2017 Sherwood Fusion Theory Conference

Finite Larmor Radius Effects at the Tokamak Edge and the associated MHD Equilibria

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1) We present a novel mechanism for producing the equilibrium potential well near the edge of a tokamak. Briefly, because of the difference in gyroradii between electrons and ions, an equilibrium electrostatic potential is generated in the presence of spatial inhomogeneity of the background plasma, which, in turn, produces a well associated with the radial electric field, Er, as observed at the edge of many tokamak experiments. We will show that this theoretically predicted Er field, which can be regarded as producing a long radial wave length zonal flow, agrees well with many recent experimental measurements on JET, NSTX and C-MOD.

2) The properties of associated gyrokinetic MHD and the schemes for achieving related equilibria will also be discussed.

Phys. Fluids **23**(10), October 1980

Ion temperature drift instabilities in a sheared magnetic field

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(Received 13 February 1980; accepted 26 June 1980)

Results from the first particle code simulations of the ion-temperature-gradient-driven instabilities in a sheared slab geometry are reported. In the linear stage of the instability, the results are in very good agreement with the theoretical calculations of the mode frequency, growth rate, and radial mode structure. Ion energy transport caused by the instability is found to be the process primarily responsible for nonlinear saturation. Enhanced fluctuations associated with marginally stable eigenmodes have been observed.

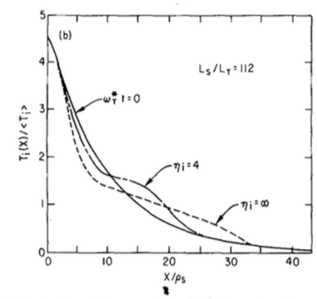

FIG. 4. Simulation results of (a) the saturation amplitude and (b) the resultant ion temperature profiles.

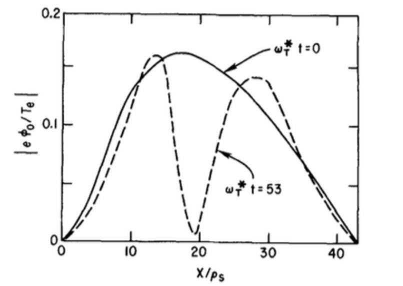

FIG. 5. Ambipolar ($k_y=0$) mode structures at the onset of simulation and at saturation for $\eta_i=\infty$, $L_S/L_T=112$.

Naitou, Tokuda and Kamimura, JCP **38**, 265 (1980): attempted to eliminate this extra charge density.

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556 Phys. Fluids **26** (2), February 1983

Gyrokinetic approach in particle simulation

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(Received 16 October 1981; accepted 20 October 1982)

A new scheme for particle simulation based on the gyrophase-averaged Vlasov equation has been developed. It is suitable for studying linear and nonlinear low-frequency microinstabilities and the associated anomalous transport in magnetically confined plasmas. The scheme retains the gyroradius effects but not the gyromotion; it is, therefore, far more efficient than conventional ones. Furthermore, the reduced Vlasov equation is also amenable to analytical studies.

- Origin of this extra ion charge density due to spatial inhomogeneity was first discussed from the gyrokinetic point of view.

- Gyrokinetic Poisson's Equation

$$\nabla^2\Phi - k_{Di}^2\frac{n_i}{n_0}(\Phi-\tilde{\Phi}) = -4\pi e(n_i + \frac{1}{2}\rho_i^2\nabla_\perp^2 n_i^{inho} - n_e)$$

- For $k_\perp\rho_i \ll 1$ $\Downarrow$

$$-k_{Di}^2(\Phi-\tilde{\Phi}) \sim \frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp^2\Phi$$

PHYSICS OF PLASMAS **16**, 044506 (2009)

On higher order corrections to gyrokinetic Vlasov–Poisson equations in the long wavelength limit

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$$\bar{n}(\mathbf{x}) = \int\left(1+\frac{1}{4}\frac{v_\perp^2}{\Omega^2}\nabla_\perp^2\right)F_{gc}(\mathbf{R})dv_\parallel d\mu,$$

$$\bar{n}(\mathbf{x}) = n + \frac{1}{2}\rho_i^2\frac{1}{T}\nabla_\perp^2 nT,$$

$$\rho_s^2\nabla_\perp^2\frac{e\phi}{T_e} = -\frac{\delta n_i}{n_0}, \qquad \frac{e\phi}{T_i} = \frac{1}{2}\frac{\kappa_{Ti}^2}{k_\perp^2}, \qquad \sim O(1)$$

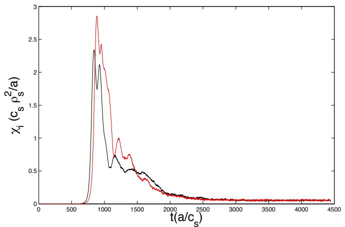
$$\frac{V_{E\times B}}{c_s} \equiv k_\perp\rho_s\frac{e\phi}{T_i} = \frac{1}{2}\frac{\kappa_{Ti}^2\rho_s^2}{k_\perp\rho_s}.$$

Linear Upshift of ITG Gradient due to Equilibrium Ion Density

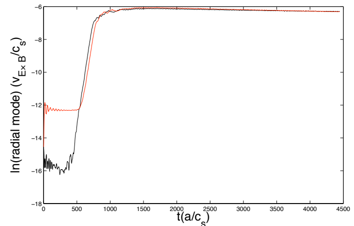
• Gyrophase-Averaged Equilibrium Ion Number Density

$$\bar{n}(\mathbf{x}) = n + \underline{\frac{1}{2}\rho_i^2 \frac{1}{T}\nabla_\perp^2 nT}$$

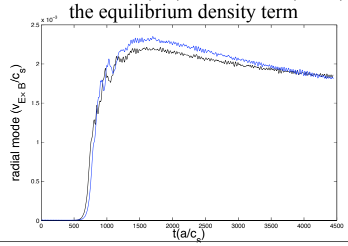
• ITG simulation using GTC* (a/rho =125)



Heat Flux with (red) and without (black) the equilibrium density term



Zonal flow amplitude with (red) and without (black) the equilibrium density term



-- Zonal flow amplitude with (blue) and without (black) the equilibrium density term

-- Similar to the Dimits Shift, which is nonlinear

-- This shift is linear

[W. W. Lee, Sherwood Conference 2014]

Calculations for the Radial Electric Field

[W. W. Lee and R. B. White, PoP 2017 (to appear)]

• Gyrokinetic Quasineutrality: $n_i \approx n_e \quad > \quad n_i^{gc} + \underline{n_i^{pol} + n_i^{inho}} = n_e^{gc}$

• Focus of Present Calculation: $n_i^{pol} + n_i^{inho} = 0 \quad \text{for} \quad n_i^{gc} = n_e^{gc}$

• Density Profile with Monogenic Particles: $\frac{n(r)}{n_0} = \frac{1}{2} - \frac{tanh[(r-r_0)/w]}{2} + n_s$

• Total Ion Density

$$n = n^{gc} + \delta n = n^{gc} + \frac{1}{2T_i}\rho_i^2\nabla_\perp^2 n^{gc}T_i = n^{gc} + \frac{\rho_i^2}{2T_i}\left(\frac{\partial^2 n^{gc}T_i}{\partial^2 r} + \frac{1}{r}\frac{\partial n^{gc}T_i}{\partial r}\right)$$

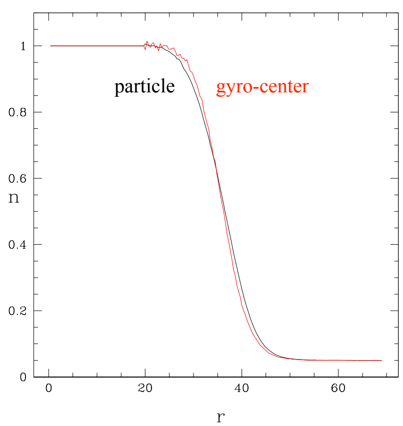
• GK Poisson's Equation: $\rho_s^2 \nabla_\perp \cdot n\nabla_\perp \frac{e\phi}{T_e} = -\delta n$

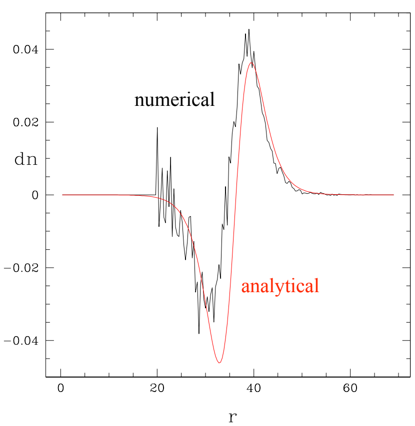
• Electric Field and potential: $E(r) = \frac{T_i}{2nre}\int_0^r sds[n''(s) + n'(s)/s]$

$$\frac{e\phi(r)}{T_i} = -\int_0^r dr' \frac{1}{2nr'}\int_0^{r'} sds[n''(s) + n'(s)/s],$$

$$enE_\perp = (1/2)(T_e/T_i)\nabla_\perp p_i \qquad p_i \equiv n^{gc}T_i$$

Particle and Gyro-Center Densities

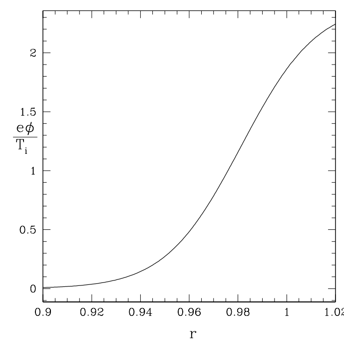


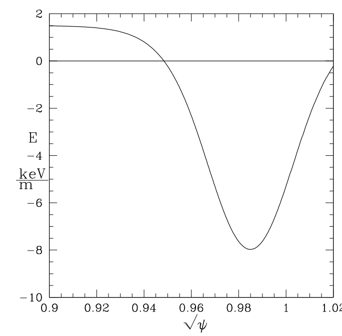


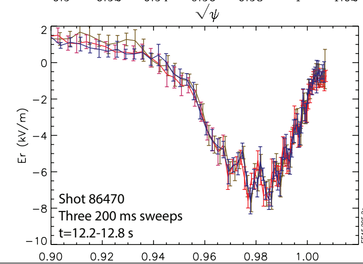
Plots of particle and gyro center densities as a function of minor radius (cm) with particle density in black and gyro-center density in red at left, and at the right the difference, shown in black, with the analytic expression, shown in red. Gyro radius ρ_i = 2.3 cm, w = 5 cm, and r_0 = 36 cm.

JET Ohmic Discharge

C.Hillesheim, E.Delabie, H.Meyer,C.F.Maggi, L.Meneses,E.Poli, and JETContributors, Phys. Rev. Letters, 116, 065002 (2016).



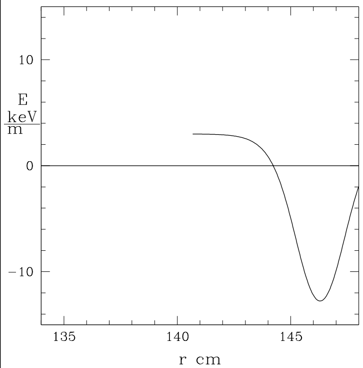


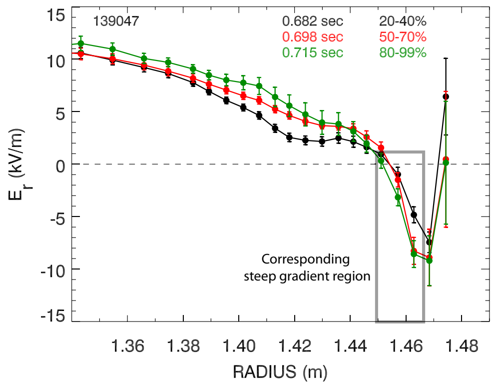


Electric potential and electric field caused by the charge density resulting from ion cyclotron motion in the density distribution and the electric field measurement from JET Ohmic discharge, shot 86470.

NSTX Discharge

A. Diallo, J. Canik, T. Goerler, S.H. Ku, G.J. Kramer, T. Osborne, P. Snyder, D.R. Smith, W. Guttenfelder, R.E. Bell, D.P. Boyle, C.S. Chang, B.P. LeBlanc, R. Maingi, M. Podesta, and S. Sabbagh, Nucl Fus, 53, 1 (2013).

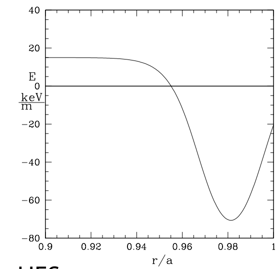


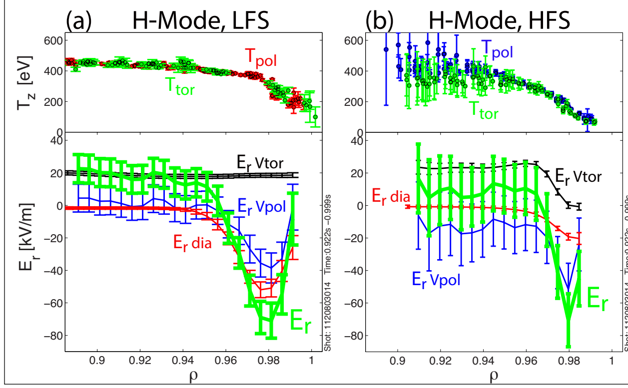


Electric field caused by the charge density resulting from ion cyclotron motion in the density distribution for a NSTX discharge, and the radial electric field observed.

C-MOD Discharge

Electric field caused by the charge density resulting from ion cylotron motion in the density distribution for a Alcator C-Mod H-mode discharge, and the observed temperature profile and radial electric field observed.



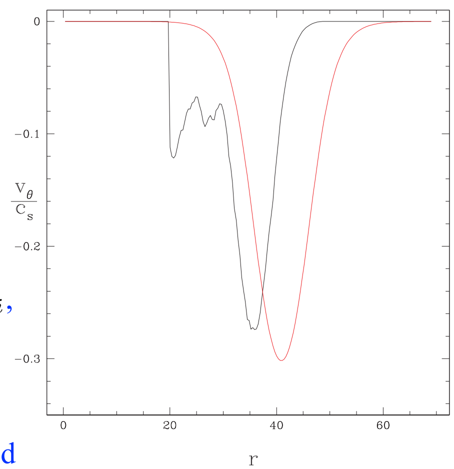


C. Theiler, R.M. Churchill, B. Lipschultz, M. Landreman, D.R. Ernst, J.W. Hughes, P.J. Catto, R.I. Parra, I.H. Hutchinson, M.L. Reinke, A.E. Hubbard, E.S. Marmar, J.T. Terry, J.R. Walk, and the Alcator C-Mod team, Nucl. Fusion 54, 083017 (2014).

Comparisons between the direct calculation of v_θ from the actual particles (black) and that from the pressure balance (red)

$$\frac{\mathbf{J}_\perp(\mathbf{x})}{enc_s} = \frac{1}{n}\int \frac{\mathbf{v}_\perp}{c_s} F(\mathbf{x},\mathbf{v})d\mathbf{v}$$

$$\frac{\mathbf{J}_\perp(\mathbf{x})}{enc_s} \approx \mathbf{b}\times\frac{\rho_s \nabla p_\perp}{nT_e}$$



- The pressure balance is calculated from the lowest order approximation in $k_\perp \rho_i$, and higher order is needed.
- It should not be confused with the poloidal velocities produced by the electric field at the tokamak edge, which comes from the charge imbalance.

Summary I

- This presentation gives a possible theoretical explanation for the formation of observed radial electric field wells at edge pedestals through finite Larmor radius (FLR) effects of the plasma particles. The well can be regarded as producing a long radial wavelength global zonal flow.
- The surprising agreement between our model, based on equilibrium profiles with no turbulence, and the experimental measurements based on steady state profiles with turbulence, should be a topic of interest in the tokamak community.
- It is possible these two totally different states are thermodynamically related through the Woltjer-Taylor State. This is the topic for the second part of the talk.

Magnetohydrodynamics for Collisionless Plasmas from the Gyrokinetic Perspective

[W. W. Lee, Phys. Plasmas **23**, 070705 (2016)]

$$\frac{n_i|_{particle}}{n_i} = 1 + \frac{1}{2}\rho_i^2 \frac{1}{p_i}\nabla_\perp^2 p_i$$

$$\mathbf{v}_{E\times B} \approx -\frac{1}{2}\hat{\mathbf{b}} \times \frac{\nabla_\perp p_i}{p_i}\frac{cT_i}{eB}$$

$$\mathbf{J}_\perp^{E\times B}(\mathbf{x}) = \sum_\alpha q_\alpha \langle \int \mathbf{v}_{E\times B}(\mathbf{R}) F_\alpha(\mathbf{R})\delta(\mathbf{R}-\mathbf{x}+\rho)\, d\mathbf{R}\, d\mu\, dv_\parallel \rangle_\varphi$$

$$\mathbf{J}_\perp = \frac{c}{B}\hat{\mathbf{b}}\times\nabla p + e n_i \frac{\rho_i^2}{2}\left[\nabla_\perp^2 \mathbf{v}_{E\times B} + \frac{\mathbf{v}_{E\times B}}{p_i}\nabla_\perp^2 p_i\right]$$

-- Difference in gyroradius effects between ions and electrons

$$\boxed{\mathbf{J}_\perp \approx \frac{c}{B}\hat{\mathbf{b}}\times(\nabla p)\left[1-\frac{1}{4}\rho_i^2\frac{\nabla_\perp^2 p}{p}\right]}$$

-- FLR modified current

$$\nabla\left[\frac{B^2}{8\pi} + p\left(1-\frac{1}{4}\rho_i^2\frac{\nabla_\perp^2 p}{p}\right)\right]\approx 0$$

- New Pressure Balance:

$$\nabla\left[\frac{B^2}{8\pi} + p\left(1-\frac{1}{4}\rho_i^2\frac{\nabla_\perp^2 p}{p}\right)\right]\approx 0$$

- From Woltjer-Taylor State of

$$\nabla\times\mathbf{B} = \alpha\mathbf{B}$$

- We obtain

$$\nabla p_\perp\left(1-\frac{1}{4}\rho_i^2\frac{\nabla_\perp^2 p_\perp}{p_\perp}\right) = 0$$

- And the solutions are: $\nabla p_\perp = 0,$

  and, at large radius, $p_\perp = exp(-2r/\rho_i).$

- It resembles the profiles used in the study for edge potential well on pp. 7.

Gyrokinetic MHD

- Fully Electromagnetic Gyrokinetic Vlasov Equation:

$$\frac{\partial F_\alpha}{\partial t} + \left[v_\parallel \mathbf{b} - \frac{c}{B_0}\nabla(\bar\phi - \frac{1}{c}\overline{\mathbf{v}_\perp\cdot\mathbf{A}_\perp})\times\hat{\mathbf{b}}_0\right]\cdot\frac{\partial F_\alpha}{\partial\mathbf{x}} - \frac{q}{m}\left[\nabla(\bar\phi - \frac{1}{c}\overline{\mathbf{v}_\perp\cdot\mathbf{A}_\perp})\cdot\mathbf{b} + \frac{1}{c}\frac{\partial\bar A_\parallel}{\partial t}\right]\frac{\partial F_\alpha}{\partial v_\parallel}=0$$

- Associated Gyrokinetic Field Equations:

$$\nabla^2\phi + \frac{\omega_{pi}^2}{\Omega_i^2}\nabla_\perp^2\phi = -4\pi\sum_\alpha q_\alpha\int\bar F_\alpha dv_\parallel d\mu$$

-- for $k_\perp^2\rho_i^2 \ll 1$

$$\nabla^2\mathbf{A} - \underline{\frac{1}{v_A^2}\frac{\partial\mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c}\sum_\alpha q_\alpha \int \mathbf{v}\bar F_\alpha dv_\parallel d\mu$$

Negligible for $\omega^2 \ll k_\perp^2 v_A^2$

$\mu_B \equiv \mu/B \approx const.$ $\quad \mu = v_\perp^2/2$

$$\mathbf{v}_p^L = -(mc^2/eB^2)(\partial\nabla_\perp\phi/\partial t)$$
$$\mathbf{v}_p^T = -(mc/eB^2)(\partial^2\mathbf{A}_\perp/\partial^2 t)$$

- Energy Conservation: $\Phi \equiv \phi - \overline{\mathbf{v}_\perp\cdot\mathbf{A}_\perp}/c$

$$\frac{d}{dt}\left\langle\int(\frac{1}{2} v_\parallel^2+\mu)(m_eF_e+m_iF_i)dv_\parallel d\mu + \frac{\omega_{ci}^2}{\Omega_i^2}\frac{|\nabla_\perp\Phi|^2}{8\pi} + \frac{|\nabla A_\parallel|^2}{8\pi}\right\rangle_{\mathbf{x}} = 0$$

- Gyrokinetic Vlasov Equation in General Geometry

[Lee and Qin, PoP (2003), Porazic and Lin, PoP (2010); Startsev et al. APS (2015)]

$$\frac{\partial F_\alpha}{\partial t} + \frac{d\mathbf{R}}{dt}\cdot\frac{\partial F_\alpha}{\partial\mathbf{R}} + \frac{dv_\parallel}{dt}\frac{\partial F_\alpha}{\partial v_\parallel} = 0$$

$$\frac{d\mathbf{R}}{dt} = v_\parallel\mathbf{b}^* + \frac{v_\perp^2}{2\Omega_{\alpha0}}\hat{\mathbf{b}}_0\times\nabla lnB_0 - \frac{c}{B_0}\nabla\bar\Phi\times\hat{\mathbf{b}}_0$$

$$\frac{dv_\parallel}{dt} = -\frac{v_\perp^2}{2}\mathbf{b}^*\cdot\nabla lnB_0 - \frac{q_\alpha}{m_\alpha}\left(\mathbf{b}^*\cdot\nabla\bar\Phi + \underline{\frac{1}{c}\frac{\partial\bar A_\parallel}{\partial t}}\right)$$

Startsev et al.

$$\Omega_{\alpha0} \equiv q_\alpha B_0/m_\alpha c$$

$$\bar\Phi \equiv \bar\phi - \overline{\mathbf{v}_\perp\cdot\mathbf{A}_\perp}/c \qquad \underline{\overline{\mathbf{v}_\perp\cdot\mathbf{A}_\perp} = -\frac{1}{2\pi}\frac{eB_0}{mc}\int_0^{2\pi}\int_0^\rho \delta B_\parallel r\,dr\,d\theta}$$

Porazic and Lin

$$\mathbf{b}^* \equiv \mathbf{b} + \frac{v_\parallel}{\Omega_{\alpha0}}\hat{\mathbf{b}}_0\times(\hat{\mathbf{b}}_0\cdot\nabla)\hat{\mathbf{b}}_0 \qquad \mathbf{b} = \hat{\mathbf{b}}_0 + \frac{\nabla\times\bar{\mathbf{A}}}{B_0}$$

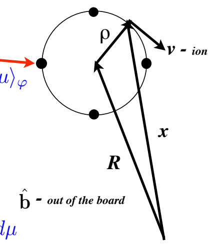
$$F_\alpha = \sum_{j=1}^{N_\alpha}\delta(\mathbf{R}-\mathbf{R}_{\alpha j})\delta(\mu-\mu_{\alpha j})\delta(v_\parallel - v_{\parallel\alpha j})$$

Gyrokinetic Current Densities

[Qin, Tang, Rewoldt and Lee, PoP **7**, 991 (2000); Lee and Qin, PoP **10**, 3196 (2003).]

$$\mathbf{J}_{gc}(\mathbf{x}) = \mathbf{J}_{\parallel gc}(\mathbf{x}) + \boxed{\mathbf{J}^M_{\perp gc}(\mathbf{x})} + \mathbf{J}^d_{\perp gc}(\mathbf{x}) + \mathbf{J}^{E\times B}_{\perp gc}(\mathbf{x})$$

$$= \sum_\alpha q_\alpha \langle \int F_{\alpha gc}(\mathbf{R})(\mathbf{v}_\parallel + \mathbf{v}_\perp + \mathbf{v}_d + \mathbf{v}_{E\times B})\delta(\mathbf{R} - \mathbf{x} + \rho)d\mathbf{R}dv_\parallel d\mu\rangle_\varphi$$



FLR calculation

$$\mathbf{v}_d = \frac{v_\parallel^2}{\Omega_\alpha}\hat{\mathbf{b}} \times (\hat{\mathbf{b}} \cdot \frac{\partial}{\partial \mathbf{R}})\hat{\mathbf{b}} + \frac{v_\perp^2}{2\Omega_\alpha}\hat{\mathbf{b}} \times \frac{\partial}{\partial \mathbf{R}} lnB$$

$$\mathbf{J}^M_{\perp gc}(\mathbf{x}) = -\sum_\alpha \nabla_\perp \times \frac{c\hat{\mathbf{b}}}{B}p_{\alpha\perp} \qquad p_{\alpha\perp} = m_\alpha \int (v_\perp^2/2)F_{\alpha gc}(\mathbf{x})dv_\parallel d\mu$$

$$\mathbf{J}^d_{\perp gc} = \frac{c}{B}\sum_\alpha \left[p_{\alpha\parallel}(\nabla \times \hat{\mathbf{b}})_\perp + p_{\alpha\perp}\hat{\mathbf{b}} \times (\nabla lnB)\right] \qquad p_{\alpha\parallel} = m_\alpha \int v_\parallel^2 F_{\alpha gc}(\mathbf{x})dv_\parallel d\mu$$

$$\mathbf{J}_{\perp gc} = \mathbf{J}^M_{\perp gc} + \mathbf{J}^d_{\perp gc} \quad = \frac{c}{B}\sum_\alpha \left[\hat{\mathbf{b}} \times \nabla p_{\alpha\perp} + (p_{\alpha\parallel} - p_{\alpha\perp})(\nabla \times \hat{\mathbf{b}})_\perp\right]$$

$$\boxed{\mathbf{J}_{\perp gc} = \frac{c}{B}\sum_\alpha \hat{\mathbf{b}} \times \nabla p_{\alpha\perp}} \qquad \text{for} \quad (\nabla \times \hat{\mathbf{b}})_\perp \approx 0$$

• Gyrokinetic MHD Equations: a reduced set of equations but in full toroidal geometry

-- For $k_\perp^2 \rho_i^2 \ll 1$ $\quad \bar{F} \to F \quad \bar{\phi} \to \phi \quad \bar{A}_\parallel \to A_\parallel \quad \overline{\mathbf{v}_\perp \cdot \mathbf{A}_\perp} \to 0$

-- Ampere's law

$$\nabla_\perp^2 A_\parallel = -\frac{4\pi}{c}J_\parallel$$

$$\nabla_\perp^2 \mathbf{A}_\perp - \cancel{\frac{1}{v_A^2}\frac{\partial^2 \mathbf{A}_\perp}{\partial t^2}} = -\frac{4\pi}{c}\mathbf{J}_\perp \qquad \text{Negligible for} \quad \omega^2 \ll k_\perp^2 v_A^2$$

$$\delta\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B} \qquad \mathbf{b} \equiv \frac{\mathbf{B}}{B}$$

-- Perpendicular Current due to the ions: $\mathbf{J}_\perp = \frac{c}{B}\mathbf{b} \times \nabla_\perp p_i$

-- Vorticity Equation: $\frac{d}{dt}\nabla_\perp^2\phi - 4\pi\frac{v_A^2}{c^2}\nabla \cdot (\mathbf{J}_\parallel + \mathbf{J}_\perp) = 0 \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi \times \mathbf{b} \cdot \nabla$

-- Ohm's law: $E_\parallel \equiv -\frac{1}{c}\frac{\partial A_\parallel}{\partial t} - \mathbf{b} \cdot \nabla\phi = -\frac{T_e}{e}\frac{1}{p_e}\frac{\partial p_e}{\partial x_\parallel} + \eta J_\parallel$

-- Equation of State: $\frac{dp_\alpha}{dt} = 0$

-- Normal modes: $\omega = \pm k_\parallel v_A$ for $E_\parallel \approx 0$ and $\mathbf{J}_\perp \approx 0$

• MHD Equilibrium

1. For a given ion pressure profile, we should obtain

$$\boxed{\nabla \cdot \mathbf{J}_\perp \approx \frac{c}{B}\nabla_\perp p_i \cdot \nabla \times \hat{\mathbf{b}} \approx 0}$$

or

$$\boxed{\nabla_\perp \left(\frac{B^2}{8\pi} + p_i\right) \approx 0}$$

2. We then solve the coupled equations of

$$\boxed{\begin{aligned} &\frac{d}{dt}\nabla_\perp^2\phi + \frac{v_A^2}{c}(\mathbf{b} \cdot \nabla)\nabla_\perp^2 A_\parallel = 0 \\ &\frac{1}{c}\frac{\partial A_\parallel}{\partial t} + \mathbf{b} \cdot \nabla\phi \approx \frac{T_e}{e}\frac{1}{p_e}\frac{\partial p_e}{\partial x_\parallel} + \eta\frac{c}{4\pi}\nabla_\perp^2 A_\parallel \end{aligned}} \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} - \frac{c}{B}\nabla\phi \times \mathbf{b} \cdot \nabla$$

3. If we look for a solution for $\phi \to 0$ which, in turn, gives $\frac{\partial A_\parallel}{\partial t} \to 0$,

this is then the equilibrium solution that satisfies the quasineutral condition of

$$\boxed{\nabla \cdot \mathbf{J}_\parallel = 0}$$

Summary II

• This set of gyrokinetic equations can indeed be used to study steady state electromagnetic turbulence to understand the physics of radial electric field wells.

• It can also recover the equilibrium MHD equations in the absence of fluctuations.

• It will be interesting to couple a 3D global EM PIC code, e.g., GTS [Wang et al., 2006] with a 3D MHD equilibrium code, e.g., SPEC [Hudson et al., 2012] for transport minimization purposes.

• A SciDAC proposal, "First Principles Based Transport and Equilibrium Module for Whole Device Modeling and Optimization" has been submitted to DoE.