1 stepped pressure equilibrium code : be00aa

1. Construction of Bessel function solution of Beltrami field.

1.0.1 Bessel function solution

1. Consider a cylinder with minor radius a, major radius $R = 1$, with given toroidal flux ψ_t and given "pinch-parameter" μ, then

$$B^\zeta = \alpha J_0(\mu r), \tag{1}$$
$$B^\theta = r^{-1}\alpha J_1(\mu r), \tag{2}$$

   where $\alpha = \psi_t \mu / 2\pi a J_1(\mu a)$.

2. The q-profile is $q = B^\zeta / B^\theta$.

3. This solution is the minimum energy solution only for $\mu a < 3.11\ldots$ [1]. At this point, a helical state is expected to emerge.

1.0.2 Cylindrical geometry

1. In cylindrical geometry, $\sqrt{g} = r$, the vector potential may be written $\mathbf{A} = A_\theta \nabla\theta + A_\zeta \nabla\zeta$ where

$$A_\theta = \sum_p a_{\theta,p} r^{2+p}, \quad A_\zeta = \sum_p a_{\zeta,p} r^{2+p}. \tag{3}$$

2. The magnetic field, $\mathbf{B} = \nabla \times \mathbf{A}$, is then

$$B^r = 0, \quad B^\theta = \frac{-\partial_r A_\zeta}{r} = -\sum_p a_{\zeta,p}(2+p) r^p, \quad B^\zeta = \frac{+\partial_r A_\theta}{r} = +\sum_p a_{\theta,p}(2+p) r^p. \tag{4}$$

$$B_r = 0, \quad B_\theta = -\sum_p a_{\zeta,p}(2+p) r^{p+2}, \quad B_\zeta = +\sum_p a_{\theta,p}(2+p) r^p. \tag{5}$$

3. The energy integrals are given

$$\mathbf{B}\cdot\mathbf{B} = \sum_{p,q} \left[a_{\zeta,p} a_{\zeta,q} (2+p)(2+q) r^{p+q+2} + a_{\theta,p} a_{\theta,q} (2+p)(2+q) r^{p+q} \right] \tag{6}$$

$$\mathbf{A}\cdot\mathbf{B} = \sum_{p,q} a_{\theta,p} a_{\zeta,q} \left[-r^{2+p}(2+q) r^q + r^{2+q}(2+p) r^p \right] = \sum_{p,q} a_{\theta,p} a_{\zeta,q} r^{2+p+q}(p-q) \tag{7}$$

$$\int_{\mathcal{V}} dv \equiv \int dr \int d\theta \int d\zeta \;\; r = (2\pi)^2 \int dr \;\; r \tag{8}$$

$$F \equiv \int_{\mathcal{V}} \mathbf{B}\cdot\mathbf{B} \;\; dv - \mu \int_{\mathcal{V}} \mathbf{A}\cdot\mathbf{B} \;\; dv \tag{9}$$
$$= \sum_{p,q} a_{\zeta,p} a_{\zeta,q} (2+p)(2+q) \int dr \;\; r^{p+q+2+1} + \sum_{p,q} a_{\theta,p} a_{\theta,q} (2+p)(2+q) \int dr \;\; r^{p+q+1} \tag{10}$$
$$-\mu \sum_{p,q} a_{\theta,p} a_{\zeta,q} (p-q) \int dr \;\; r^{p+q+2+1}. \tag{11}$$

$$\frac{\partial F}{\partial a_{\theta,p}} = \sum_q a_{\zeta,q}(2+p)(2+q) \left. \frac{r^{p+q+2}}{p+q+2} \right|_0^a - \mu \sum_q a_{\zeta,q}(p-q) \left. \frac{r^{2+q+p+2}}{2+q+p+2} \right|_0^a \tag{12}$$

be00aa.h last modified on 2011-11-01 ;

[1] J. B Taylor. Relaxation and magnetic reconnection in plasmas. *Rev. Mod. Phys.*, 58:741, 1986.