



# Advanced MHD models of anisotropy, flow and chaotic fields

M. J. Hole<sup>1</sup>, M. Fitzgerald<sup>1,4</sup>, G. Dennis<sup>1</sup>, Zhisong Qu<sup>1</sup>,  
S. Hudson<sup>2</sup>, R. L. Dewar<sup>1</sup>, D. Terranova<sup>3</sup>, L. C. Appel<sup>4</sup>,  
P. Franz<sup>3</sup>, G. von Nessi<sup>1</sup>, B. Layden<sup>1</sup>

[1] Australian National University, ACT 0200, Australia

[2] Princeton Plasma Physics Laboratory, New Jersey 08543, U.S.A.

[3] Consorzio RFX, Padua, Italy

[4] EURATOM/CCFE Fusion Assoc., Culham Science Centre, Abingdon, Oxon OX14 3DB, UK

**42<sup>nd</sup> EPS Conference on Plasma Physics**

21-26 June 2015

**Funding Acknowledgement:** Australian Research Council, DIISRTE

**Support Acknowledgement:** Luca Guazzotto (University of Rochester), David Pretty (ANU), Rob Akers, Ken McClements, David Muir (CCFE)

# MHD with rotation & anisotropy

- Inclusion of anisotropy and flow in equilibrium MHD equations

[R. Iacono, et al Phys. Fluids B 2 (8). 1990]

$$\nabla \cdot (\rho \mathbf{v}) = 0, \quad \rho \mathbf{v} \cdot \nabla \mathbf{v} = \mathbf{J} \times \mathbf{B} - \nabla \cdot \bar{\mathbf{P}}, \quad \nabla \cdot \mathbf{B} = 0$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla \times (\mathbf{v} \times \mathbf{B}) = 0,$$

$$\bar{\mathbf{P}} = p_{\perp} \bar{\mathbf{I}} + \Delta \mathbf{B} \mathbf{B} / \mu_0, \quad \Delta = \frac{\mu_0 (p_{\parallel} - p_{\perp})}{B^2}$$

# MHD with rotation & anisotropy

- Inclusion of anisotropy and flow in equilibrium MHD equations

[R. Iacono, et al Phys. Fluids B 2 (8). 1990]

$$\nabla \cdot (\rho \mathbf{v}) = 0, \quad \rho \mathbf{v} \cdot \nabla \mathbf{v} = \mathbf{J} \times \mathbf{B} - \nabla \cdot \bar{\mathbf{P}}, \quad \nabla \cdot \mathbf{B} = 0$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla \times (\mathbf{v} \times \mathbf{B}) = 0,$$

$$\bar{\mathbf{P}} = p_{\perp} \bar{\mathbf{I}} + \Delta \mathbf{B} \mathbf{B} / \mu_0, \quad \Delta = \frac{\mu_0 (p_{\parallel} - p_{\perp})}{B^2}$$

- Frozen flux condition + axis-symmetry + neglect poloidal flow  $\Rightarrow$

$$\mathbf{v} = -R \phi'_E(\psi) \mathbf{e}_{\phi} = R \Omega(\psi) \mathbf{e}_{\phi}$$

Equilibrium eqn becomes:

$$\nabla \cdot \left[ (1 - \Delta) \left( \frac{\nabla \psi}{R^2} \right) \right] = - \frac{\partial p_{\parallel}}{\partial \psi} - \rho H'(\psi) + \rho \frac{\partial W}{\partial \psi} - \frac{F(\psi) F'(\psi)}{R^2 (1 - \Delta)} + R^2 \rho \Omega(\psi) \Omega'(\psi)$$

$$F = R B_{\phi} \quad H(\psi) = W_M(\rho, B, \psi) - \frac{1}{2} [R \phi'_E(\psi)]^2$$

Guiding-centre/MHD/double-adiabatic

# MHD with rotation & anisotropy

- Inclusion of anisotropy and flow in equilibrium MHD equations

[R. Iacono, et al Phys. Fluids B 2 (8). 1990]

$$\nabla \cdot (\rho \mathbf{v}) = 0, \quad \rho \mathbf{v} \cdot \nabla \mathbf{v} = \mathbf{J} \times \mathbf{B} - \nabla \cdot \bar{\mathbf{P}}, \quad \nabla \cdot \mathbf{B} = 0$$

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}, \quad \nabla \times (\mathbf{v} \times \mathbf{B}) = 0,$$

$$\bar{\mathbf{P}} = p_{\perp} \bar{\mathbf{I}} + \Delta \mathbf{B} \mathbf{B} / \mu_0, \quad \Delta = \frac{\mu_0 (p_{\parallel} - p_{\perp})}{B^2}$$

- Frozen flux condition + axis-symmetry + neglect poloidal flow  $\Rightarrow$

$$\mathbf{v} = -R \phi'_E(\psi) \mathbf{e}_{\phi} = R \Omega(\psi) \mathbf{e}_{\phi}$$

Equilibrium eqn becomes:

$$\nabla \cdot \left[ (1 - \Delta) \left( \frac{\nabla \psi}{R^2} \right) \right] = -\frac{\partial p_{\parallel}}{\partial \psi} - \rho H'(\psi) + \rho \frac{\partial W}{\partial \psi} - \frac{F(\psi) F'(\psi)}{R^2 (1 - \Delta)} + R^2 \rho \Omega(\psi) \Omega'(\psi)$$

$$F = R B_{\phi} \quad H(\psi) = W_M(\rho, B, \psi) - \frac{1}{2} [R \phi'_E(\psi)]^2$$

Guiding-centre/MHD/double-adiabatic

- If two temperature Bi-Maxwellian model chosen

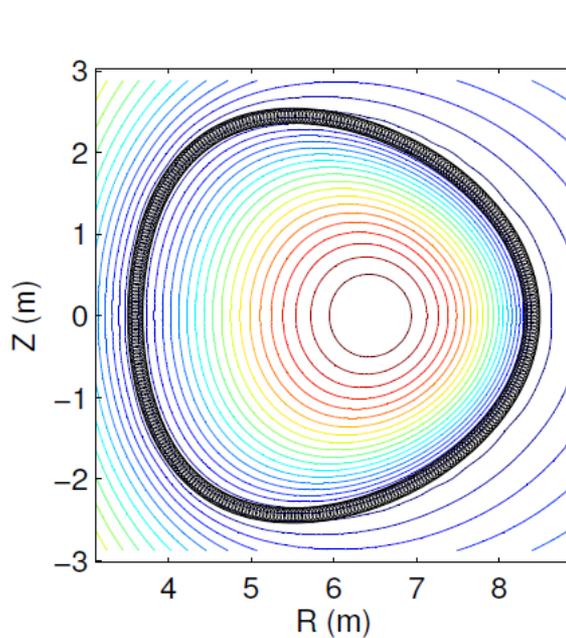
$$p_{\parallel}(\rho, B, \psi) = \frac{k_B}{m} \rho T_{\parallel}(\psi) \quad p_{\perp}(\rho, B, \psi) = \frac{k_B}{m} \rho T_{\perp}(\psi) = \frac{k_B}{m} \rho T_{\parallel}(\psi) \frac{B}{B - \theta(\psi) T_{\parallel}}$$

Set of 5 profile constraints  $\{F(\psi), \Omega(\psi), H(\psi), T_{\parallel}(\psi), \theta(\psi)\}$

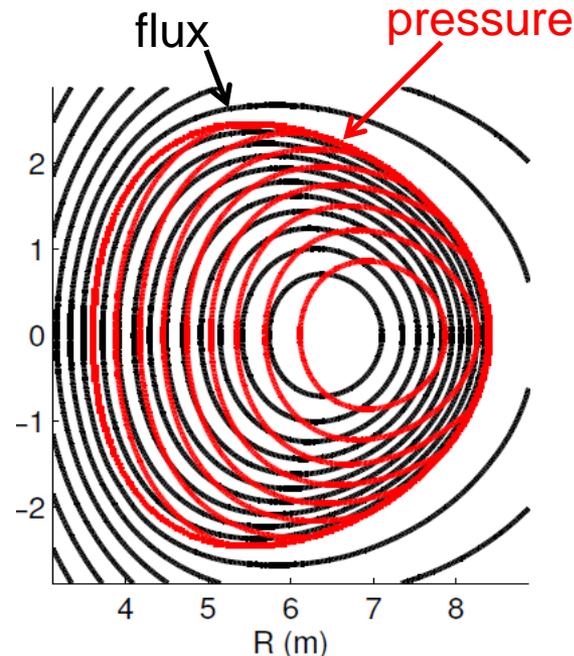
# EFIT TENSOR: reconstruction code

- Adds kinetic constraints to magnetic-only constraints of EFIT
- Soloviev benchmarks computed for isotropic, anisotropic and flow  
**(Reveals  $J_\phi$  sensitive to heat transport constraints)**
- Installed for both MAST and JET

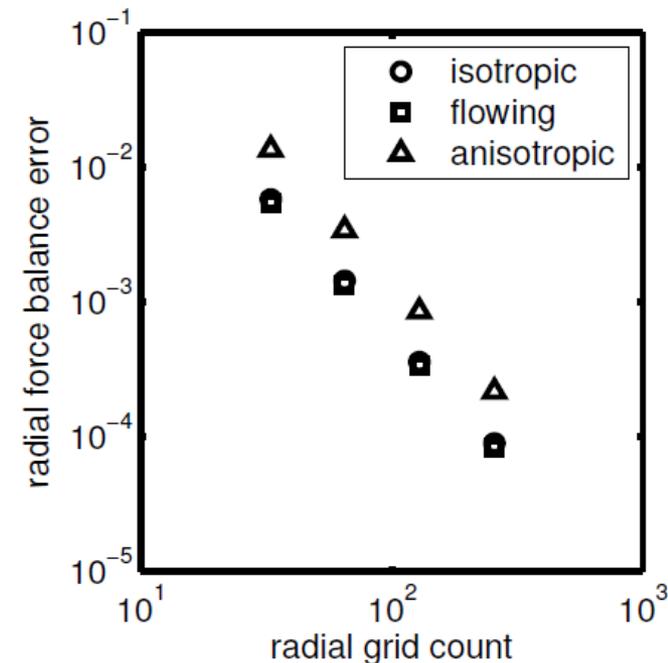
[Fitzgerald, Appel, Hole, Nucl. Fusion **53** (2013) 113040]



Soloviev:  
 $\beta_t=0.07$

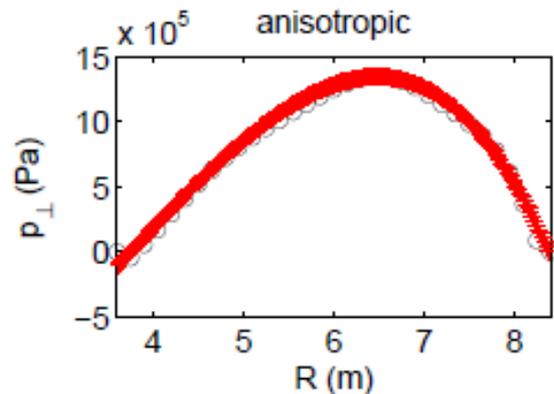
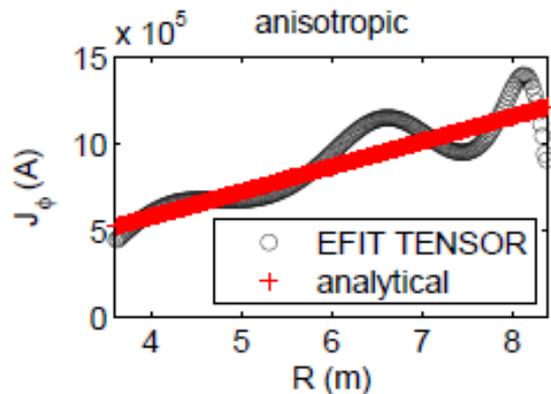
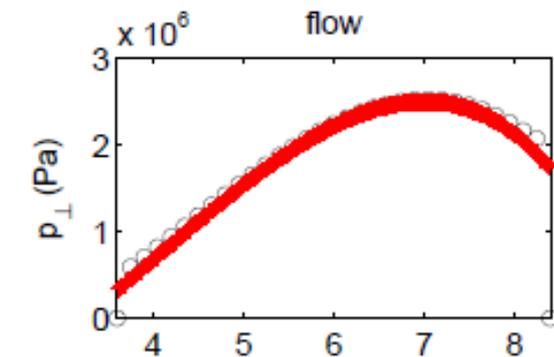
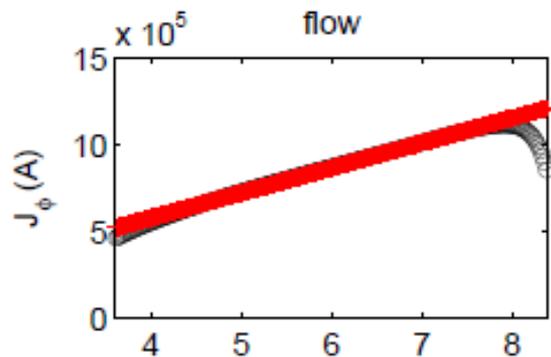
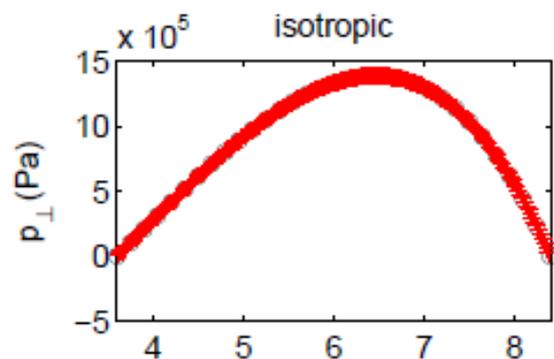
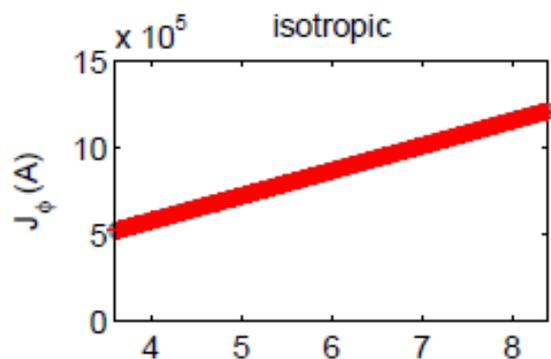


Extended Soloviev:  
 $\beta_t=0.07$ ,  $M_\phi=0.8$ ,  $\Delta=-0.004$ ,



Solution Convergence

# $J_\phi$ a strong function of transport model



e.g. ITER-like plasma

$\epsilon$	0.4
$\sigma$	1
$\tau$	1
$R_0$	6 m
$B_0$	5 T
$\alpha$	-3
$\rho_0$	$1 \times 10^{-7}$
$\Omega_0$	0 or $7 \times 10^5 \text{ rad s}^{-1}$
$\Delta_0$	0 or $4 \times 10^{-3}$
$I_p$	16 MA
$q^*$	1.6
$\beta_p$	1.0
$\beta_T$	0.07

$$p_\perp / p_\parallel \sim 1.06$$

**Extended Soloviev**

$$p_\perp(R, B, \psi) = \frac{1}{2} \rho_0 \Omega_0^2 R^2 - \frac{\Delta_0}{2} B^2 + \sigma_0 p_S(\psi)$$

$$p_\parallel(R, B, \psi) = \frac{1}{2} \rho_0 \Omega_0^2 R^2 + \frac{\Delta_0}{2} B^2 + \sigma_0 p_S(\psi)$$

**EFIT TENSOR**

$$p_\perp(\rho, B\psi) = \frac{k_B}{m} \rho T_\perp(\psi)$$

$$p_\parallel(\rho, B\psi) = \frac{k_B}{m} \rho T_\parallel(\psi)$$

# HELENA+ATF

Zhisong Qu

- Companion code written to enable stability studies.
- Can be used to study how equilibrium changes with anisotropy

$$J_\varphi = \underbrace{R \frac{B_p^2}{B^2} \left( \frac{\partial p_\parallel}{\partial \Psi} \right)_B}_{p_\parallel} + \underbrace{R \frac{B^2 - B_p^2}{B^2} \left( \frac{\partial p_\perp}{\partial \Psi} \right)_B}_{p_\perp} + \underbrace{\frac{1 - \Delta}{2R} \left( \frac{\partial (RB_\varphi)^2}{\partial \Psi} \right)_B}_{\text{toroidal field}} - \underbrace{R \nabla \cdot \frac{\Delta \nabla \Psi}{R^2}}_{\text{nonlinear}}$$

# HELENA+ATF

Zhisong Qu

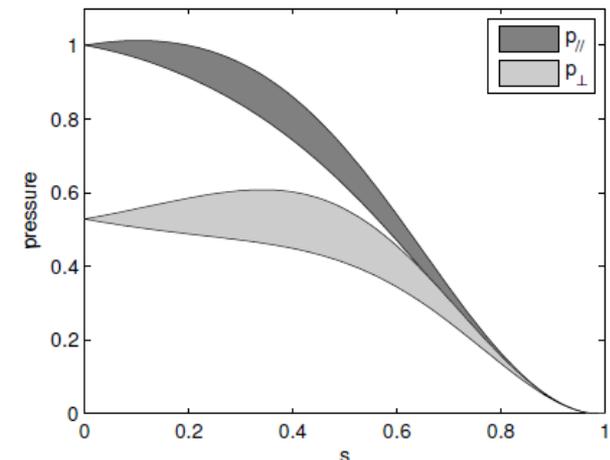
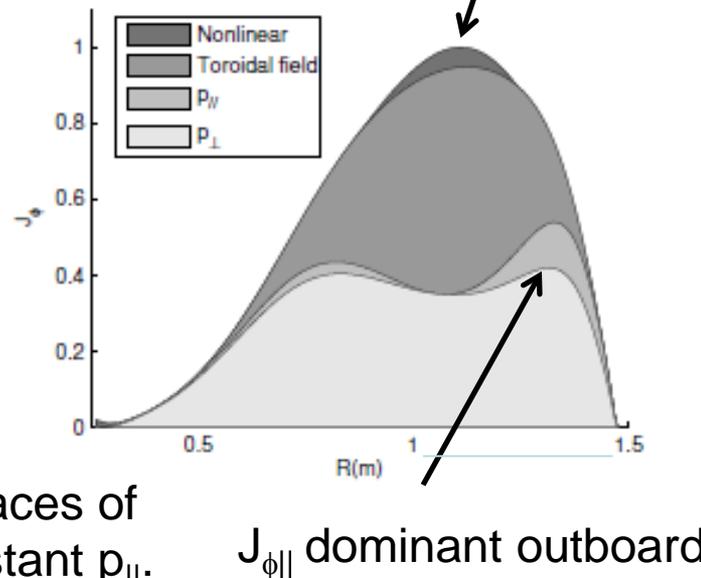
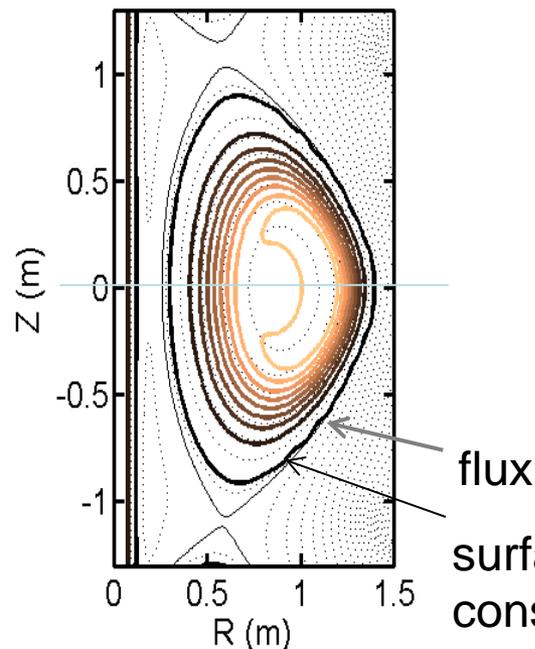
- Companion code written to enable stability studies.
- Can be used to study how equilibrium changes with anisotropy

$$J_\phi = \underbrace{R \frac{B_p^2}{B^2} \left( \frac{\partial p_{\parallel}}{\partial \Psi} \right)_B}_{p_{\parallel}} + \underbrace{R \frac{B^2 - B_p^2}{B^2} \left( \frac{\partial p_{\perp}}{\partial \Psi} \right)_B}_{p_{\perp}} + \underbrace{\frac{1 - \Delta}{2R} \left( \frac{\partial (RB_\phi)^2}{\partial \Psi} \right)_B}_{\text{toroidal field}} - \underbrace{R \nabla \cdot \frac{\Delta \nabla \Psi}{R^2}}_{\text{nonlinear}}$$

MAST-like equilibrium

$J_\phi$  components

$J_{\phi \text{nl}}$  core localised

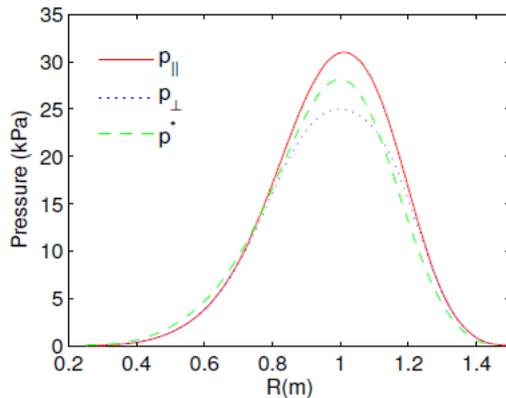


$$s \propto \sqrt{r}$$

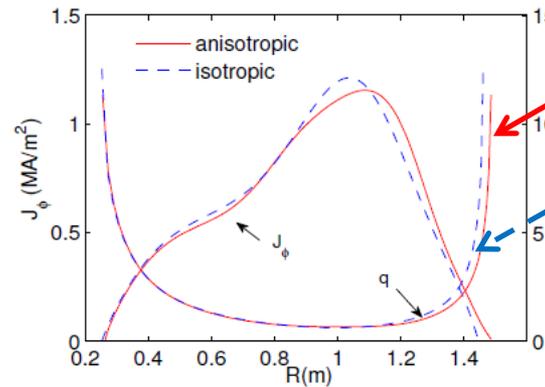
# Anisotropy modifies poloidal current

Zhisong Qu

- EFIT TENSOR reconstructions of MAST #18696 at 290ms
  - Anisotropic:  $p_{\parallel}$  and  $p_{\perp}$  constrained to values from TRANSP
  - Isotropic:  $p^* = (p_{\parallel} + p_{\perp})/2$



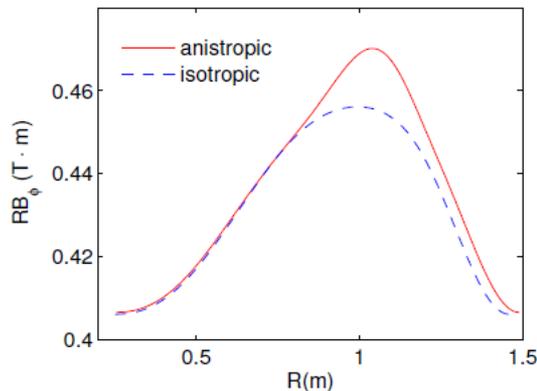
(a)



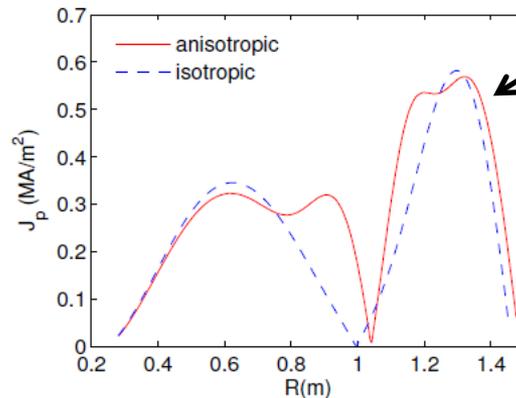
(b)

$p_{\parallel}/p_{\perp} \approx 1.25$  (anisotropic)

$p^* = (p_{\parallel} + p_{\perp})/2$  (isotropic)



(c)



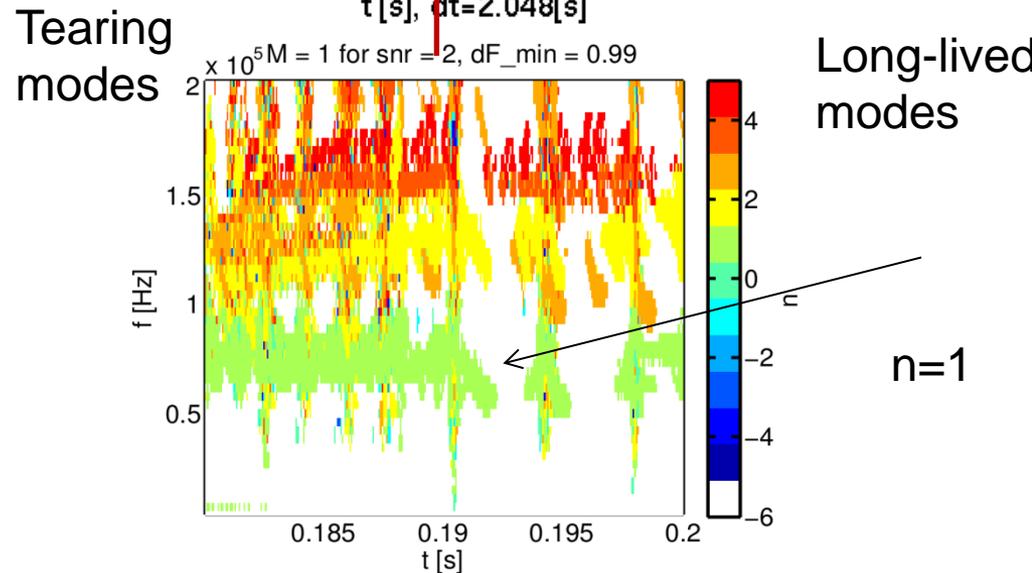
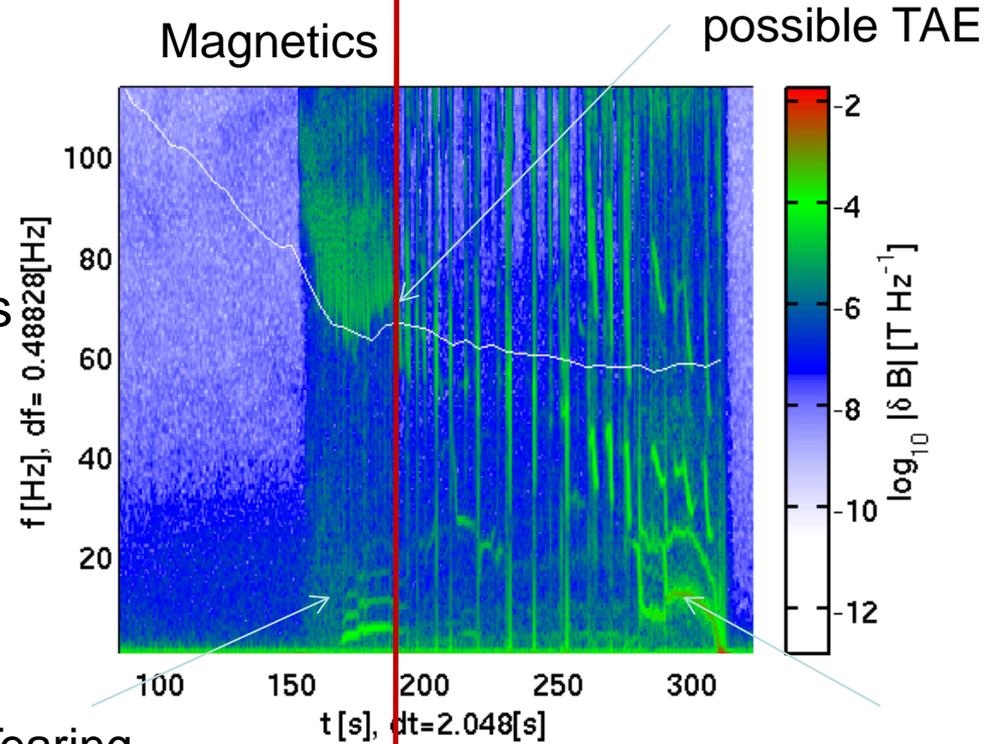
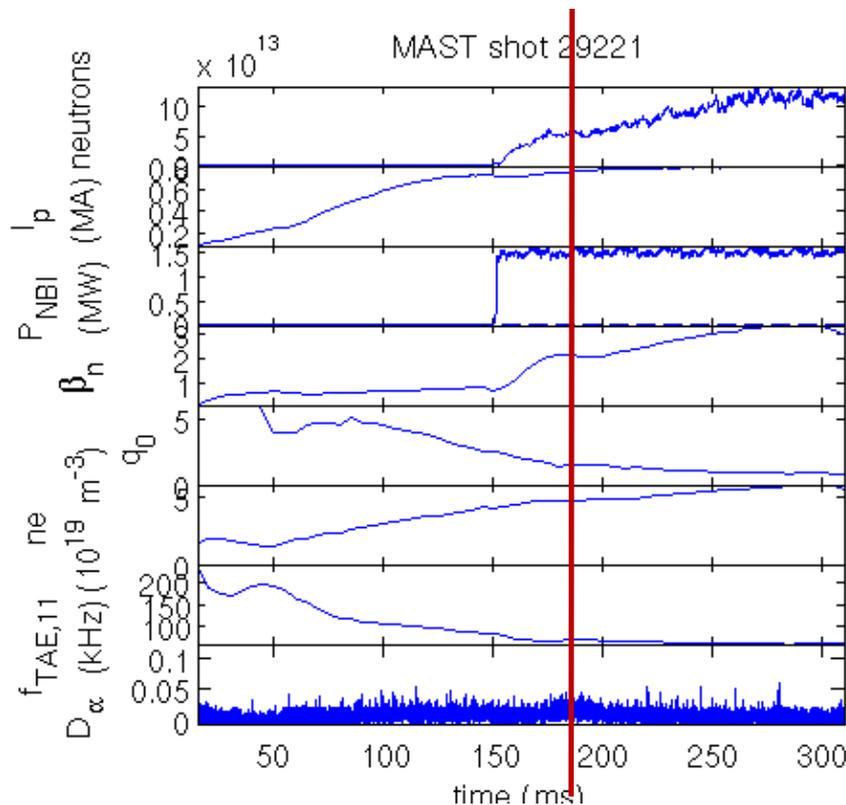
(d)

Most significant difference in  $J_p$  which can effect change in stability

$$\mu_0 \mathbf{J}_p = \nabla(RB_{\phi}) \times \nabla\psi$$

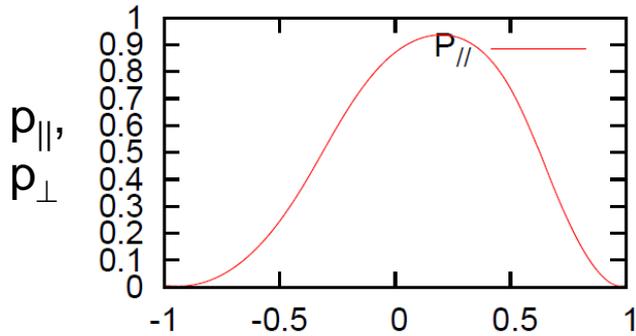
# Anisotropy on MAST: #29221

- MAST #29221
- 1.6MW NB heating
- $I_p = 0.9\text{MA}$ ,  $\beta_n \sim 3$
- Magnetics shows TAEs, tearing modes fishbones, long-lived modes

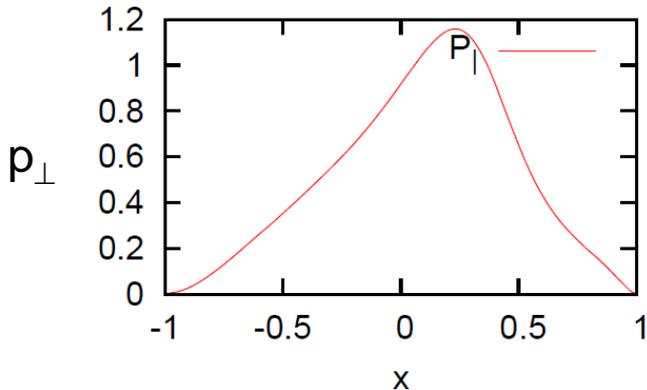
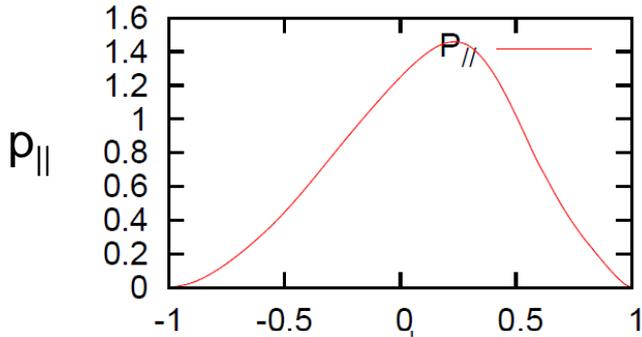


# Beam + thermal population: $p_{\parallel} / p_{\perp} \approx 1.7$

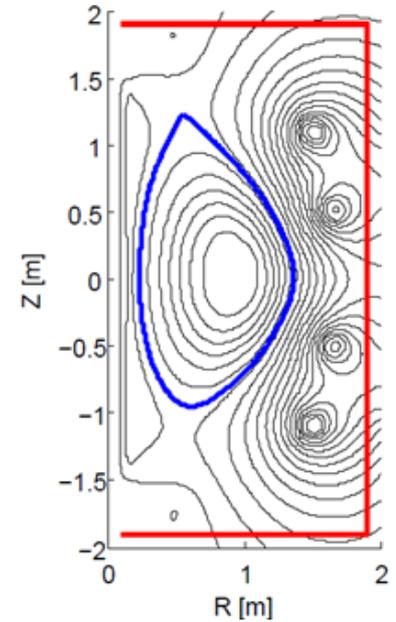
HELENA+ATF / EFIT TENSOR:  $p^* = (p_{\parallel} + p_{\perp})/2$  (isotropic)



HELENA+ATF / EFIT TENSOR:  $p_{\parallel}, p_{\perp}$  (anisotropic)

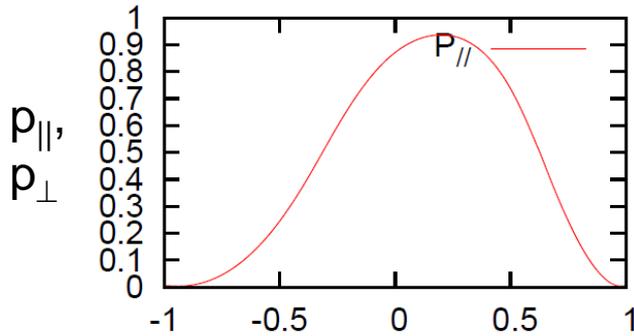


$p_{\parallel} / p_{\perp} = 1.7$  at  $s=0.5$  outboard

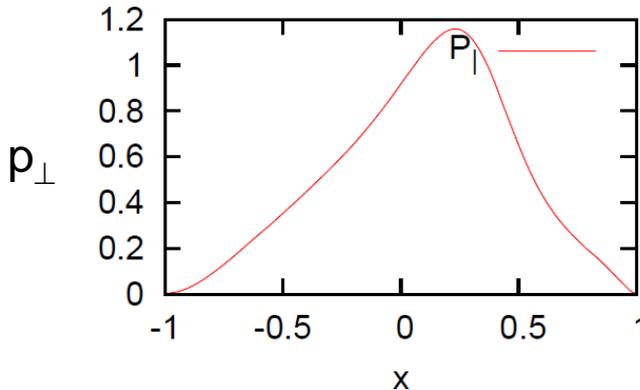
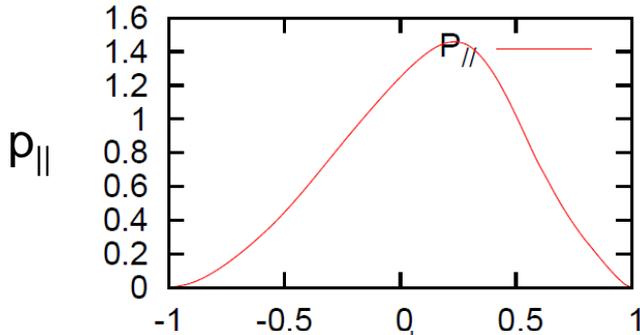


# Beam + thermal population: $p_{\parallel} / p_{\perp} \approx 1.7$

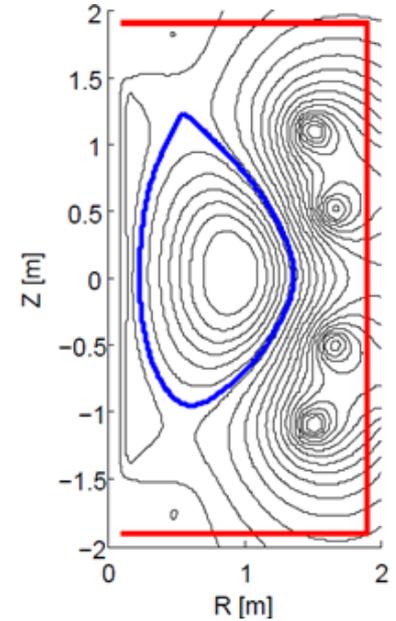
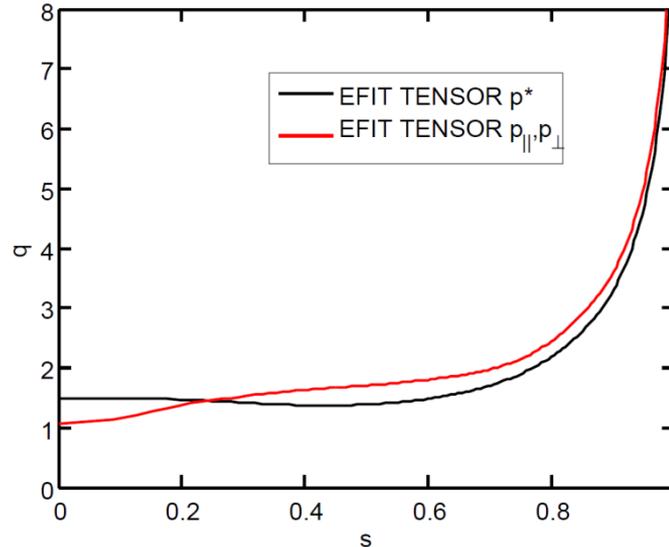
HELENA+ATF / EFIT TENSOR:  $p^* = (p_{\parallel} + p_{\perp})/2$  (isotropic)



HELENA+ATF / EFIT TENSOR:  $p_{\parallel}, p_{\perp}$  (anisotropic)



$p_{\parallel}/p_{\perp} = 1.7$  at  $s=0.5$  outboard



- What is the impact on stability due to this  $q$  profile?

# Stability: New single adiabatic model

- Compressional

- *Double-adiabatic (CGL)*

- Collisionless,  $p_{\parallel}$  and  $p_{\perp}$  do **independent** work
    - No streaming particle heat flow
    - Does not reduce to MHD in the isotropic limit

- *New Single adiabatic (SA) model*

- $p_{\parallel}$  and  $p_{\perp}$  doing **joint** work
    - Account for the isotropic part of the perturbation
    - Can reduce to MHD in isotropic limit  
[Fitzgerald, Hole, Qu, PPCF **57** (2015) 025018 ]

$$\frac{d}{dt} \left( \frac{p_{\perp}}{\rho B} \right) = 0,$$

$$\frac{d}{dt} \left( \frac{p_{\parallel} B^2}{\rho^3} \right) = 0.$$

$$\tilde{P} \rightarrow \tilde{p}I + \tilde{\pi}$$

$$\text{Tr } \nabla \cdot \tilde{Q} \rightarrow 0$$

$$\text{Tr } \tilde{\pi} \rightarrow 0$$

# Stability: New single adiabatic model

- Compressional

- *Double-adiabatic (CGL)*

- Collisionless,  $p_{\parallel}$  and  $p_{\perp}$  do **independent** work
    - No streaming particle heat flow
    - Does not reduce to MHD in the isotropic limit

$$\frac{d}{dt} \left( \frac{p_{\perp}}{\rho B} \right) = 0,$$

$$\frac{d}{dt} \left( \frac{p_{\parallel} B^2}{\rho^3} \right) = 0.$$

- *New Single adiabatic (SA) model*

- $p_{\parallel}$  and  $p_{\perp}$  doing **joint** work
    - Account for the isotropic part of the perturbation
    - Can reduce to MHD in isotropic limit  
[Fitzgerald, Hole, Qu, PPCF **57** (2015) 025018 ]

$$\tilde{P} \rightarrow \tilde{p}I + \tilde{\pi}$$

$$\text{Tr } \nabla \cdot \tilde{Q} \rightarrow 0$$

$$\text{Tr } \tilde{\pi} \rightarrow 0$$

- Incompressional

$$p_{\parallel 1} = -\xi_n \left[ \frac{\partial p_{\parallel}}{\partial n} - (p_{\parallel} - p_{\perp}) \frac{\partial \ln B}{\partial n} \right] \quad p_{\perp 1} = -\xi_n \left[ \frac{\partial p_{\perp}}{\partial n} - (2p_{\perp} + \hat{c}) \frac{\partial \ln B}{\partial n} \right]$$

\*A B Mikhailovskii, Instabilities in a confined plasma, IOP publishing (1998)

# Stability: New single adiabatic model

- Compressional

- *Double-adiabatic (CGL)*

- Collisionless,  $p_{\parallel}$  and  $p_{\perp}$  do **independent** work
    - No streaming particle heat flow
    - Does not reduce to MHD in the isotropic limit

$$\frac{d}{dt} \left( \frac{p_{\perp}}{\rho B} \right) = 0,$$

$$\frac{d}{dt} \left( \frac{p_{\parallel} B^2}{\rho^3} \right) = 0.$$

- *New Single adiabatic (SA) model*

- $p_{\parallel}$  and  $p_{\perp}$  doing **joint** work
    - Account for the isotropic part of the perturbation
    - Can reduce to MHD in isotropic limit  
[Fitzgerald, Hole, Qu, PPCF **57** (2015) 025018 ]

$$\tilde{P} \rightarrow \tilde{p}I + \tilde{\pi}$$

$$\text{Tr } \nabla \cdot \tilde{Q} \rightarrow 0$$

$$\text{Tr } \tilde{\pi} \rightarrow 0$$

- Incompressional

$$p_{\parallel 1} = -\xi_n \left[ \frac{\partial p_{\parallel}}{\partial n} - (p_{\parallel} - p_{\perp}) \frac{\partial \ln B}{\partial n} \right] \quad p_{\perp 1} = -\xi_n \left[ \frac{\partial p_{\perp}}{\partial n} - (2p_{\perp} + \hat{c}) \frac{\partial \ln B}{\partial n} \right]$$

\*A B Mikhailovskii, Instabilities in a confined plasma, IOP publishing (1998)

- Implemented in CSCAS (CSMIS-A) and MISHKA (MISHKA-A)

[Qu, Hole, Fitzgerald, PPCF submitted 09/02/2015]

# MISHKA-A agrees with Bussac criterion

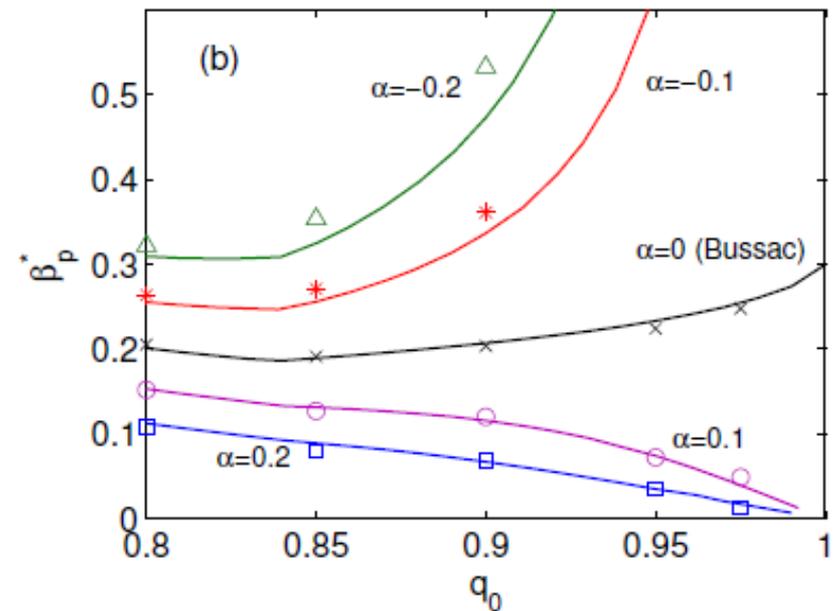
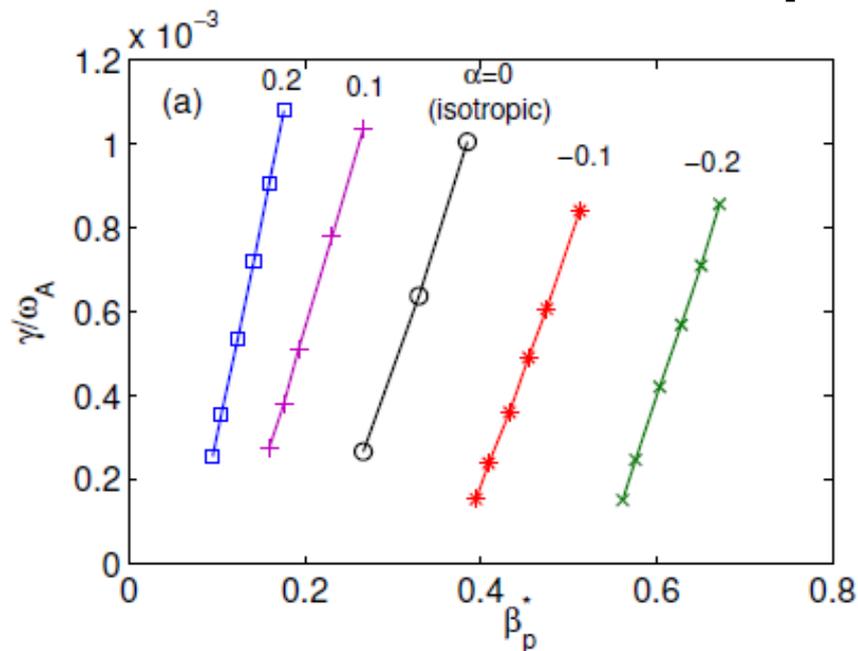
- Benchmark result Generalised Bussac condition: marginal stability of  $n=1$  internal kink for  $\varepsilon=0.1$ , circular cross section

$$\frac{\langle p_{\perp} \rangle}{\langle p_{\parallel} \rangle} = \frac{1}{1 - \alpha(1 - \psi_n)}$$

Bussac condition = solid lines

MISHKA-A = points

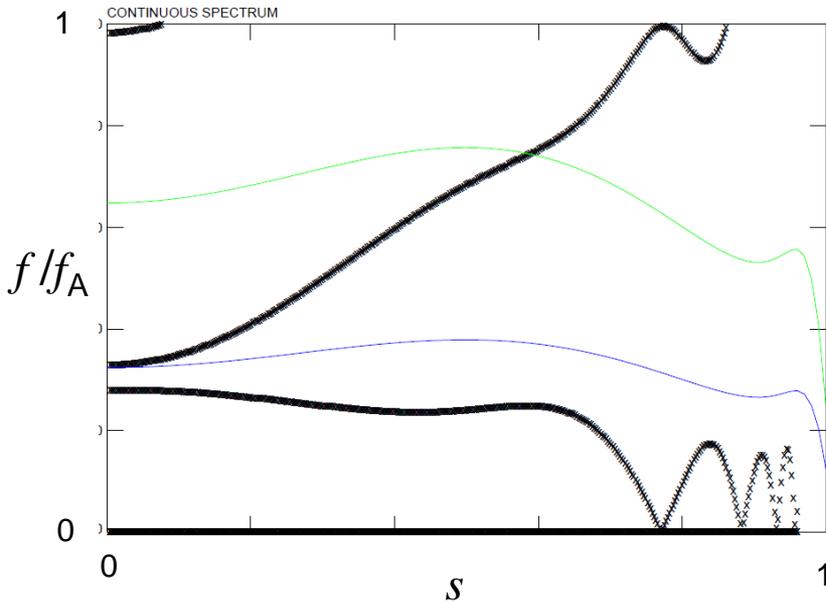
[Qu, Hole, Fitzgerald, PPCF submitted 09/02/2015]



# Incompressible continuum for MAST

isotropic

$n=1, \gamma=0$



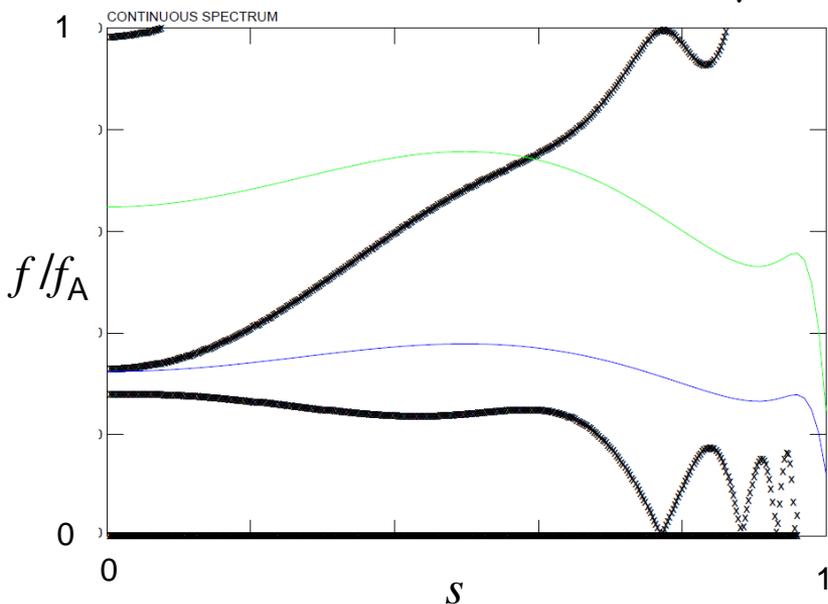
$$R_{mag} = 0.914$$

$f_A$  at magnetic axis = 280kHz

# Incompressible continuum for MAST

isotropic

$n=1, \gamma=0$

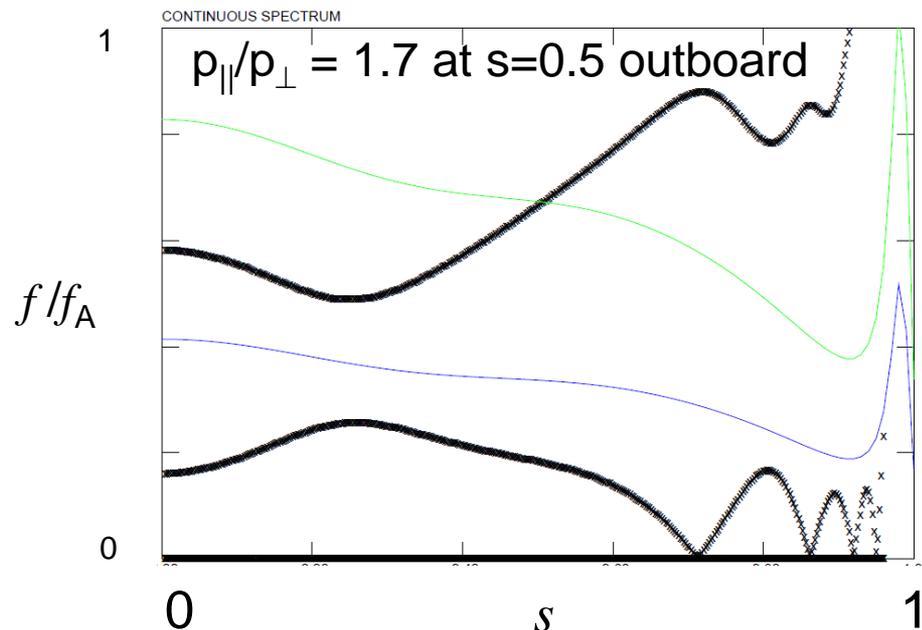


$$R_{mag} = 0.914$$

$$f_A \text{ at magnetic axis} = 280\text{kHz}$$

anisotropic

$n=1, \gamma=0$



$$R_{mag} = 0.928$$

$$f_A \text{ at magnetic axis} = 260\text{kHz}$$

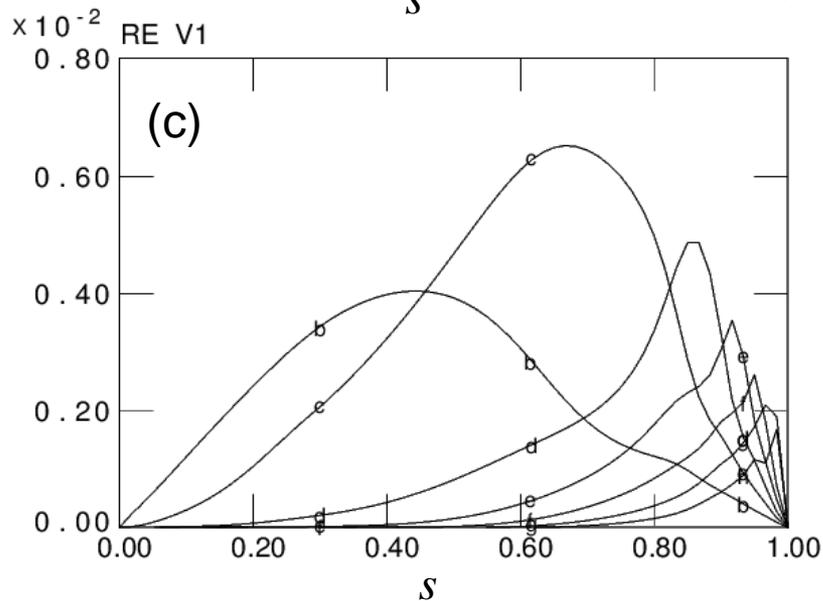
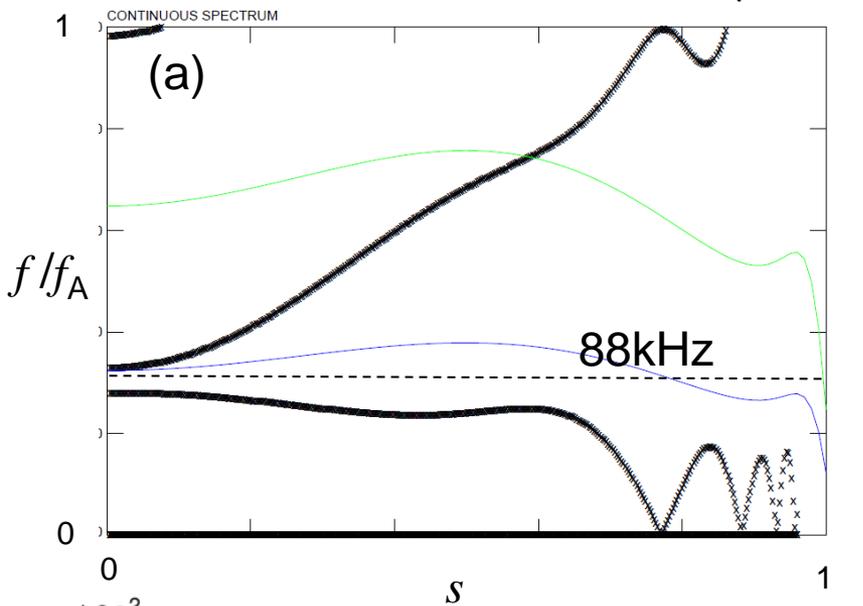
isotropic  $\Delta f_{TAE} <$  anisotropic  $\Delta f_{TAE}$

$\Rightarrow$  anisotropic modes likely to have less continuum damping

# mode profile broader with anisotropy

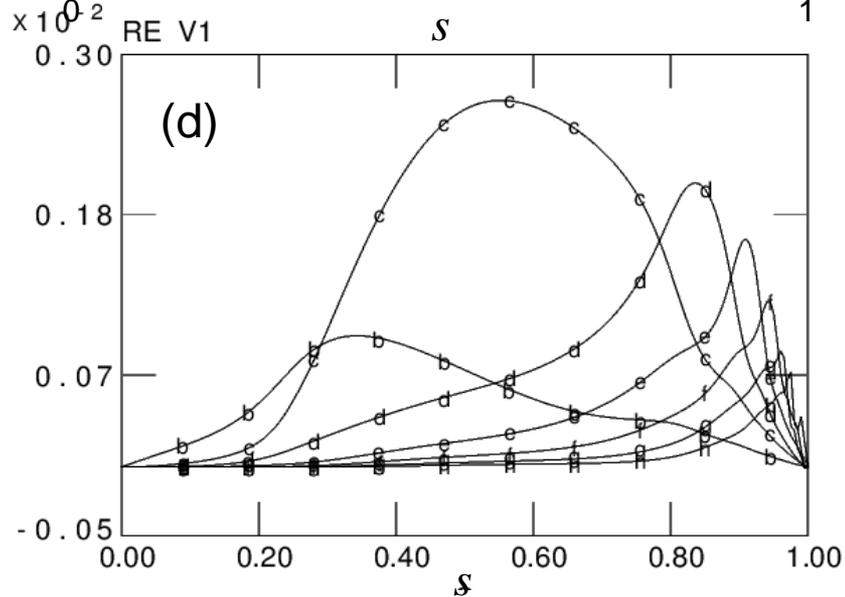
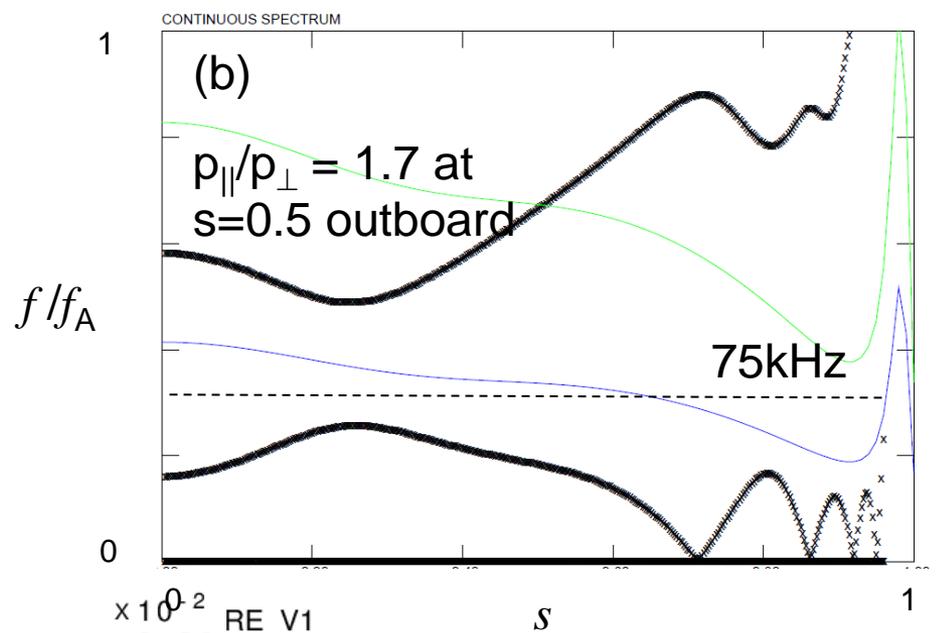
isotropic

$n=1, \gamma=0$



anisotropic

$n=1, \gamma=0$



# Ongoing work in Anisotropy and Flow

- What is the impact of different radial structure on anomalous transport?
  - Couple EFIT TENSOR, MISHKA-A to wave-particle interaction code HAGIS for self-consistent evolution
- Explore the impact of anisotropy and flow on a wide range of MAST plasma conditions

G. Bowden, A. Könies: Implemented complex contour algorithm into CKA to compute continuum damping in 3D



# 3D equilibria in toroidal plasmas

- Simplest model to approximate global, macroscopic force-balance is magnetohydrodynamics (MHD).

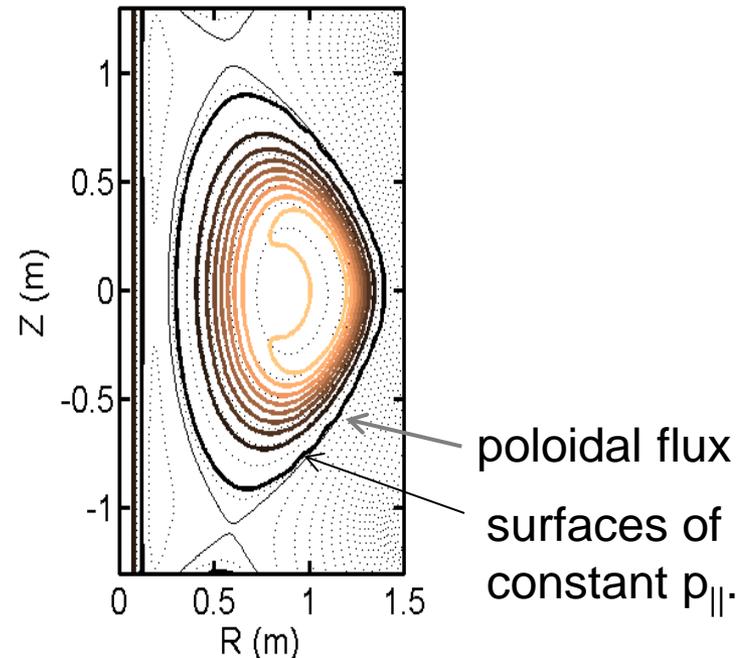
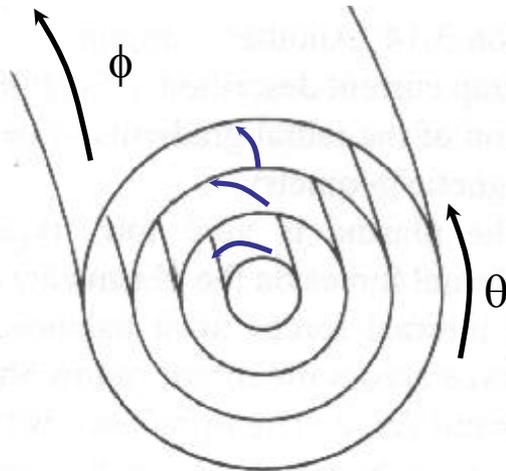
$$\nabla p = \mathbf{J} \times \mathbf{B}, \quad \nabla \times \mathbf{B} = \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0$$

# 3D equilibria in toroidal plasmas

- Simplest model to approximate global, macroscopic force-balance is magnetohydrodynamics (MHD).

$$\nabla p = \mathbf{J} \times \mathbf{B}, \quad \nabla \times \mathbf{B} = \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0$$

- Toroidal symmetry  $\Rightarrow$  field lies in nested flux surfaces

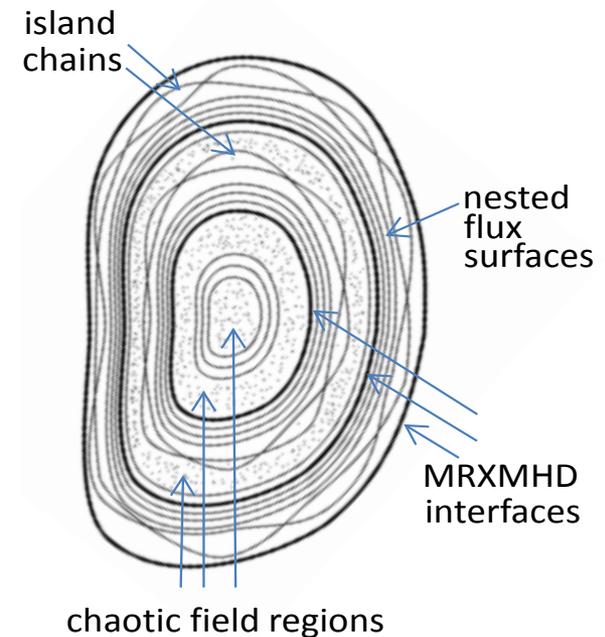


# 3D equilibria in toroidal plasmas

- Simplest model to approximate global, macroscopic force-balance is magnetohydrodynamics (MHD).

$$\nabla p = \mathbf{J} \times \mathbf{B}, \quad \nabla \times \mathbf{B} = \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0$$

- Non-axisymmetric  $\Rightarrow$  field does **not** lie in nested flux surfaces **unless** surface currents allowed.

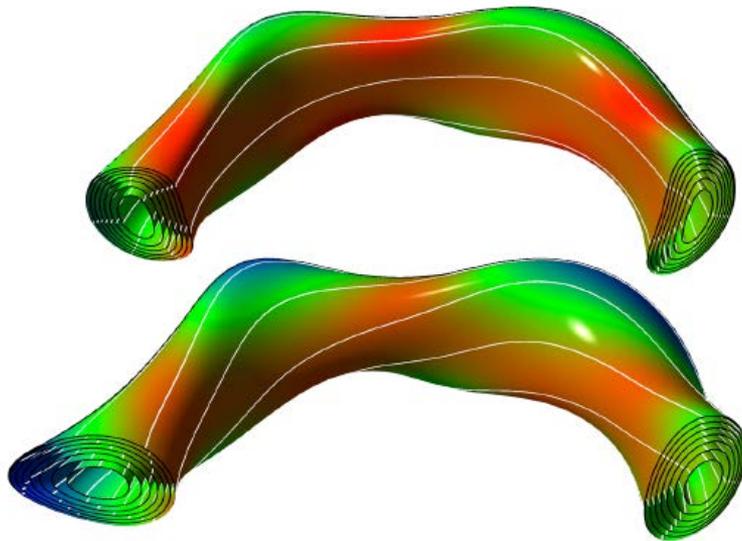


# 3D equilibria in toroidal plasmas

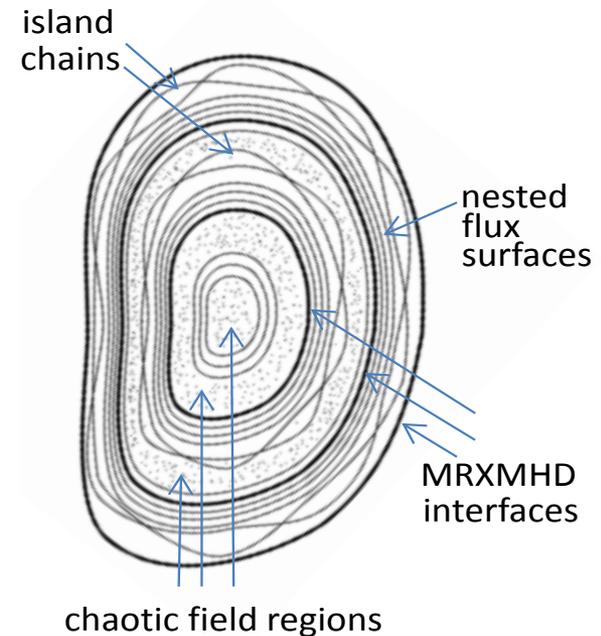
- Simplest model to approximate global, macroscopic force-balance is magnetohydrodynamics (MHD).

$$\nabla p = \mathbf{J} \times \mathbf{B}, \quad \nabla \times \mathbf{B} = \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0$$

- Non-axisymmetric  $\Rightarrow$  field does **not** lie in nested flux surfaces **unless** surface currents allowed.
- Existing 3D solvers (e.g. VMEC) assume nested flux surfaces.



[CTH stellarator, Hanson et al, IAEA 2012]



# Generalised Taylor Relaxation:

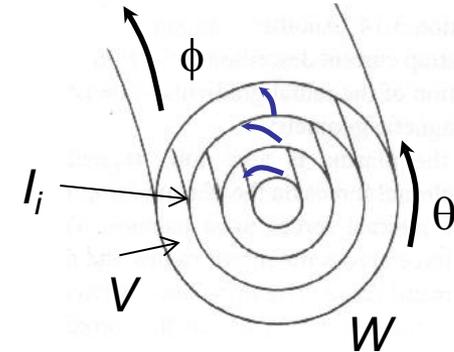
## Multiple Relaxed Region MHD (MRXMHD)

R. L. Dewar

- Assume each invariant tori  $I_i$  act as ideal MHD barriers to relaxation, so that Taylor constraints are localized to subregions.

New system comprises:

- $N$  plasma regions  $P_i$  in relaxed states.
- Regions separated by ideal MHD barrier  $I_i$ .
- Enclosed by a vacuum  $V$ ,
- Encased in a perfectly conducting wall  $W$



$$W_i = \int_{R_i} \left( \frac{B_i^2}{2\mu_0} + \frac{P_i}{\gamma - 1} \right) d\tau^3$$

$$H_i = \int_V (\mathbf{A}_i \cdot \mathbf{B}_i) d\tau^3$$

Seek minimum energy state:

$$F = \sum_{l=1}^N (W_l - \mu_l H_l / 2)$$

$$P_l : \quad \nabla \times \mathbf{B} = \mu_l \mathbf{B}$$

$$P_l = \text{constant}$$

$$I_l : \quad \mathbf{B} \cdot \mathbf{n} = 0$$

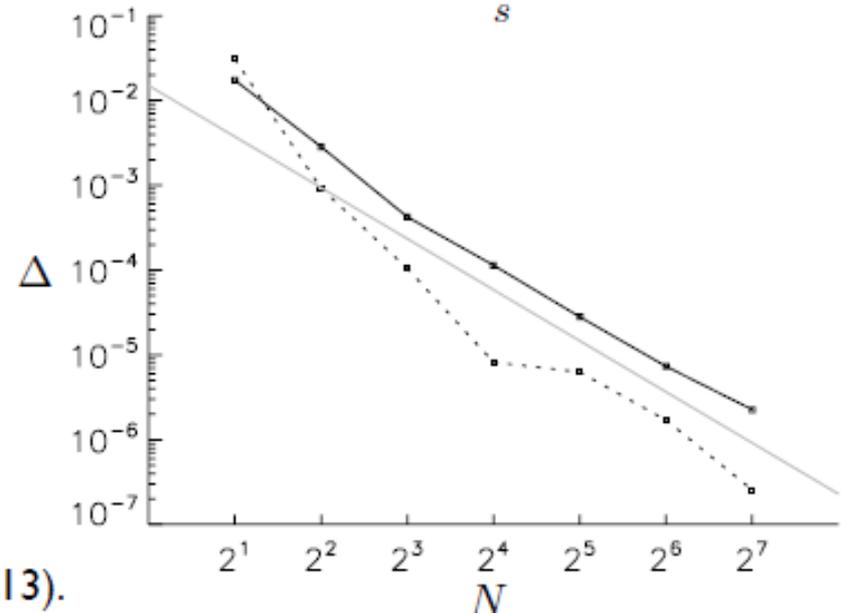
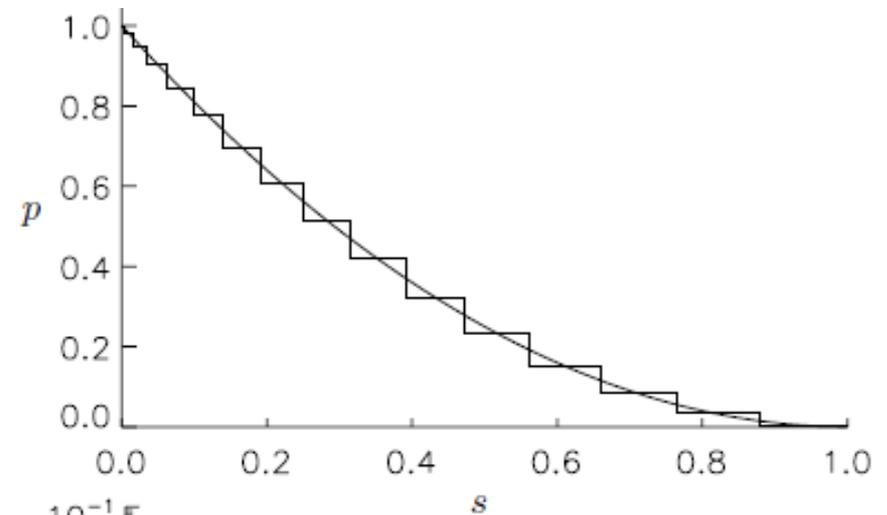
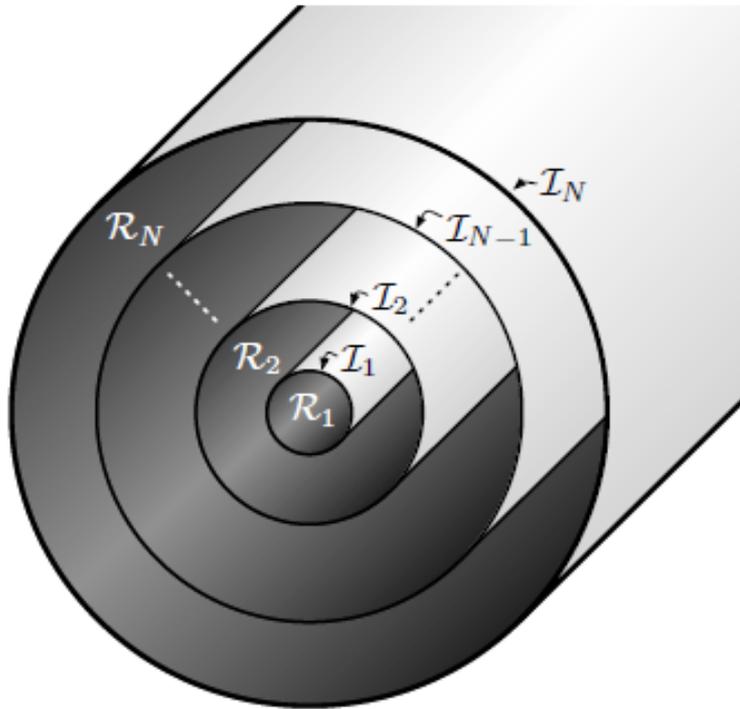
$$[[P_l + B^2 / (2\mu_0)]] = 0$$

$$V : \quad \nabla \times \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

$$W : \quad \mathbf{B} \cdot \mathbf{n} = 0$$

# MRXMHD approaches ideal MHD as $N \rightarrow \infty$



[1] G. Dennis et al., *Phys. Plasmas* **20**, 032509 (2013).

# Stepped Pressure Equilibrium Code, SPEC

[Hudson et al Phys. Plasmas 19, 112502 (2012)]

Hudson

## Vector potential is discretised using mixed Fourier & finite elements

- Coordinates  $(s, \vartheta, \zeta)$
- Interface geometry  $R_i = \sum_{l,m,n} R_{lmn} \cos(m\vartheta - n\zeta)$ ,  $Z_i = \sum_{l,m,n} Z_{lmn} \sin(m\vartheta - n\zeta)$
- Exploit gauge freedom  $\mathbf{A} = A_\vartheta(s, \vartheta, \zeta) \nabla \vartheta + A_\zeta(s, \vartheta, \zeta) \nabla \zeta$
- Fourier  $A_\vartheta = \sum_{m,n} \alpha(s) \cos(m\vartheta - n\zeta)$
- Finite-element  $a_\vartheta(s) = \sum_i a_{\vartheta,i}(s) \rho(s)$

## & inserted into constrained-energy functional $F = \sum_{l=1}^N (W_l - \mu_l H_l / 2)$

- Derivatives wrt  $\mathbf{A}$  give Beltrami field  $\nabla \times \mathbf{B} = \mu \mathbf{B}$
- Field in each annulus computed independently, distributed across multiple cpu's
- Field in each annulus depends on enclosed toroidal flux, poloidal flux, interfaces  $\xi$

## Force balance solved using multi-dimensional Newton method

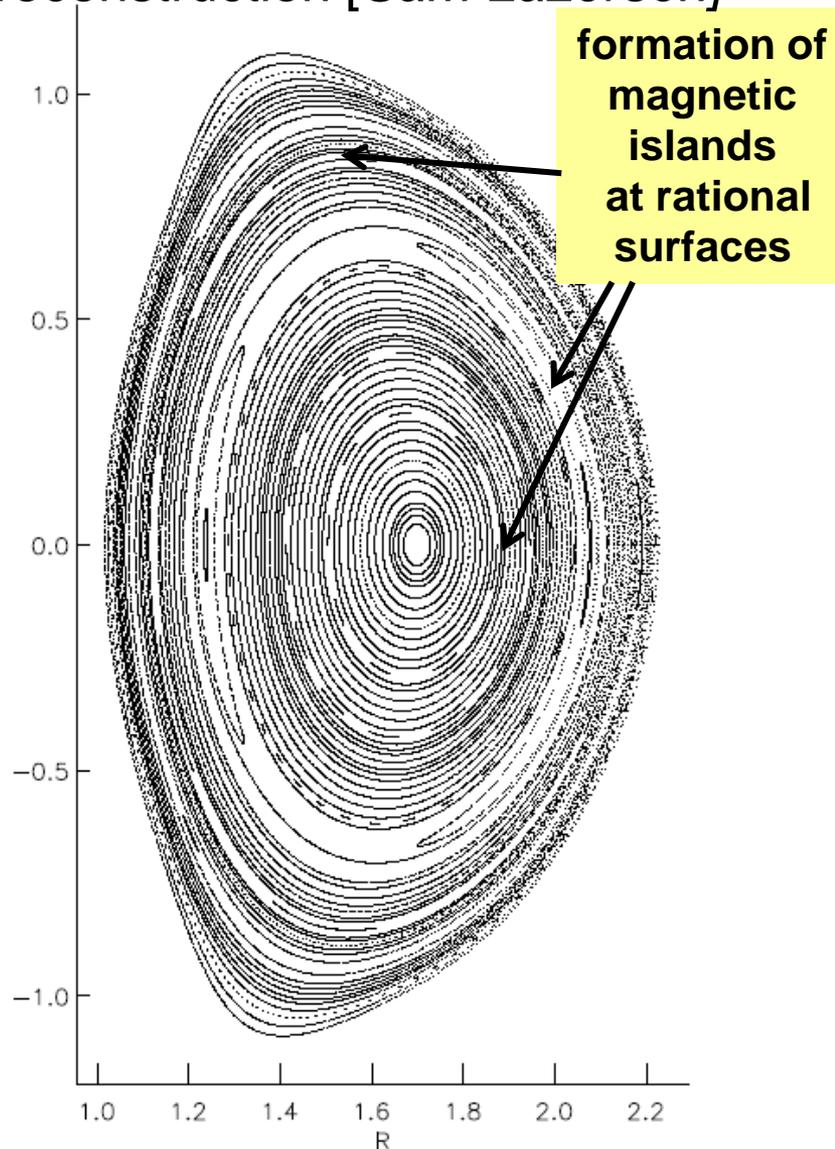
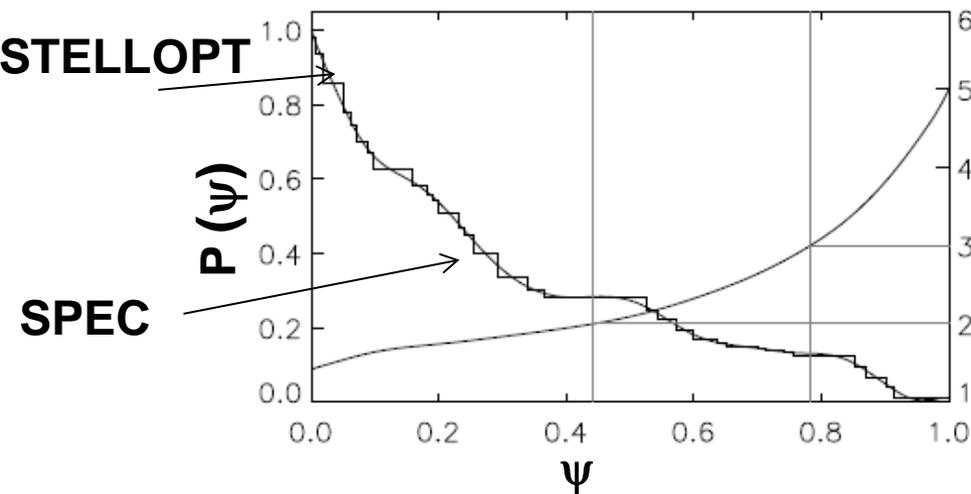
- Interface geometry adjusted to satisfy force balance  $\mathbf{F}[\xi] = \{ \llbracket p + B^2 / 2 \rrbracket_{m,n} \} = 0$
- Angle freedom constrained by spectral condensation,
- Derivative matrix  $\nabla F[\xi]$  computed in parallel using finite difference

# Example: DIID with $n=3$ applied error field

[Hudson et al Phys. Plasmas 19, 112502 (2012)]

Hudson

- 3D boundary,  $p$ ,  $q$ -profile from STELLOPT reconstruction [Sam Lazerson]
- Irrational interfaces chosen to coincide with pressure gradients.

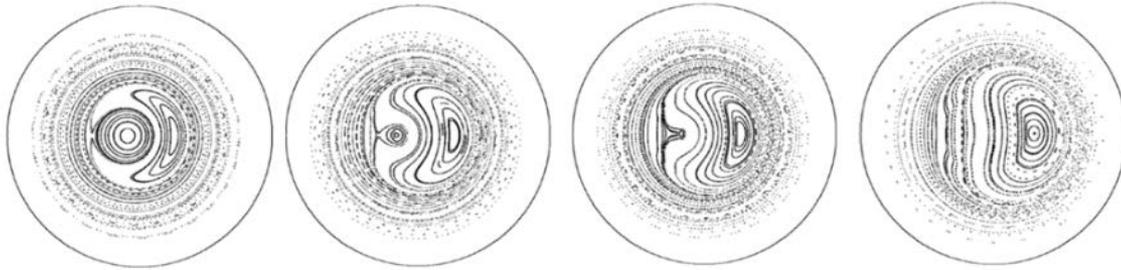


- Island formation is permitted
- No rational “shielding currents” included in calculation.

# Spontaneously formed helical states

Dennis, Hudson, Terranova, Dewar, Hole, Escande

- The quasi-single helicity state is a stable helical state in RFP: becomes purer as current is increase



Double-Helical  
Axis state

Increasing current

Single Helical  
Axis state

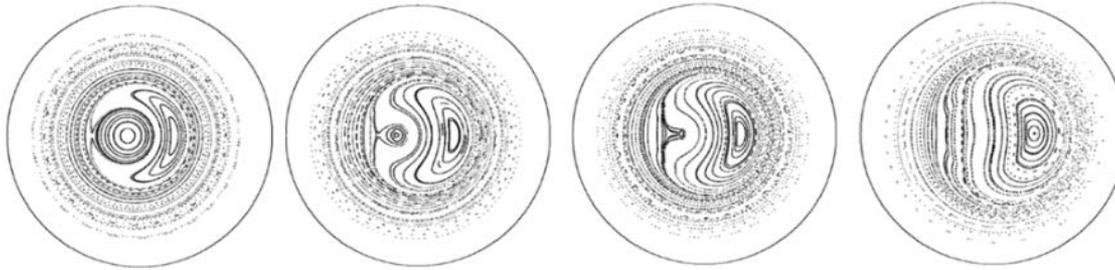
“Experimental” Poincaré plot

[Fig. 6 of P. Martin et al., Nuclear Fusion 49, 104019 (2009)]

# Spontaneously formed helical states

Dennis, Hudson, Terranova, Dewar, Hole, Escande

- The quasi-single helicity state is a stable helical state in RFP: becomes purer as current is increase



“Experimental” Poincaré plot

[Fig. 6 of P. Martin et al., Nuclear Fusion 49, 104019 (2009)]

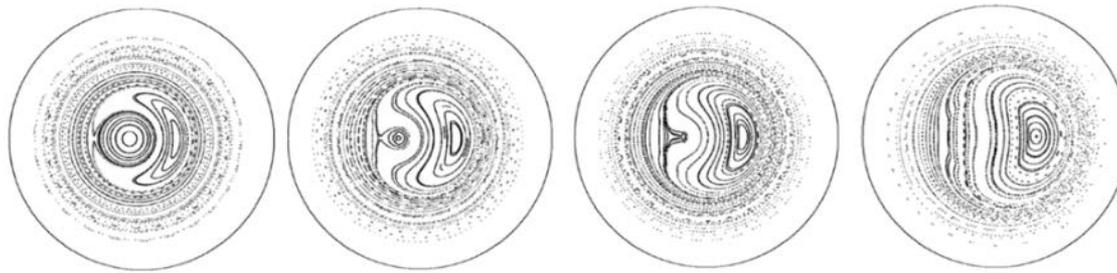
Double-Helical Axis state  $\xrightarrow{\text{Increasing current}}$  Single Helical Axis state

- Ideal MHD with assumed nested flux surfaces can **not** model the DAX state

# Spontaneously formed helical states

Dennis, Hudson, Terranova, Dewar, Hole, Escande

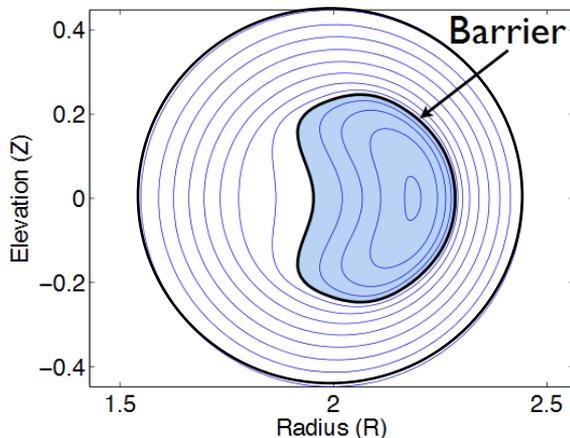
- The quasi-single helicity state is a stable helical state in RFP: becomes purer as current is increase



“Experimental” Poincaré plot

[Fig. 6 of P. Martin et al., Nuclear Fusion 49, 104019 (2009)]

Double-Helical Axis state  $\xrightarrow{\text{Increasing current}}$  Single Helical Axis state

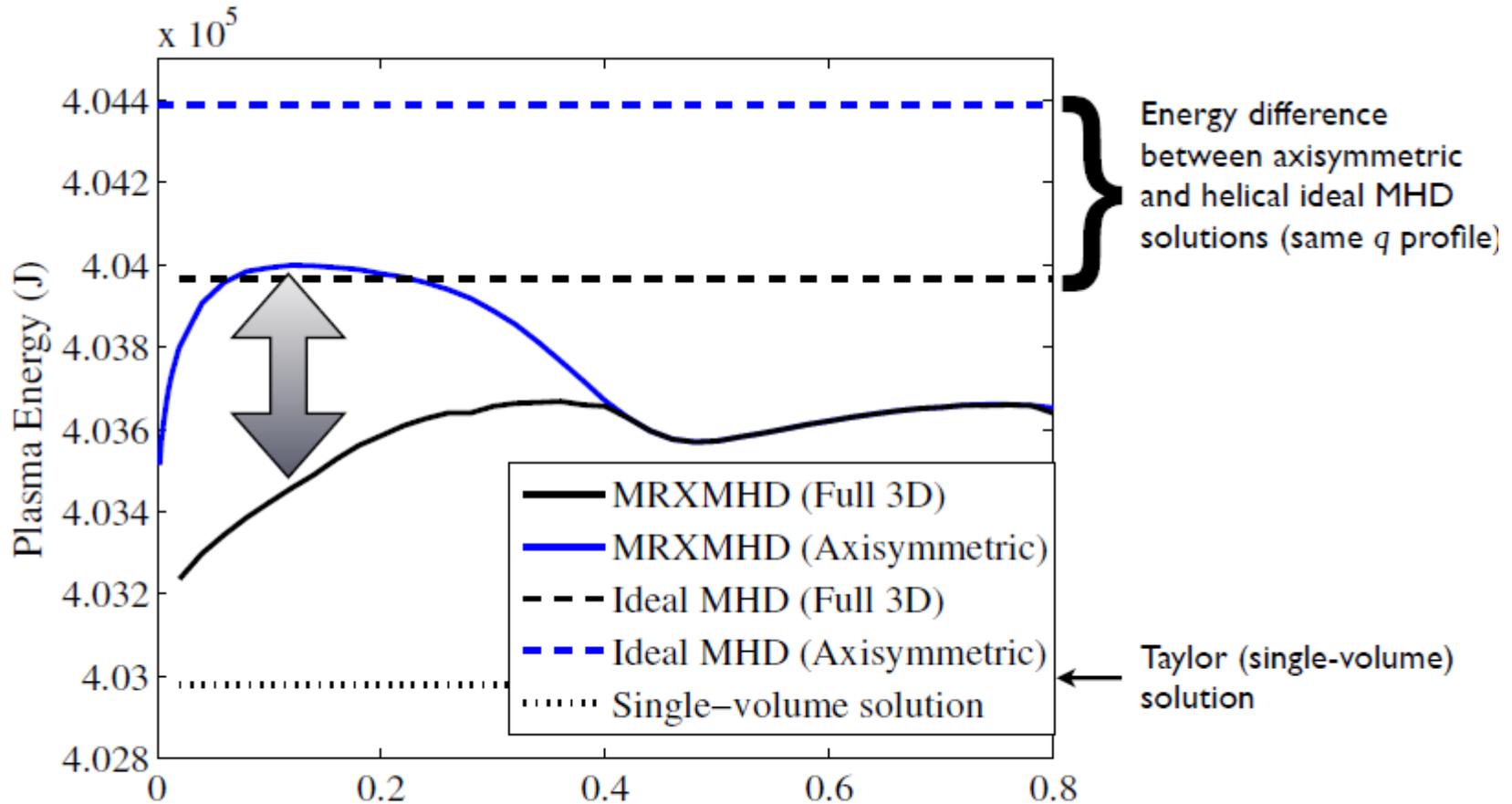


- Ideal MHD with assumed nested flux surfaces can **not** model the DAX state
- Might MRXMHD with 2 barriers offer a minimal description to describe DAX and SHAX states in the RFP?
- Model RFX-mod QSH state by a 2-interface minimum energy MRXMHD state.

[G. R. Dennis *et al* , Phys. Rev. Lett. **111**, 055003, 2013]

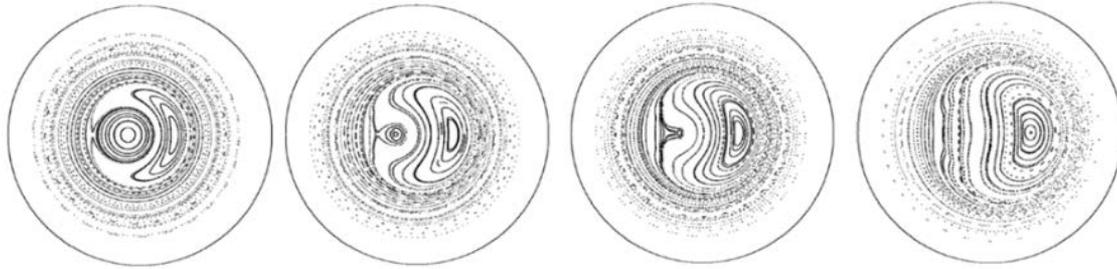
# Plasma is a minimum energy state

- RFP bifurcated state has lower energy (preferred) than comparable axis-symmetric state



Ideal MHD flux surface chosen as ideal barrier

# Spontaneously formed helical states

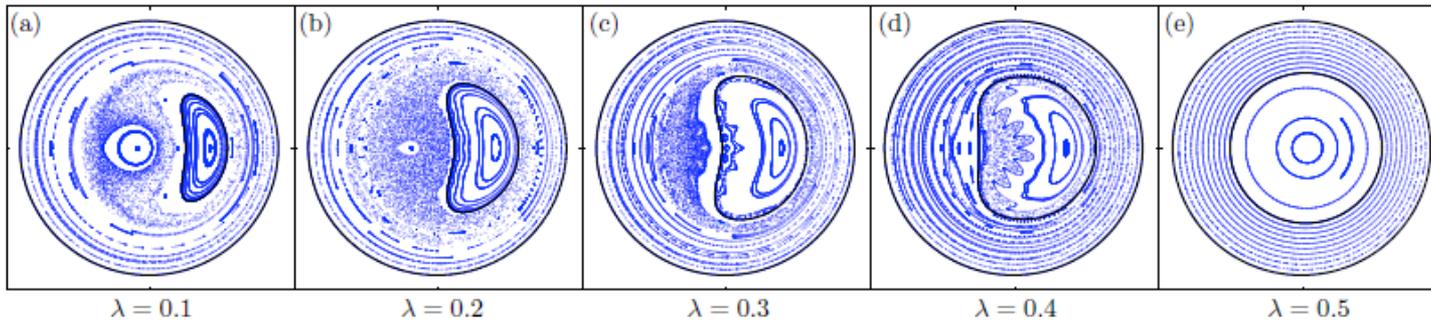


“Experimental” Poincaré plot

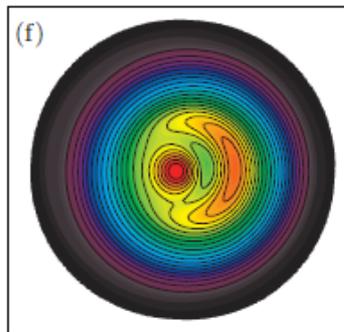
[Fig. 6 of P. Martin et al., Nuclear Fusion 49, 104019 (2009)]

Double-Helical  
Axis state

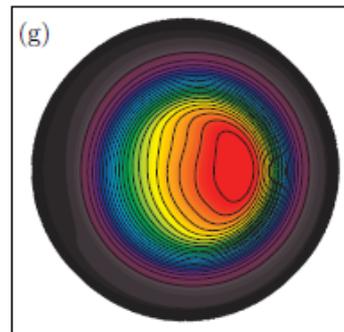
Single Helical  
Axis state



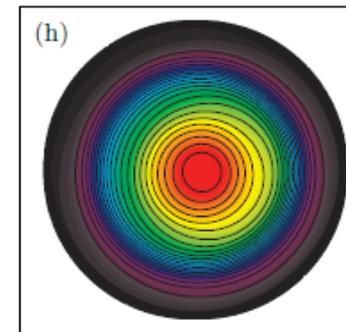
MRXMHD  
Poincaré plot  
G. R. Dennis  
PRL



DAx state



SHAx state



Axisymmetric  
multiple-helicity state

Soft X-ray data

# Recent progress in MRxMHD

- Extended MRxMHD to include non-zero plasma flow and plasma anisotropy  
[G.R. Dennis, S.R. Hudson, R.L. Dewar, M.J. Hole, Phys. Plas. **21**, 042501 (2014)]  
[G.R. Dennis, S.R. Hudson, R.L. Dewar, M.J. Hole, Phys. Plas. **21**, 072512 (2014)]
- Generalized straight field line coordinates concept to fully 3D plasmas  
[R. L. Dewar, S. R. Hudson, A. Gibson, *Plasma Phys. Control. Fusion*, **55**, 014004, 2013]
- Related helical bifurcation of a Taylor relaxed state to a tearing mode  
[Z. Yoshida and R. L. Dewar , *J. Phys. A: Math. Theor.* **45**, 365502, 2012]  
[M. J. Hole, R. Mills, S. R. Hudson and R. L. Dewar Nucl. Fusion 49 (2009) 065019]
- Developed techniques to establish pressure jump a surface can support.  
[M. McGann, ANU PhD thesis, 2013]

# Recent progress in MRxMHD

- Developed “plasmoids”, representing partial magnetic island chains  
[R. L. Dewar *et al*, Phys. Plas. **20**, 082103, 2013.]
- Computed the high- $n$  stability of a pressure discontinuity in a 3D plasma.  
[D. Barmaz, ANU Masters Thesis 2011]
- Related ghost surfaces and isotherms in chaotic fields  
[S. R. Hudson and J. Breslau, Phys. Rev. Let., **100**, 095001, 2008]

# Conclusions: Anisotropy and Flow

- Added anisotropy and toroidal flow to equilibrium reconstruction code EFIT TENSOR, and HELENA+ATF
- Developed new single adiabatic stability model which includes anisotropy and flow, reduces to ideal MHD as anisotropy and flow reduced
- Implemented Single Adiabatic CGL and incompressible stability treatments into continuum code CSMIS and stability code MISHKA-A
- Shown anisotropy changes the radial structure of TAE modes.
- *Does it change (wave-particle) anomalous transport?*
- *Explore the impact of anisotropy and flow on a wide range of MAST plasma conditions*



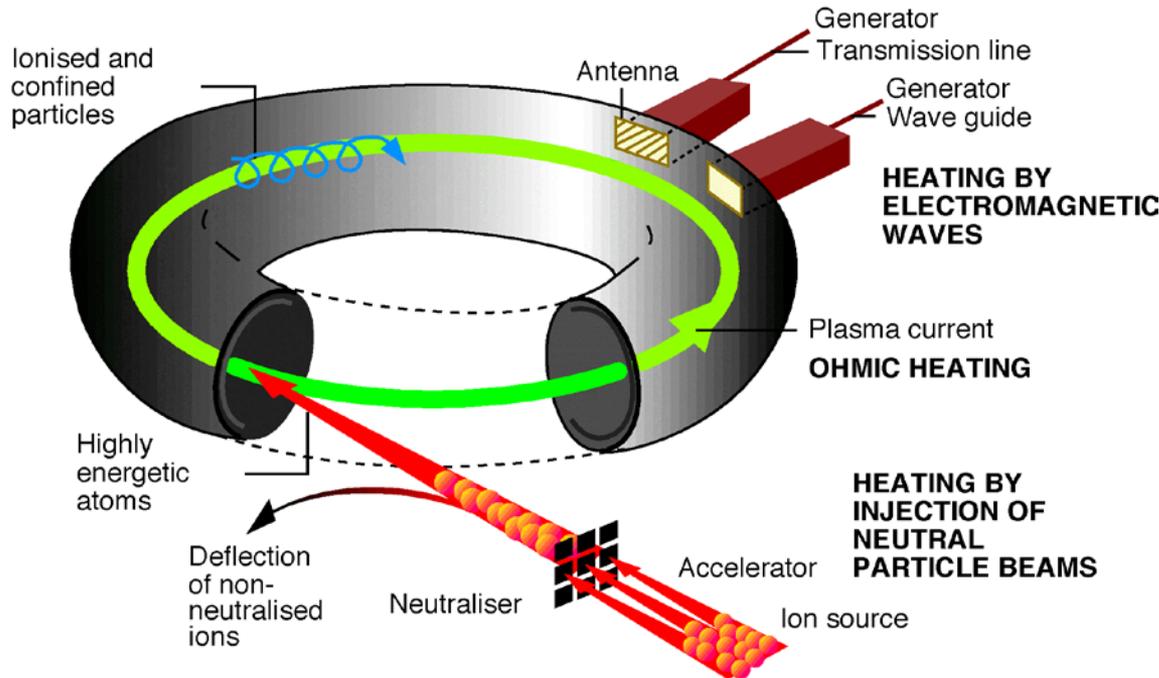
# Conclusions: MRxMHD

- Introduced/ motivated multi-region relaxed MHD, and SPEC 3D MHD code
- Demonstrated application of SPEC to describe DIIID plasma with an applied error field
- Applied MRxMHD to reverse field pinch, explained transition from a double helical-axis to single helical axis state as a sequence of minimum energy MRxMHD states.
- *Extend SPEC to free boundary, including vacuum region and external conductors*
  - *Enables calculation of stability to external modes and response due to Resonant Magnetic Perturbation (RMP) coils*
- *Explain helical states in MAST (e.g. long-lived modes), and sawtooth reconnection cycle*
- *Address stability of chaotic field configurations*



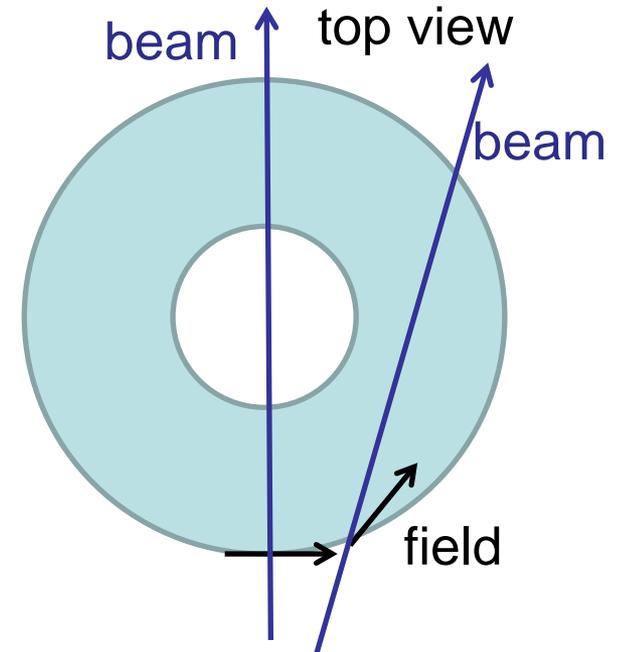
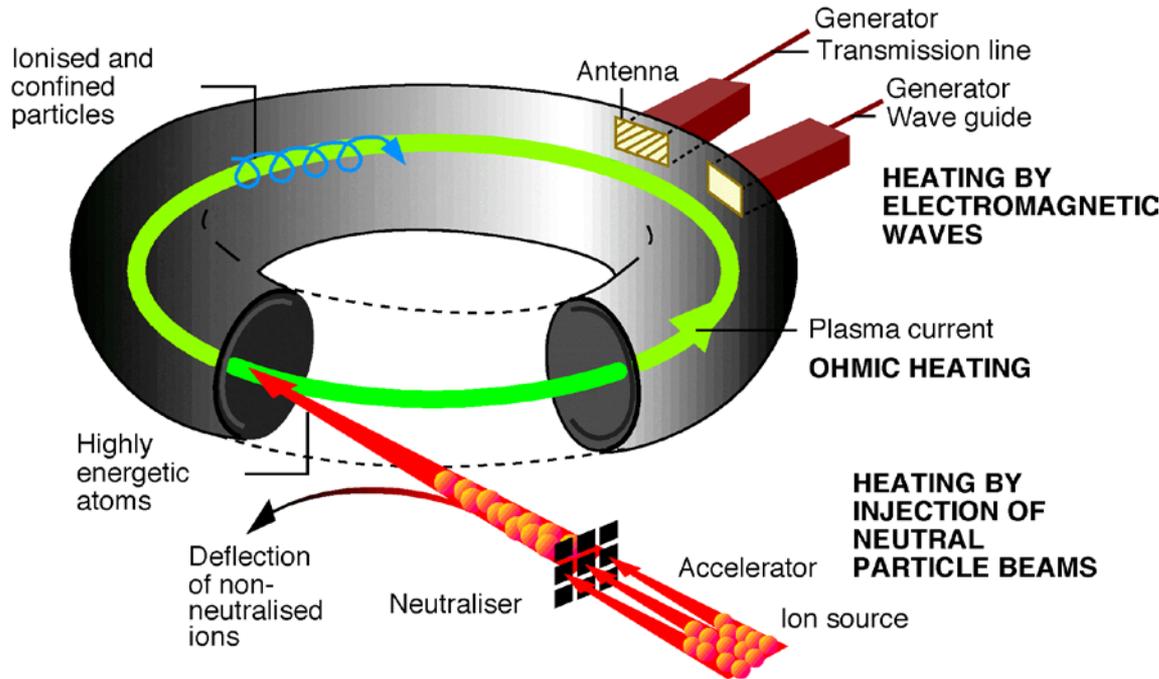
# “MHD with anisotropy in velocity, pressure”

- Pressure different parallel and perpendicular to field due mainly to *directed* neutral beam injection



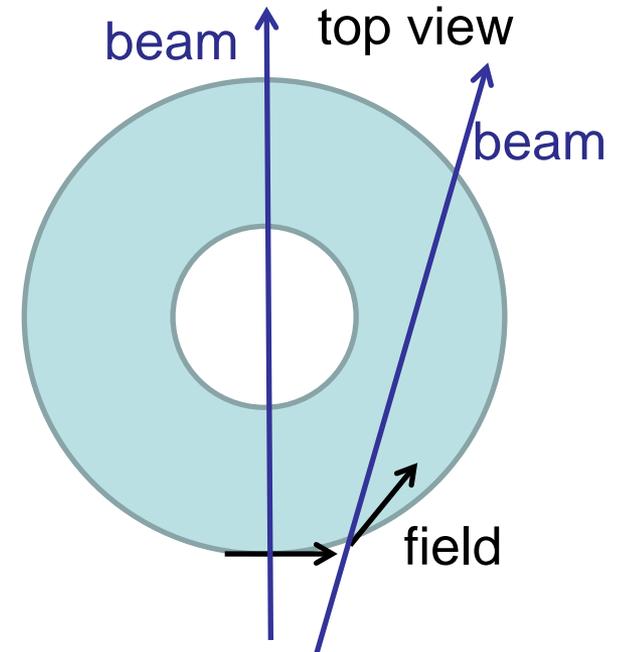
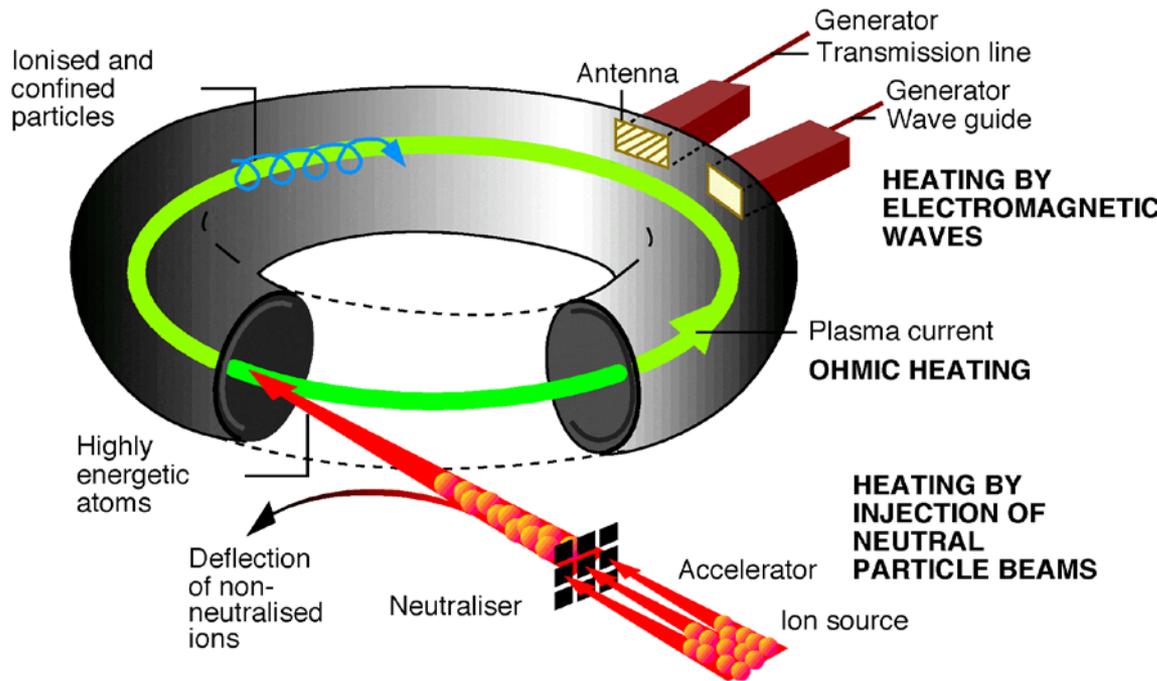
# “MHD with anisotropy in velocity, pressure”

- Pressure different parallel and perpendicular to field due mainly to *directed* neutral beam injection



# “MHD with anisotropy in velocity, pressure”

- Pressure different parallel and perpendicular to field due mainly to *directed* neutral beam injection



⇒ Pressure is a tensor

$$\bar{\mathbf{P}} = p_{\perp} \bar{\mathbf{I}} + \Delta \mathbf{B}\mathbf{B} / \mu_0, \quad \Delta = \frac{\mu_0 (p_{\parallel} - p_{\perp})}{B^2}$$

# Expected impact of anisotropy

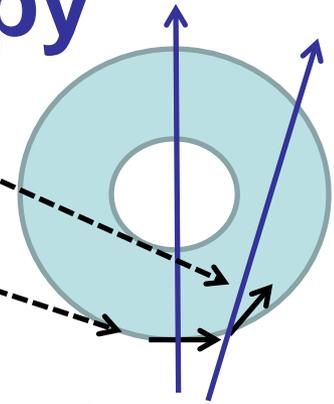
- Small angle  $\theta_b$  between beam, field  $\Rightarrow p_{\parallel} > p_{\perp}$
- Beam orthogonal to field,  $\theta_b = \pi/2 \Rightarrow p_{\perp} > p_{\parallel}$
- If  $p_{\parallel}$  sig. enhanced by beam,  $p_{\parallel}$  surfaces distorted and displaced inward relative to flux surfaces

[Cooper et al, Nuc. Fus. 20(8), 1980]

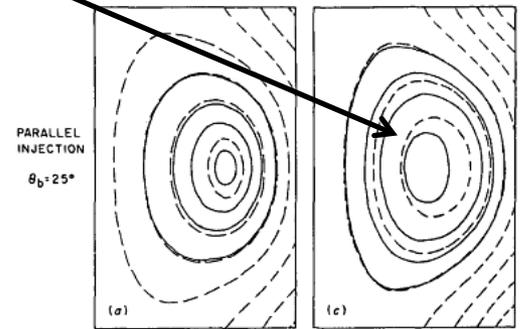
- If  $p_{\perp} > p_{\parallel}$ , an increase will occur in centrifugal shift :

[R. Iacono, A. Bondeson, F. Troyon, and R. Gruber, Phys. Fluids B 2 (8). August 1990]

- Obtain  $p_{\perp}$  and  $p_{\parallel}$  from moments of distribution function, computed by TRANSP



Peaked pressure profile      Broad pressure profile



Parallel pressure contours (solid)      Flux surfaces (dashed)

[M J Hole, G von Nessi, M Fitzgerald, K G McClements, J Svensson, PPCF 53 (2011) 074021]

# VMEC / SPEC comparison reveals chaos

Different toroidal cross-sections at  $\lambda = 0.4$

