Accurate calculation of Singular Current Densities in non-linear, ideal MHD equilibrium solutions

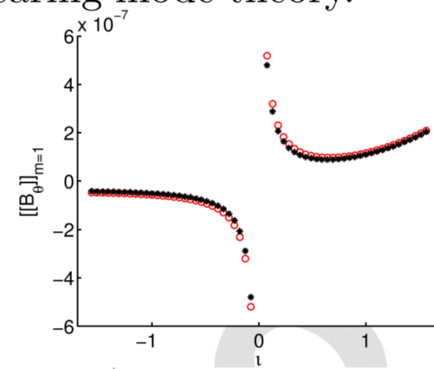
Max-Planck-Institut fur Plasmaphysik (Dr. J. Loizu) & Princeton Plasma Physics Laboratory (Dr. S.R. Hudson)

[PROBLEM] For decades, singular current densities have been predicted in ideal 3D MHD equilibria,

- From $\nabla \cdot \mathbf{j} = \nabla \cdot (\sigma \mathbf{B} + \mathbf{j}_\perp) = 0$, where $\mathbf{j}_\perp \equiv \mathbf{B} \times \nabla p / B^2$, derive $\mathbf{B} \cdot \nabla \sigma = -\nabla \cdot \mathbf{j}_\perp$.
- Solution for $\sigma = \dfrac{\mathbf{j} \cdot \mathbf{B}}{B^2}$, assuming $\mathbf{B} = \nabla\psi \times \nabla\theta + \iota(\psi)\nabla\zeta \times \nabla\psi$, is $\sigma_{m,n} = \underbrace{\dfrac{g_{m,n}(\psi)\, p'}{\iota\, m - n}}_{\text{Pfirsch-Schlüter}} + \underbrace{\Delta_{m,n}\, \delta(\psi - \psi_s)}_{\delta-\text{function}}$
- Conventional 3D, nonlinear, MHD equilibrium codes (such as NSTAB, VMEC) cannot resolve these currents because of numerical methods that assume continuous functions. . .

[IMPORTANCE] It is essential to resolve these singularities for (i) accurate equilibrium calculations, and these currents play a vital role in both (ii) linear stability theory *and* (iii) tearing mode theory.

[ADVANCEMENT] The STEPPED PRESSURE EQUILIBRIUM CODE (SPEC) [Hudson et al., PoP 19, 112502 (2012)] *does* allow for discontinuities and *can* resolve the singular currents!



Shown (below) is a sequence of equilibria [Loizu et al., PoP, 2015], where an island is successively "shielded" by ideal currents on so-called "ideal"-interfaces which are brought closer to the rational surface; and shown (right) is the resonant pressure-driven $1/x$ current-density, [SPEC (red squares); analytic (black stars)], in excellent agreement.