knot!



e.g. a colored trefoil knot

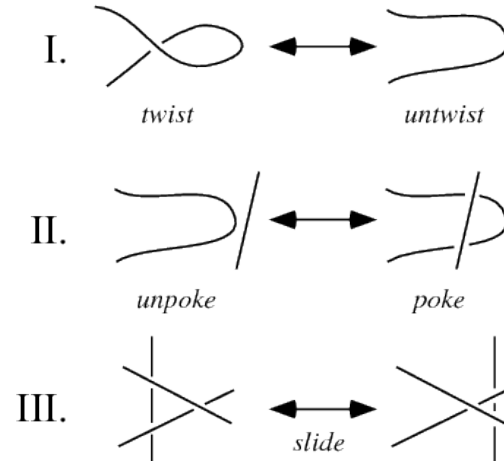
Mathematically, a knot,  $K: S^1 \rightarrow S^3$ , is an *embedding* of the circle,  $S^1$ , into real space,  $S^3 \equiv R^3$ .

Two knots,  $K_0$  and  $K_1$ , are *ambient isotopic* if there exists a continuous, one-parameter family of homeomorphisms of  $S^3$ ,  $h_t$ , such that  $h_0 \circ K_0 = K_0$  and  $h_1 \circ K_0 = K_1$

Equivalently,

Two knots are **not** ambient isotopic if one knot **cannot** be deformed into the other knot without introducing cuts or having the knot passing through itself.

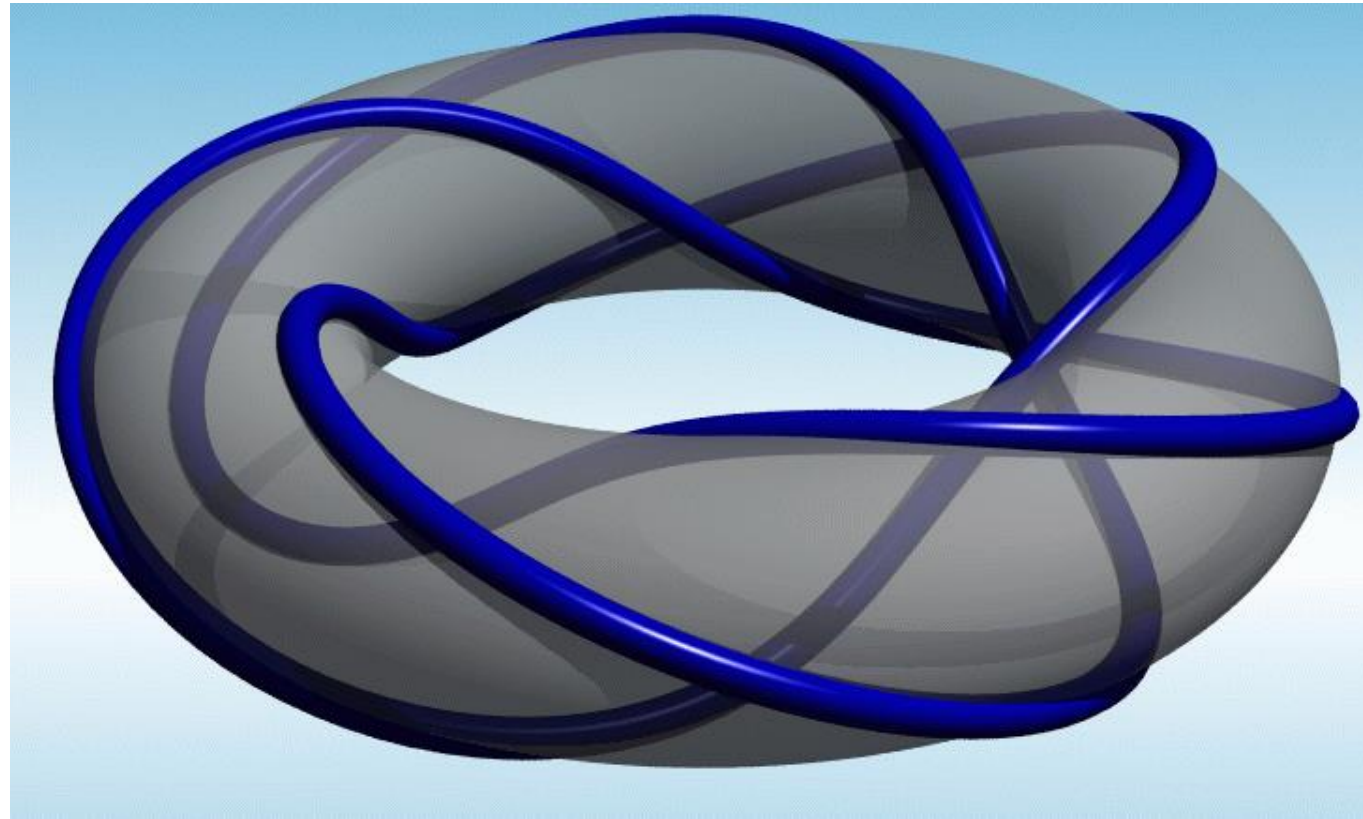
Reidemeister moves



A  $(p, q)$  torus knot, with  $p, q$  co-prime, wraps  $p$  times around poloidally and  $q$  times around toroidally on a torus.

$$\begin{aligned}x(\zeta) &= R(\zeta) \cos(p\zeta), & \text{where } R(\zeta) &= R_0 + r \cos(q\zeta), \text{ and } R_0, r \text{ are constants} \\y(\zeta) &= R(\zeta) \sin(p\zeta), \\z(\zeta) &= -r \sin(q\zeta),\end{aligned}$$

is a  $(p, q)$  torus-knot.





A suitably placed set of circular current coils  
can produce a magnetic field  
with an axis in the shape of a knot.

Let  $\mathbf{x}(\zeta) = x(\zeta) \mathbf{i} + y(\zeta) \mathbf{j} + z(\zeta) \mathbf{k}$  be a closed curve in real space,  
closed:  $\mathbf{x}(\zeta + 2\pi) = \mathbf{x}(\zeta)$

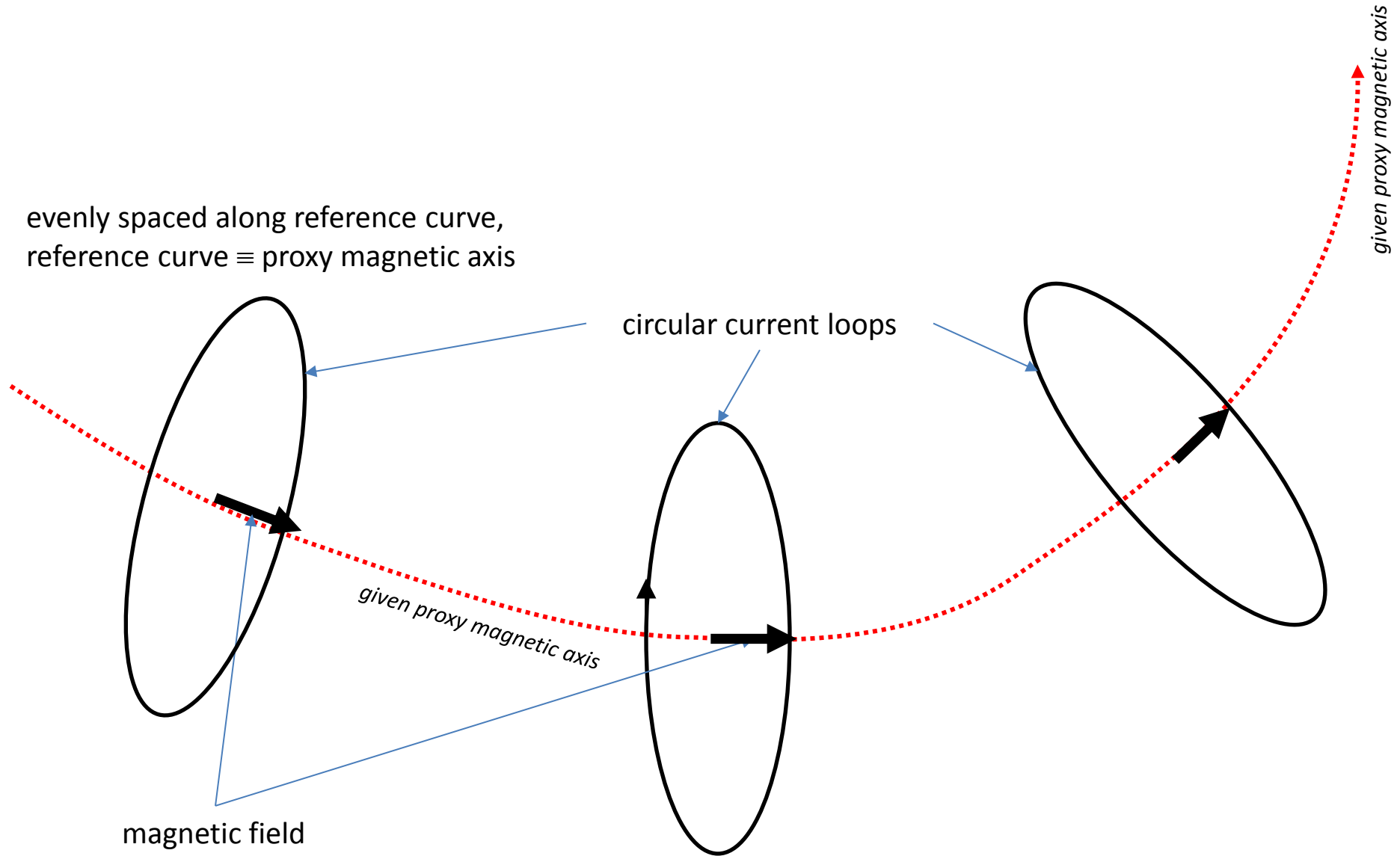
tangent  $\mathbf{t} \equiv \mathbf{x}'/|\mathbf{x}'|$ , normal  $\mathbf{n} \equiv \mathbf{t}'/\kappa$ , bi-normal  $\mathbf{b} \equiv \mathbf{t} \times \mathbf{n}$ ,  $\mathbf{b}' \equiv -\tau \mathbf{n}$ ,  $\kappa \equiv$  curvature,  $\tau \equiv$  torsion

Place a set of  $i = 1, \dots, N$  circular coils, radius  $a$ , unit current, equally spaced in  $\zeta$ ,  
e.g.  $\bar{\mathbf{x}}_i(\mathcal{G}) = \mathbf{x}(\zeta_i) + a \cos(\mathcal{G}) \mathbf{n} + a \sin(\mathcal{G}) \mathbf{b}$

Adjust the orientation of each coil so that the total magnetic is tangential to  $\mathbf{x}(\zeta)$ ,

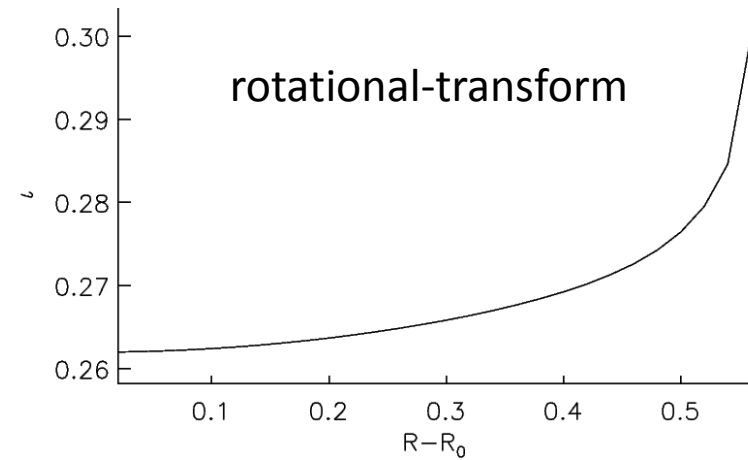
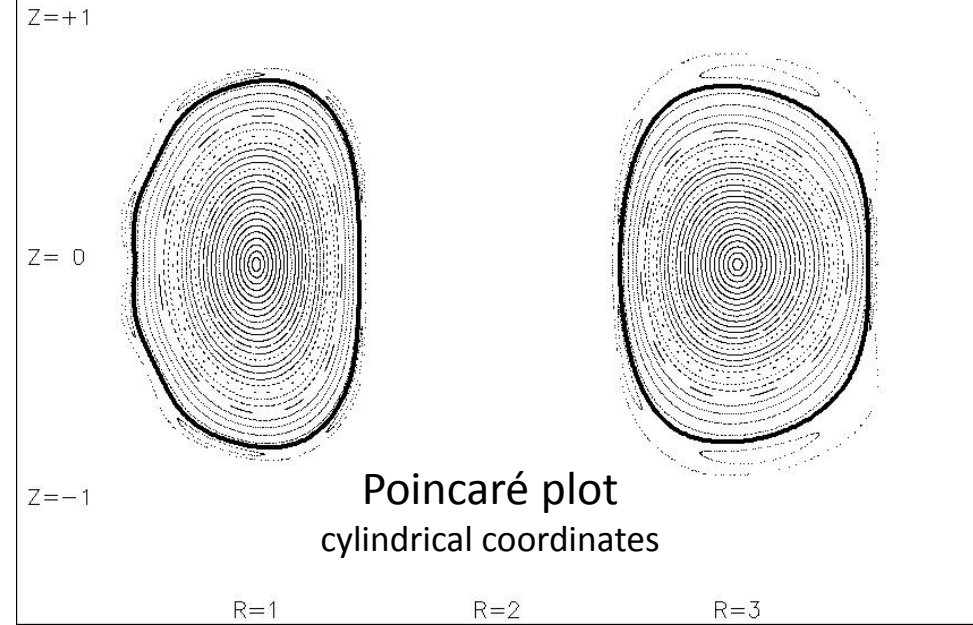
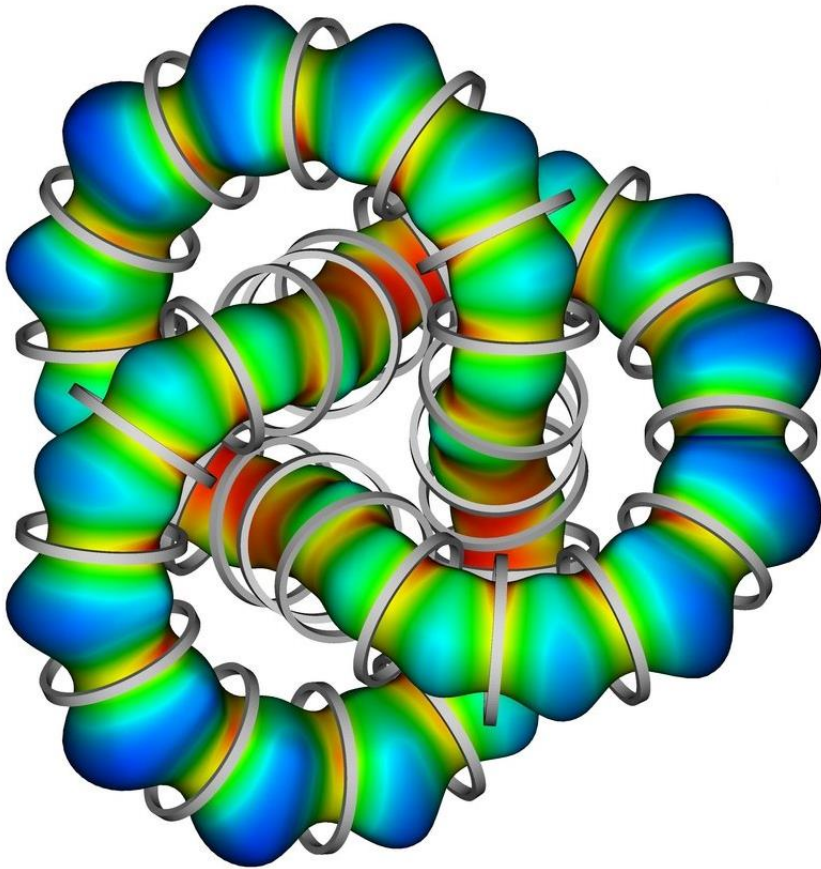
i.e.  $\sum_{coils} \mathbf{B}_{coil}(\mathbf{x}_i) \times \mathbf{t}_i = 0$  for all  $i = 1, \dots, N$

# The orientation of a set of circular coils is adjusted to produce the required magnetic axis.

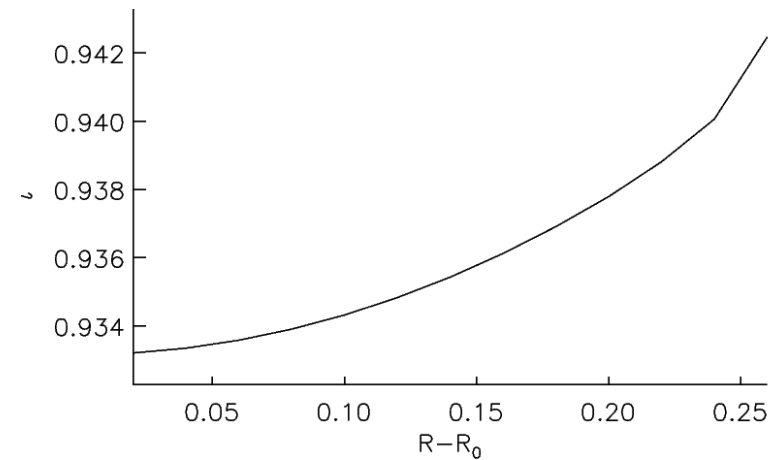
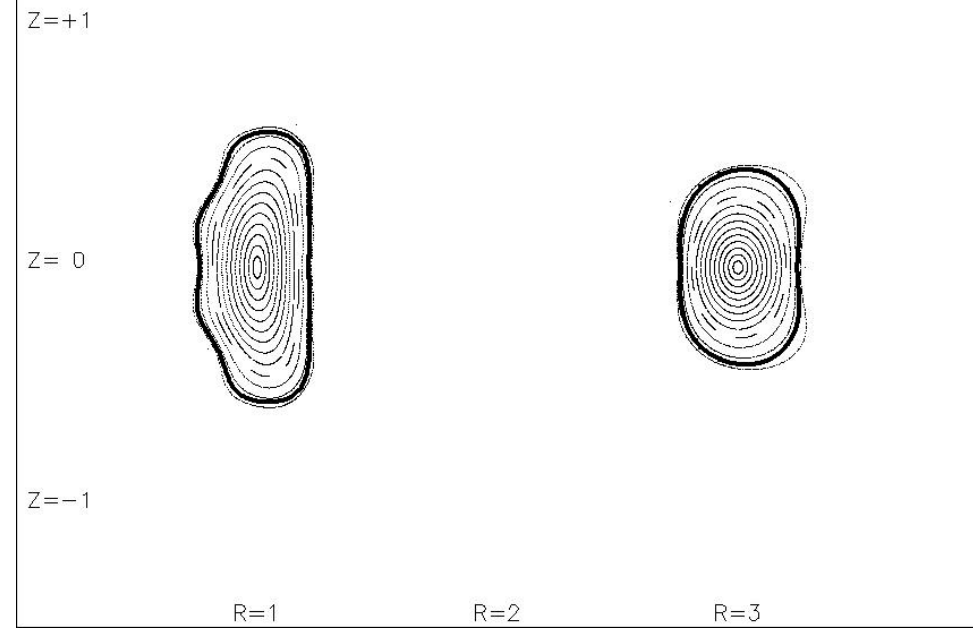


# Example: (2,3) torus knotatron

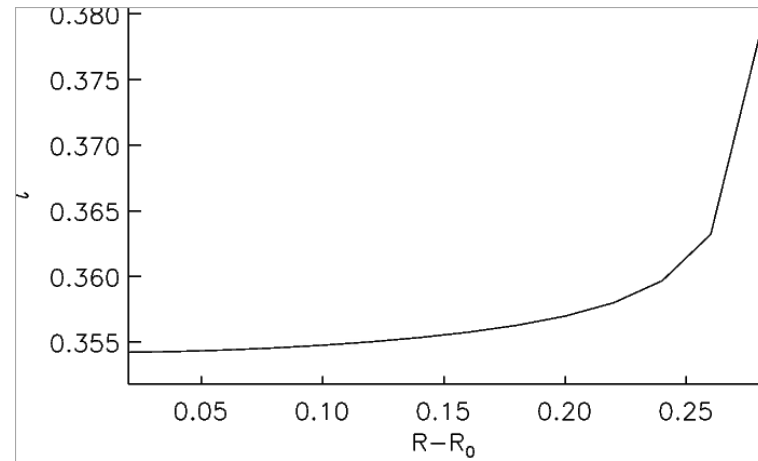
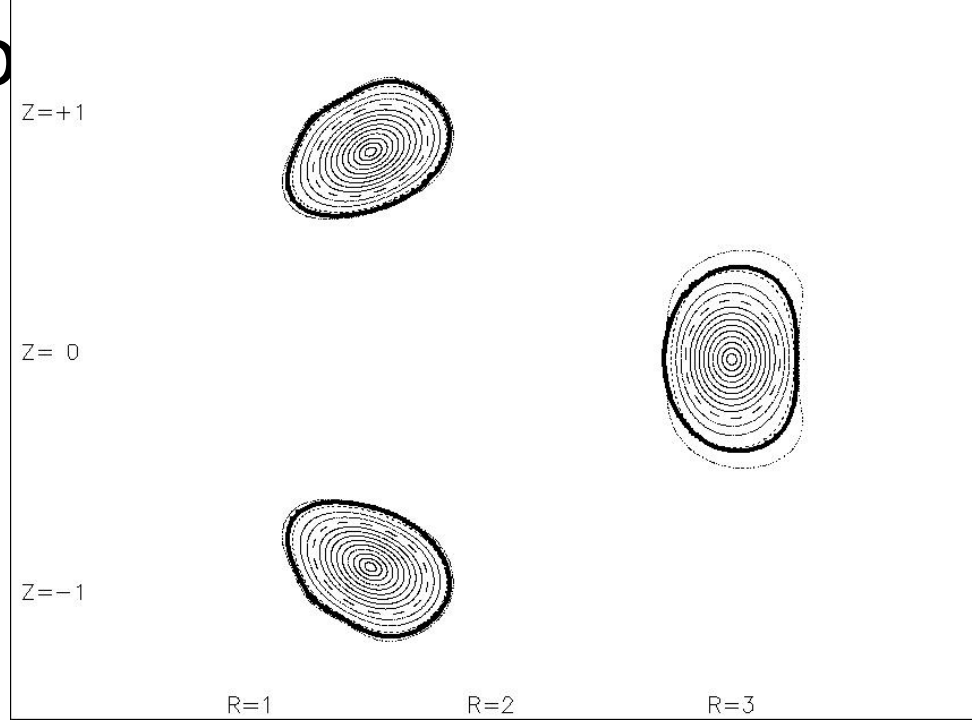
A flux surface in a (2,3) torus-knotatron  
with 36 circular coils.  
The color indicates  $|B|$ .



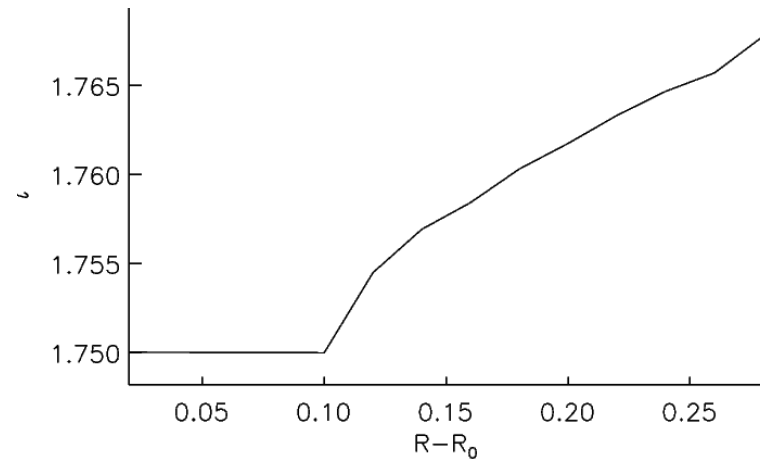
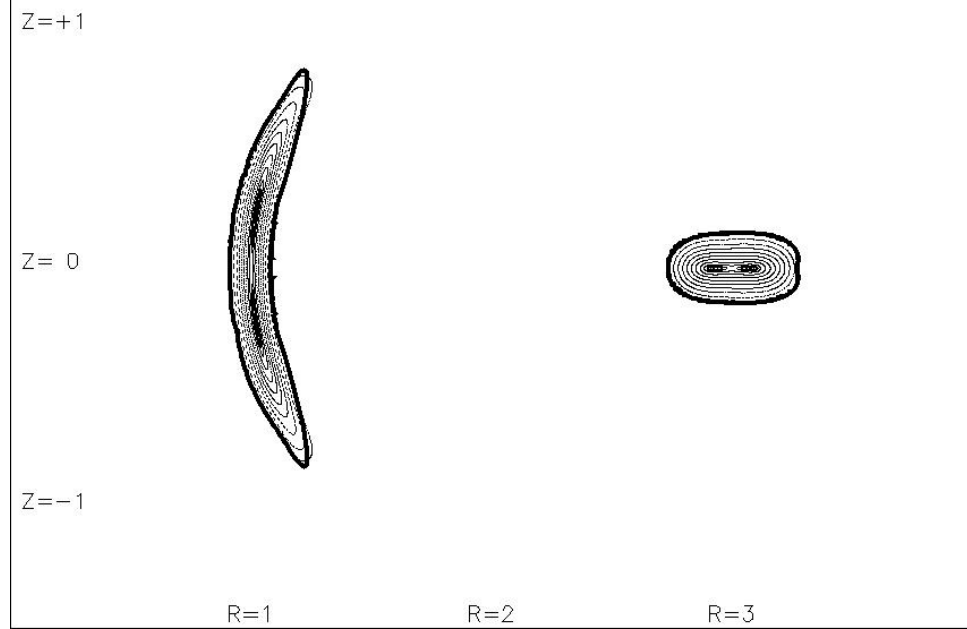
# Example: (2,5) torus knotatron



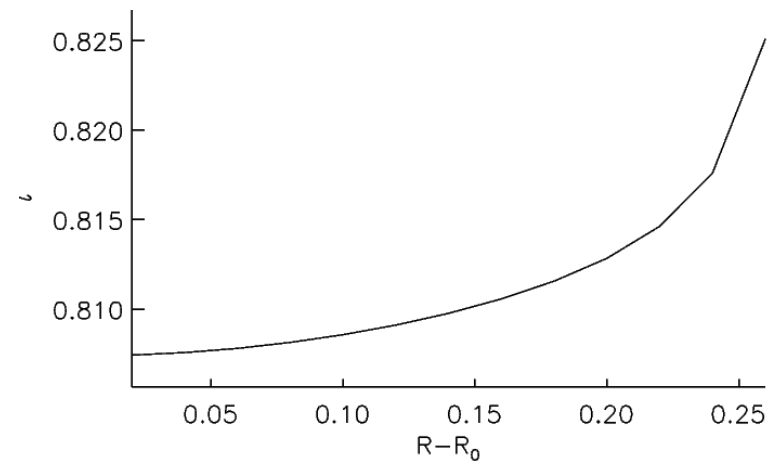
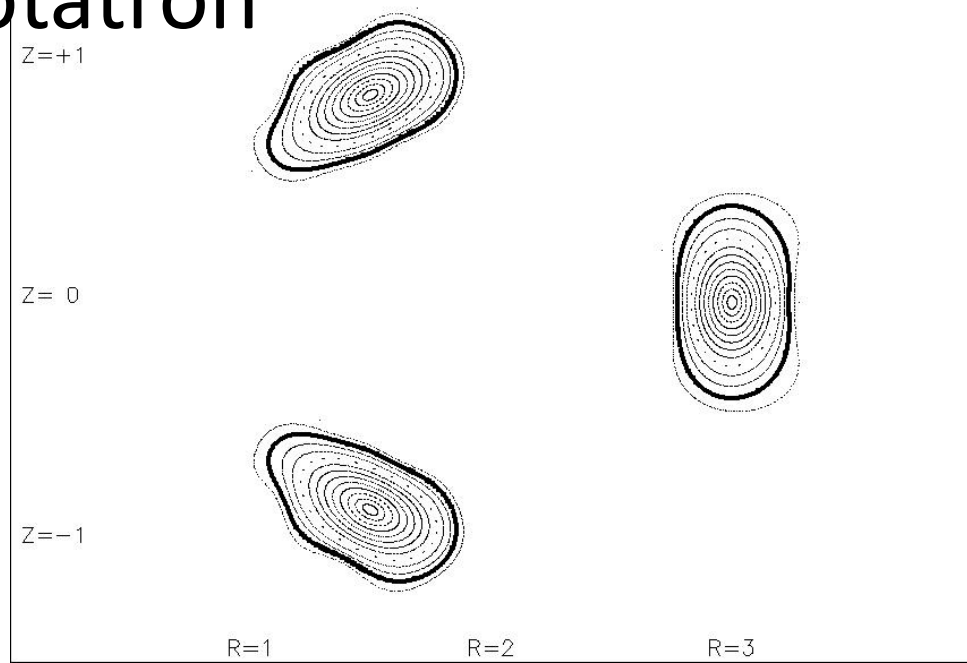
# Example: (3,5) torus knot



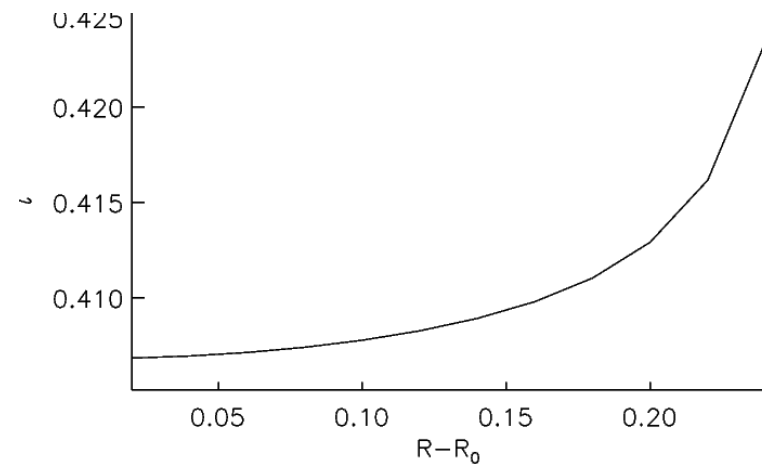
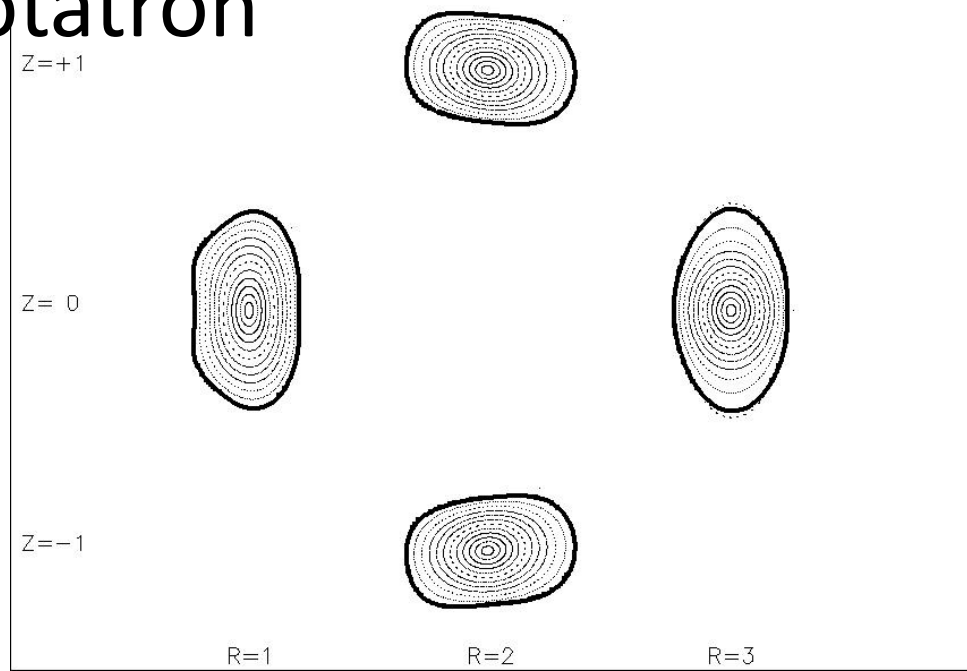
# Example: (2,7) torus knotatron



# Example: (3,7) torus knotatron

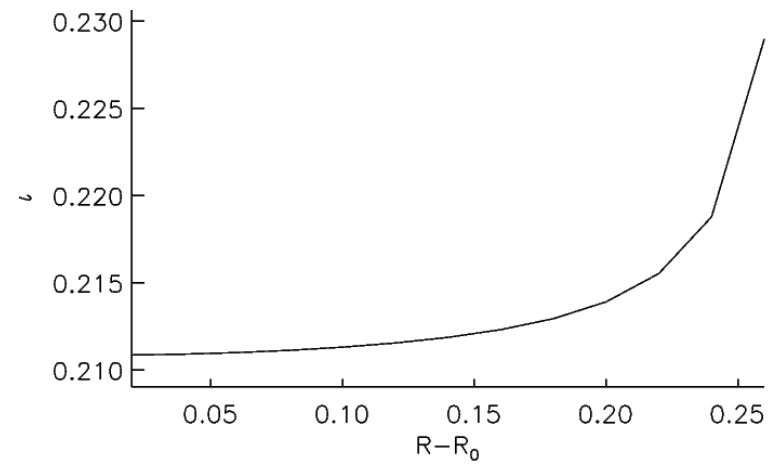
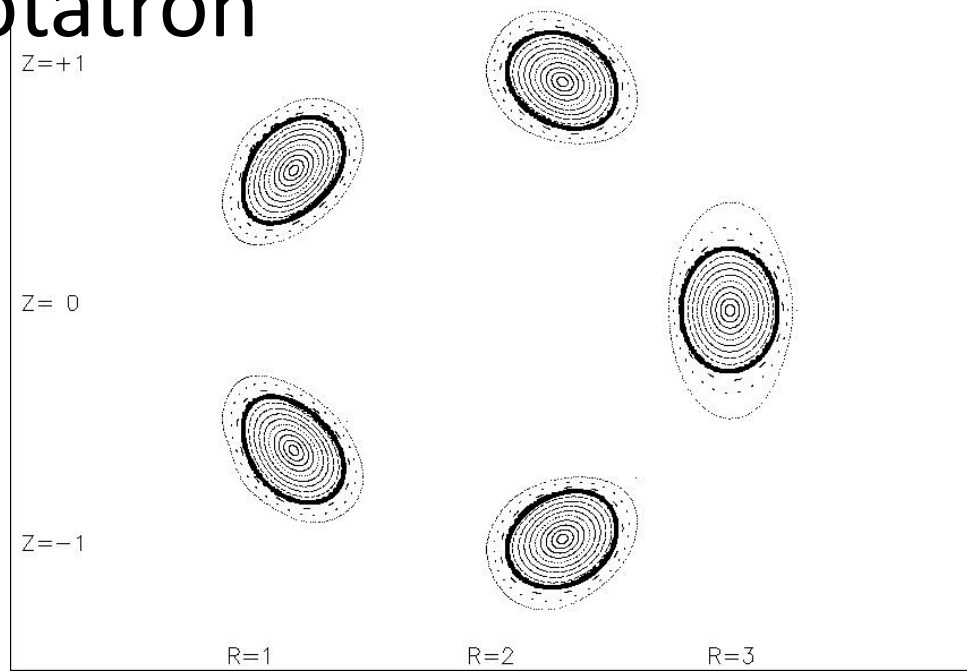


# Example: (4,7) torus knotatron

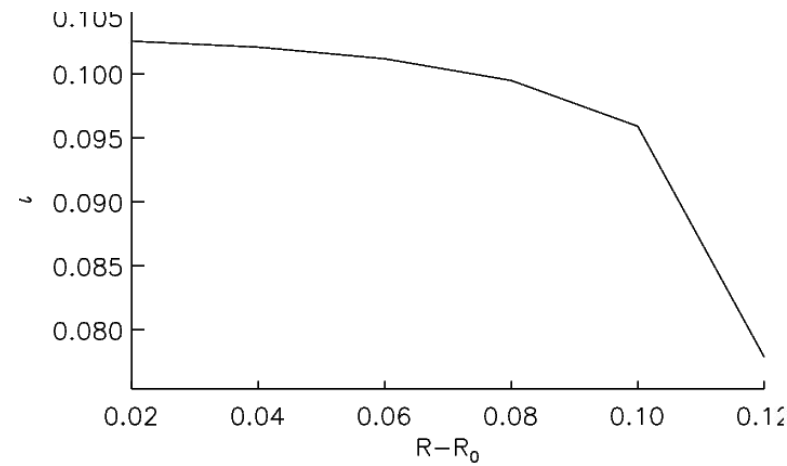
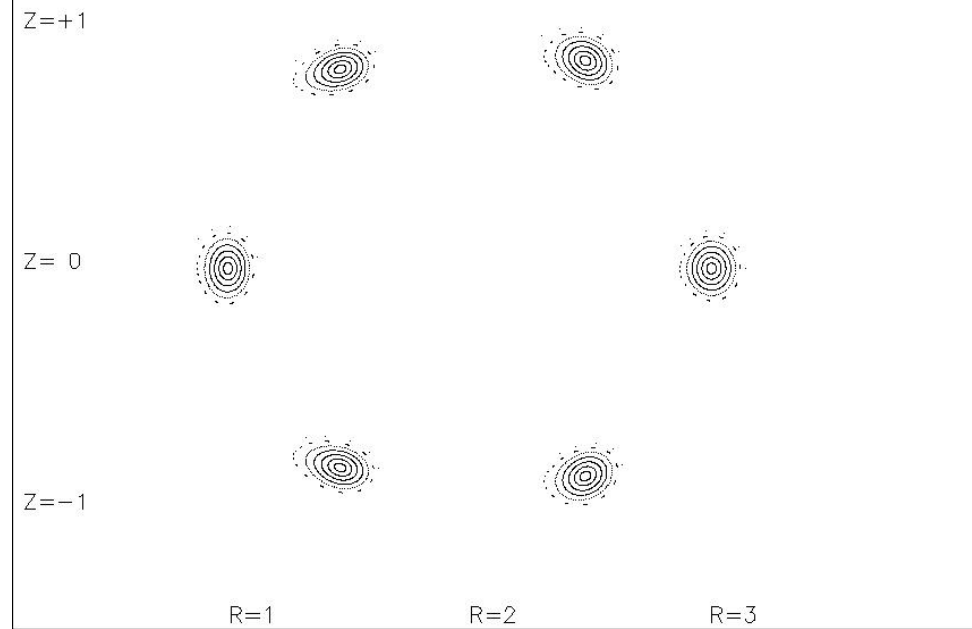




# Example: (5,7) torus knotatron



# Example: (6,7) torus knotatron



The knotatron is a new class of stellarator.  
Both tokamaks & stellarators are *unknotatrons*.

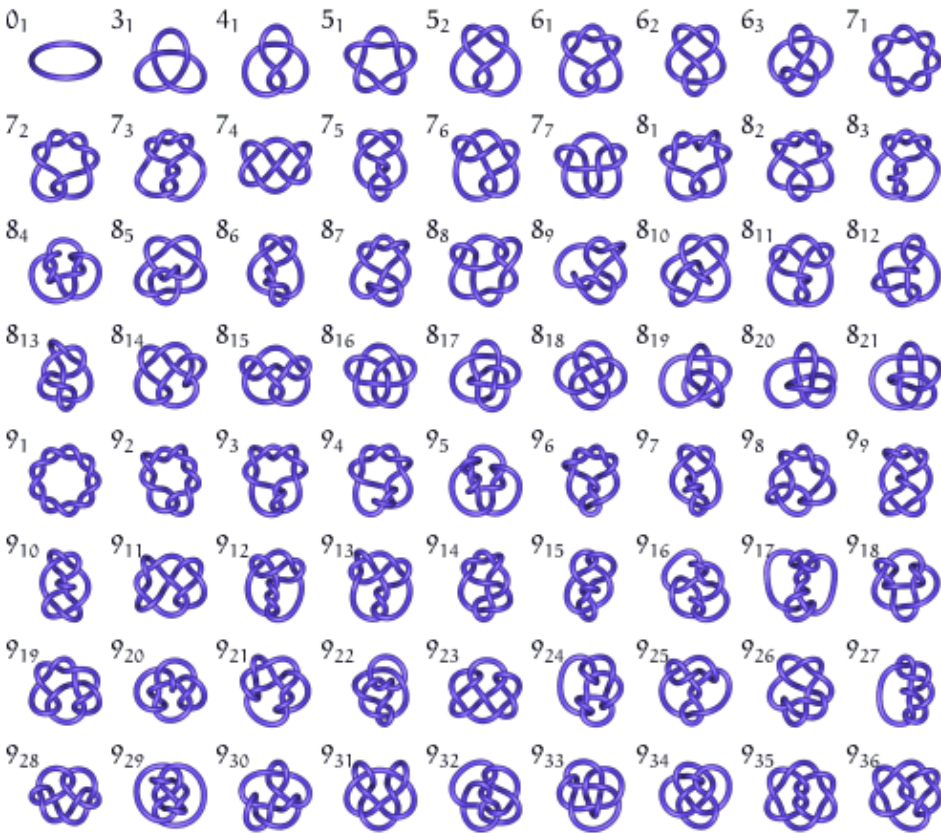
A knotatron is a magnetic confinement device with a magnetic axis that is ambient isotopic to a knot.

The confining magnetic field in a knotatron is produced by currents external to the plasma. Thus, the knotatron is a new example of a stellarator.

Tokamaks, conventional stellarators have magnetic axes that are ambient isotopic to the circle. The circle is ambient isotopic to the trivial knot, which is called the *unknot*.

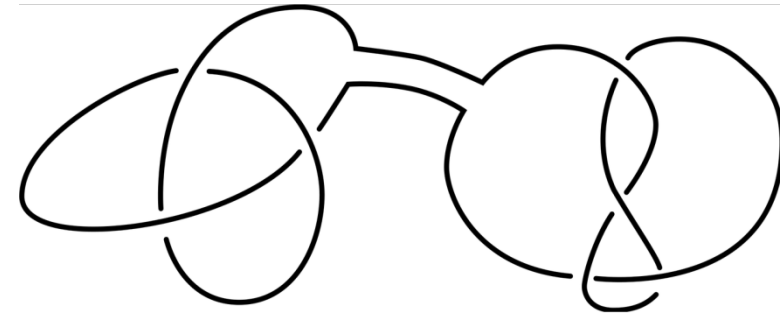
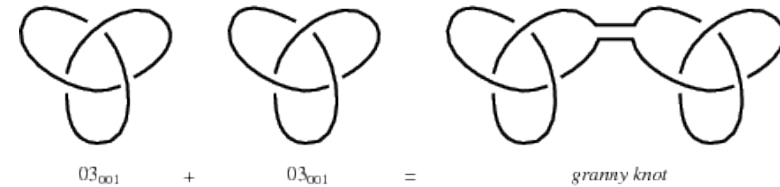
Tokamaks, conventional stellarators are *unknotatrons*.

There is a seemingly infinite variety of knots.  
Composite knots are formed from simple knots.



and many more . . . . .

composition of knots



Is there a knot that is optimal for confinement?



# Does the knotatron have advantages?

It is not known if knotatrons have advantages over conventional stellarators.

Knotatrons will probably have stability and transport properties similar to stellarators.

Modern stellarators must be carefully designed to have favorable properties.

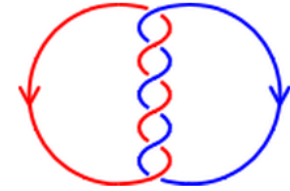
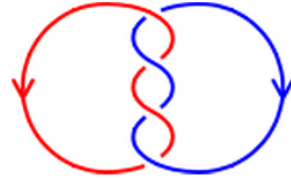
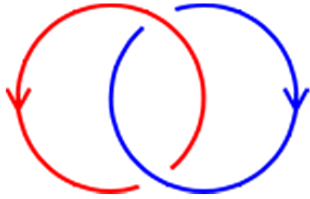
A greater variety of geometrical shapes are allowed in the knotatron class.

Equilibrium, stability and transport studies will be needed to explore the properties of knotatrons.

Stellarator design algorithms could be used to search for knotatrons with favorable properties.

## Is there a quasi-symmetric knotatron?

The helicity integral measures the “linked-ness” of a magnetic field.



How many times do two closed curves link each other?

$$\text{Gauss linking number} = -\frac{1}{4\pi} \oint \oint \frac{\mathbf{r}}{r^3} \times d\mathbf{y} \cdot d\mathbf{x}, \quad \text{where } \mathbf{r} \equiv \mathbf{y} - \mathbf{x}$$

How ‘linked’ is a magnetic field?

$$\begin{aligned} \text{Helicity}^{\text{q}} &= -\frac{1}{4\pi} \int \int \frac{\mathbf{r}}{r^3} \times \mathbf{B}(\mathbf{y}) \cdot \mathbf{B}(\mathbf{x}) d^3x d^3y \\ &= \int \mathbf{A} \cdot \mathbf{B} d^3x \end{aligned} \quad \text{Coulomb gauge vector potential } \mathbf{A}(\mathbf{x}) = -\frac{1}{4\pi} \int \frac{\mathbf{r}}{r^3} \times \mathbf{B}(\mathbf{y}) d^3y$$

Does the theory of knots and links play a role in plasma confinement?

Taylor Relaxation: weakly resistive plasmas will relax to minimize the energy, but the plasma cannot easily “unlink” itself i.e. constraint of conserved helicity

# The figure-eight stellarator

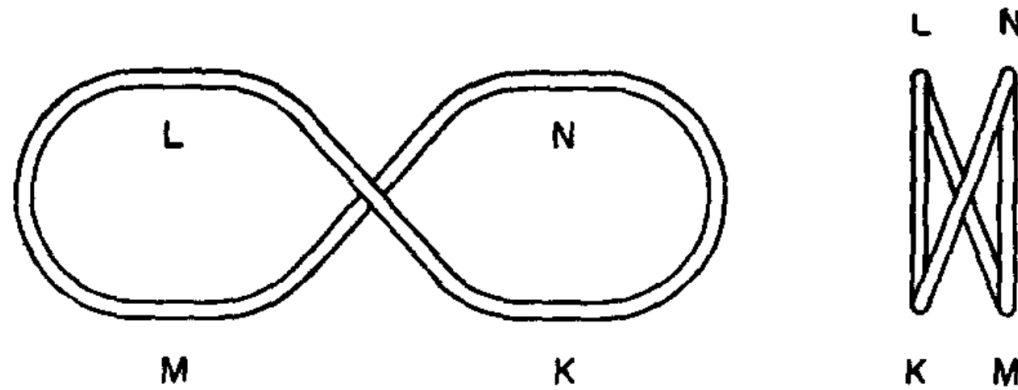


FIG. 2. Top and end views of a figure-eight stellarator.  
Spitzer, 1958

