Electron Thermal Transport in the Madison Symmetric Torus

Ph.D. Defense
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Thanks to:
The MST group at UW-Madison
in collaboration with UCLA
and RPI
Motivation and Outline

Motivation: To measure the coefficients of electron particle and thermal transport with sufficient accuracy to make valid comparisons to theoretical models (R.R.).

• “Standard” MST plasmas w/ sawteeth: $T_e$, $n_e$, $T_i$, $j$
• Using MSTFit to calculate $P_{\parallel}$ and $\Pi_e$
• The RFP as a test bed for stochastic magnetic transport
• DEBS modeling of MST Standard plasmas
• Electron thermal conductivity: measured v. R.R. modeled
• Comparisons to other MST plasmas.
• Summary
The Madison Symmetric Torus Reversed-Field Pinch

Magnetic diagnostics:
Radial field:
32 coils around the poloidal gap

Mode spectra:
64 sets of 3 orthogonal coils toroidally distributed
16 sets of 3 orthogonal coils poloidally distributed
8 sets of 3 orthogonal coils poloidally distributed

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>minor radius</td>
<td>0.52 m</td>
</tr>
<tr>
<td>major radius</td>
<td>1.5 m</td>
</tr>
<tr>
<td>plasma current</td>
<td>&lt; 0.6 MA</td>
</tr>
<tr>
<td>loop voltage</td>
<td>~ 10 V</td>
</tr>
<tr>
<td>toroidal volt-sec</td>
<td>&lt; 2 V-s</td>
</tr>
<tr>
<td>pulse length</td>
<td>&lt; 80 ms</td>
</tr>
<tr>
<td>electron temperature</td>
<td>&lt; 1500 eV</td>
</tr>
<tr>
<td>ion temperature</td>
<td>&lt; 500 eV</td>
</tr>
<tr>
<td>electron density</td>
<td>&lt; 2 x 10^{13} cm^{-3}</td>
</tr>
<tr>
<td>beta</td>
<td>~ 5 - 20%</td>
</tr>
<tr>
<td>energy confinement time</td>
<td>&lt; 10 ms</td>
</tr>
</tbody>
</table>
Poloidal Projection of Diagnostic Locations

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“Standard” MST Plasma Discharges

13-nov-2000 Shot 69

- Typical plasma parameters:
  - “Standard” w/ full PFN
  - $I_p \sim 375$ kA
  - $n_e \sim 1.1 \times 10^{13}$ cm$^{-3}$
  - $F \sim -0.22$
  - Deuterium

- Sawtooth period $\sim 5.5$ ms

- Time of interest: $\sim 15$ ms, i.e. early in the discharge but during plasma “flattop”
Shots are ensembled wrt. the sawtooth crash.

- Plasma shots are ensembled for 2 reasons:
  - Want an “average” measure of quantities, i.e. smooth out the fluctuations
  - Need ~400 shots to get $T_e(r,t)$ from Thomson sct.

- This 6 ms time window about the crash is subdivided into 12 time slices, every 0.5 ms.
  - High time resolution signals (e.g. FIR) are computed as 0.1 ms ensembles.
  - Raw TS data is ensembled in 0.5 ms bins.
Density and Temperature Profile Evolution

MSTfit splines to 16 chord Thomson Scattering

MSTfit Abel inverted 11 chord FIR Interferometry*

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*in collaboration with S. Terry, W. Ding
Comparison of Raw TS Data to MSTFit Splines

$t = -1.75$ ms

$t = -0.25$ ms

$t = +0.75$ ms
Select Time Slices of $T_e(r)$ and $n_e(r)^*$

*in collaboration with S. Terry, W. Ding
Ion Temperature Profile Evolution

MSTFIT splines to 6 chord Rutherford Scattering*

- Ions are anomalously heated at the sawtooth crash to temperatures above the electron temperature.
- Must be some other mechanism for ion heating than e-i collisions.

*in collaboration with J. Reardon
Anomalous ion heating at the sawtooth crash may be due to current profile relaxation.

MSTFit reconstructions of the equilibrium profiles for Standard (solid) and Non-Reversed plasmas (dotted) between sawteeth.
MSTFit* and Measurement of j(r)

- MSTFit is an equilibrium reconstruction code that solves the Grad-Shafranov equation in toroidal geometry, constrained by experimental data.

- To calculate j(r): MSE, Faraday rotation, Mirnov coils, flux loops, etc.

*in collaboration with J. Anderson
P\(_\square\)(r,t) is found from the equilibrium fields.*

From Poynting’s theorem:

\[ P_{\square} = \frac{dW}{dt} = \nabla P_{\text{Poynt.}} \nabla W_{\text{mag}} = \nabla \left( \frac{1}{\mu_0} \oint_S (E \cdot B) \cdot da \right) \frac{\partial}{\partial t} \left( \frac{1}{\mu_0} (\nabla E^2 + B^2 / \mu_0) \right) dV \]

MSTFit solves the Grad-Shafranov Eqn. to find toroidal and poloidal flux functions, \( R, S(r) \): \( J(r), B(r) \)

From 12 equilibrium solutions through the sawtooth cycle, using Maxwell’s Eqns. and finite difference time derivatives:

\[ B = \nabla \nabla A \quad E = \nabla \frac{\partial A}{\partial t} \quad E_{\square} = \nabla \nabla A_{\square} = \nabla \left( \frac{1}{2} \frac{\partial E}{\partial t} \right) \quad E_{\square} = \nabla \frac{\partial A_{\square}}{\partial t} = \nabla \left( \frac{1}{R} \frac{\partial B^2}{\partial t} \right) \]

With \( E(r, t_{\text{avg}}) \) and \( B(r, t) \), the \( P_{\square}(r,t) \) is calculated without measuring \( Z_{\text{eff}}(r,t) \) or modeling the resistivity.

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Using Ohm’s law and neoclassical resistivity implies a $Z_{\text{eff}} \sim 3.5$. 

For comparison, simple Ohm’s law with neoclassical resistivity:

$$E(r) = \square^* (r) j(r)$$

$$\square^* (n_e(r), T_e(r), Z_{\text{eff}}(r)) = \square_{\text{neocl}}(n_e(r), T_e(r), Z^*_{\text{eff}}(r))$$

$$P = \square(\vec{E} \cdot \vec{j}) \frac{\partial V}{\partial \rho} \square \square^* \vec{j} \cdot \vec{j} \frac{\partial V}{\partial \rho} = \square (\vec{j}^2) \frac{\partial V}{\partial \rho} \square$$

Equating, leads to an implied $Z^*_{\text{eff}}(r)$:

At the wall an average $Z_{\text{eff}} \sim 3.5$ is needed to balance the input power.

$$P_{\text{in}} = (I_p V_p + \frac{R_0}{a} I_p V_p) \hat{W}_{\text{mag}} \square (I_p V_p \frac{R_0 B_0}{2 \rho g} V_{\text{tg}}) \hat{W}_{\text{mag}}$$

$V_{\text{loop}} I_p$ (PFM)

With flat $Z_{\text{eff}}(r) = 2$
The MHD dynamo is partially accounted for.

From Poynting’s theorem (again):

\[ E \cdot j = \frac{1}{2} \frac{\partial}{\partial t} \int \left( E^2 + \frac{1}{\mu_0} B^2 \right) + \frac{1}{\mu_0} \int \left( E \cdot \nabla \times B \right) \]

Expanding in terms of a fluctuation average over mean and fluctuating quantities:

\[ < \tilde{v} \cdot \tilde{B} > \] EMF is in this term. Tearing mode (e.g.) induced current fluctuations are not accounted for.

\[ < p_{q0} > = \bar{E} \cdot \bar{j} + < \tilde{E} \cdot \tilde{j} > = \frac{1}{2 \mu_0} \frac{\partial \tilde{B}^2}{\partial t} + \frac{1}{\mu_0} \left( \bar{E} \cdot \nabla \times \bar{B} \right) + \frac{1}{2 \mu_0} \frac{\partial \tilde{B}^2}{\partial t} + \frac{1}{\mu_0} < \nabla \times (\bar{E} \times \bar{B}) > \]

Calculated with multiple MSTFit equilibria.

Dynamo fluctuations result in an EMF which sums with the inductive E to drive current in the resistive plasma. The effects of this current are measured in our diagnostics. Those measurements are used in MSTFit to find the total B and j. Then E is calculated from finite difference time derivatives.
Measuring $\dot{Q}_e$ requires measurements of the local power balance.

\[
\frac{\partial}{\partial t} \left( \frac{3}{2} n_e T_e \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( r Q_e \right) + S_{E,e}
\]

\[
Q_e = \frac{1}{r} \left[ S_{E,e} - \frac{\partial}{\partial t} \left( \frac{3}{2} n_e T_e \right) \right]
\]

\[
Q_e = Q_e^{\text{conv}} + Q_e^{\text{cond}} = \frac{5}{2} \left[ n_e \right] T_e \left[ \frac{\partial}{\partial r} n_e \right] r T_e
\]

\[
S_{E,e} = P_{\parallel} - P_{ei} - P_{ez} - P_{\text{rad}} - e \vec{E} \cdot \vec{E}_r
\]

\[
P_{\text{rad}} = C_{\text{rad}} \left( \frac{r}{a} \right)^8 P_{\parallel}
\]

\[
P_{ei} = \frac{m_e}{m_i} n_i n_e e^4 \ln(\Lambda) \frac{4 \varepsilon_0}{4 \varepsilon_0} \left( T_e - T_i \right) \sqrt{2 / \pi^3 m_e T_e^3}
\]
Radial Electric Field

The radial electric field was calculated from the ion momentum balance equation, and compared to measurements from the HIBP.

![Graphs showing radial electric field and peak values over rho and time](image)

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*in collaboration with J. Lei, D. Demers, U. Shah*
Electron thermal conductivity profiles indicate an edge transport barrier.

- During Standard plasmas, $X_e$ in the core and mid-radius regions is $\sim 400 \text{ m}^2/\text{s}$.
- In the edge, a transport barrier is inferred, and $X_e$ drops an order of magnitude to $\sim 40 \text{ m}^2/\text{s}$.
- The thermal conductivity is reduced after the sawtooth crash, as compared to before.
- The edge transport barrier shows little change over the sawtooth cycle.
The magnetic fluctuation amplitude is not sufficient to explain the observed electron transport evolution.
The RFP is a good test bed for stochastic transport.

- Large (m=0,1) magnetic tearing-mode islands are present in the MST.
- Overlap of these islands leads to stochastic field-line wandering.
- Regions of stochastic field are confirmed in field-line tracing simulations. (DEBS/MAL)
Rechester-Rosenbluth Model of Stochastic Transport

In 1978, Rechester and Rosenbluth expanded the work of Callen in, “Electron Heat Transport in a Tokamak with Destroyed Magnetic Surfaces.”

They suggest that for stochastic magnetic field lines, $\nabla_e$ for radial heat transport is given by:

$$D_{st}(r) \nabla_R \nabla_R \frac{|b_{m,n}(r)|^2}{B^2(r)} \nabla_m \nabla_n$$

$$D_{st}(r_{m,n}) \nabla_L \nabla L_{ef} \frac{|\tilde{b}_r(r_{m,n})|^2}{B^2(r_{m,n})}$$

$$L_{ef} = L_{AC}^{[l]} + L_{mfp}^{[l]}$$

This is valid when the “stochasticity parameter” $s >> 1$. They state:

“If $s > 1$, then magnetic surfaces are destroyed in the region between $r_{m,n}$ and $r_{m',n'}$, and the field lines wander ergodically. $s=1$ corresponds to overlapping islands of different helicity. The transition region is very complicated, and we will be discussing mainly the case of high stochasticity, $s >> 1$, with dense rational surfaces.”

$$s = \frac{1}{2} \frac{(w_{m,n} \nabla w_{m',n'})}{|r_{m,n} \nabla r_{m',n'}|}$$

$$w_{m,n} = 4 \sqrt{\frac{r_{m,n} (b_z)_{m,n}}{nB \nabla \frac{\partial A}{\partial r} |_{r_{m,n}}}}$$

$L_{ef} \sim 0.7 - 1.0$ m in MST ($R=1.5$ m) from DEBS/MAL simulations.
DEBS Simulations of Standard MST Plasmas*

- DEBS is an initial value 3-D code that solves the normalized non-linear resistive MHD equations in doubly periodic cylindrical geometry:

\[
\nabla (\frac{\partial v}{\partial t} + Sv \cdot \nabla v) = S j \nabla B + P_m \Delta^2 v
\]

- The equilibrium fields and resistivity profile were used to initialize the DEBS run with \( S \approx 10^6 \).

- This value of \( S \) is within a factor of 4 of the experiment, and many of the observed dynamics are demonstrated by the simulation.

*in collaboration with J.C. Wright
DEBS reproduces the MHD activity for S~10^6.

- The edge magnetic fluctuation level (normalized by the axial field) measured in MST Standard plasmas is very similar to the level simulated by DEBS for S~10^6.

- Since \( t_{\text{res}} \sim 0.5-1.0 \) s, the simulated sawtooth period is \( \sim 3-6 \) ms, which agrees very well with the experimental sawtooth period of 6 ms.
Toroidal mode spectrum is slightly “off” in DEBS.

- Too much energy (relatively) resides in the DEBS m=0 fluctuations, particularly at very low toroidal mode number.
- The spectral fall-off with increasing n-mode number is not quite right.
- Averaging over multiple simulations could make the spectral dependence nicer.
DEBS yields radial fluctuation eigenfunctions.

- DEBS yields eigenfunctions of $b_r$, $b_\parallel$, and $b_z$. $b_\parallel$ and $b_z$ can be compared to MST measurements at the wall to determine the normalization factor for $b_r$.

- Once the $b_r$ eigenfunctions are established the magnetic island widths, the stochasticity parameter, and the thermal conductivity can be calculated.
Magnetic islands overlap in Standard plasmas.

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*in collaboration with B. Hudson
\( n_e \) and \( n_{RR} \) agree where stochasticity is high.

- In the **core** the stochasticity is low and the fluctuation amplitude overestimates the thermal transport.
- In the **mid-radius** region, where resonant surfaces are closely packed, the stochasticity is high. In this region the stochastic transport is in good agreement with the measured transport.
- The **edge** transport governs the overall confinement because of the implied transport barrier there.
Volume-averaged $\square_e$ v. $\square_{RR}$ shows good agreement.

Over the sawtooth cycle, (core & mid radius)-averaged $\square_e$ and $\square_{RR}$ scale linearly, excluding the single time point following the sawtooth crash.

$\langle X_{RR} \rangle$ ~ $\pi^* L_{eff}^* (b/B)^2 v_{te}$ (m$^2$/s)

$t=+0.25$ ms

time (ms)

normalized fluctuation level

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March 20th, 2002
Volume-averaged $\bar{D}_e$ scales with field stochasticity.

Through the discharges, the field stochasticity appears to be a good qualitative measure of the transport characteristics of the core & mid-radius regions of the plasma.

This holds across different discharge conditions.

During Standard plasmas the time point following the crash (filled circle) has a stochasticity lower than implied from fluctuation level alone, since the q-shear has increased from the current profile relaxation.
# A Summary of 0-D Transport Properties

<table>
<thead>
<tr>
<th></th>
<th>FIR</th>
<th>TS</th>
<th>RS</th>
<th>MSE</th>
<th>$E_E$</th>
<th>$E_{tot}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F</strong>&lt;br&gt; kA</td>
<td>$I_p$&lt;br&gt; x10$^{13}$ cm$^3$</td>
<td>$\bar{n}_e$&lt;br&gt; eV</td>
<td>$T_e$&lt;br&gt; eV</td>
<td>$T_i$&lt;br&gt; T</td>
<td>$B_0$</td>
<td></td>
</tr>
<tr>
<td>PPCD</td>
<td>-0.97</td>
<td>383</td>
<td>1.10</td>
<td>825</td>
<td>410</td>
<td>0.395</td>
</tr>
<tr>
<td>Stan</td>
<td>-0.22</td>
<td>375</td>
<td>1.10</td>
<td>310</td>
<td>240</td>
<td>0.347</td>
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<tr>
<td>N.rev</td>
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<td>374</td>
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<td>350</td>
<td>190</td>
<td>0.306</td>
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<tr>
<td><strong>F&gt;0</strong>&lt;br&gt; +0.02</td>
<td>390</td>
<td>1.00</td>
<td>250</td>
<td>225</td>
<td>0.329</td>
<td>1.25</td>
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<tr>
<td><strong>F&gt;&gt;0</strong>&lt;br&gt; +0.03</td>
<td>386</td>
<td>1.03</td>
<td>250</td>
<td>225</td>
<td>0.310</td>
<td>0.75</td>
</tr>
</tbody>
</table>
Comparisons of Magnetic Fluctuations

- The reversal surface is at the wall for F=0 plasmas, stabilizing m=0 magnetic fluctuations.

- A corresponding reduction in the m=0 mode amplitudes is observed, which decreases even further for F>0.

- During PPCD plasmas there is an overall reduction in magnetic fluctuations.
The energy confinement time increases by ~30% for F=0 plasmas, then decreases as F is raised above zero.

PPCD plasmas exhibit the highest confinement and $\beta$.

Total $\beta$ is roughly equal for Standard and Non-Reversed plasmas, but is reduced when F>0.
Volume-Averaged $\bar{c}_e$ for Various Discharges

- Away from the sawtooth crash PPCD and F=0 plasmas have the lowest thermal conductivity, consistent with the measurements of $\bar{c}_e$.

- The edge transport barrier is absent in F>0 plasmas.

- At the sawtooth crash, $\bar{c}_e$ reaches a very high value for F=0 and F>0 plasmas.

This may be explained by j(r) observations: j(r) does not relax as much during F=0 plasmas (recall ion heating), hence q(r) does not peak, and the stochasticity is not reduced at the crash. Thus fluctuation increases at the crash are more directly linked to enhanced transport.
\( \bar{\Delta}_e \) obeying \( \bar{\Delta}_RRe \) imposes a lower limit on \( \bar{\Delta}_i \)

Averaging over the core, \( \langle \bar{\Delta}_RRe \rangle \sim 1.3 \langle \bar{\Delta}_e \rangle \), so we might expect
\[ \langle \bar{\Delta}_i \rangle \sim \langle \bar{\Delta}_RRi \rangle = \langle D_{st} v_{ti} \rangle \]
As a lower limit, we can approximate:
\[ \bar{\Delta}_i \sim \bar{\Delta}_e (m_e T_i / m_i T_e)^{1/2} \]
Allowing an estimate of ion energy balance in the core to examine “anomalous” ion heating: It’s small everywhere except when magnetic fluctuations are big, just after the sawtooth crash.
A Note on Electron Diffusion.

Electron Diffusion suggests Ambipolarity

\[ e_c = D_e n_e \left( \frac{n_e}{T_e} \right)^{1/2} + eE_r \frac{T_e}{T_e} \]

Measured Radial Electric Field agrees with calculations from Ion Mom. Balance

- HIBP
- Implied XRRe
- Measured Xe
- Inferred Xi
- Measured De

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A Note on Electron Diffusion (cont.)

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Summary

• 1\textsuperscript{st} time that sufficient diagnostics were available to characterize the equilibrium behavior of MST plasmas over a sawtooth crash.
  • Allowed profiles of \( I_e \) to be calculated.

• In the core, DEBS simulations confirm that the magnetic islands are essentially isolated, which implies a low field stochasticity despite large fluctuations, and which results in low electron thermal transport.

• In the mid-radius region \( I_e \sim D_{st} V_{te} \) and scales correspondingly with increasing fluctuations and stochasticity.

• In the edge, a transport barrier is implied, which persists throughout the sawtooth cycle and governs the overall confinement.

• \( i \) bounded by stochastic transport implies an anomalous ion heating power (Dynamo?) which agrees qualitatively with the fluctuation increase at the sawtooth crash.

• Electron diffusivity is much lower than electron conductivity, and of the same order as ion conductivity and ion diffusivity, suggesting that particle flux remains ambipolar even though heat flux is not.
$b_n$ increases at the crash, as measured at the wall.
$B_{m,n}$ is extracted from the wall signals.

\[ \tilde{\mathbf{B}}(n) = \sqrt{\tilde{B}_p^2(n) + \tilde{B}_T^2(n) + \tilde{B}_r^2(n)} \]

\[ 0 = \tilde{\mathbf{B}} \cdot (\square \square \tilde{B}) = \frac{1}{mR} (\tilde{B}_T \square \tilde{B}_p) = \frac{1}{mR} (\tilde{B}_T \square \tilde{B}_p) \]

\[ \tilde{B}_T(m,n) = \frac{n}{mR} \tilde{B}_p(m,n) \]

\[ \tilde{\mathbf{B}}(m,n) = \sqrt{1 + \frac{na}{mR} \tilde{B}_p(m,n)} \]

\[ \tilde{B}_p(n) = \tilde{B}_p(m,n) = \tilde{B}_p(0,n) + \tilde{B}_p(1,n) + \tilde{B}_p(2,n) + \ldots \]

\[ \tilde{\mathbf{B}}(m = 1,n) \]

\[ \tilde{\mathbf{B}}(m = 0,n) \]

\[ \tilde{\mathbf{B}}(\text{other},n) \]

\[ \sqrt{\tilde{B}_r^2(n) + \tilde{B}_p^2(n)} \]

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March 20th, 2002
Deuterium was the working gas for all experiments.