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# Dynamical Formulation of Multi-region Relaxed MHD (MRxMHD)

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Recently [1] a new hydromagnetic dynamical model, Multi-region Relaxed Magnetohydrodynamics (MRxMHD), has been proposed as a dynamical generalization of Taylor relaxation theory appropriate to toroidally confined plasmas. This alternative to ideal MHD models the plasma-magnetic field system as consisting of arbitrarily many regions (i.e. it is a “waterbag” model), each containing compressible Euler fluid and Taylor-relaxed magnetic field, separated by interfaces in the form of flexible ideal-MHD current sheets. Internal self-consistency is guaranteed by deriving its evolution equations from Hamilton’s Principle of Stationary Action, using the MHD Lagrangian but replacing the local, “microscopic” constraints of ideal MHD, which freeze in magnetic flux and entropy, with invariance of “macroscopic” entropy and magnetic helicity integrals over each relaxation region, enforced using Lagrange multipliers.
The resulting Euler-Lagrange equations are Beltrami equations, $\nabla\times\mathbf{B} = \mu\mathbf{B}$, for the magnetic field, and compressible, isothermal Euler fluid equations. (As the magnetic field is force-free in each region, it is coupled to the plasma only at the interfaces between the regions.) The formalism has great flexibility in matching any given pressure and current profile as each region has different temperature and current factor.
As a first step in understanding the scope of dynamical MRxMHD, two simple time evolution problems are investigated in double-region slab geometries. Periodic toroidal and poloidal boundary conditions are used to relate the models to the outer regions of typical tokamak or reversed-field pinch plasmas:

*Eigenmode spectrum* [2]
The lowest and second-lowest eigenvalues in plasmas, in some cases unstable to tearing and kink-tearing modes, are calculated using *plane* radial boundaries and a single rippled interface, which supports the MHD wave perturbations. Very near marginal stability the lowest mode obtained using the incompressible approximation to the kinetic energy normalization of the present study is shown to correspond to the eigenvalues found in previous studies where all mass was artificially loaded onto the interfaces.

*Penetration of a Resonant Magnetic Perturbation* [3]
The adiabatic limit of MRxMHD is illustrated using a sheared-magnetic-field slab model with *rippled* radial boundaries and a single plane interface carrying a resonantly excited current sheet, in a geometry similar to that of the Hahm-Kulsrud-Taylor model problem. In this geometry a *linear* Grad-Shafranov equation applies, even at finite ripple amplitude. The adiabatic switching on of boundary ripple excites a shielding current sheet opposing reconnection at a resonant surface. The perturbed magnetic field as a function of ripple amplitude is calculated by invoking conservation of magnetic helicity in the two regions separated by the current sheet. At low ripple amplitude “half islands” appear on each side of the current sheet, locking the rotational transform at the resonant value. Above a critical amplitude these islands disappear and the rotational transform develops a discontinuity across the current sheet.

*Relaxed fluid dynamics* [4]
When a toroidal region’s boundaries are fixed, the only time-independent solution of the compressible Euler fluid equations that is robust against arbitrary 3-D changes in the boundary shapes is a solution of the Beltrami-like equation $\nabla\times\mathbf{v} = \mu_{\rm fl}\exp(-v^2/C_s^2)\,\mathbf{v}$, where $C_s$ is the ion sound speed in the region. This suggests a further development in the MRxMHD formalism, appropriate to timescales longer than sound propagation times across and around the relaxation regions, where full Euler fluid dynamics is replaced by the *relaxed fluid approximation*. In this approximation the velocity field is a solution of the above Beltrami-like equation in each region plus a small correction field to take into account motions of the boundaries.

References

[1] R.L. Dewar, Z. Yoshida, A. Bhattacharjee and S. R. Hudson, *Variational formulation of relaxed and multi-region relaxed magnetohydrodynamics*, J. Plasma Phys. **81**, 515810604 (2015)
[2] R.L. Dewar, L.H. Tuen and M.J. Hole, *Spectrum of multi-region-relaxed magnetohydrodynamic modes in topologically toroidal geometry*, Plasma Phys. Control. Fusion **59**, 044009 (2016)
[3] R.L. Dewar, S. R. Hudson, A. Bhattacharjee and Z. Yoshida, *Multi-region relaxed magnetohydrodynamics in plasmas with slowly changing boundaries — resonant response of a plasma slab*, Phys. Plasmas **24**, 042507 (2016)
[4] N. Sato and R.L. Dewar, in preparation.