Fluid Models for burning and 3D plasmas: complementing the kinetic paradigm

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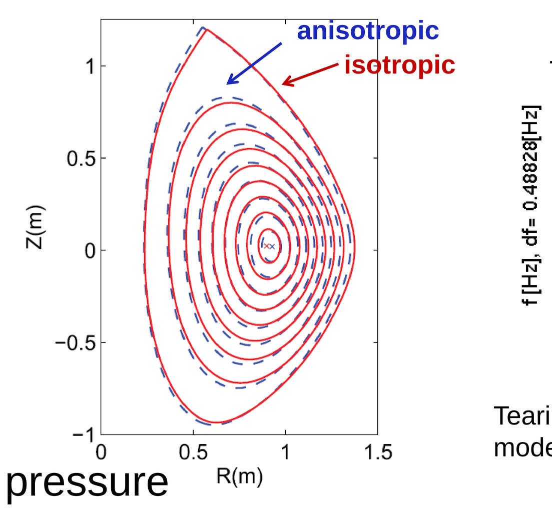
A1. Equilibrium with flow, anisotropy

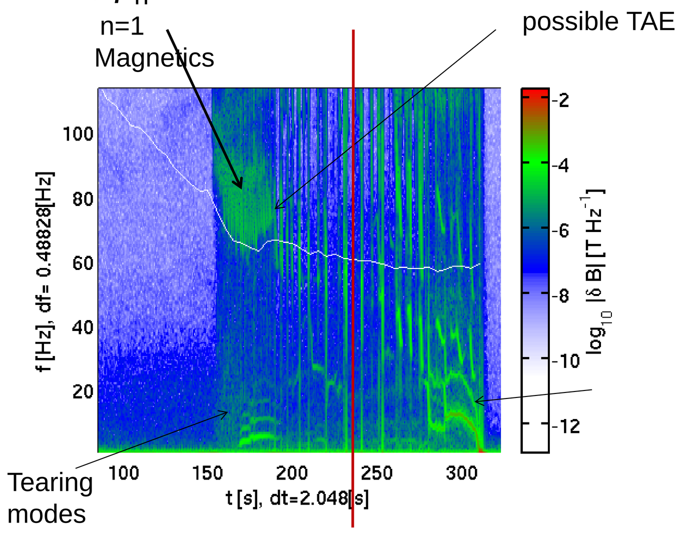
- Inclusion of anisotropy and flow in equilibrium MHD equations [R. Iacono, et al Phys. Fluids B 2 (8). 1990]
- Implemented in EFIT TENSOR for equilibrium reconstruction [Fitzgerald, Appel, Hole, Nucl. Fusion **53** (2013) 113040]
- Implemented in HELENA-ATF for stability studies [Qu, Fitzgerald, Hole, PPCF **56** (2014) 075007]

A2. Anisotropy on MAST: #29221

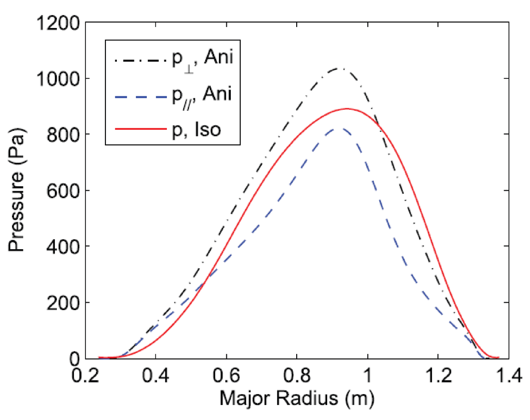
- 1.6MW NB heating, I_p = 0.9MA, β_n~3

Flux surfaces

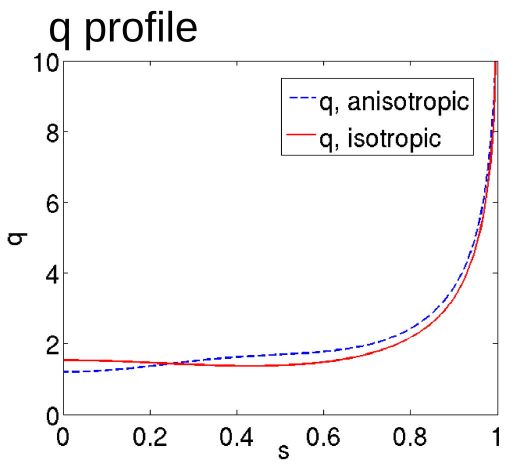




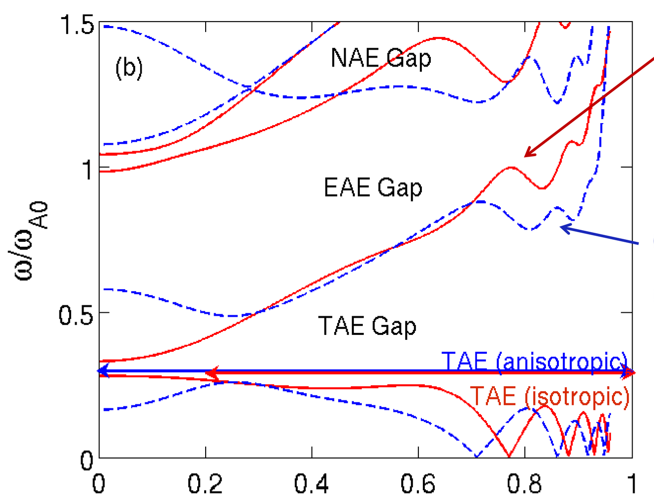
pressure



q profile



n=1 incompressible continuum



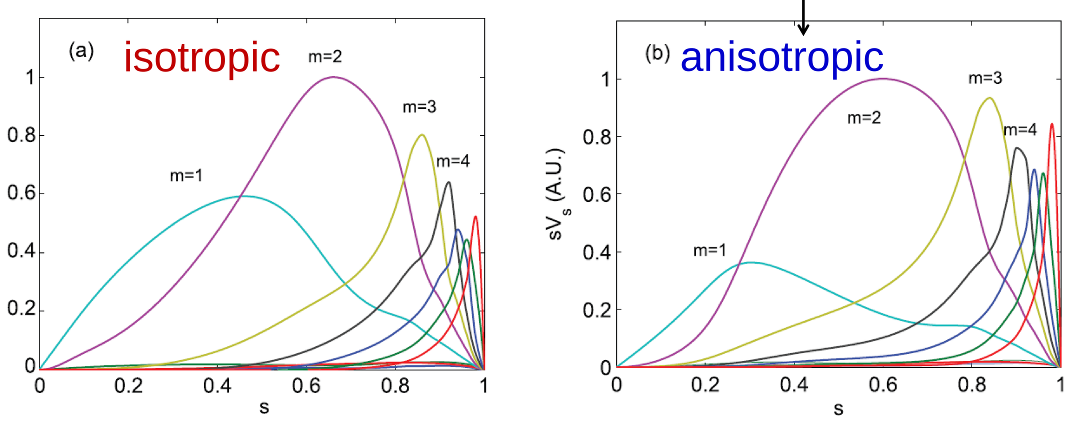
Computed with CSMISH (isotropic)

Computed with CSMISH-A (anisotropic)

[Qu Z.S., Hole M.J. and Fitzgerald M. 2015 *PPCF* **57** 095005]

n = 1 global TAE radial structure

MISHKA-A



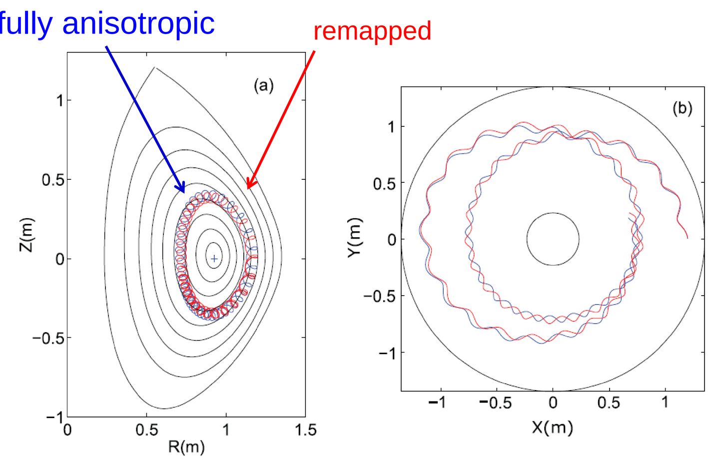
A3. Wave-particle Stability

- Rigorous approach: implement anisotropic equations of motion in HAGIS [Pinches *et al*., Comput. Phys. Comm. (1998)]

Approximate approach:

1. Compute anisotropic equilibrium with HELENA+ATF.
2. Calculate $\langle J_\phi \rangle$ and $\langle p^* \rangle$, input as "isotropic" into HELENA.
3. Rescale total current s.t. $q(\psi)$ matches HELENA+ATF.

- Calculate passing (and trapped) orbits for MAST #29221 for fully anisotropic and remapped equilibria.
- Similar guiding-centre trajectories, < 1% difference in poloidal orbit frequency ⇒ approach will be a good approximation.



Fast ion orbits calculated using CUEBIT for MAST #29221 for ions with E = 21keV and =0.4

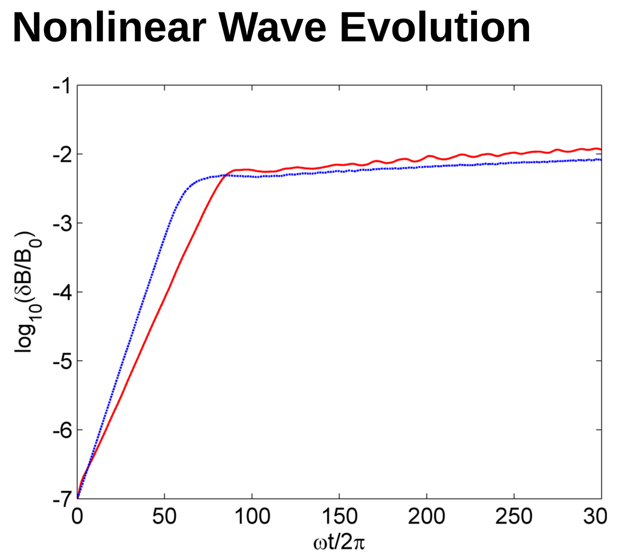
Distribution function:

$$f(E,s,\Lambda) = f(E)f(s)f(\Lambda)$$

Slowing down — Guassian — $\delta(\Lambda)$

n_f / n_0 = 14.5%

Nonlinear Wave Evolution



Growth rate 35% larger in anisotropic case : Resonance maps show larger anisotropic spatial drive

Saturation amplitude 22% larger in isotropic case. : Isotropic saturation amplitude larger due to bounce frequency

- **Differences in isotropic and anisotropic equilibrium and mode structure ⇒ differences in resonant regions, growth rates and saturation amplitudes.**

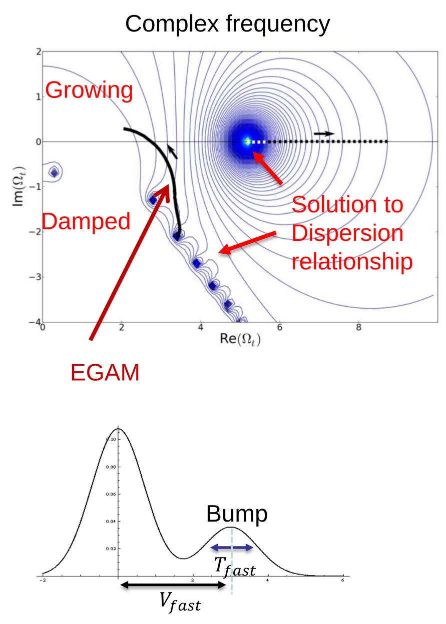
B1. Zoo of GAMS

	Conventional GAMs	ICRH driven (E?)GAMs	NBI driven EGAMs
Frequency	$\sim\sqrt{\frac{7}{4}T_i + T_e + O(\frac{1}{q^2})}$	Same as conventional GAMs	Determined by fast ions (can be half of the thermal GAM)
Drive	Nonlinear interaction with turbulence	ICRH trapped fast ions, positive dF/dE	NBI passing fast ions, positive dF/dE
Localization	Local, edge	Core	Global
Observation	Nearly all machines	JET	DIII–D, ASDEX–U, LHD, HL–2A

B2. EGAMs

- Existing hybrid theory: fluid bulk, kinetic fast ions. Driven unstable by inverse Landau damping [Fu, G.Y. PRL. **101**, 185002 (2008).]
- Girardo PoP 2014 find unstable mode emerges from Landau poles. *Excited Landau poles?*
- We find unstable mode exist even when beam is cold (small thermal spread), and wave drive is negligible.
- Fluid treatment valid for cold fast ions.
- Could fluid theory describe the mode? → *Simpler, better understanding*

[Zarzoso, D. *et al.* Nucl. Fusion **54**, 103006 (2014).]
[Girardo, J.-B. *et al.* Phys. Plasmas **21**, 092507 (2014).]



B3. Fluid model of EGAM

[Qu, Z.S. *et al.* Phys. Rev. Lett. **116**, 095004 (2016).]

- Large aspect ratio, circular cross section : (r, θ, φ)
- Electrostatic perturbations ($\tilde{\boldsymbol{B}} = 0$)

Electrons	Thermal ions	Fast ions
No flow $n_e = n_i + n_{fast}$ T_e	No flow n_i T_i	$\boldsymbol{V_0} = V_{fast}\boldsymbol{b}$ n_{fast} $P_{\parallel fast}, P_{\perp fast}$

- Thermal/Fast ion response : the double-adiabatic (CGL) closure

$$\frac{dp_{\parallel s}}{dt} = -p_{\parallel s}\nabla\cdot\mathbf{v_s} - 2p_{\parallel s}\mathbf{b}\cdot(\mathbf{b}\cdot\nabla\mathbf{v_s})$$

$$\frac{dp_{\perp s}}{dt} = -2p_{\perp s}\nabla\cdot\mathbf{v_s} + p_{\perp s}\mathbf{b}\cdot(\mathbf{b}\cdot\nabla\mathbf{v_s})$$

Why CGL?

- It can give the right thermal GAM frequency $\sim\sqrt{\frac{7}{4}T_i + T_e + O(\frac{1}{q^2})}$
- Heat flow is not important in our case (mode frequency far from thermal frequency of thermal/fast ions)

- Electrons : isothermal response to the field

- The momentum equation for thermal/fast ions, electrons

$$m_s n_s\left(\frac{\partial\tilde{\boldsymbol{V}}_s}{\partial t} + \frac{\tilde{n}_s}{n_s}\boldsymbol{V}_s\cdot\nabla\boldsymbol{V}_s + \boldsymbol{V}_s\cdot\nabla\tilde{\boldsymbol{V}}_s + \tilde{\boldsymbol{V}}_s\cdot\nabla\boldsymbol{V}_s\right)$$
$$= n_s e(-\nabla\tilde{\Phi} + \tilde{\boldsymbol{V}}_s\times\boldsymbol{B}) - \nabla\cdot\tilde{\boldsymbol{P}}.$$

- Add up species and use quasi-neutrality: $\nabla\cdot J = 0$
  - Keep all terms (finite orbit width): Global dispersion equation

$E\times B$ drift — Parallel velocity — Magnetic drift velocity — Perturbed velocity

$$\tilde{V} = \tilde{V}_E\frac{B_0}{B}\hat{\theta} + \sum_{m=-2}^{m=2}(\tilde{V}_{\parallel m}\boldsymbol{b} + \cancel{\tilde{V}_{dm}\boldsymbol{b}\times\boldsymbol{\kappa}})e^{im\theta}$$

- Drop $\tilde{V}_{dm}$ (zero orbit width): Local dispersion equation

B4. Local dispersion relationship

$$D(\omega) = 1 - (1-\alpha)\frac{\omega^2_{\mathrm{GAM}}}{\omega^2} - \alpha G(\omega) \qquad \alpha = n_{fast}/n_{total}$$

Bulk response (thermal GAM) — Fast ion response

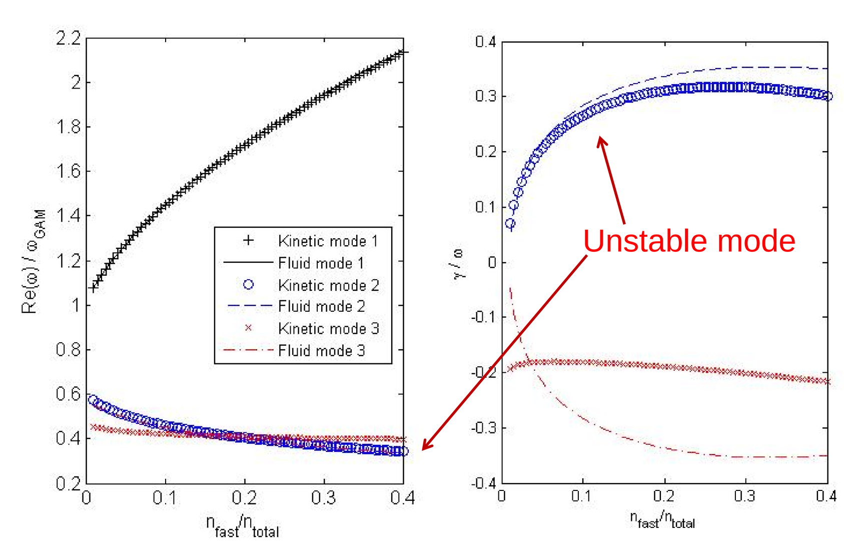
- For a bump on tail distribution with small beam thermal spread
  - **Doppler shift** of the wave in the static frame of fast ions

$$G(\omega) \approx \frac{\frac{3}{2}\omega_b^2 q^2}{\omega^2 - \omega_b^2} + \frac{\omega_b^4 q^2}{(\omega^2 - \omega_b^2)^2}$$

ω_b: fast ion transit frequency

- Three roots are presented, depends on the relationship between ω_b, ω_{GAM}, q and fast ion density.

$$\omega_b < \omega_{GAM}:\ \omega_b = 0.58\omega_{GAM},\ q = 4,\ T_{fast}/T_i = 0.25$$



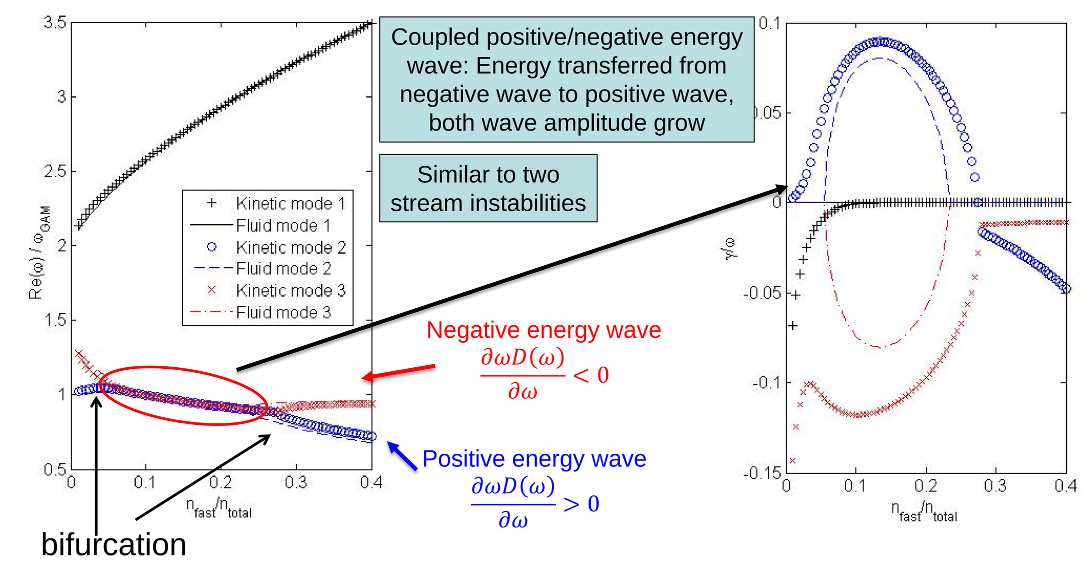
Fluid Model

zero turn-on threshold in α

$$\gamma \approx \frac{q\omega_b^2\sqrt{\alpha}}{2\sqrt{\omega_{GAM}^2 - \omega_b^2}}$$

But no wave–particle interaction in fluid theory
⇒ Drive does *not* come from wave-particle interaction

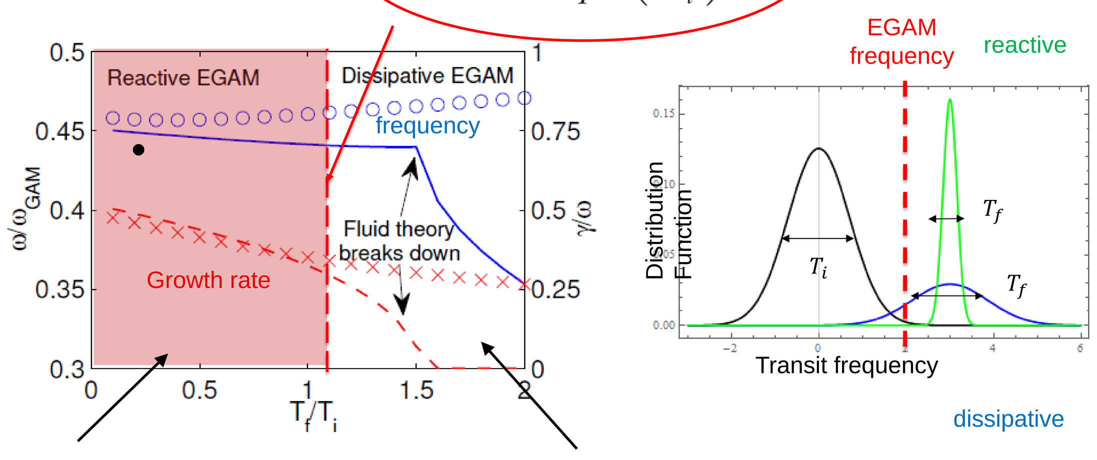
$$\omega_b > \omega_{GAM}:\ \omega_b = 1.76\omega_{GAM},\ q = 2,\ T_{fast}/T_i = 1$$



B5.Reactive/dissipative EGAMs

- Reactive (energy conserved) EGAM [Qu PRL 2016]
- Dissipative (energy not conserved, e.g. wave-particle driven) EGAMs [Fu PRL 2008]

To be reactive: $|\omega - \omega_b| > \frac{1}{qR}\left(\frac{2T_f}{m_i}\right)^{1/2}$

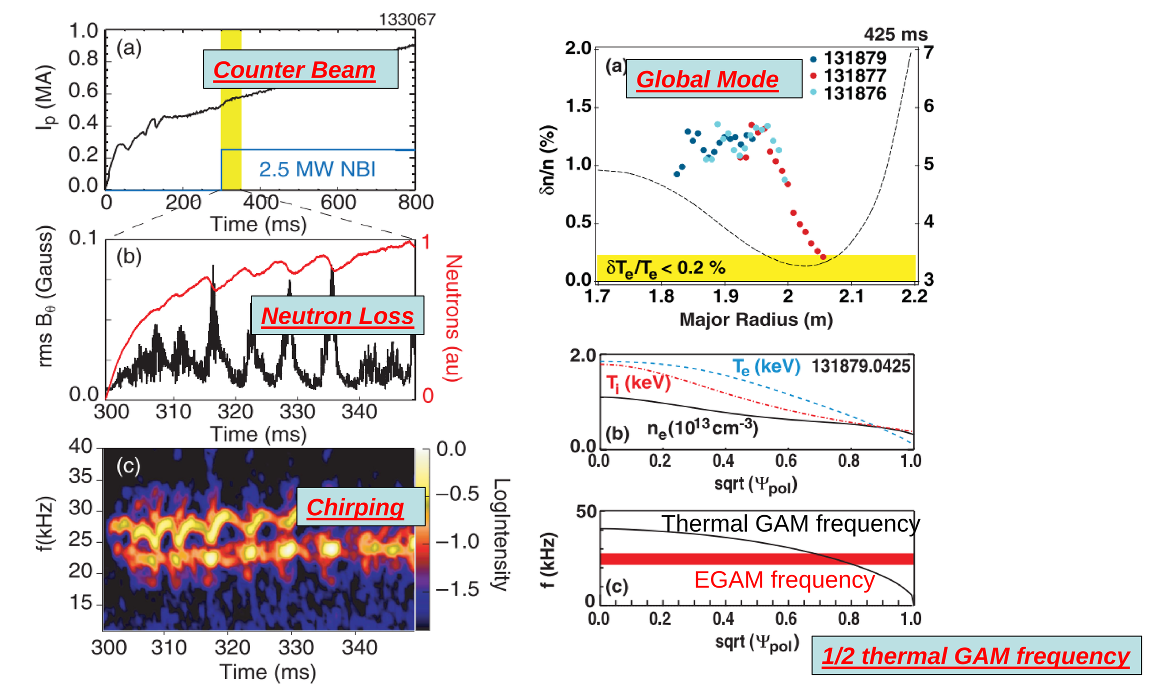


Reactive EGAM
Two-stream instabilities

Dissipative EGAM
Bump-on tail instabilities

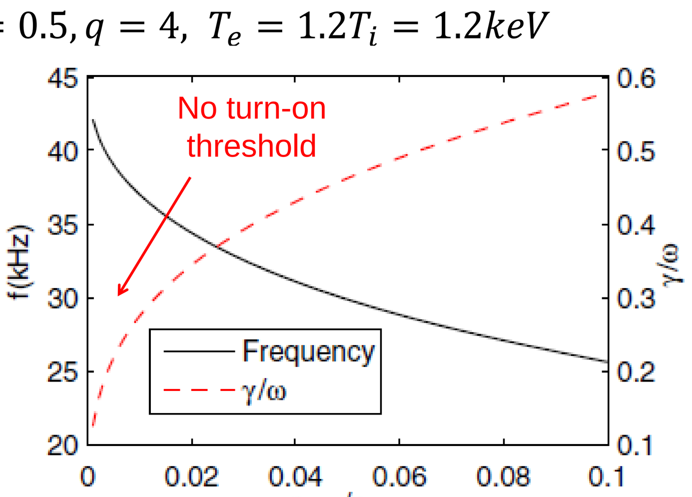
B6. Application to EGAM in DIII-D

[Nazikian, R. *et al.* Plasma. Phys. Rev. Lett. **101**, 185001 (2008).]



- Instant turn on (~1ms) of the mode, much faster than slowing down time (~a few 10ms).

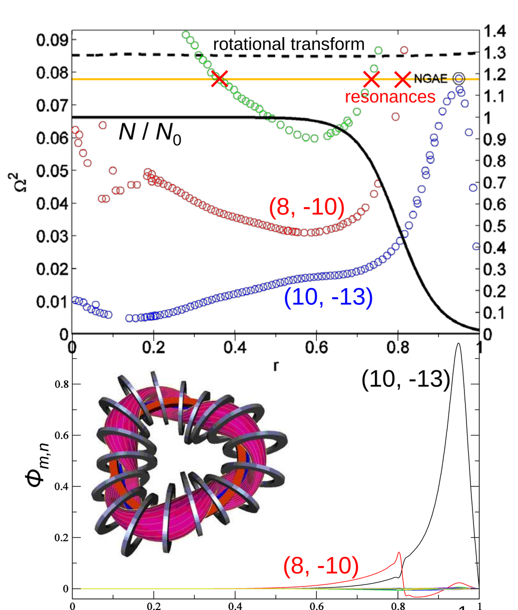
- $F(E,\Lambda) = \delta(E - E_0)\delta(\Lambda - \Lambda_0)$
- For DIII-D conditions
  - $E_0 = 75keV, \Lambda_0 = 0.5, q = 4,\ T_e = 1.2T_i = 1.2keV$



C1. 3D Continuum Damping

- Commonly calculated based on limit of vanishing resistive / kinetic effects
- Demonstrated perturbative treatment does not agree with the accepted result in the limit of small damping. [Bowden, Könies, Hole, Gorelenkov, Dennis, PoP, 21, 052508 (2014)]
- Developed singular finite element technique to compute continuum damping. [Bowden, M. J. Hole, PoP, 22, 022116 (2015)]
- Implementation of complex contour technique into MHD code CKA. **First calculation of continuum damping in 3D for realistic configurations.** [Bowden, Hole, Könies, PoP, 22, 092114 (2015)]

Complex contour example: continuum damping of m=10, n=-13 NGAE due to resonance with m=8, n=-10 continuum branch



Convergence of damping to γ/ω ≈5.8×10^{-3} on 100 × 60 ×20 mesh

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