1 stepped pressure equilibrium code : fu01aa

1. Evaluates volume integrals. In the annular regions, the volume integrals are given by the following:

$$
\begin{aligned}
& \int_{s_{l-1}}^{s_l} ds \ \oint\!\!\oint d\theta d\zeta \ \sqrt{g} \ B^s B^s g_{ss} \\
= \ & \sum_{i=1}^{N_i} \sum_{j,k} \int_{s_{i-1}}^{s_i} ds \ (-m_j A_{\zeta,l,j} - n_j A_{\theta,l,j})(-m_k A_{\zeta,l,k} - n_k A_{\theta,l,k}) \left\langle s_j \left| \sqrt{g}^{-1} g_{ss} \right| s_k \right\rangle \\
= \ & m_j \ A_{\zeta,l,j,p,i-1+l} \ m_k \ A_{\zeta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \ \varphi_{l,p} \varphi_{r,q} \left\langle s_j \left| \sqrt{g}^{-1} g_{ss} \right| s_k \right\rangle \\
+ \ & m_j \ A_{\zeta,l,j,p,i-1+l} \ n_k \ A_{\theta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \ \varphi_{l,p} \varphi_{r,q} \left\langle s_j \left| \sqrt{g}^{-1} g_{ss} \right| s_k \right\rangle \\
+ \ & n_j \ A_{\theta,l,j,p,i-1+l} \ m_k \ A_{\zeta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \ \varphi_{l,p} \varphi_{r,q} \left\langle s_j \left| \sqrt{g}^{-1} g_{ss} \right| s_k \right\rangle \\
+ \ & n_j \ A_{\theta,l,j,p,i-1+l} \ n_k \ A_{\theta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \ \varphi_{l,p} \varphi_{r,q} \left\langle s_j \left| \sqrt{g}^{-1} g_{ss} \right| s_k \right\rangle
\end{aligned}
\tag{1}
$$

$$
\begin{aligned}
& \int_{s_{l-1}}^{s_l} ds \ \oint\!\!\oint d\theta d\zeta \ \sqrt{g} \ B^s B^\theta g_{s\theta} \\
= \ & \sum_{i=1}^{N_i} \sum_{j,k} \int_{s_{i-1}}^{s_i} ds \ \ (-m_j A_{\zeta,l,j} - n_j A_{\theta,l,j})(-A'_{\zeta,l,k}) \left\langle s_j \left| \sqrt{g}^{-1} g_{s\theta} \right| c_k \right\rangle \\
= \ & m_j A_{\zeta,l,j,p,i-1+l} A_{\zeta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \ \varphi_{l,p} \varphi'_{r,q} \left\langle s_j \left| \sqrt{g}^{-1} g_{s\theta} \right| c_k \right\rangle \\
+ \ & n_j A_{\theta,l,j,p,i-1+l} A_{\zeta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \ \varphi_{l,p} \varphi'_{r,q} \left\langle s_j \left| \sqrt{g}^{-1} g_{s\theta} \right| c_k \right\rangle
\end{aligned}
\tag{2}
$$

$$
\begin{aligned}
& \int_{s_{l-1}}^{s_l} ds \ \oint\!\!\oint d\theta d\zeta \ \sqrt{g} \ B^s B^\zeta g_{s\zeta} \\
= \ & \sum_{i=1}^{N_i} \sum_{j,k} \int_{s_{i-1}}^{s_i} ds \ \ (-m_j A_{\zeta,l,j} - n_j A_{\theta,l,j})(A'_{\theta,l,k}) \left\langle s_j \left| \sqrt{g}^{-1} g_{s\zeta} \right| c_k \right\rangle \\
= \ & -m_j A_{\zeta,l,j,p,i-1+l} A_{\theta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \ \varphi_{l,p} \varphi'_{r,q} \left\langle s_j \left| \sqrt{g}^{-1} g_{s\zeta} \right| c_k \right\rangle \\
+ \ & -n_j A_{\theta,l,j,p,i-1+l} A_{\theta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \ \varphi_{l,p} \varphi'_{r,q} \left\langle s_j \left| \sqrt{g}^{-1} g_{s\zeta} \right| c_k \right\rangle
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
& \int_{s_{l-1}}^{s_l} ds \ \oint\!\!\oint d\theta d\zeta \ \sqrt{g} \ B^\theta B^\theta g_{\theta\theta} \\
= \ & \sum_{i=1}^{N_i} \sum_{j,k} \int_{s_{i-1}}^{s_i} ds \ (-A'_{\zeta,l,j})(-A'_{\zeta,l,k}) \left\langle c_j \left| \sqrt{g}^{-1} g_{\theta\theta} \right| c_k \right\rangle \\
= \ & A_{\zeta,l,j,p,i-1+l} A_{\zeta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \ \varphi'_{l,p} \varphi'_{r,q} \left\langle c_j \left| \sqrt{g}^{-1} g_{\theta\theta} \right| c_k \right\rangle
\end{aligned}
\tag{4}
$$

$$
\int_{s_{l-1}}^{s_l} ds \ \oint\!\!\oint d\theta d\zeta \ \sqrt{g} \ B^\theta B^\zeta g_{\theta\zeta}
$$

$$
\begin{aligned}
&= \sum_{i=1}^{N_i}\sum_{j,k}\int_{s_{i-1}}^{s_i} ds \;\; (-A'_{\zeta,l,j})(A'_{\theta,l,k}) \left\langle c_j \left| \sqrt{g}^{-1} g_{\theta\zeta} \right| c_k \right\rangle \\
&= -A_{\zeta,l,j,p,i-1+l} A_{\theta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \;\; \varphi'_{l,p}\varphi'_{r,q} \left\langle c_j \left| \sqrt{g}^{-1} g_{\theta\zeta} \right| c_k \right\rangle
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
&\int_{s_{l-1}}^{s_l} ds \;\; \oint\!\!\oint d\theta d\zeta \;\; \sqrt{g} \;\; B^\zeta B^\zeta g_{\zeta\zeta} \\
&= \sum_{i=1}^{N_i}\sum_{j,k}\int_{s_{i-1}}^{s_i} ds \;\; (A'_{\theta,l,j})(A'_{\theta,l,k}) \left\langle c_j \left| \sqrt{g}^{-1} g_{\zeta\zeta} \right| c_k \right\rangle \\
&= A_{\theta,l,j,p,i-1+l} A_{\theta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \;\; \varphi'_{l,p}\varphi'_{r,q} \left\langle c_j \left| \sqrt{g}^{-1} g_{\zeta\zeta} \right| c_k \right\rangle
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
&\int_{s_{l-1}}^{s_l} ds \;\; \oint\!\!\oint d\theta d\zeta \;\; \sqrt{g} \;\; \mathbf{A}\cdot\mathbf{B} \\
&= \sum_{i=1}^{N_i}\int_{s_{i-1}}^{s_i} ds \;\; \oint\!\!\oint d\theta d\zeta \;\; [-A_\theta \partial_s A_\zeta + A_\zeta \partial_s A_\theta] \\
&= \sum_{i=1}^{N_i}\sum_{j,k}\int_{s_{i-1}}^{s_i} ds \;\; \left[-A_{\theta,l,j} A'_{\zeta,l,k} + A_{\zeta,l,j} A'_{\theta,l,k}\right] \langle c_j \,|1|\, c_k \rangle \\
&= -A_{\theta,l,j,p,i-1+l} A_{\zeta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \;\; \varphi_{l,p}\varphi'_{r,q} \langle c_j \,|1|\, c_k \rangle \\
&\quad + A_{\zeta,l,j,p,i-1+l} A_{\theta,l,k,q,i-1+r} \int_{s_{i-1}}^{s_i} ds \;\; \varphi_{l,p}\varphi'_{r,q} \langle c_j \,|1|\, c_k \rangle
\end{aligned} \tag{7}
$$

2. In the innermost volume, there are additional radial factor $s^{m/2}$, but these are incorporated into the `igss`, `igst`, `igsz`, `igtt`, `igtz`, `igzz`, and `igoo`.

3. The above integrals are returned in

   `volume(0,lvol)=`$\int_{\mathcal{V}} dv$

   `volume(1,lvol)=`$\int_{\mathcal{V}} dv \mathbf{B}\cdot\mathbf{B}$

   `volume(2,lvol)=`$\int_{\mathcal{V}} dv \mathbf{A}\cdot\mathbf{B}$

4.

$$
F_l \equiv \frac{P_l}{\gamma-1} V_l^{1-\gamma} + \int_{\mathcal{V}_l} \frac{B^2}{2} dv. \tag{8}
$$

fu01aa.h last modified on 2011-11-17 ;