Progress on Nonlinear Resistive MHD Code Verification Problems with M3D-C1

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Outline

1. Linear resistive MHD test problems for JA-2

2. Nonlinear resistive MHD test problems for JA-2

3. Nonlinear resistive MHD test problem with resistive wall and error fields

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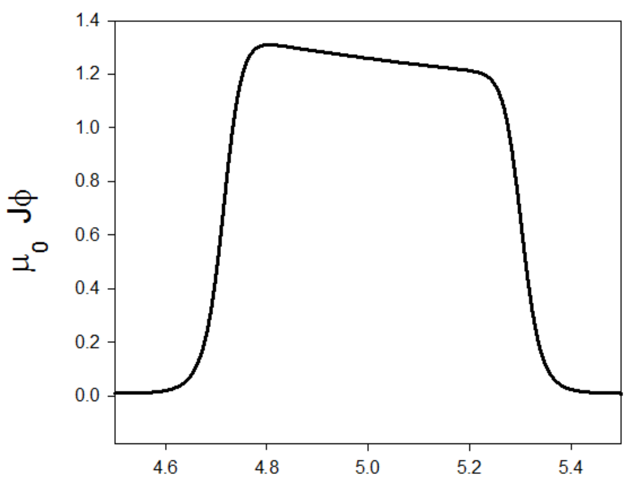
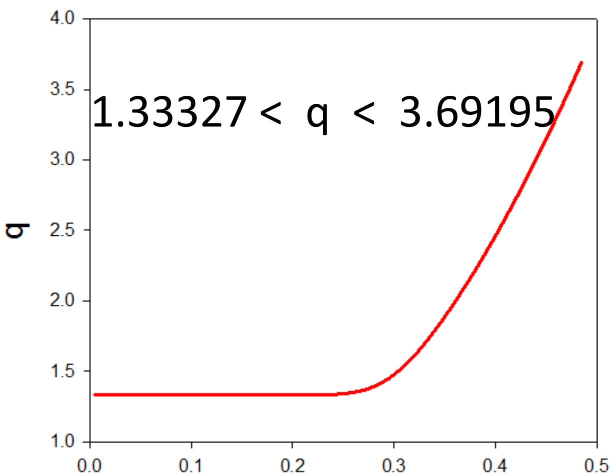
Progress on ITPA JA-2 benchmark with M3D-C1

We started with a cylindrical equilibrium with a given q(r) profile. This was then converted to a toroidal A=10 equilibrium with the same q(ψ). This was unstable to both n=1 and n=2 modes.

$$q(r) = 1.33\left[1+\left(r^2/.354\right)^4\right]^{1/4}$$



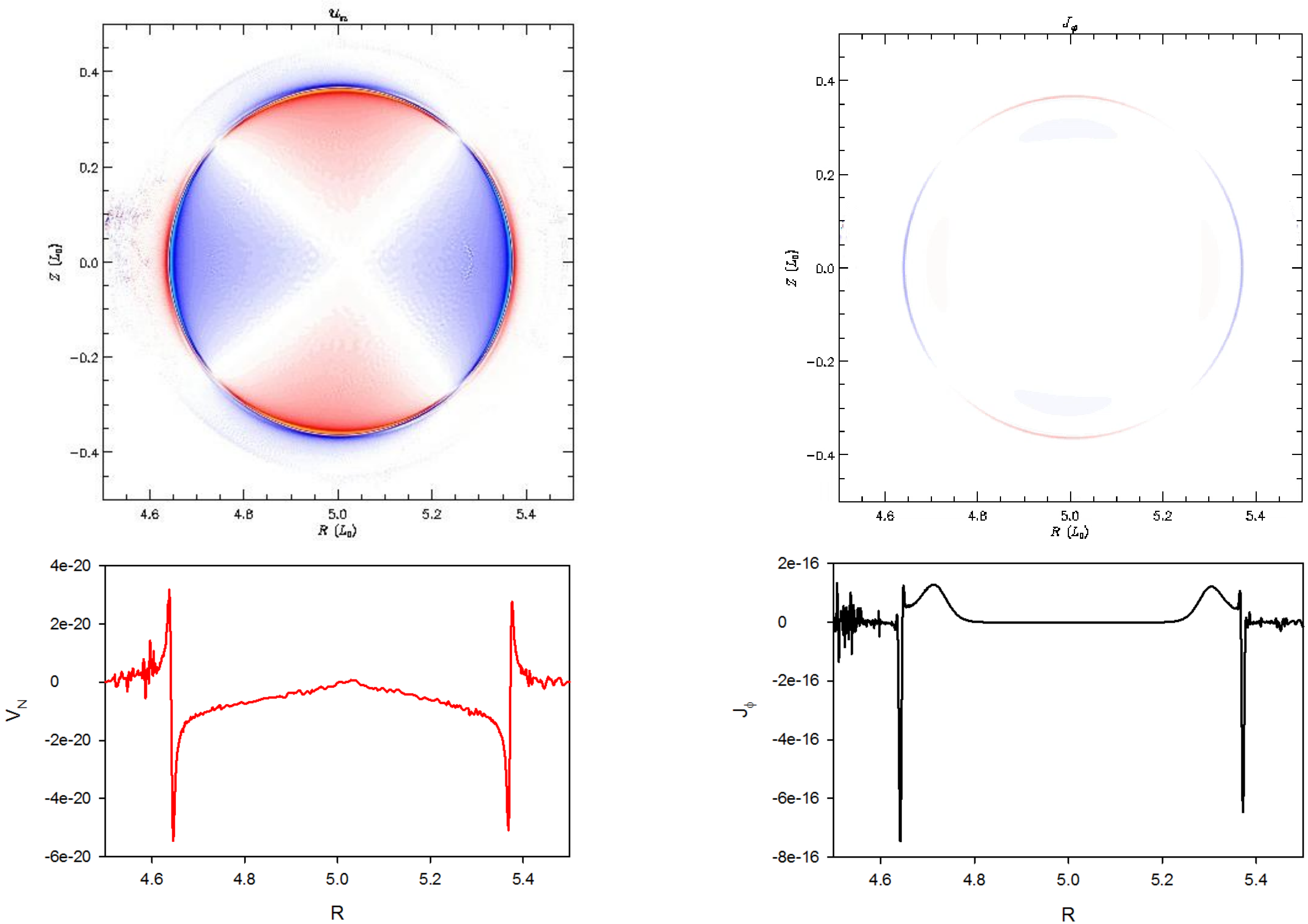


Equilibrium parameters (SI units): R=5, a=0.5, B_T = 4.2, $n_0 = 10^{20}$

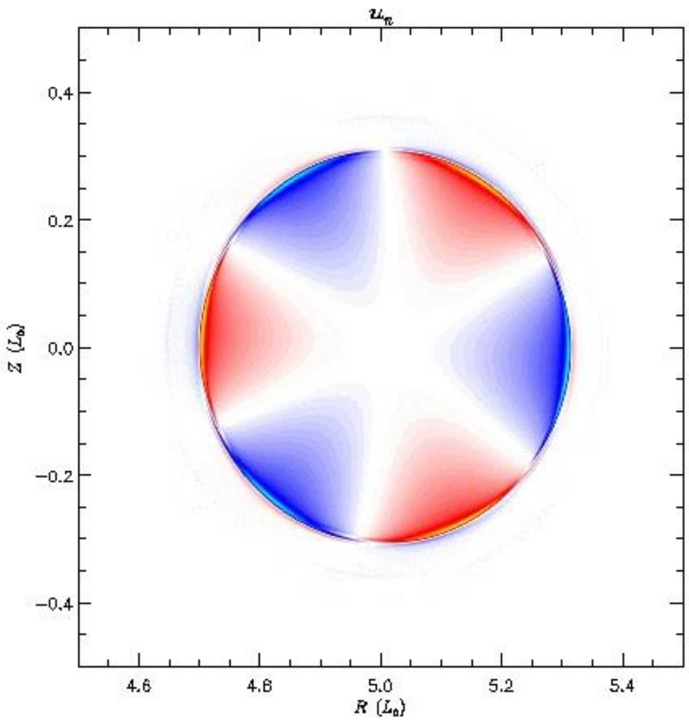
$$\eta = 0.21\times10^{-6}\eta_0, \qquad \nu = 10^{-8}\nu_0, \qquad t_0 = \left[\mu_0 n_0 M_i\right]^{1/2} = 4.58\times10^{-7}$$

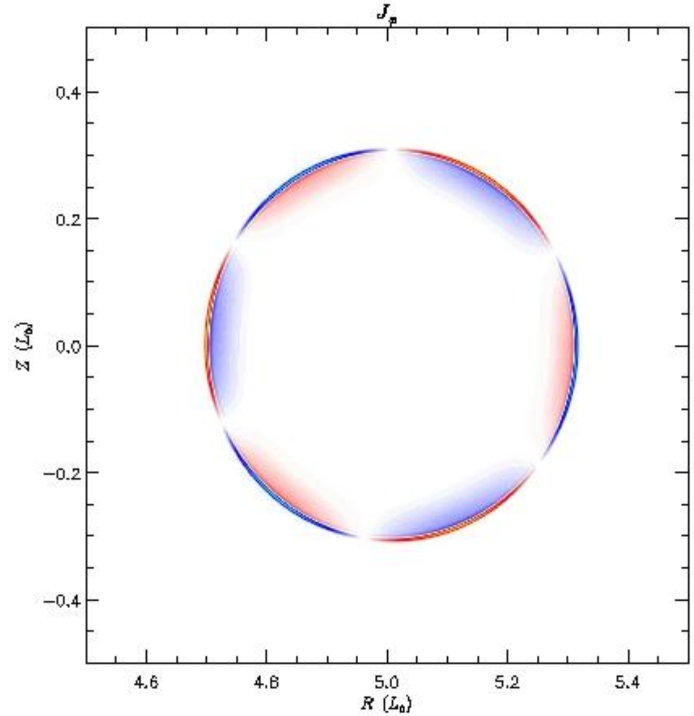
$$\eta_0 = \left[\frac{\mu_0}{n_0 M_i}\right]^{1/2} = 7.52, \qquad \nu_0 = \left[\frac{n_0 M_i}{\mu_0}\right]^{1/2} = 0.132, \qquad S \equiv \frac{a^2 B_T}{R}\frac{\eta_0}{\eta} = 10^6$$

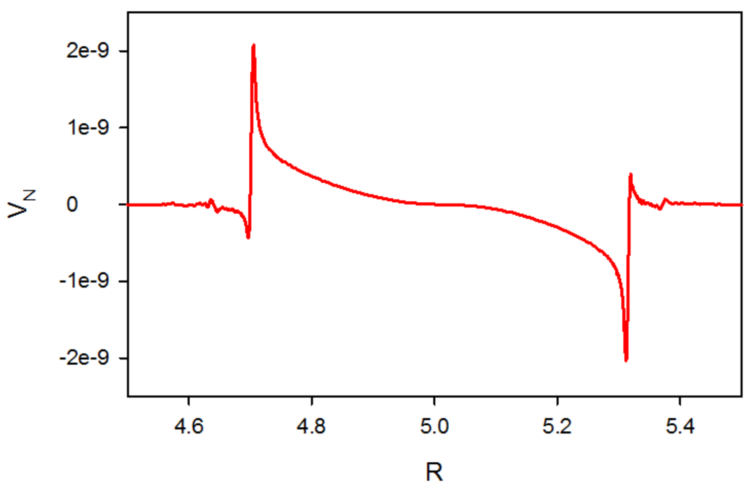
2. Normal displacement and perturbed current eigenfunctions for n=1 mode

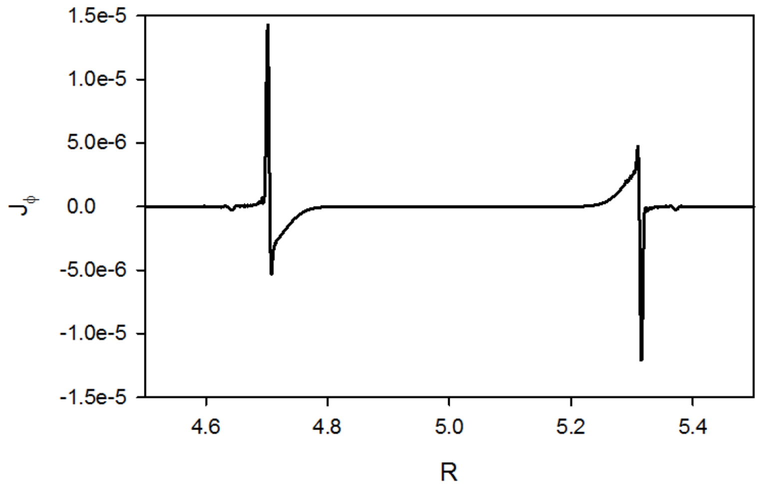


3. Normal displacement and perturbed current eigenfunctions for n=2 mode

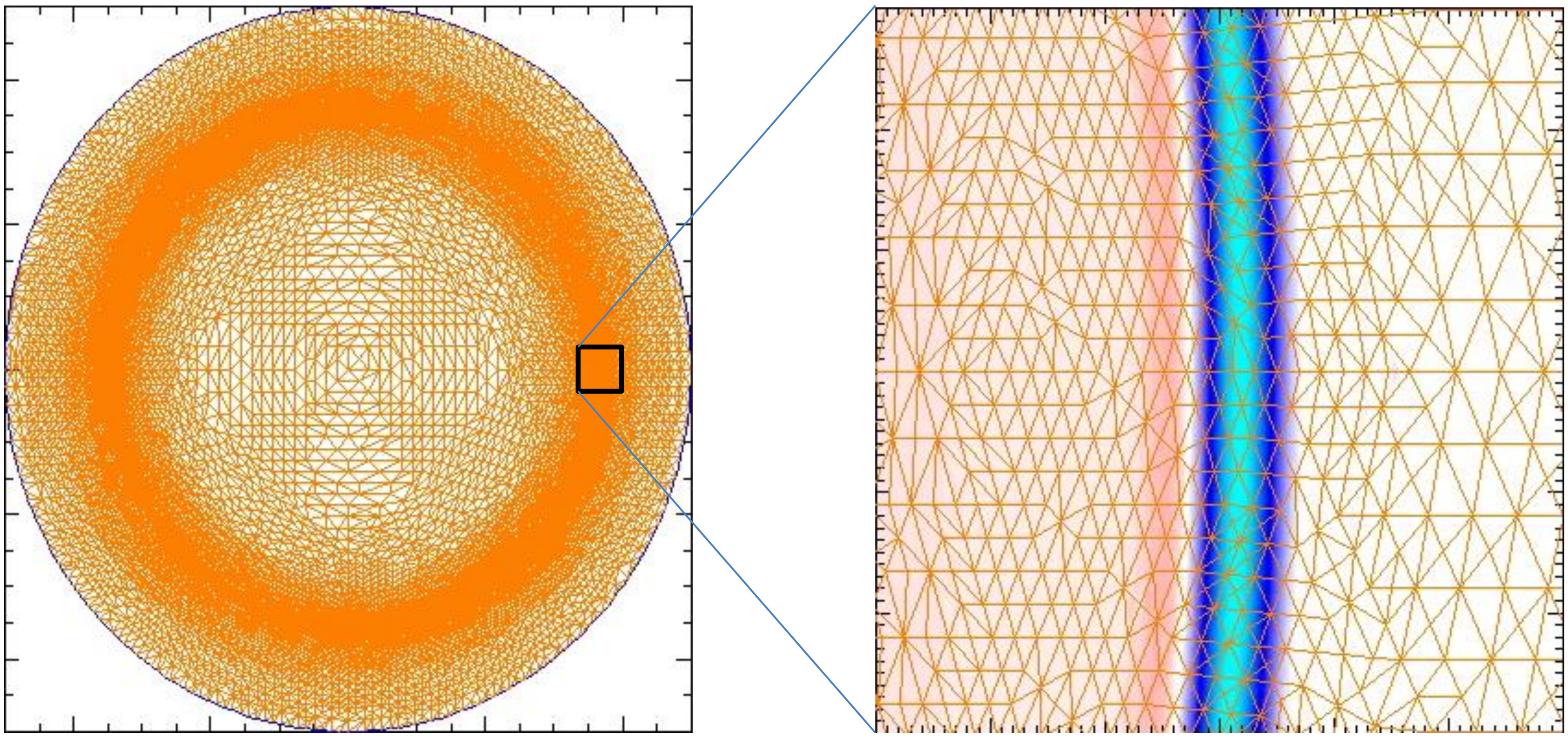






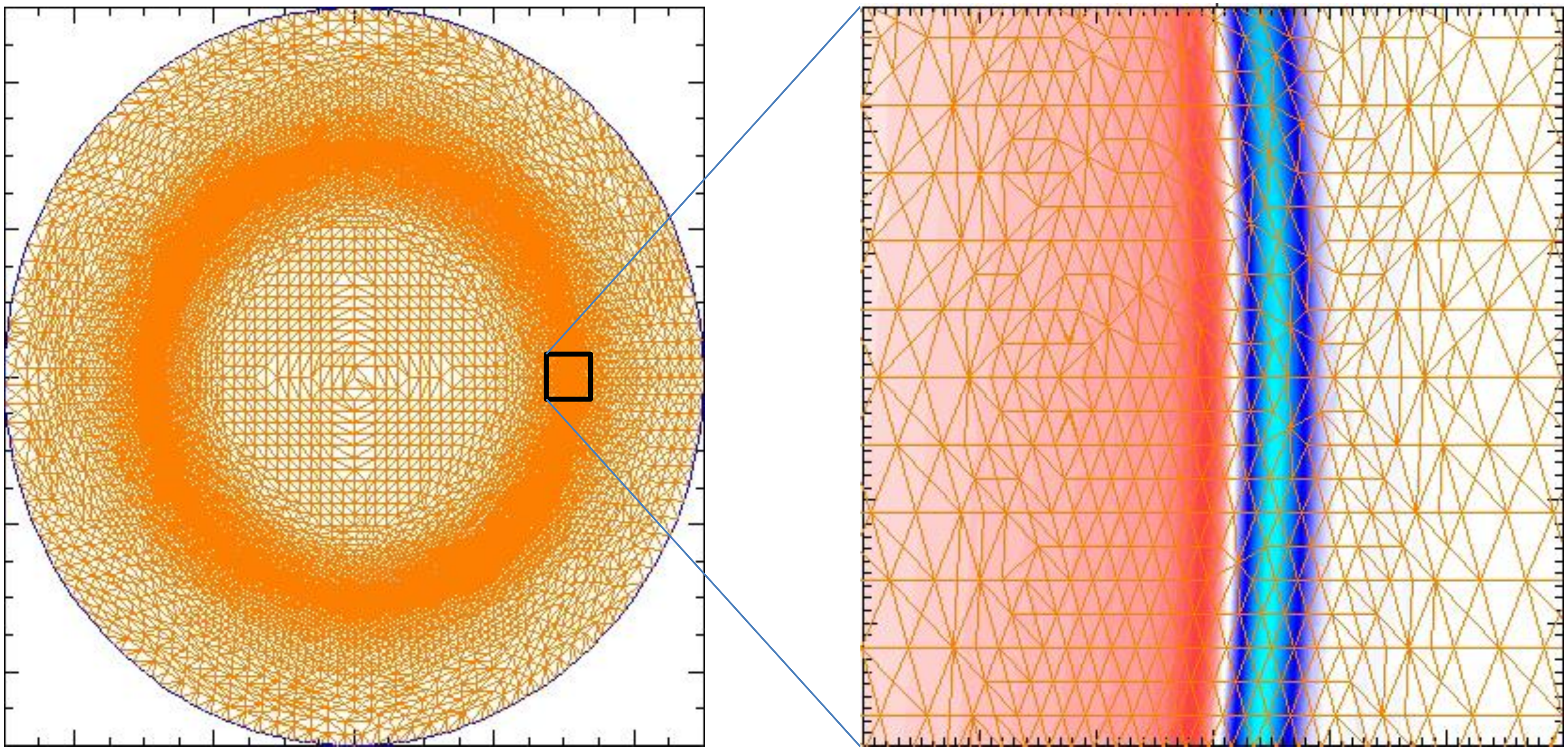


4. Adapted mesh used for n=1 mode



Mesh size varies from 0.002 to 0.03

5. Adapted mesh used for n=2 mode



Mesh size varies from 0.002 to 0.03

6. Convergence study for the S=10^6, β=0, A=10 equilibrium

DT	ΔX_{min}	$\gamma_{n=1} t_H$	$\gamma_{n=2}\, t_H$
5 τ_A	0.002	**1.64 E-3**	**7.19 E-3**
5 τ_A	0.004	1.70 E-3	7.87 E-3
10 τ_A	0.002	1.65 E-3	7.29 E-3

$$\tau_H \equiv \frac{R}{B_T}\left[\mu_0 n_0 M_i\right]^{1/2}$$

7. Dependence on geometry and beta

n=1

geom	β	$\gamma\tau_H$
A=10	0	1.64 E-3
A=10	10^{-6}	1.13 E-3
Cyl	0	1.49 E-3
Cyl	10^{-6}	1.49 E-3

n=2

geom	β	$\gamma\tau_H$
A=10	0	7.19 E-3
A=10	10^{-6}	5.00 E-3
Cyl	0	2.89 E-3
Cyl	10^{-6}	2.00E-3

- n=2 is always more unstable than n=1
- Finite pressure is stabilizing in torus, not so much in cylinder
- Toroidal geometry is more unstable than cylinder, especially for n=2

8. Test of growth rate with different numerical options:

Cylindrical test case:

p_0 = 1.E-4, dt = 2.0, μ = 1.e-5, η =1.e-5 (S = 2.1 x 10^4), uniform mesh Δx ~ 0.03

isplitstep=1							isplitstep=0
	impmod=0			impmod=1			
	16	32	linear	16	32	linear	linear
n=1	.0063	.0063	.0063	.0070	.0070	.0071	.0072
n=2	.0048	.0048	.0048	.0064	.0064	.0064	.0066

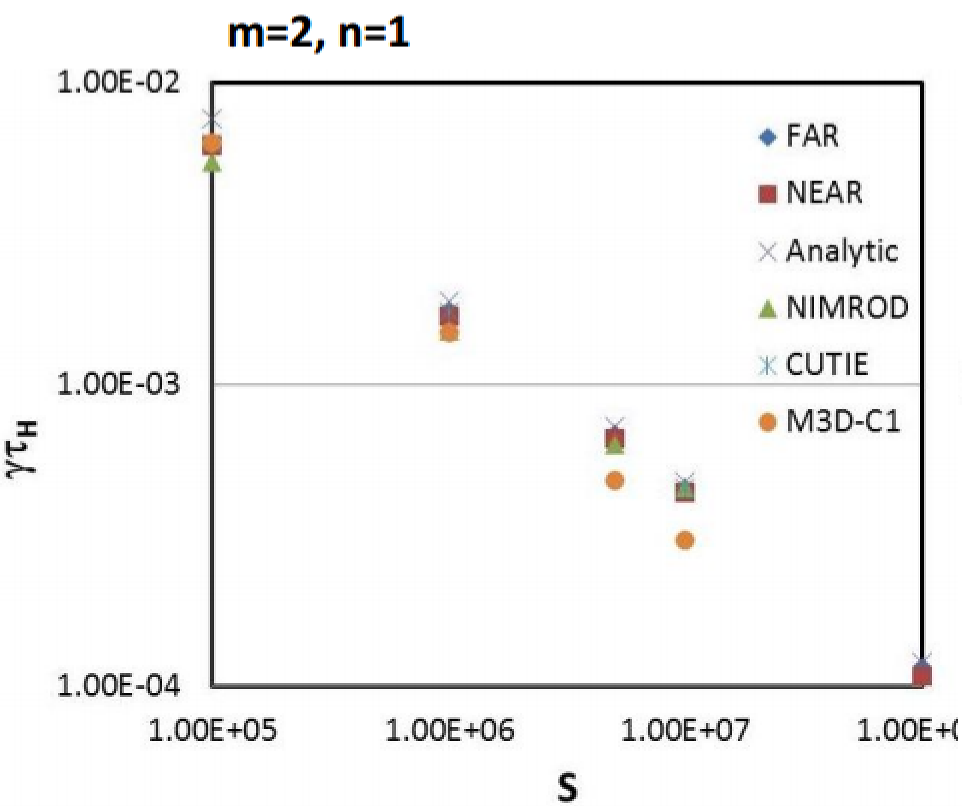
Conclusions:

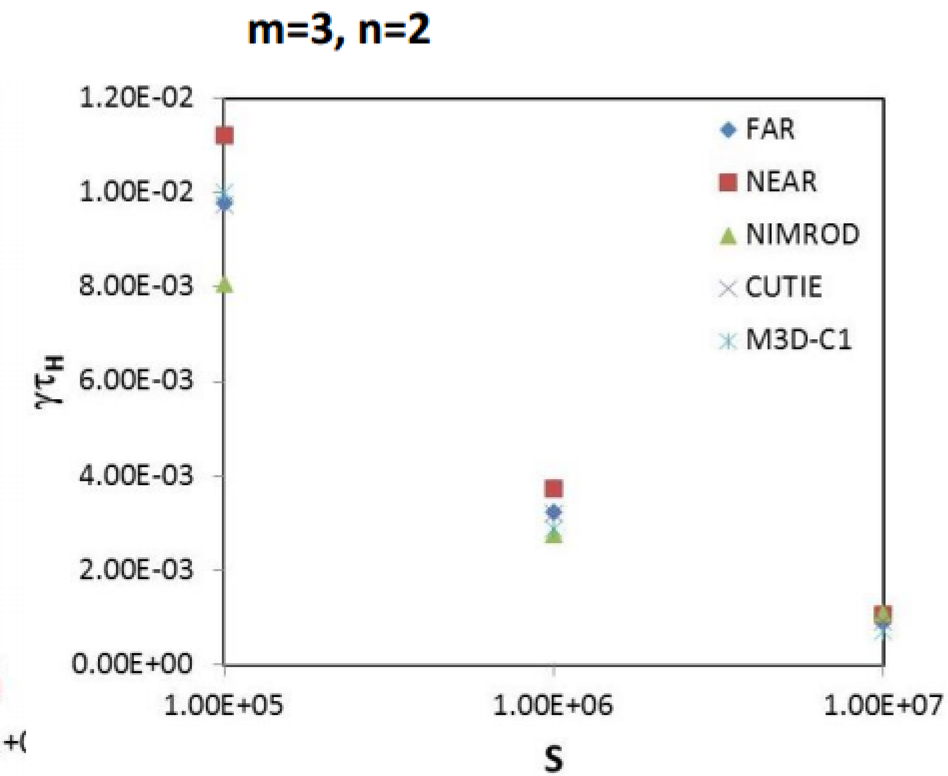
1. isplitstep=0 is most accurate, but only viable for linear runs
2. impmod = 1 (Caramana advance) more accurate than impmod=0
3. Good convergence in toroidal mode number for low-n modes (1 and 2)

For explanation of impmod=0,1, see: Ferraro and Jardin, JCP 228 (2009) 7742-7770

Linear Benchmarking

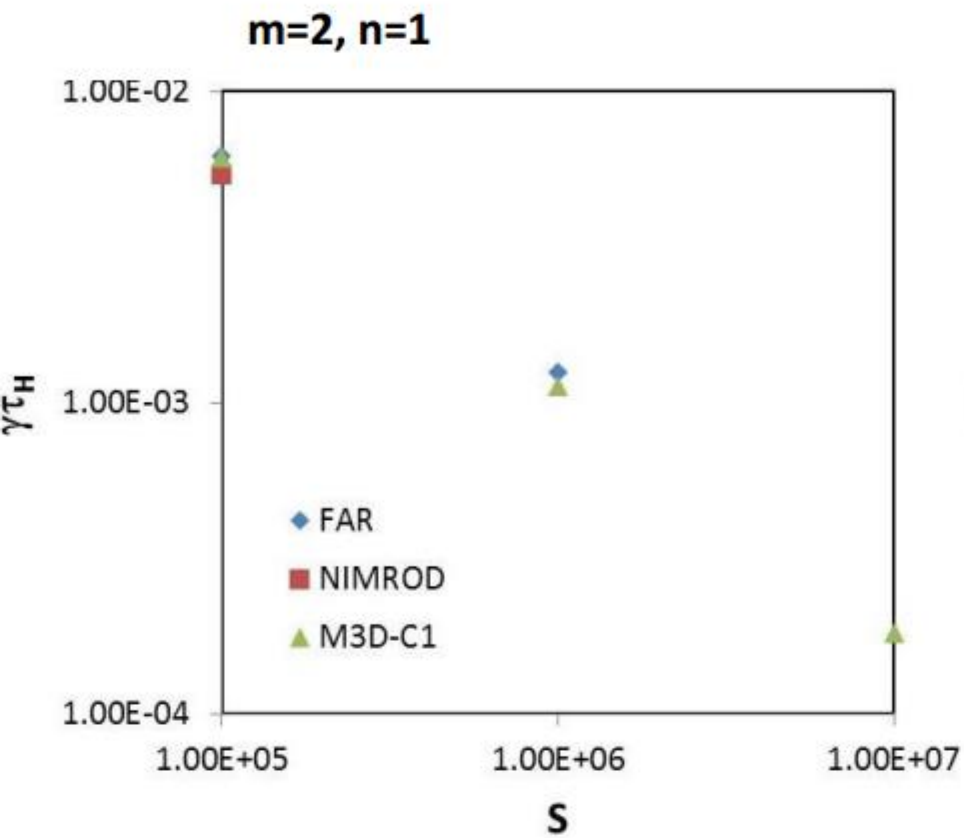
Cylindrical $q=1.33(1 + (r/0.595)^8)^{0.25}$ and $\beta=0$

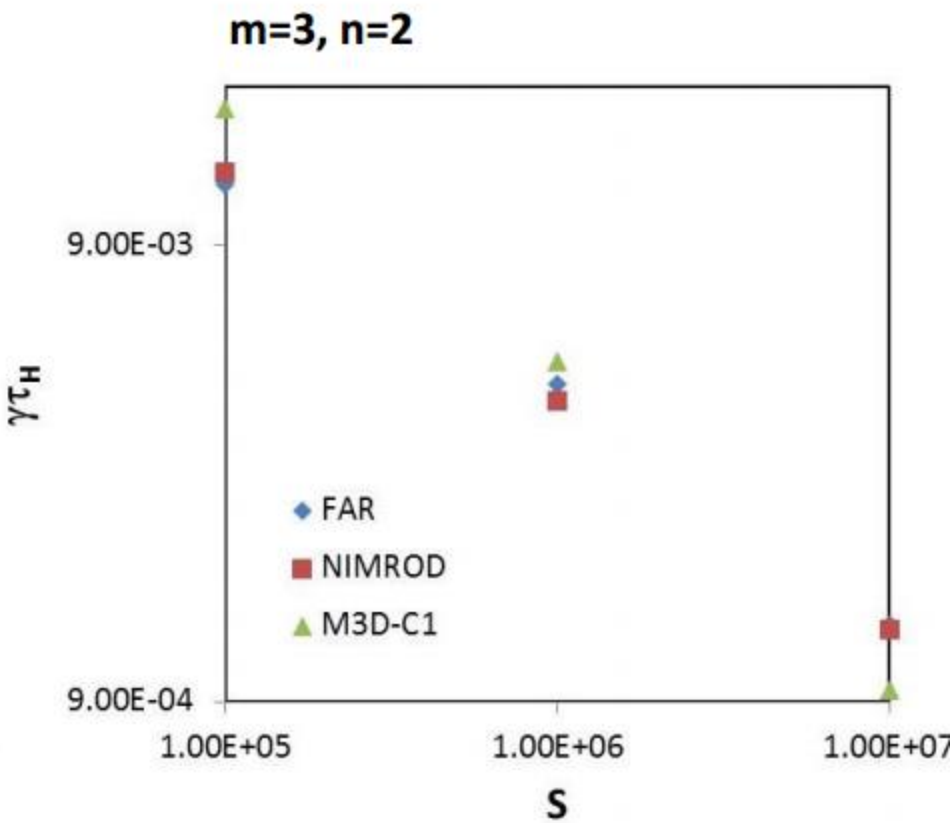




Linear Benchmarking

R/a=10, $q=1.33(1 + (r/0.595)^8)^{0.25}$ and $\beta_0=1.1\times10^{-7}$





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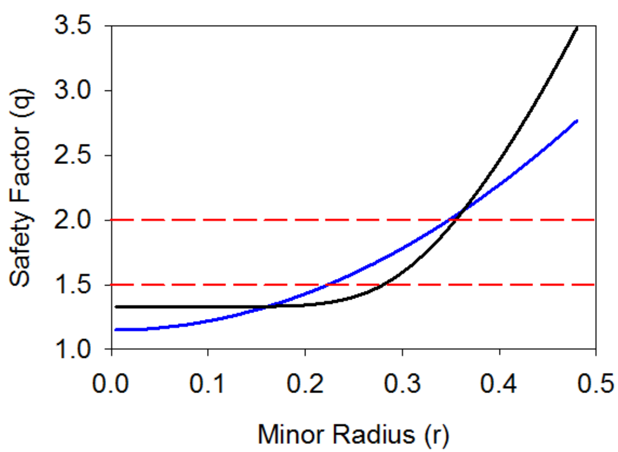
Non-Linear Runs looked at Two Equilibria

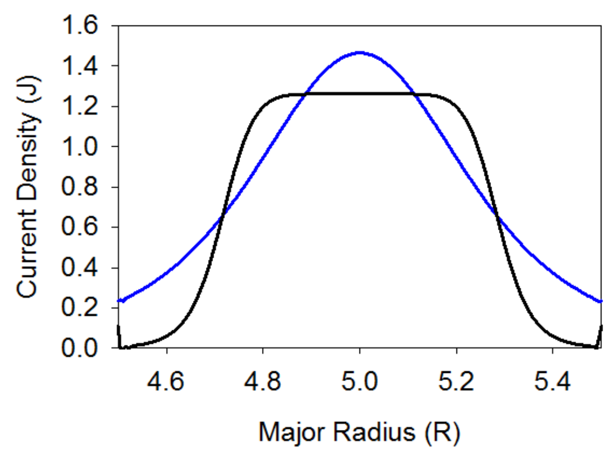
$$q(r) = 1.33\left[1 + \left(\frac{r^2}{.354}\right)^4\right]^{1/4}$$

- **Two-Mode** Case (same as linear)
- n=1 and n=2 unstable
- Cylinder, A=3, A=10 torus
- Linear and Nonlinear

$$q(r) = 1.15\left[1 + \left(\frac{r^2}{.6561}\right)\right]$$

- **One-Mode** Case
- Only n=1 unstable
- Cylinder, A=3, A=10 torus
- Linear and Nonlinear





Non-Linear Results

1. **Two-Mode case: Ran Nonlinear A=10 Torus numvar=3**
   - $S = 2 \times 10^4$ $\nu/\eta = 1$
   - Ran both 32 and 16 toroidal planes (with hermit cubic elements)

In both cases, islands continue to grow and becomes totally stochastic by Time Slice 30. (t=1500) Very similar results for N=16 and N=32 plane cases. Error α N^4

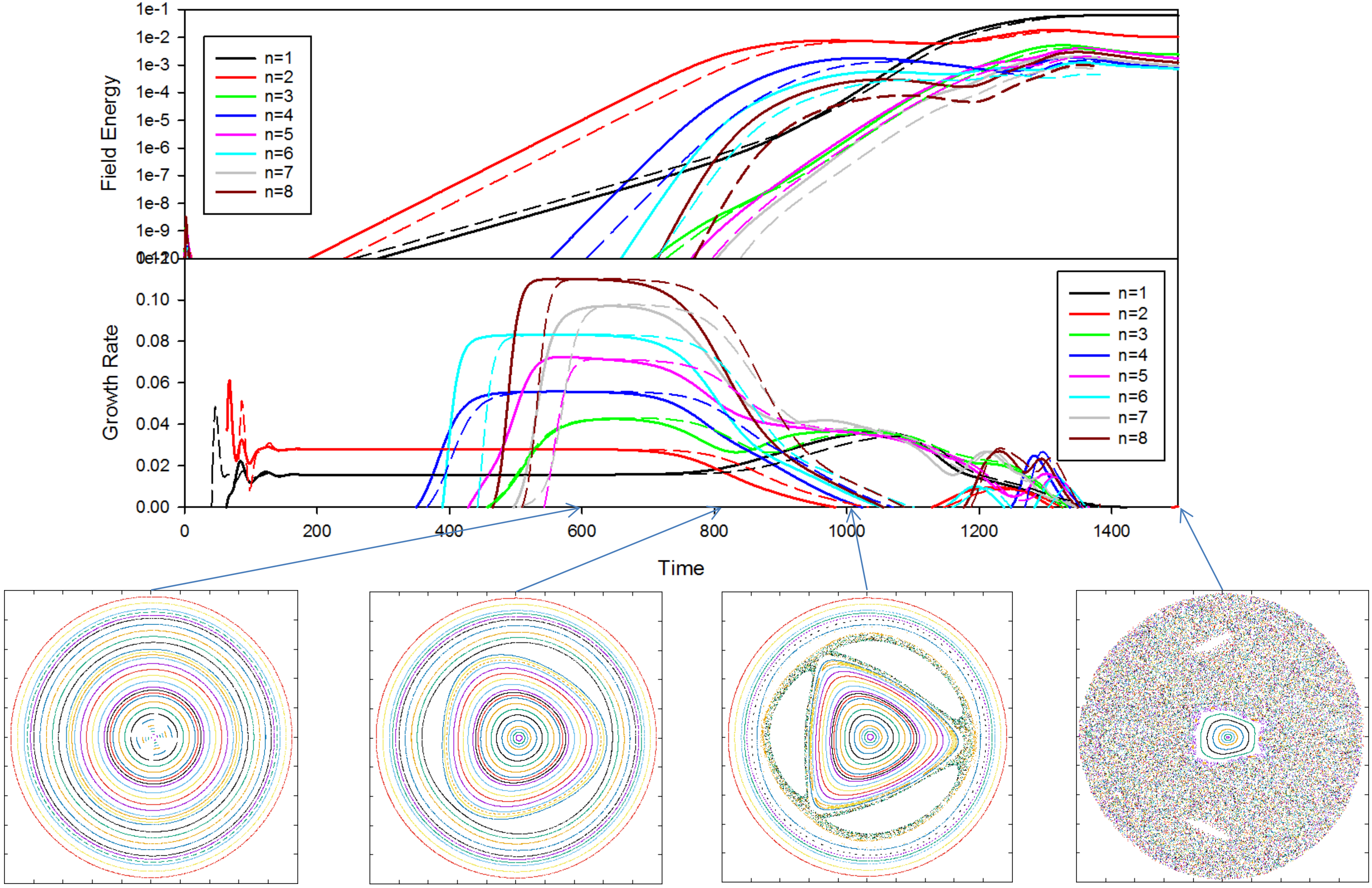
2. **One-Mode Case**: Ran Cylindrical geometry case to saturation with reduced MHD
   - $S = 10^5$, $\nu/\eta = .047$

Much slower island growth than above. Island saturates at about W/a = .14 at t=50,000

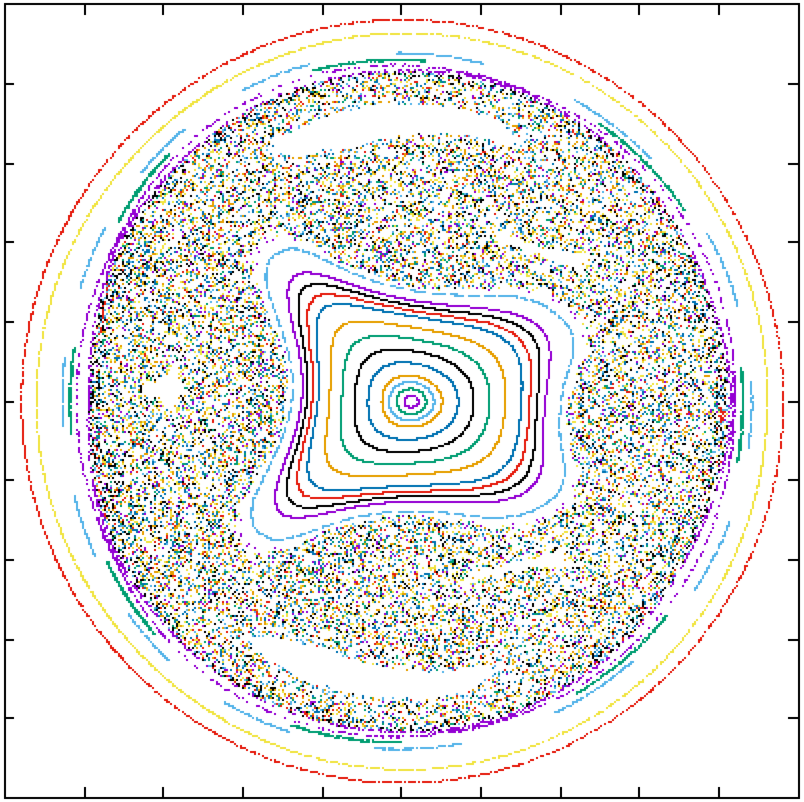
- Now running A=10 toroidal cases with full MHD

Mode Growth vs time for **Two-mode case**

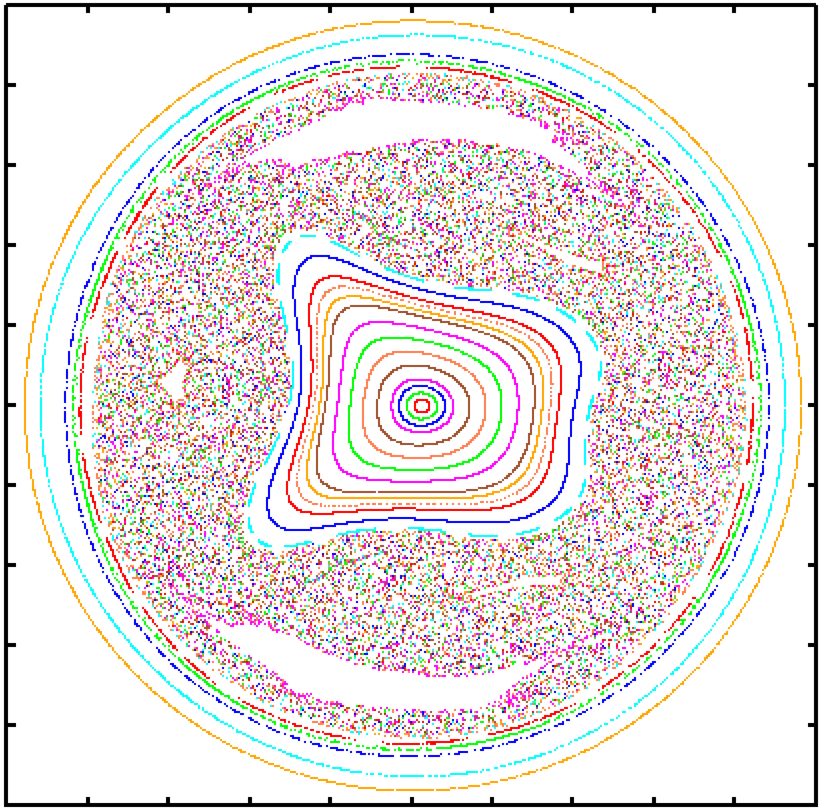
Comparison of 16 planes (solid) and 32 planes (dashed)



Comparison of surfaces at time t=1200 for **Two-Mode case** with 16 and 32 Planes

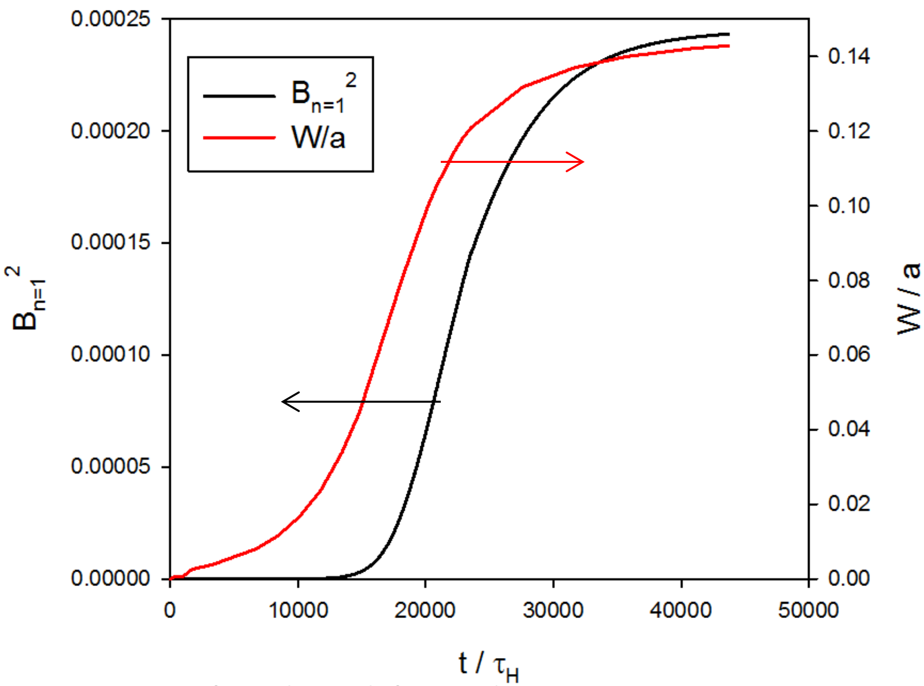


16 Planes

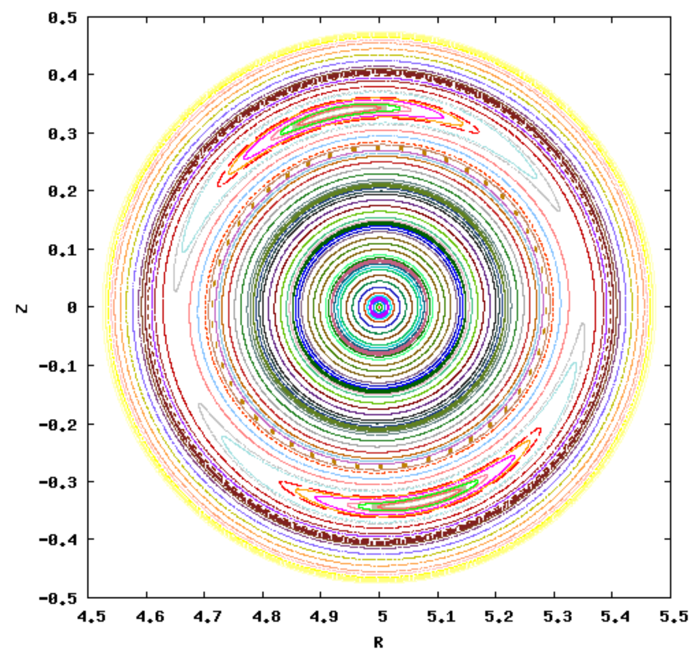


32 Planes

Single Unstable Mode Case in a cylinder



Island width and magnetic energy in n=1 harmonic vs time.



Poincare plot at final time.

SI Units: R=5, a=0.5, B_T = 4.2, n_0 = 10^{20}

$$q(r) = 1.15\left[1 + \left(\frac{r}{0.81}\right)^2\right]$$

$$S = \frac{a^2 B_T}{\eta R}\left[\frac{\mu_0}{n_0 M_i}\right]^{1/2} = 10^5, \quad \tau_H \equiv \frac{R}{B_T}\left[\mu_0 n_0 M_i\right]^{1/2}, \quad \nu/\eta = .047$$

N=0 (axisymmetric) equilibrium not advanced in time.

Low $\beta < 10^{-7}$

/tigress/sjardin/JA-2/One_Mode/Cylinder

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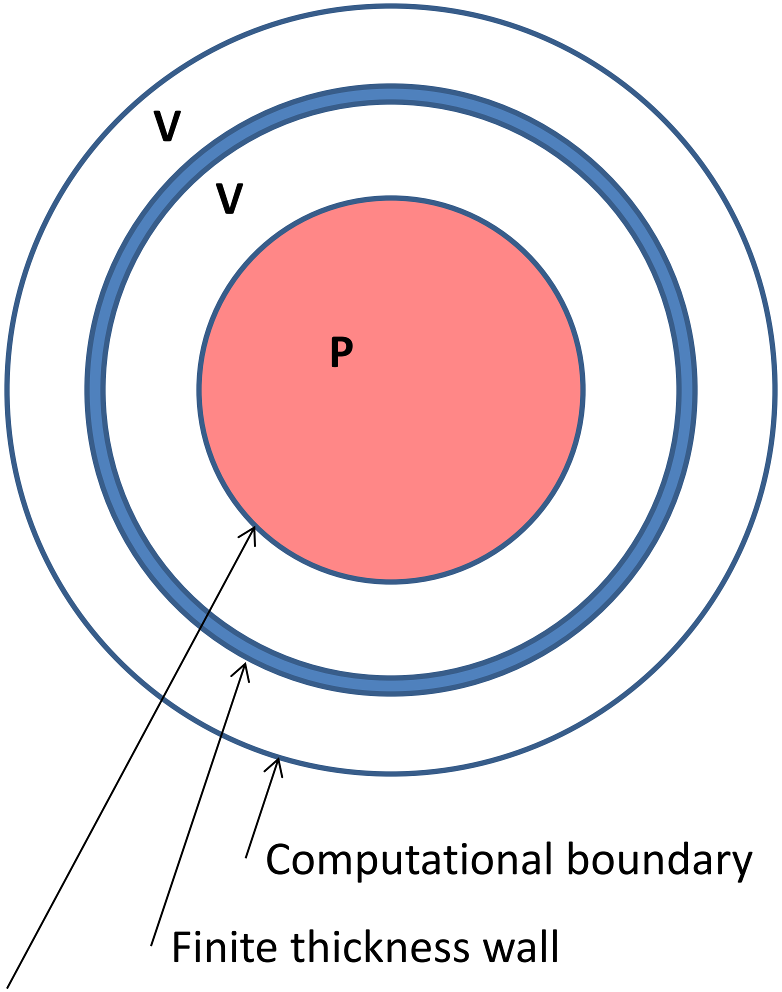
Model Cylindrical Equilibrium



- Poloidal flux low order polynomial in r^2
- Current density vanishes at a
- $q_0 < q < q_a$

$$\Psi(r)=\begin{cases} \dfrac{B_T r^2}{2q_0 R}\left[1-(r/a)^2\left(1-\dfrac{3}{2}\dfrac{q_0}{q_a}\right)+\dfrac{1}{3}(r/a)^4\left(1-2\dfrac{q_0}{q_a}\right)\right] & r<a \\ \dfrac{a^2 B_T}{6q_0 R}\left[1+\dfrac{5}{2}\dfrac{q_0}{q_a}+6\dfrac{q_0}{q_a}\ln(r/a)\right] & r\geq a \end{cases}$$

$$J(r)=\frac{B_T}{6q_0 R}\left[1-4(r/a)^2\left(1-\frac{3}{2}\frac{q_0}{q_a}\right)+3(r/a)^4\left(1-2\frac{q_0}{q_a}\right)\right]$$

$$q(r)=\begin{cases} q_0\left[1-2(r/a)^2\left(1-\dfrac{3}{2}\dfrac{q_0}{q_a}\right)+(r/a)^4\left(1-2\dfrac{q_0}{q_a}\right)\right]^{-1} & r<a \\ q_a(r/a)^2 & r\geq a \end{cases}$$

Stationary State Equilibrium equations

Uniform density: $T \sim p$; Spitzer resistivity: $\eta \sim p^{-3/2}$

Constant loop voltage: $\eta J \sim (\eta_0 p_0^{3/2}) p^{-3/2} J \sim V_L$

$$\Longrightarrow \boxed{p = p_0 \left[1 - (r/a)^2 \left(4 - 6\frac{q_0}{q_a} \right) + (r/a)^4 \left(3 - 6\frac{q_0}{q_a} \right) \right]^{2/3}}, \qquad V_L = B_T \eta_0 p_0^{3/2} / 6 q_0 R$$

$$\nabla p = \mathbf{J} \times \mathbf{B} \qquad \Longrightarrow \boxed{\frac{1}{2}\frac{d}{dr^2} B_z^2 = \frac{-\dfrac{dp}{dr^2} + \dfrac{B_z^2}{(Rq)^2}\left(1 - \dfrac{r^2}{q}\dfrac{dq}{dr^2}\right)}{1 + r^2/(Rq)^2}}, \qquad B_z(a) = B_T$$

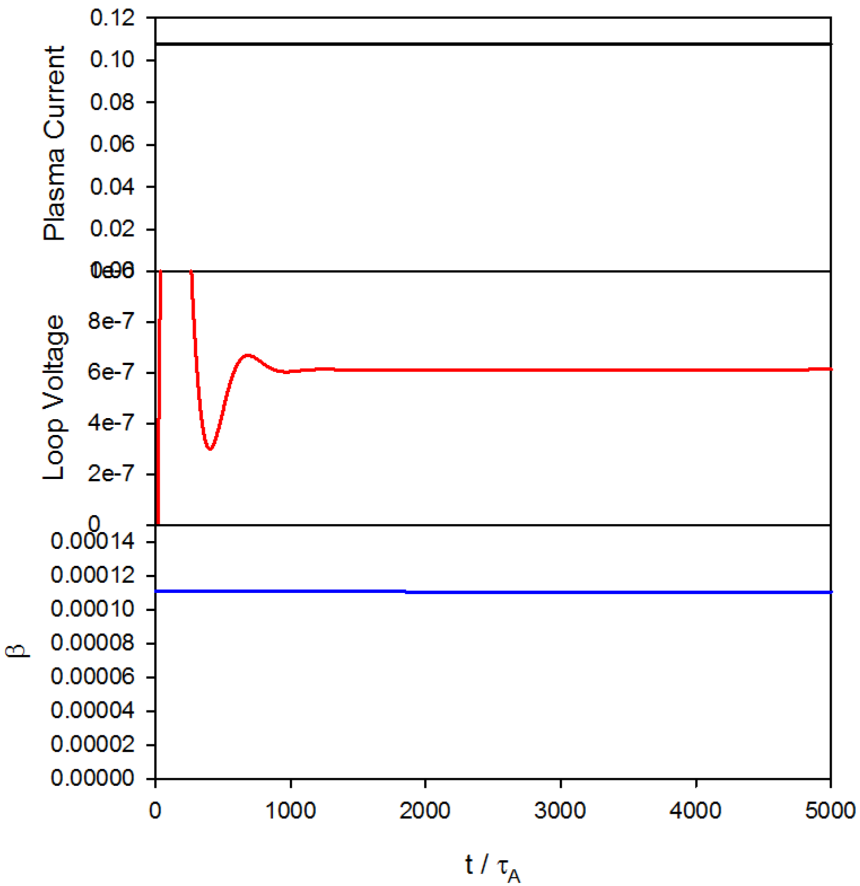
Stationary pressure profile: $\dfrac{1}{r}\dfrac{d}{dr}\left(r\kappa \dfrac{dp}{dr} \right) = -\eta J^2 = -V_L J$

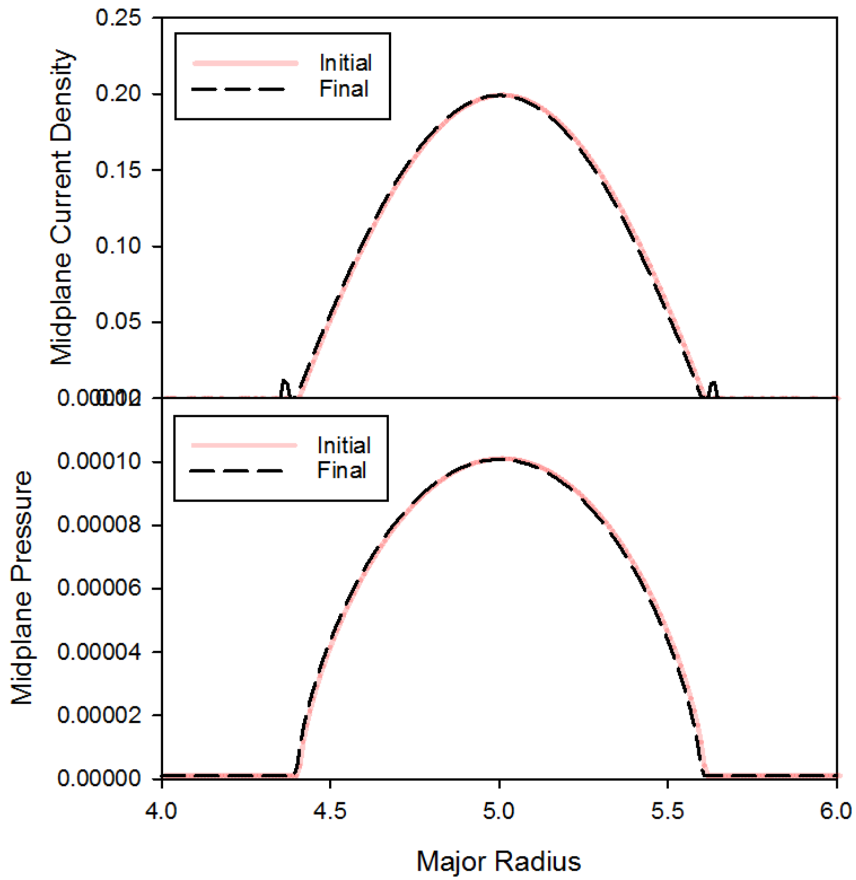
$$\Longrightarrow \quad \kappa \frac{dp}{dr} = -\frac{V_L B_T}{6 q_0 R}\frac{1}{r}\int_0^r r \left[1 + (r/a)^2 \left(-4 + 6\frac{q_0}{q_a} \right) + (r/a)^4 \left(3 - 6\frac{q_0}{q_a} \right) \right] dr$$

$$\boxed{\kappa = \kappa_0 \frac{\left[1 - 2(r/a)^2 \left(1 - \frac{3}{2}\frac{q_0}{q_a} \right) + (r/a)^4 \left(1 - 2\frac{q_0}{q_a} \right) \right] \left[1 - 4(r/a)^2 \left(1 - \frac{3}{2}\frac{q_0}{q_a} \right) + 3(r/a)^4 \left(1 - 2\frac{q_0}{q_a} \right) \right]^{1/3}}{\left[\left(1 - \frac{3}{2}\frac{q_0}{q_a} \right) - \frac{3}{2}(r/a)^2 \left(1 - 2\frac{q_0}{q_a} \right) \right]}}$$

$$\kappa_0 = \frac{3}{192} V_L B_T a^2 / q_0 R p_0 \left(1 - \frac{3}{2}\frac{q_0}{q_a} \right) \qquad q_0, q_a, B_T, a, R, \eta_0, p_0 \quad \Longrightarrow \quad V_L, \kappa_0$$

2D Nonlinear Run reaches stationary state





Linear Resistive Stability of Model Cylindrical Equilibrium with no wall (free boundary)

$$q(r)=\begin{cases} q_0\left[1-2(r/a)^2\left(1-\dfrac{3}{2}\dfrac{q_0}{q_a}\right)+(r/a)^4\left(1-2\dfrac{q_0}{q_a}\right)\right]^{-1} & r<a \\ q_a(r/a)^2 & r\geq a \end{cases}$$

q_0

q_a

	1.01	1.11	1.33	1.67	1.80	2.01
2.4						
2.6						
2.8						
3.0						
3.2						
3.4						
3.6						
3.8						
4.0						
4.2						
4.4						
4.6						
4.8						
5.0						
5.2						
5.4						
5.6						
5.8						

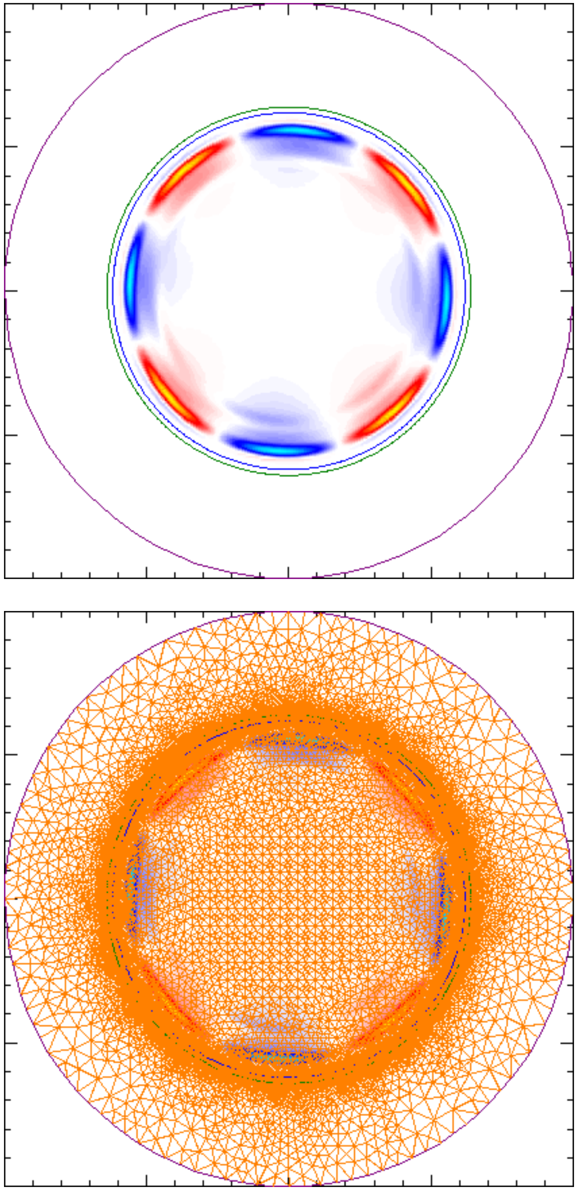
	2/1 most unstable
	3/1 most unstable
	4/1 most unstable
	Stable to all modes

monotonic solutions for:

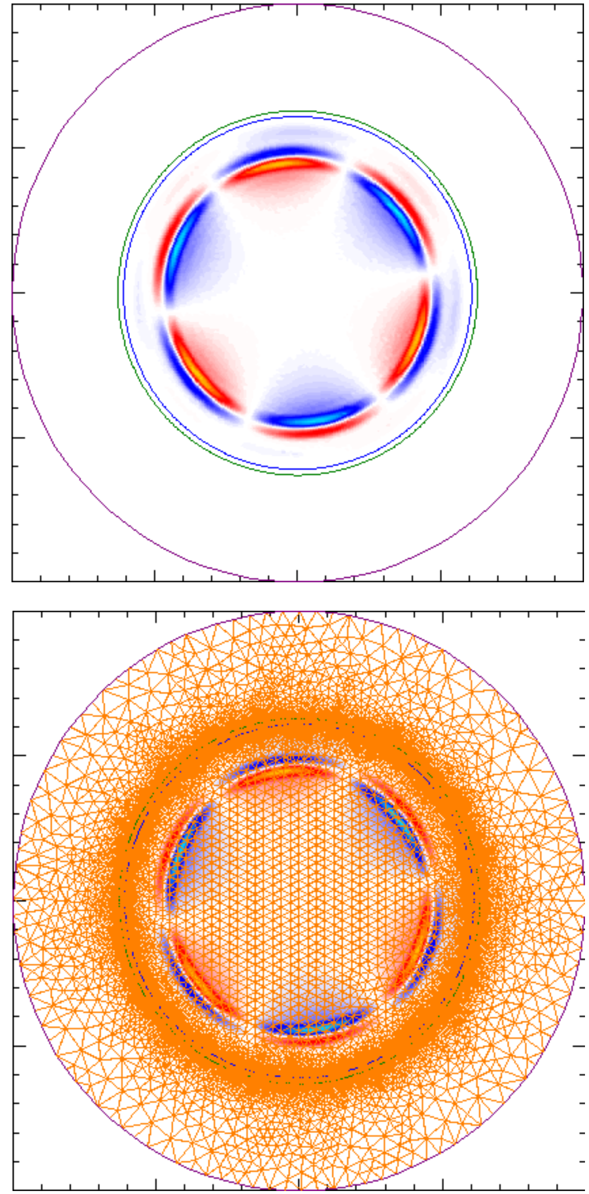
$$3q_0 > q_a > \frac{3}{2}q_0$$

Examine this case with a nearby wall

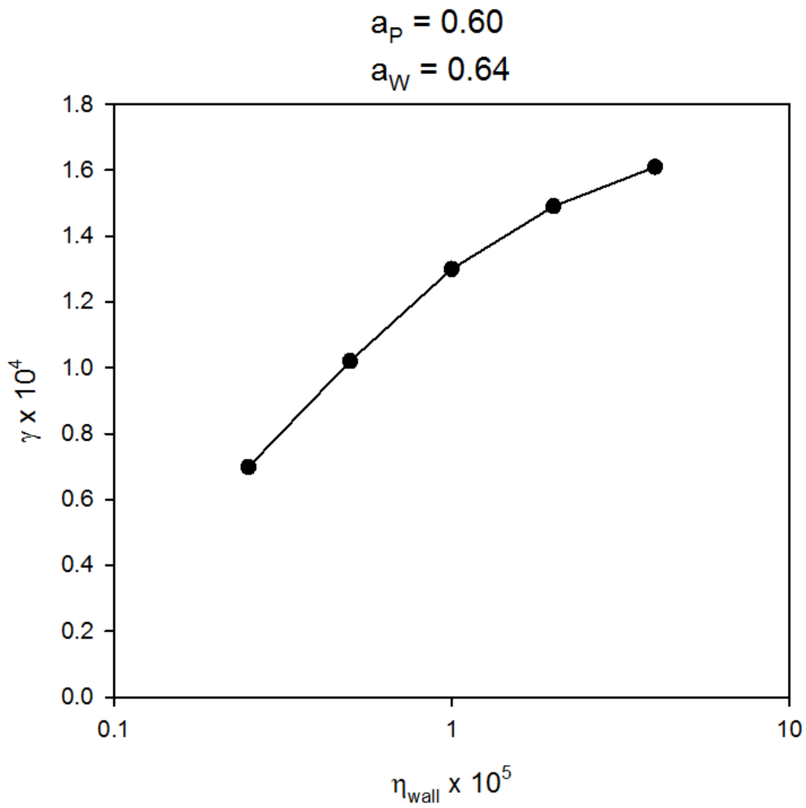
Wall at r/a = 0.64: unstable
(4/1) mode dominates

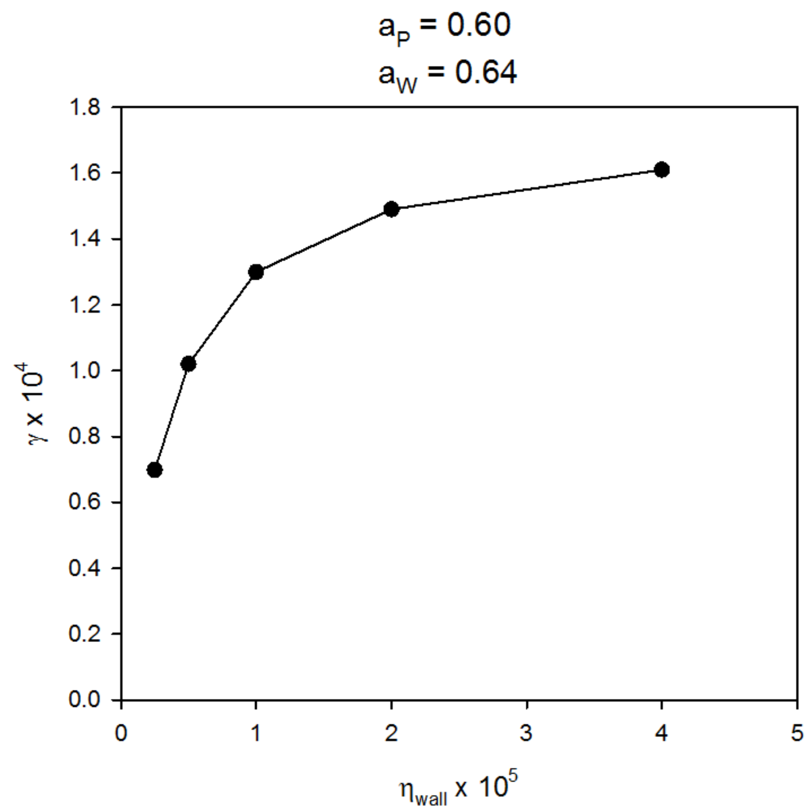


Wall at r/a = 0.62: stable!
(3/1) mode dominates



Growth rate of mode depends on wall resistivity





Next Steps

- Add sheared toroidal velocity with torque input (also to equilibrium equation)
- Add error field
- Study interaction of island with resistive wall and error field, and compare with theoretical results for mode locking
- Extend to Torus

References:
[1] Fitzpatrick, R., Nuclear Fusion **33** (1993) p. 1049, Section (3)
[2] Fitzpatrick, R., Nuclear Fusion **33** (1993) p. 1049, Section (6)
[3] Waelbroeck, F. and Fitzpatrick, R. Phys. Rev. Lett. **78** (1997) p. 1703
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[8] De Bock, M.F.M. et al Nucl. Fusion 48 (2008) 015007 .