# pc00ab

briefly

[called by: .] [calls: .]

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### 1.0.1 Energy functional

1. The energy functional is

$$F \equiv \sum_{l=1}^{N} \int_{\mathcal{V}} \left( \frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv, \tag{1}$$

where $N \equiv$`Nvol` is the number of interfaces.

2. Assuming that the toroidal and poloidal fluxes, $\psi_t$ and $\psi_p$, the helicity, $\mathcal{K}$, the helicity multiplier, $\mu$, and/or the interface rotational-transforms, $\iota$, are appropriately constrained, the Beltrami fields in each volume depend only the geometry of the adjacent interfaces. So, the energy functional is assumed to be a function of "position", i.e. $F = F(R_{l,j}, Z_{l,j})$.

3. Introducing a ficitious time, $t$, the position may be advanced according to

$$\begin{aligned} \frac{\partial R_j}{\partial t} &\equiv -\frac{\partial}{\partial R_j} \sum_{l=1}^{N} \int \left( \frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv, \\ \frac{\partial Z_j}{\partial t} &\equiv -\frac{\partial}{\partial Z_j} \sum_{l=1}^{N} \int \left( \frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv. \end{aligned} \tag{2}$$

4. There remain degrees of freedom in the angle representation of the interfaces.

### 1.0.2 Spectral energy minimization

5. Consider variations which do not affect the geometry of the surfaces,

$$\delta R = R_\theta \, u, \tag{3}$$

$$\delta Z = Z_\theta \, u, \tag{4}$$

where $u$ is a angle variation.

6. The corresponding variation in each of the Fourier harmonics is

$$\delta R_j \equiv \oint\!\!\oint d\theta d\zeta \;\; R_\theta \; u \; \cos\alpha_j, \tag{5}$$

$$\delta Z_j \equiv \oint\!\!\oint d\theta d\zeta \;\; Z_\theta \; u \; \sin\alpha_j, \tag{6}$$

7. Following Hirshman et al., introducing the normalized spectral width

$$M \equiv \frac{\sum_j (m_j^p + n_j^q)(R_{l,j}^2 + Z_{l,j}^2)}{\sum_j (R_{l,j}^2 + Z_{l,j}^2)}, \tag{7}$$

8. Using the notation

$$N \equiv \sum_j \lambda_j (R_{l,j}^2 + Z_{l,j}^2), \tag{8}$$

$$D \equiv \sum_j (R_{l,j}^2 + Z_{l,j}^2), \tag{9}$$

where $\lambda_j \equiv m_j^p + n_j^q$, the variation in the normalized spectral width is

$$\delta M = (\delta N - M \delta D)/D. \tag{10}$$

9. For tangential variations,

$$\delta N = 2 \oint\oint d\theta d\zeta \;\; d\theta d\zeta \; u \left( R_\theta \sum_j \lambda_j R_j \cos\alpha_j + Z_\theta \sum_j \lambda_j Z_j \sin\alpha_j \right), \tag{11}$$

$$\delta D = 2 \oint\oint d\theta d\zeta \;\; d\theta d\zeta \; u \left( R_\theta \sum_j R_j \cos\alpha_j + Z_\theta \sum_j Z_j \sin\alpha_j \right). \tag{12}$$

10. The "tangential spectral-width descent direction" is thus

$$\frac{\partial u}{\partial t} = -\left[ R_\theta \sum_j (\lambda_j - M) R_j \cos\alpha_j / D + Z_\theta \sum_j (\lambda_j - M) Z_j \sin\alpha_j / D \right]. \tag{13}$$

11. This suggests that position should be advanced according to

$$\frac{\partial R_j}{\partial t} \equiv -\frac{\partial}{\partial R_j} \sum_{l=1}^{N} \int \left( \frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv - [R_\theta (R_\theta X + Z_\theta Y)]_j, \tag{14}$$

$$\frac{\partial Z_j}{\partial t} \equiv -\frac{\partial}{\partial Z_j} \sum_{l=1}^{N} \int \left( \frac{p}{\gamma - 1} + \frac{B^2}{2} \right) dv - [Z_\theta (R_\theta X + Z_\theta Y)]_j, \tag{15}$$

where $X \equiv \sum_j (\lambda_j - M) R_j \cos\alpha_j / D$ and $Y \equiv \sum_j (\lambda_j - M) Z_j \sin\alpha_j / D$.

### 1.0.3 numerical implementation

12. The spectral condensation terms,

$$R_\theta (R_\theta X + Z_\theta Y) = \sum_{j,k,l} m_j m_k (\lambda_l - M) R_j (+R_k R_l \sin\alpha_j \sin\alpha_k \cos\alpha_l - Z_k Z_l \sin\alpha_j \cos\alpha_k \sin\alpha_l)/D, \tag{16}$$

$$Z_\theta (R_\theta X + Z_\theta Y) = \sum_{j,k,l} m_j m_k (\lambda_l - M) Z_j (-R_k R_l \cos\alpha_j \sin\alpha_k \cos\alpha_l + Z_k Z_l \cos\alpha_j \cos\alpha_k \sin\alpha_l)/D, \tag{17}$$

are calculated using triple angle expressions: IT IS VERY LIKELY THAT FFTs WOULD BE FASTER!!!