Workshop on Integrated Simulations for MFES
Category D

A Multiphysics and Multiscale Coupling of Microturbulence with MHD Equilibria

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Abstract

We propose to couple

1. GTS [1], a gyrokinetic turbulence code, based on the newly developed electromagnetic capability [2], with
2. SPEC [3], an MHD equilibrium code,

for the purpose of self-consistently obtaining a new magnetic configuration which reduces the anomalous transport due to microturbulence.

The proposed iterative scheme, which requires the codes to "talk to each other", is based on a recent realization [4] connecting the gyrokinetic Vlasov-Maxwell equations with the MHD equilibrium equations via the gyrokinetic vorticity equation and Ohm's law.

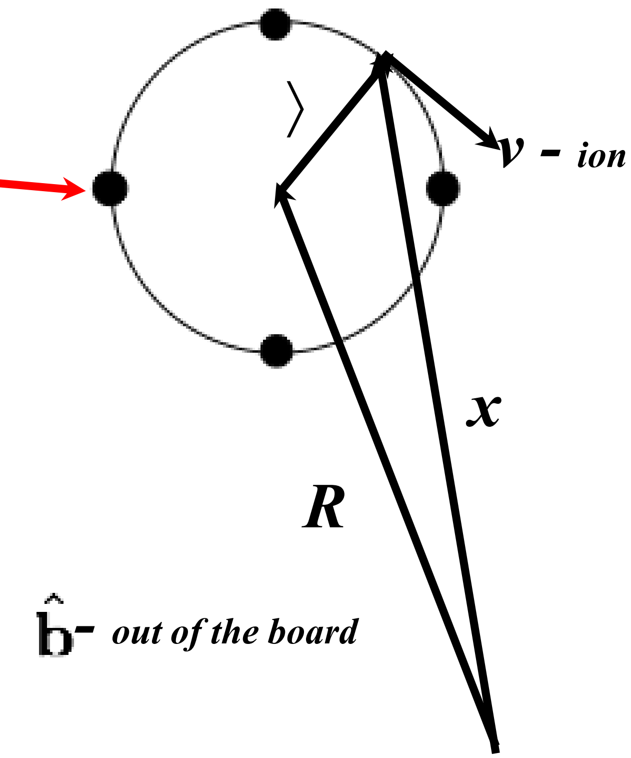
[1] W. X. Wang et al., PoP 13, 092505 (2006)
[2] E. A. Startsev et al., Sherwood Conference, NYC, NY (2015)
[3] S. R. Hudson et al., PoP 19, 112502 (2012)
[4] W. W. Lee, Sherwood Conference, NYC, NY (2015).

Gyrokinetic Current Densities

[Qin, Tang, Rewoldt and Lee, PoP **7**, 991 (2000); Lee and Qin, PoP **10**, 3196 (2003).]

$$\mathbf{J}_{gc}(\mathbf{x}) = \mathbf{J}_{\parallel gc}(\mathbf{x}) + \boxed{\mathbf{J}^{M}_{\perp gc}(\mathbf{x})} + \mathbf{J}^{d}_{\perp gc}(\mathbf{x})$$

$$= \sum_\alpha q_\alpha \langle \int F_{\alpha gc}(\mathbf{R})(\mathbf{v}_\parallel + \mathbf{v}_\perp + \mathbf{v}_d)\delta(\mathbf{R} - \mathbf{x} + \rho) d\mathbf{R} dv_\parallel d\mu \rangle_\varphi$$



FLR calculation

$\mathbf{v} \equiv (\mathbf{v}_\parallel, \mathbf{v}_\perp)$ - particle velocity

$$\mathbf{v}_d = \frac{v_\parallel^2}{\Omega_\alpha}\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}})\hat{\mathbf{b}} + \frac{v_\perp^2}{2\Omega_\alpha}\hat{\mathbf{b}} \times \frac{\partial}{\partial \mathbf{R}} lnB$$ - particle drifts

$$\mathbf{J}^{M}_{\perp gc}(\mathbf{x}) = -\sum_\alpha \nabla_\perp \times \frac{c\hat{\mathbf{b}}}{B} p_{\alpha\perp} \qquad p_{\alpha\perp} = m_\alpha \int (v_\perp^2/2) F_{\alpha gc}(\mathbf{x}) dv_\parallel d\mu$$

$$\mathbf{J}^{d}_{\perp gc} = \frac{c}{B}\sum_\alpha \left[p_{\alpha\parallel}(\nabla \times \hat{\mathbf{b}})_\perp + p_{\alpha\perp}\hat{\mathbf{b}} \times (\nabla lnB)\right] \qquad p_{\alpha\parallel} = m_\alpha \int v_\parallel^2 F_{\alpha gc}(\mathbf{x}) dv_\parallel d\mu$$

$$\mathbf{J}_{\perp gc} = \mathbf{J}^{M}_{\perp gc} + \mathbf{J}^{d}_{\perp gc} \quad = \frac{c}{B}\sum_\alpha \left[\hat{\mathbf{b}} \times \nabla p_{\alpha\perp} + (p_{\alpha\parallel} - p_{\alpha\perp})(\nabla \times \hat{\mathbf{b}})_\perp\right]$$

$$p_\alpha = p_{\alpha\parallel} = p_{\alpha\perp}$$

$$\boxed{\mathbf{J}_{\perp gc} = \frac{c}{B}\sum_\alpha \hat{\mathbf{b}} \times \nabla p_\alpha}$$

- **Gyrokinetic MHD Equations:** a reduced set of equations but in full toroidal geometry

-- Ampere's law

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c} J_\parallel$$

$$\nabla_\perp^2 \mathbf{A}_\perp - \underline{\frac{1}{v_A^2}\frac{\partial^2 \mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c}\mathbf{J}_\perp$$

Negligible for $\omega^2 \ll k_\perp^2 v_A^2$

$$\delta\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B} \qquad \mathbf{b} \equiv \frac{\mathbf{B}}{B}$$

-- Pressure Driven Current: $\mathbf{J}_\perp = \frac{c}{B}\sum_\alpha \mathbf{b} \times \nabla p_\alpha$

-- Vorticity Equation: $\frac{d}{dt}\nabla_\perp^2 \phi - 4\pi\frac{v_A^2}{c^2}\nabla \cdot (\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0 \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi \times \mathbf{b} \cdot \nabla$

-- Ohm's law: $E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b} \cdot \nabla\phi = \eta J_\parallel \rightarrow 0$

-- Equation of State: $\frac{dp_\alpha}{dt} = 0$

-- Normal modes: $\omega = \pm k_\parallel v_A$

• MHD Equilibrium

1. For a given pressure profile, we obtain the pressure driven current from

$$\boxed{\mathbf{J}_\perp = \frac{c}{B}\sum_\alpha \mathbf{b}\times\nabla p_\alpha}$$

2. We then solve the coupled equations of

$$\boxed{\begin{aligned} &\frac{d}{dt}\nabla_\perp^2\phi + \frac{v_A^2}{c}(\mathbf{b}\cdot\nabla)\nabla_\perp^2 A_\parallel - 4\pi\frac{v_A^2}{c^2}\nabla\cdot\mathbf{J}_\perp = 0 \\ &E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b}\cdot\nabla\phi = 0 \end{aligned}}$$

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi\times\mathbf{b}\cdot\nabla$$

3. If we look for a solution for $\phi \to 0$ which, in turn, gives $\frac{\partial A_\parallel}{\partial t} \to 0$,

this is then the equilibrium solution that satisfies the quasineutral condition of

$$\boxed{\nabla\cdot(\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0}$$

4. The GK vorticity equation retains all the toroidal physics, different than Strauss‘ equation [PF 77]

5. Perpendicular current is consisted of both a divergent free diamagnetic current and a magnetic drift current. Only the latter was originally included in Lee and Qin [PoP, 2003].

6. $\phi \approx 0$ also corresponds to the condition when fluctuations associated microturbulence cease to exist.

The Scheme

1. Use SPEC to give basic magnetic configuration to GTS

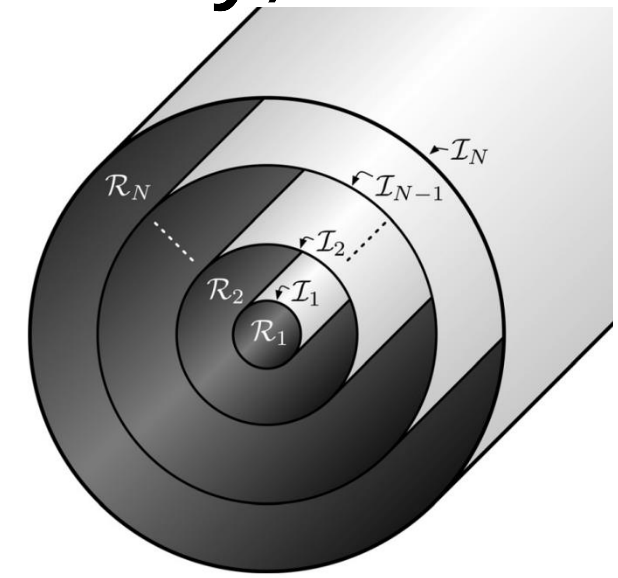
2. Use GTS to study microturbulence and to produce perturbed pressure and current

3. Give these information back to SPEC

4. Use SPEC to give a new magnetic configuration to GTC and so on ....

Since we use the nonlinearly modified profiles at every iteration and the equilibrium solutions are supposed to mimic the fluctuation free states, we should expect the system to evolve gradually to a state where fluctuations become less.

MHD equilibria ≡ constrained, minimum-energy states with given pressure, boundary, ...



1. SPEC minimizes the global plasma energy, $W \equiv \int_{\mathcal{V}} \left(\frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv$ where the pressure, $p(\psi)$, etc. is a given function of toroidal flux, ψ.

2. The volume integral is partitioned (and parallelized), $\int_{\mathcal{V}} dv \equiv \sum_{i=1}^{N} \int_{\mathcal{V}_i} dv$

3. The simplest constraint is conserved helicity: $H_i \equiv \int_{\mathcal{V}_i} \mathbf{A} \cdot \mathbf{B}\, dv = H_{i,o} = const.$ in each $\mathcal{V}_i$
   and the ideal-constraint: $\delta \mathbf{B} = \nabla \times (\boldsymbol{\xi} \times \mathbf{B})$ at each $\mathcal{I}_i$

4. The multi-region, relaxed MHD (MRxMHD) energy functional is
   $F \equiv \sum_{i=1}^{N} \left[W_i - \frac{\mu_i}{2} \left(H_i - H_{i,o} \right) \right]$ [Hole, et al. JPP, 72:1167, 2006]
   The equilibrium state satisfies $\nabla \times \mathbf{B} = \mu \mathbf{B}$ in each $\mathcal{V}_i$ and $[[p + B^2/2]] = 0$ across each $\mathcal{I}_i$.

5. If $N = 1$, recover globally-relaxed, "Taylor" state.
   If $N \to \infty$, recover globally-ideal, $\nabla p = \mathbf{j} \times \mathbf{B}$ [Dennis et al. PoP, 20:032509, 2013]
   If N is finite, flat pressure and islands at resonances; pressure jumps at arbitrarily many 'KAM' surfaces.

6. SPEC [Hudson et al. PoP, 19:112502, 2012] is the only equilibrium code that simultaneously
   (1) is based on an energy functional (2) computes magnetic field consistent with given pressure profile
   (3) can accurately compute singular currents in ideal-MHD equilibria [Loizu et al. PoP, 22:022501, 2015]
   (4) allows for partially relaxed fields, magnetic islands, chaos, (5) is parallelized.