# Simulation of microtearing turbulence in NSTX and scaling with collisionality

**Walter Guttenfelder[1]**

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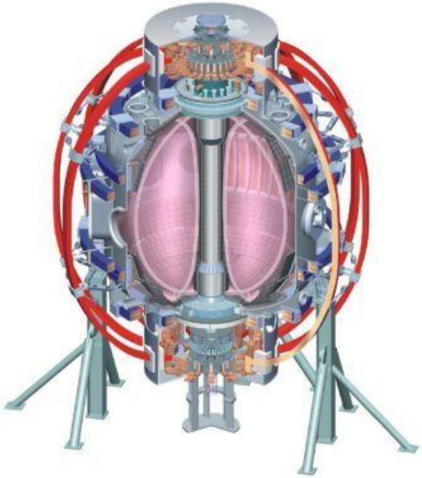

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# Overview

- Experimental motivation - favourable $\Omega_i \tau_{E,th} \sim \nu_*^{-(0.8\text{-}0.95)}$ dependence in STs
  - Microtearing modes found to be unstable in experimental $\nu_*$ scans, $\gamma_{lin} \sim \nu_e$

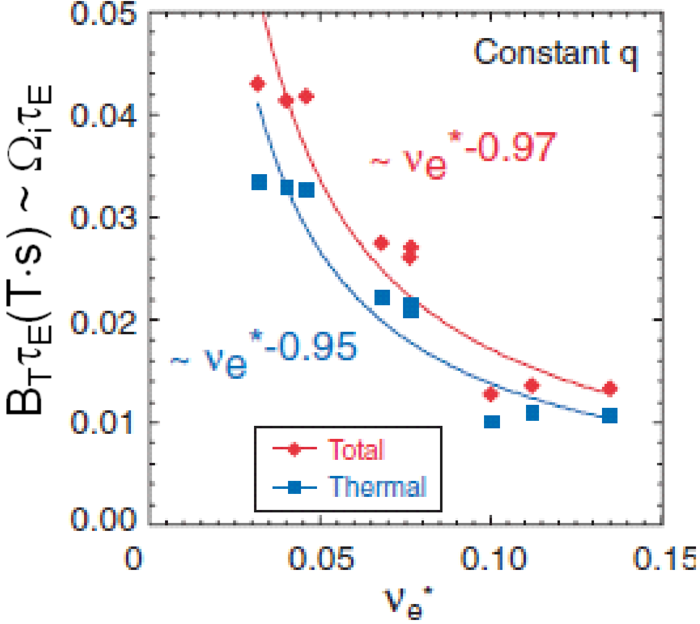
- Linear microtearing properties for high-$\beta$ NSTX discharges
  - Electromagnetic, electron drift mode with narrow resonant current layer, $\Delta_j < \rho_s$
  - Non-monotonic dependence on $\nu^{e/i}/\omega$, threshold in $\beta_e$ and $\nabla T_e$
  - $Z_{eff}$ (and s/q) scaling distinct from ETG

- Non-linear simulations
  - Necessary to “resolve” (distinguish) each simulated rational surface
  - Transport is experimentally significant and dominated by magnetic “flutter”

- Scaling of non-linear transport
  - Predicted $\chi_{e,sim} \sim \nu_e^{1.1}$ close to experimental trend
  - “Stiff” with $\nabla T_e$ but suppressible by experimental levels of E×B shear

# Experimental motivation - strong collisionality scaling in STs

NSTX (no-Lithium)



NSTX* (Kaye et al., Nucl. Fusion 2007)

$$\boxed{\Omega\tau_E^{th} \sim \nu_{*e}^{-0.95}}$$

MAST (Valovič et al., Nucl. Fusion 2011)

$$\boxed{\Omega\tau_E^{th} \sim \nu_{*e}^{-0.82}}$$

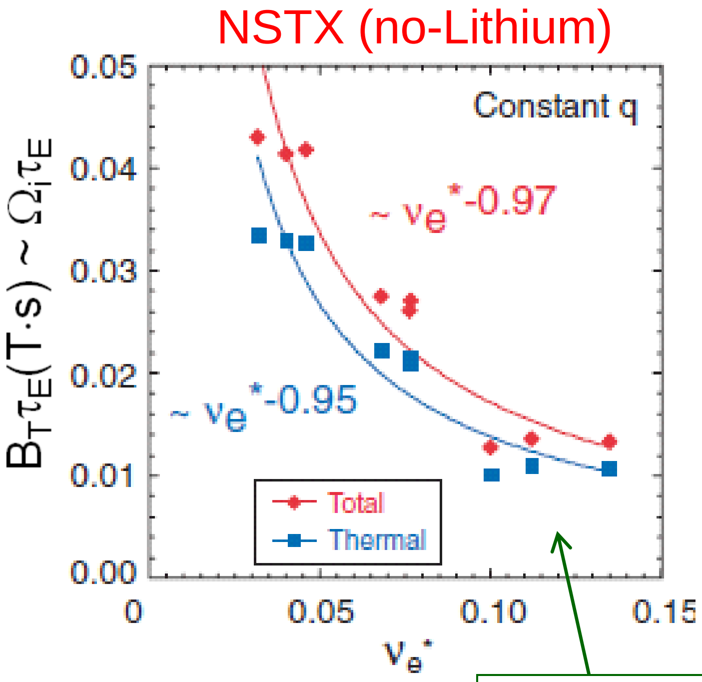
ITER (PIPB, Doyle et al., Nucl.Fusion 2007)

$$\boxed{\Omega\tau_E^{th,04(2)} \sim \nu_{*e}^{-0.2}}$$

*Y. Ren, TI-2.2
S. Kaye, PP-9.30 (Thurs. PM)
S. Gerhardt, YI-2.2 (Fri. AM)

- Ion transport is neoclassical, consistent with strong toroidal flow and flow shear
- What is the cause of anomalous electron thermal transport?
- Will favorable $\tau_E$ scaling hold at lower $\nu_*$ envisioned for next generation ST (high heat flux, CTF, …)?

# Experimental motivation - strong collisionality scaling in STs



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**Following simulations based on a single NSTX high-$\nu_*$ discharge**
**$B_T$=0.35T, $I_p$=0.7 MA, $P_{NBI}$=4 MW, $n_e \approx 6\times10^{19}$ m$^{-3}$, $T_e(0)$~1 keV**

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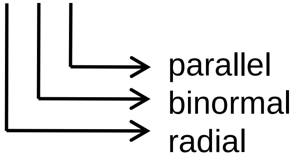
- What is the cause of anomalous electron thermal transport?
- Will favorable $\tau_E$ scaling hold at lower $\nu_*$ envisioned for next generation ST (high heat flux, CTF, …)?

# GYRO* used for gyrokinetic simulations

- Eulerian solver of gyrokinetic-Maxwell equations, evolving $\delta f(E,\lambda,r,\alpha,\theta)$



$$\delta f \sim \delta\hat{f}(r,\theta)e^{-in\alpha} \qquad \alpha = \phi + \nu(r,\theta) \approx \phi - q(r)\theta$$

$$k_\theta \doteq \frac{nq}{r}$$

High-aspect ratio, low $\beta$ limit

- Kinetic ions (D+C) and electrons, general equilibrium
- Fully collisional & electromagnetic ($\delta A_{||}$, $\delta B_{||}$) (both important in NBI heated ST)
- Freedom to include toroidal flow and flow shear (important in NBI heated ST)
- Can use experimental profile variations, T(r), n(r), q(r), etc… (likely important in ST, $\rho_s/a\sim1/100$, $\rho_s/L\sim1/40$)

*J. Candy & R.E. Waltz, Phys. Rev. Lett. **91**, 045001 (2003); J. Comp. Physics **186**, 545 (2003); https://fusion.gat.com/theory/Gyro
J. Candy, Phys. Plasmas Control. Fusion **51**, 105009 (2009); E.A. Belli & J. Candy, Phys. Plasmas **17**, 112314 (2010).

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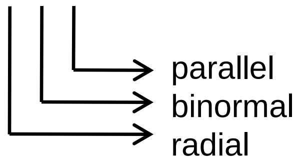

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- *All following <u>linear</u> calculations performed in the local flux-tube limit (periodic BC’s) without toroidal flow & shear*

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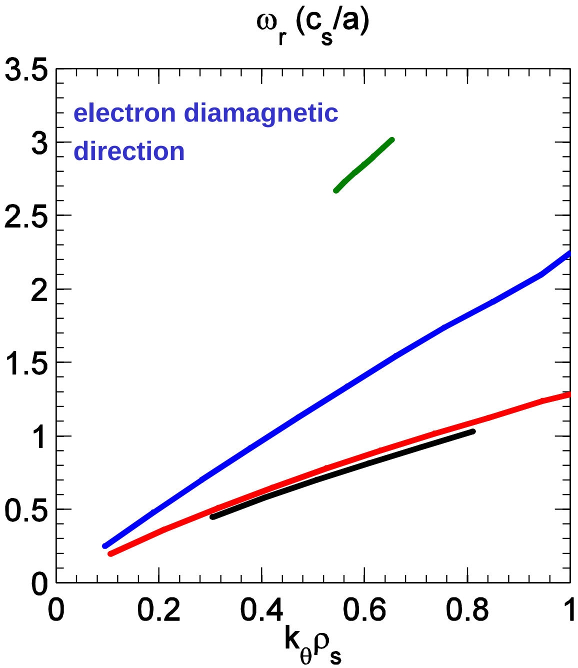
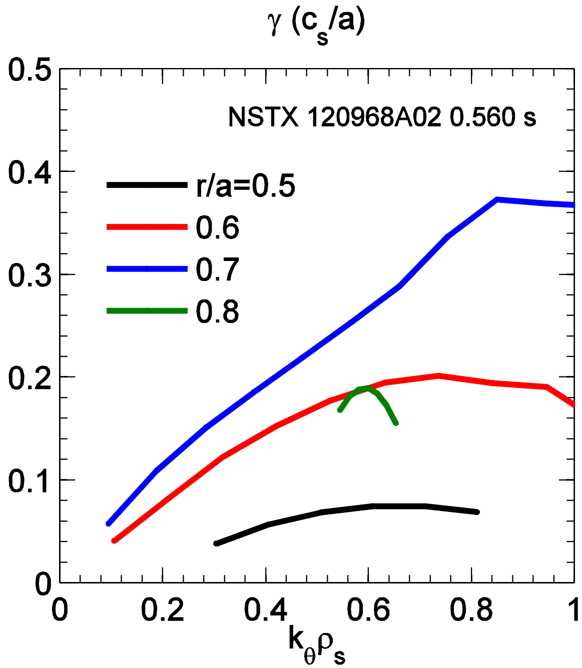
# Microtearing modes found to be unstable in many high $\nu_*$ discharges

- Microtearing dominates over r/a=0.5-0.8, $k_\theta\rho_s<1$ ($n\approx5$-$70$)
- Real frequencies in electron diamagnetic direction, $\omega \approx \omega_{*e} = (k_\theta\rho_s)\cdot(a/L_n+a/L_{Te})\cdot(c_s/a)$
- ETG mostly stable due to larger $Z_{eff}\approx3$, $(R/L_{Te})_{crit,ETG}\sim(1+Z_{eff}T_e/T_i)$

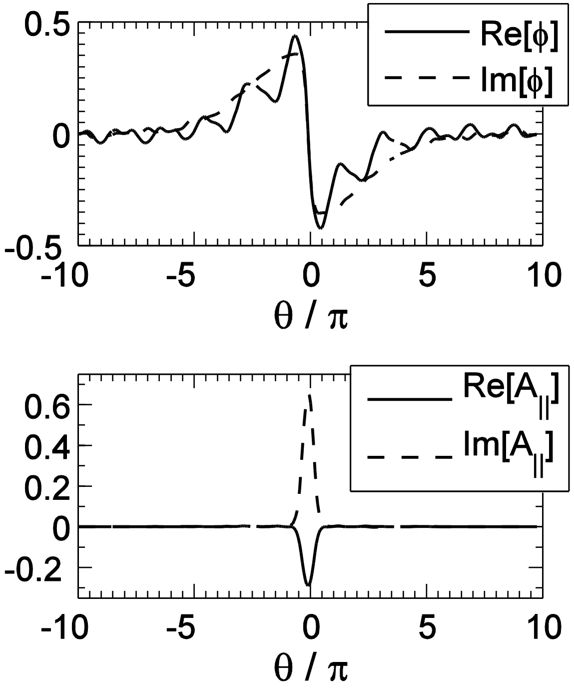
real frequencies



growth rates



eigenfunctions in "ballooning" space



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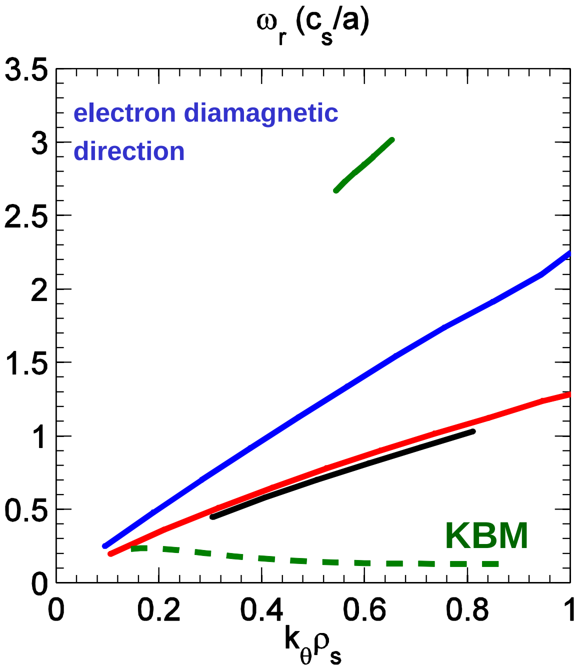
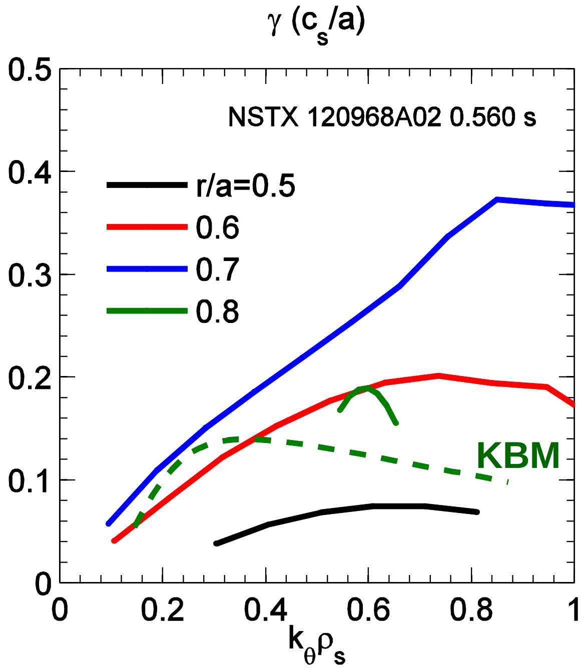
- ETG mostly stable due to larger $Z_{eff}\approx3$, $(R/L_{Te})_{crit,ETG}\sim(1+Z_{eff}T_e/T_i)$
- KBM competes farther out ($r/a\geq0.8$) where $\alpha_{MHD}=-q^2R\beta'$ much larger (larger q, $a/L_n$)

real frequencies



growth rates



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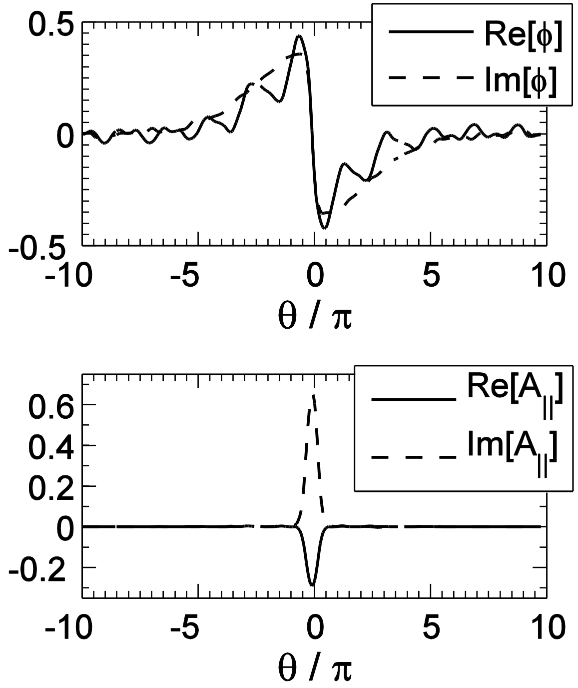


*Following calculations mostly for r/a=0.6*

# Linear microtearing instability

- High-m tearing mode around a rational $q(r_0)=m/n$ surface ($k_{||}(r_0)=0$)
  (Classical tearing mode stable for large m, $\Delta' \approx -2m/r < 0$)
- Driven by $\nabla T_e$ with time-dependent parallel thermal force* $\Rightarrow$ *requires e-i collisions*

### Conceptual linear picture

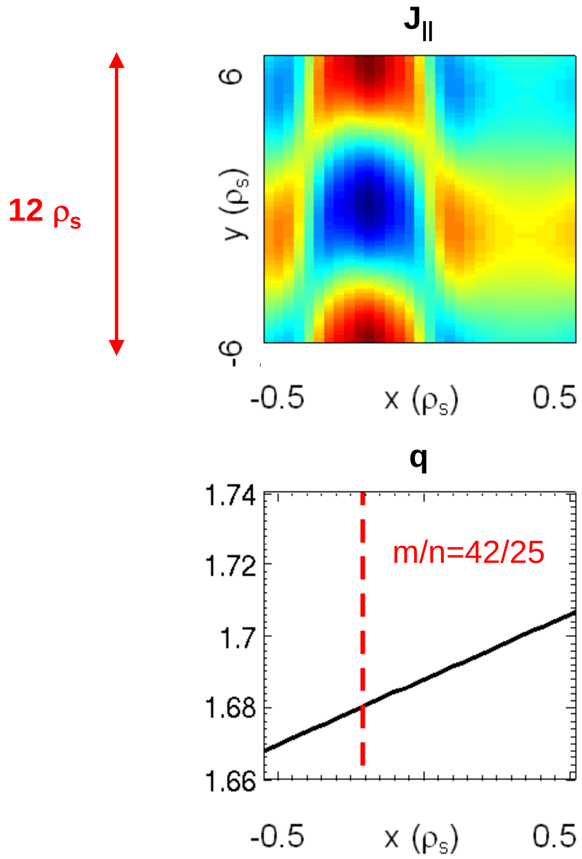
- Imagine helically resonant (q=m/n) $\delta B_r$ perturbation $$\delta B_r \sim \cos(m\theta - n\varphi)$$
- $\delta B_r$ leads to radially perturbed field line, finite island width $$w = 4\left(\frac{\delta B_r}{B}\frac{rR}{n\hat{s}}\right)^{1/2}$$
- $\nabla T_e$ projected onto field line gives parallel gradient $$\nabla_{||} T_{e0} = \frac{\vec{B}\cdot\nabla T_{e0}}{B} = \frac{\delta B_r}{B}\underline{\nabla T_{e0}}$$
- Parallel thermal force ($R_{T||} \sim -\alpha(\omega) n_e \nabla_{||} T_e$) drives parallel electron current that reinforces $\delta B_r$ via Amperes's law $$k_\perp^2 \rho_s^2 \hat{A}_{||} = \frac{\beta_e}{\underline{2}}\hat{j}_{||} \quad , \quad B_r = i k_\theta A_{||}$$
- Instability requires sufficient $\nabla T_e$, $\beta_e$, $\nu_e$, and time dependence ($\omega$) important

*e.g. Hazeltine et al., Phys. Fluids 18, 1778 (1975); Gladd et al., Phys. Fluids 23, 1182 (1980);
D'Ippolito et al., Phys. Fluids 23, 771 (1980); M. Rosenberg et al., Phys. Fluids 23, 2022 (1980).

# Linear mode structure in perpendicular plane illustrates key microtearing mode features

- Narrow resonant current channel (≈0.3$\rho_s$≈1.4 mm) centered on rational surface

**x-y perpendicular plane ($\theta$=0)**

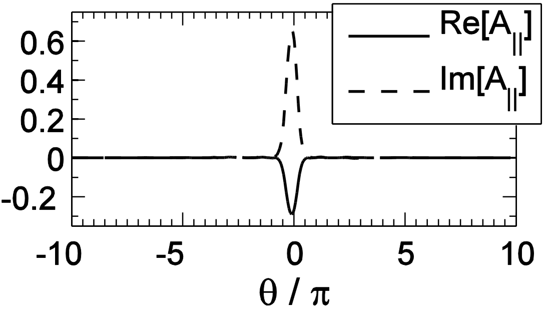


# Linear mode structure in perpendicular plane illustrates key microtearing mode features

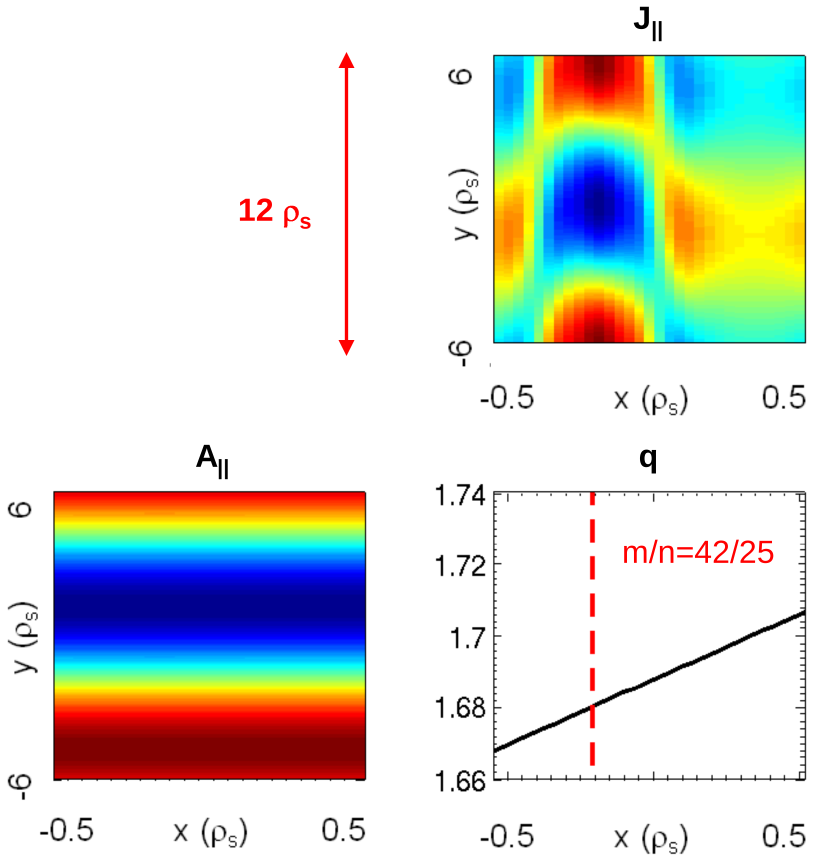
- Narrow resonant current channel ($\approx 0.3\rho_s \approx 1.4$ mm) centered on rational surface
- Finite $\langle A_{||} \rangle_\theta$ (resonant tearing parity), strongly ballooning

**“ballooning” space**

$$k_r(\theta) = \hat{s} k_\theta (\theta - \theta_0)$$



**x-y perpendicular plane (θ=0)**



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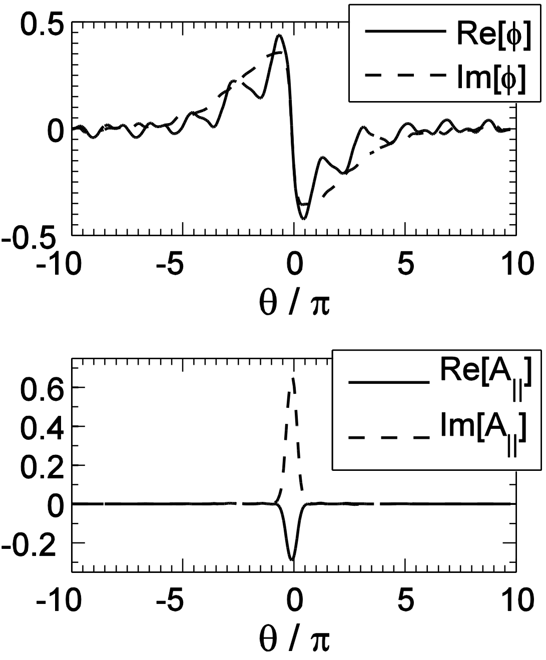
- Finite $\langle A_{||}\rangle_\theta$ (resonant tearing parity), strongly ballooning
- Narrow $n_e$ & $T_e$ perturbations
- Nearly unmagnetized/adiabatic ion response $\Rightarrow \dfrac{\tilde{n}}{n_0} \approx -Z_{eff}\left(\dfrac{e\tilde{\varphi}}{T_i}\right)$

**"ballooning" space**

$$k_r(\theta) = \hat{s}k_\theta(\theta-\theta_0)$$



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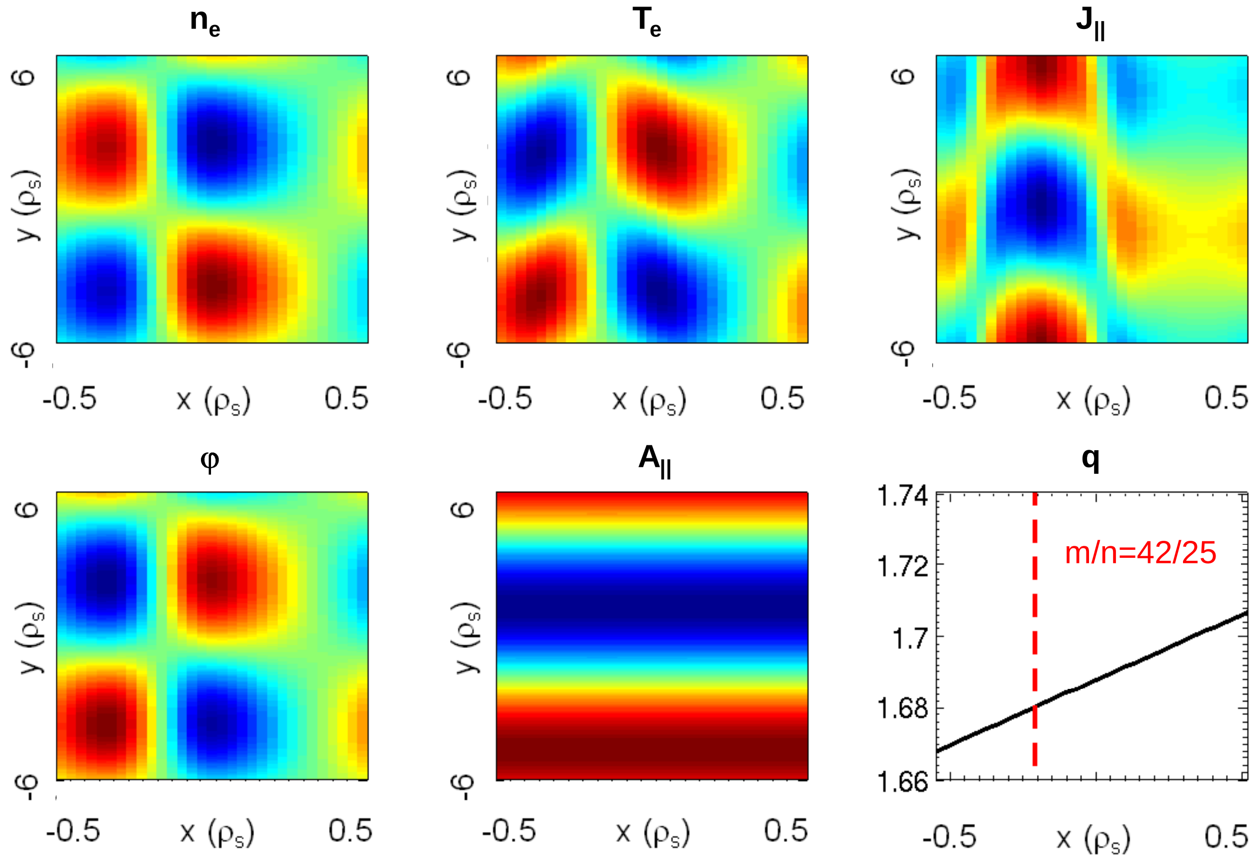
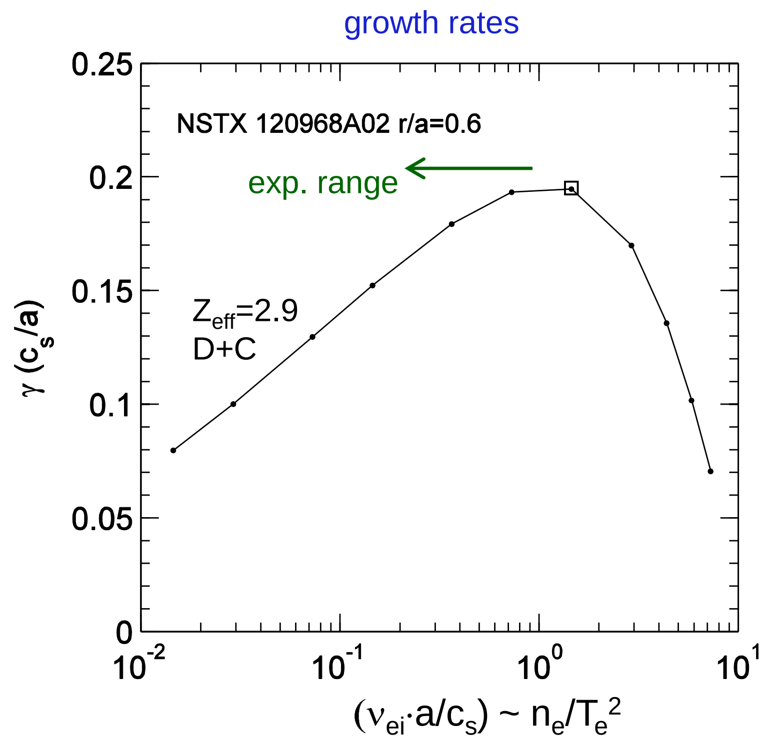


# A distinguishing feature of the microtearing mode is the non-monotonic dependence on $\nu^{e/i}/\omega$

- Peak $\gamma$ occurs for $\nu^{e/i}/\omega = Z_{eff} \cdot \nu_{ei}/\omega \sim 1\text{-}6$, similar to slab calculations (Gladd et al., 1980)
- $\gamma$ decreases with $\nu_e$ in experimental range, qualitatively consistent with confinement scaling



$$\boxed{\nu^{e/i} = Z_{eff}\nu_{ei} \propto Z_{eff}\frac{n_e}{T_e^{3/2}}}$$

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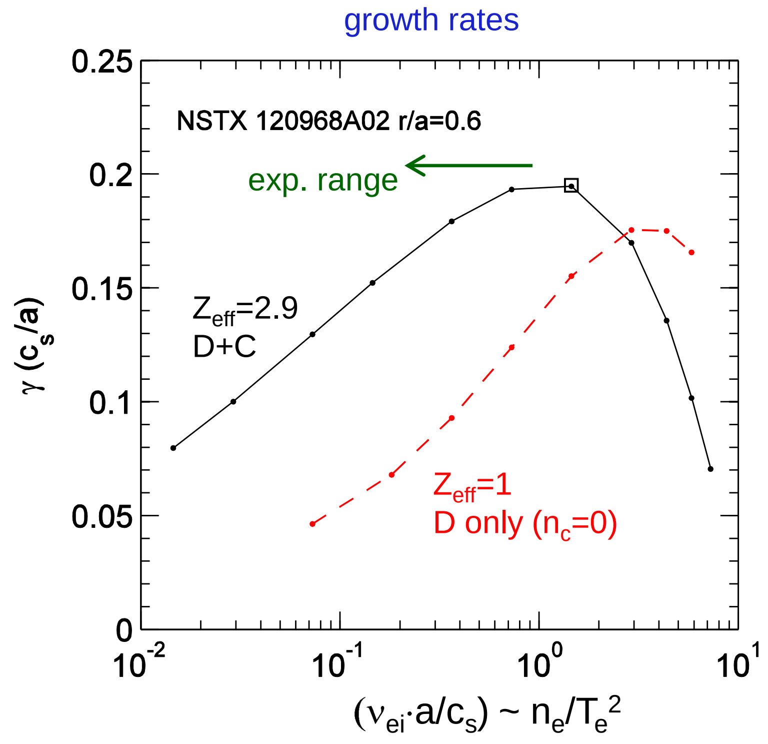
- $\gamma$ decreases with $\nu_e$ in experimental range, qualitatively consistent with confinement scaling
- In addition to shifting peak in $\nu^{e/i}/\omega$, $Z_{eff}$ can enhance instability through shielding potential (from adiabatic ion response, $\delta n_i \sim -Z_{eff}\delta\varphi/T_i$ )



$$\nu^{e/i} = Z_{eff}\nu_{ei} \propto Z_{eff}\frac{n_e}{T_e^{3/2}}$$

Jenko et al. (2001)

- $Z_{eff}$ (and s/q)* dependence opposite to ETG expectations

$$\left(\frac{R}{L_{Te}}\right)^{ETG}_{crit} \sim \left(1+Z_{eff}\frac{T_e}{T_i}\right)\left(1.3+1.9\frac{s}{q}\right)(\cdots)$$

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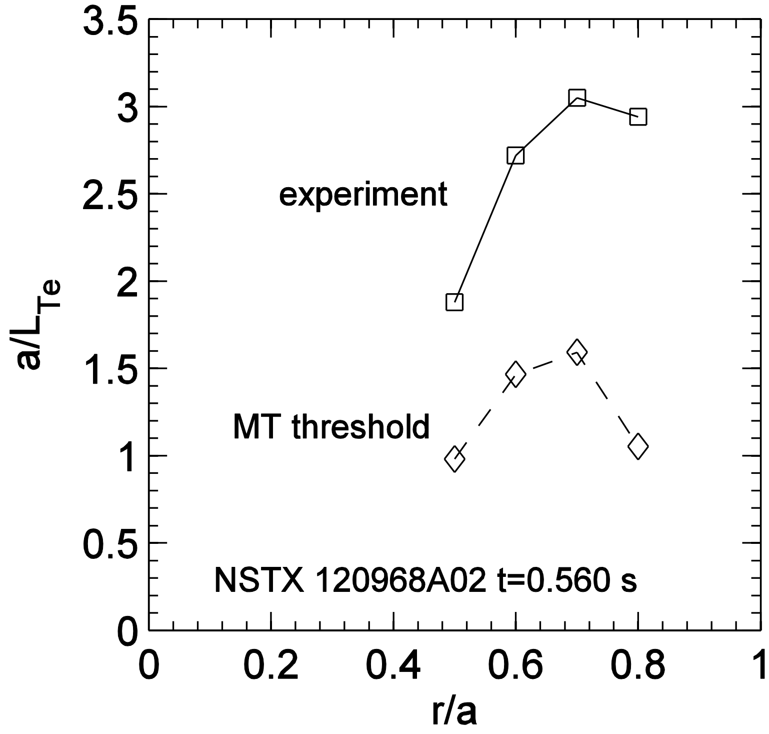
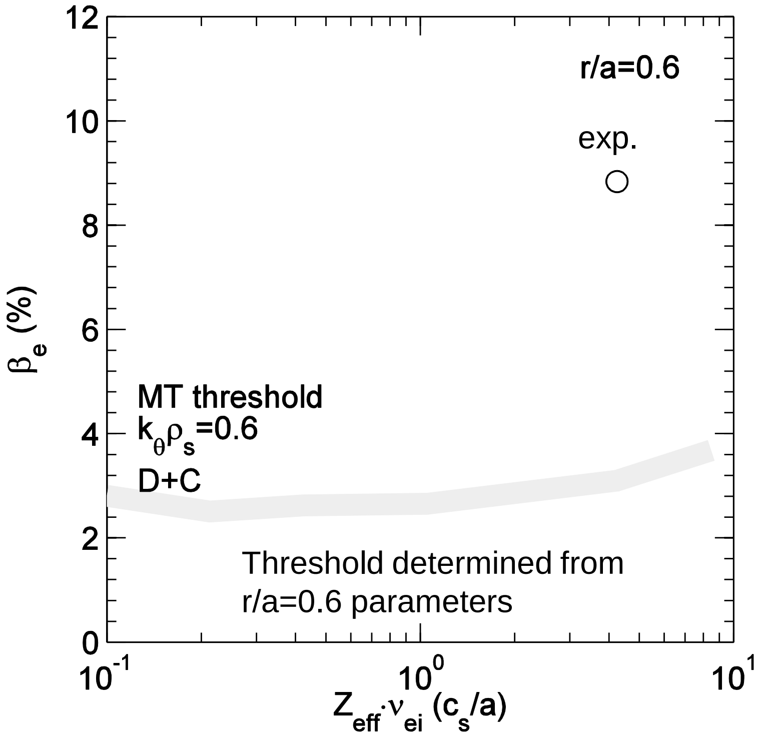
* Guttenfelder et al., *Scaling of linear microtearing stability for a high collisionality NSTX discharge*, submitted to Phys. Plasmas (Oct, 2011)

# Microtearing instability exhibits thresholds in electron temperature gradient and beta

- In this high-$\nu_*$ discharge, $a/L_{Te}$ and $\beta_e$ are 2-3× larger than linear thresholds





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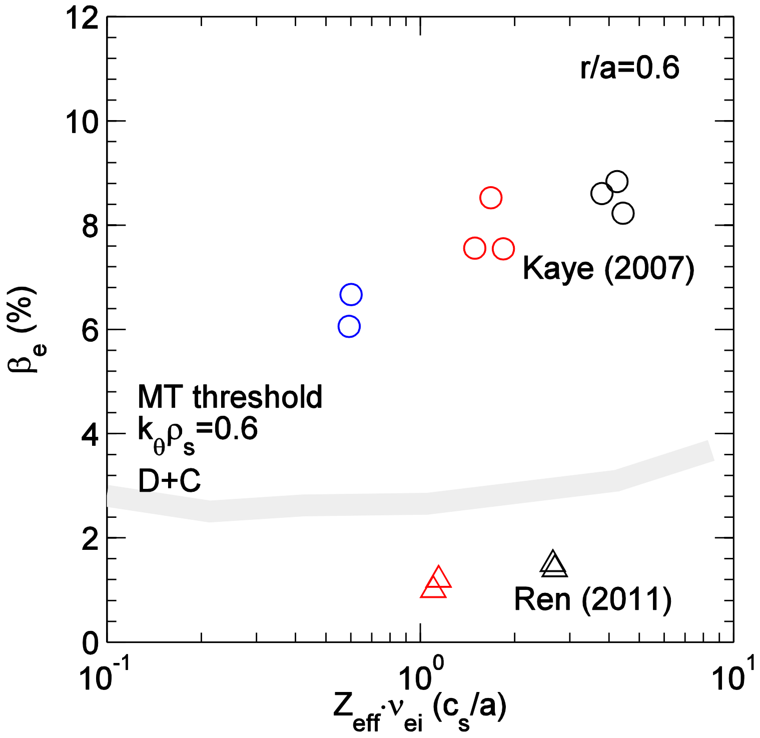
# NSTX has studied electron transport for a range of beta and collisionality

- Ren (2011) $\nu_*$ experiment (invited talk TI-2.2) performed at lower $\beta_e$ compared to Kaye (2007) (lower density and NBI power)



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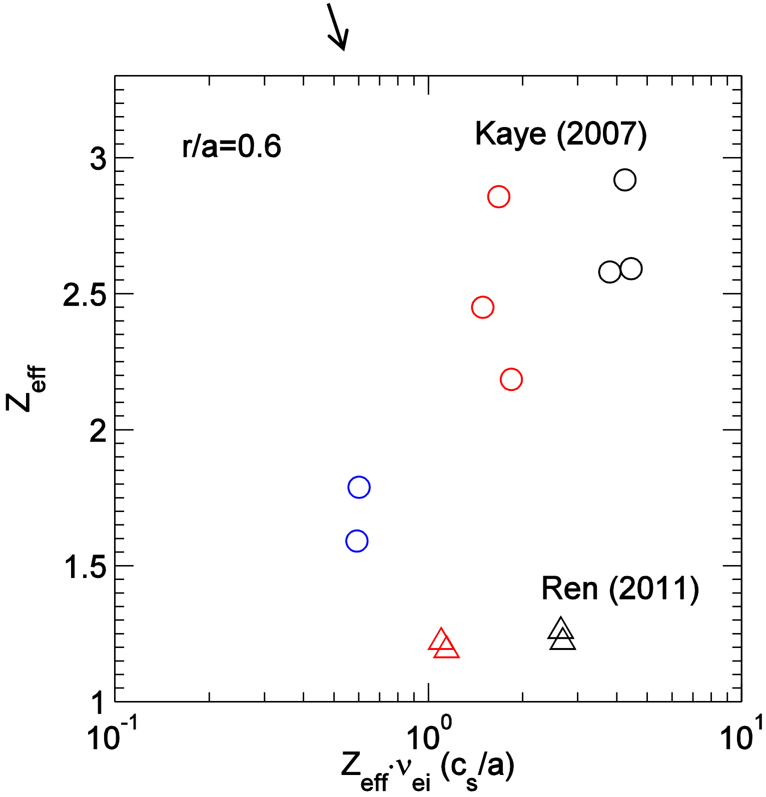
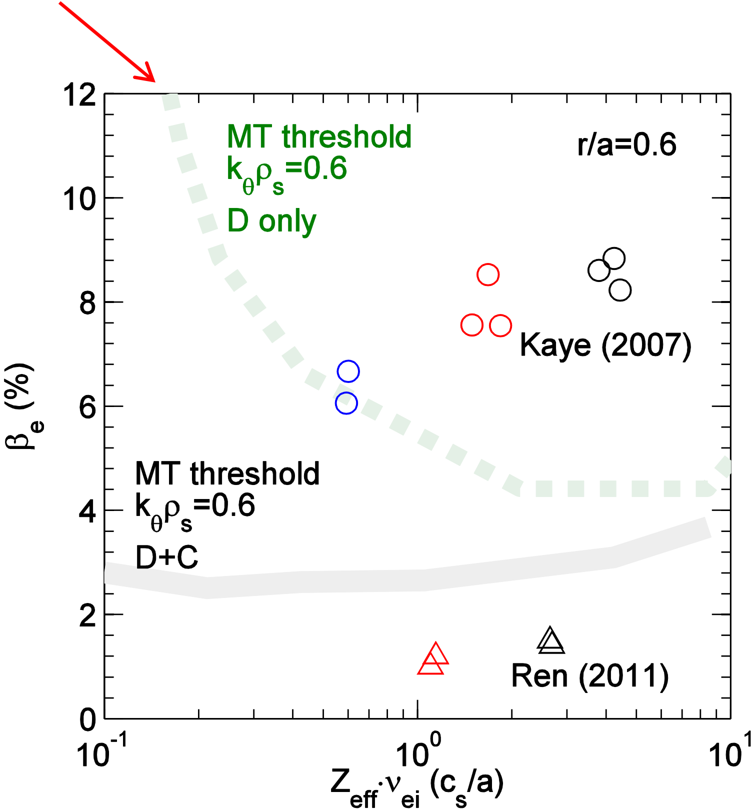
- Also at lower $Z_{eff}$ – increase in MT threshold, but smaller ETG threshold





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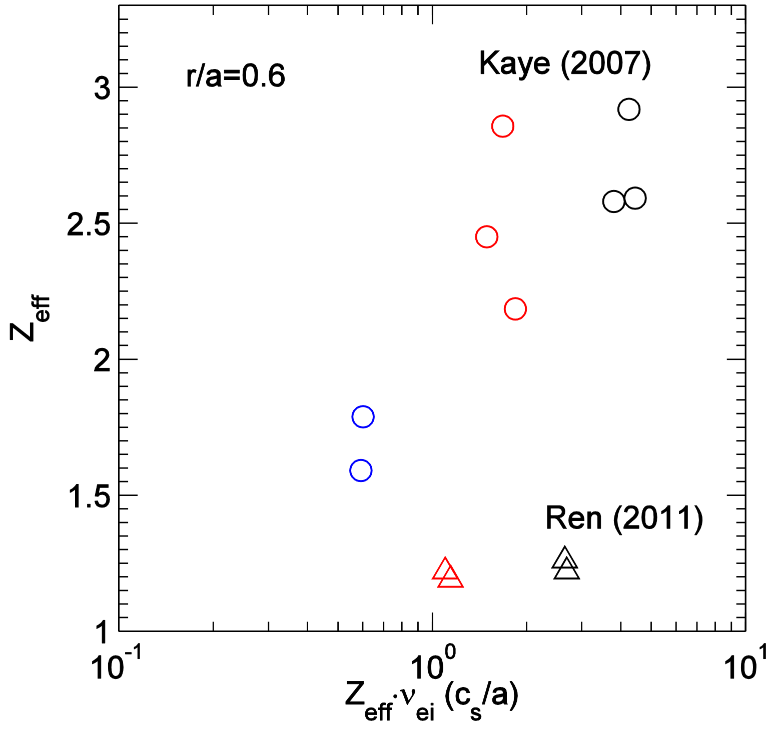

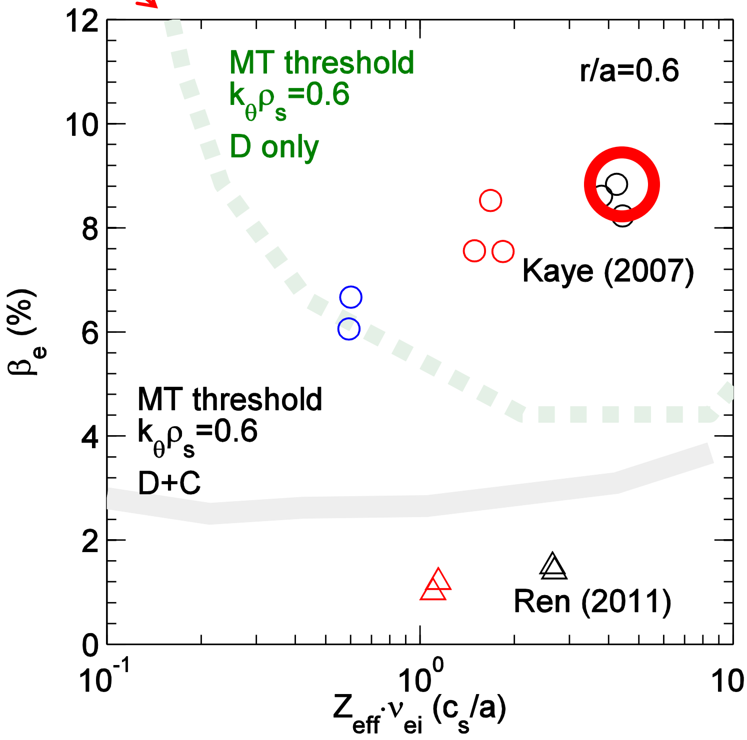

- Nonlinear simulations run for high-$\beta_e$, high-$\nu_e$ where only microtearing unstable

# First nonlinear microtearing simulations in NSTX*

- Simulations where only microtearing unstable, no ETG (NSTX 120968, r/a=0.6)
  - Electromagnetic ($\varphi$, $A_{||}$) and collisional ($\nu_e$)
  - Varying E×B shear (mostly $\gamma_E=0$)
  - Deuterium only (but $Z_{eff}$ in collision operator)
  - "Local" → no profile variation in equilibrium quantities
  - $k_\theta\rho_s=[0,0.105,0.21,\ldots]$, same as $n=[0,5,10,\ldots]$

$L_x \times L_y = 80 \times 60\rho_s$

$n_x \times n_y = 400 \times 8$ ($\Delta x = 0.2\ \rho_s$)

$n_\theta=14$ (parallel mesh points)

$n_E=8$, $n_\lambda=12 \times 2$ (velocity space)

<u>120968 r/a=0.6 surface</u>

| | |
|---|---|
| $a/L_{Te}=2.73$ | $a/L_n=-0.83$ |
| $q=1.69$ | $s=1.75$ |
| $\kappa=1.7$ | $\delta=0.13$ |
| $T_e/T_i=1.05$ | $Z_{eff}=2.9$ |
| $\beta_e=8.8\%$ | $\nu_{ei}=1.46\ c_s/a$ |
| $(\beta_{e,unit}=2.5\%)$ | |

**Acknowledgements: NERSC & OLCF (INCITE award FUS023)**

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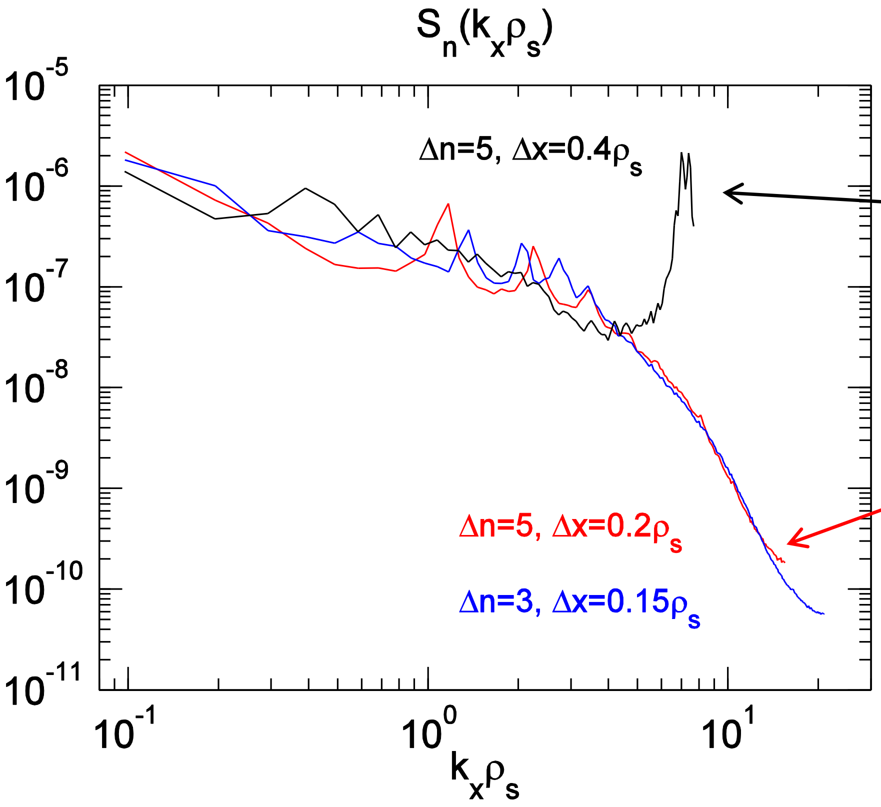
* W. Guttenfelder et al., Phys. Rev. Lett. **106**, 155004 (2011)

# Fine radial resolution required to obtain decaying nonlinear spectra

- Unphysical pile-up at high-k with insufficient resolution (Δx=0.4 $\rho_s$)
- Smoothly decaying turbulent spectrum with better resolution (Δx=0.2 $\rho_s$, Δx=0.15 $\rho_s$)



Poor resolution – unphysical pile-up at high-k

Better resolution – smoothly decaying turbulent spectrum

- Similar high-k pile-up observed in first careful attempts of GS2 MAST simulations – for more discussion see Applegate Ph. D. thesis (2007, Imperial College London)

# Fine radial resolution required to distinguish *linear* resonant layers of fastest growing mode

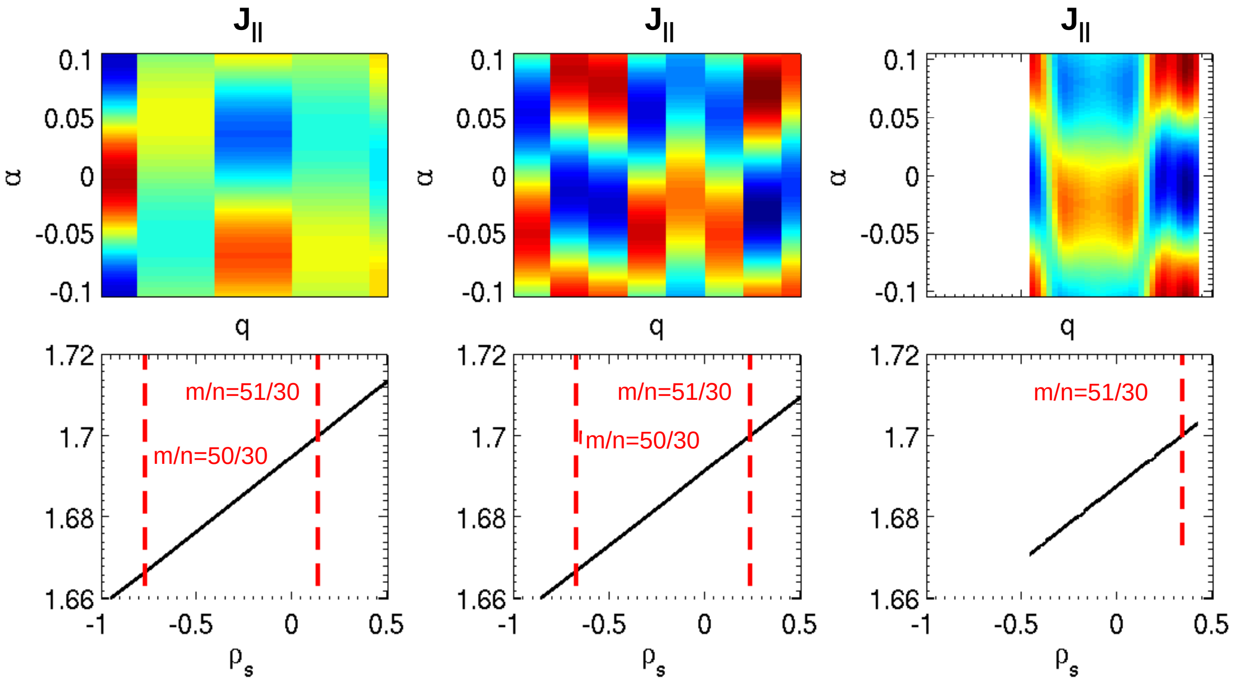
Linear calculation using box width and resolution of nonlinear simulations

“Typical” linear flux-tube calculation

**$L_x = 80\ \rho_s = 88\ \Delta r_{rat}$**
nx = 200
**$\Delta x = 0.4\ \rho_s$**

**$L_x = 80\ \rho_s = 88\ \Delta r_{rat}$**
nx = 400
**$\Delta x = 0.2\ \rho_s$**

**$L_x = 0.9\ \rho_s = \Delta r_{rat}$**
nx = 32
**$\Delta x = 0.03\ \rho_s$**

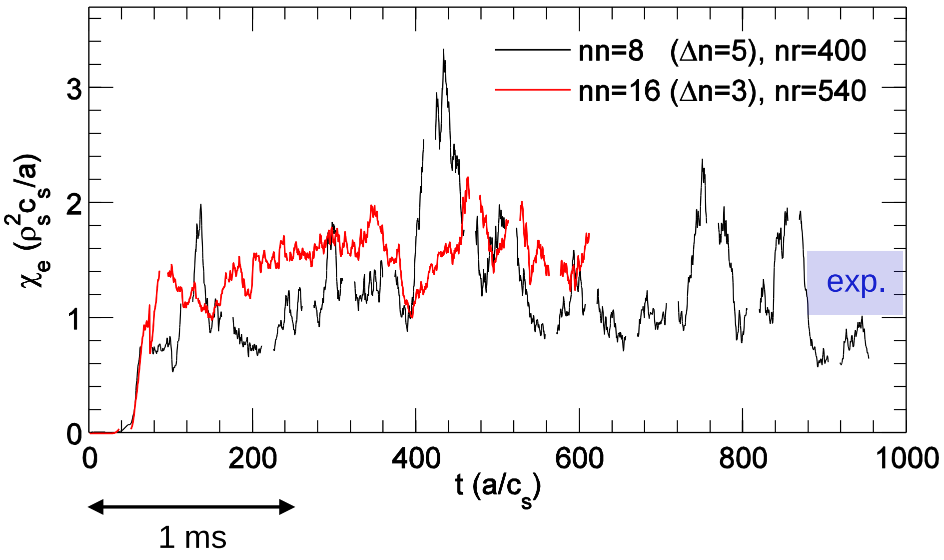


Rough rule-of-thumb: $\Delta x \leq \min[\Delta r_{rat}]/4$ **$\Delta x/\rho_s \leq 1/(4\cdot\max[k_\theta\rho_s]\cdot s)$**

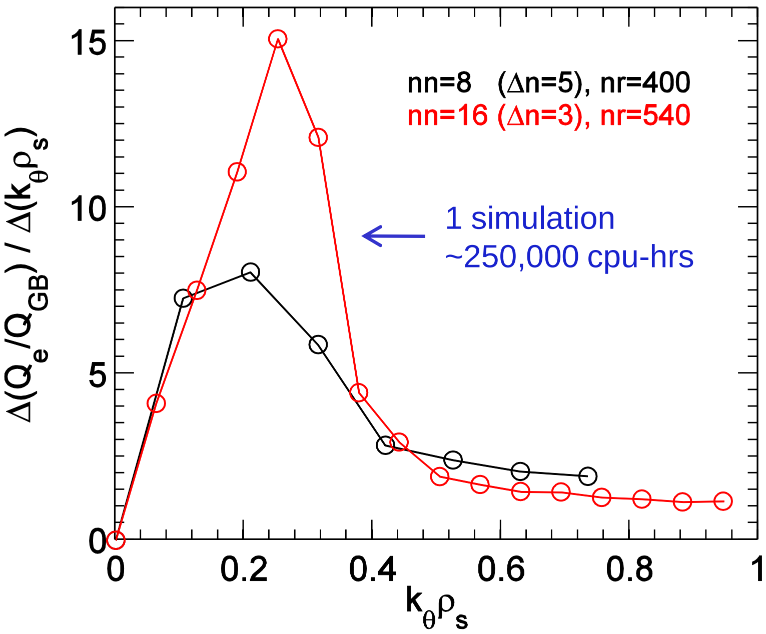
# Predicted electron thermal transport comparable to experiment

- Simulated transport (1.2 $\rho_s^2 c_s/a$, 6 $m^2/s$) comparable to experimental transport (1.0-1.6 $\rho_s^2 c_s/a$)
- Well defined peak in transport spectra ($k_\theta \rho_s \approx 0.2$), downshifted from maximum $\gamma_{lin}$ ($k_\theta \rho_s \approx 0.6$)
- Slowly decaying tail - predicted transport increases ~25% with higher resolution

Transport time series



Fractional transport spectra

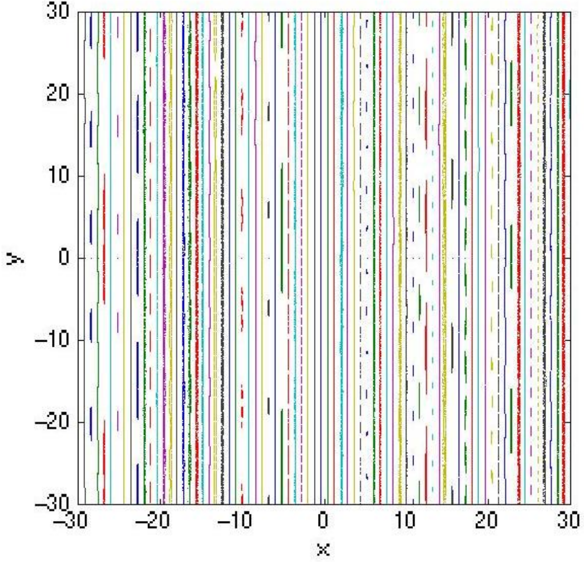


- Negligible particle, momentum, or ion thermal transport

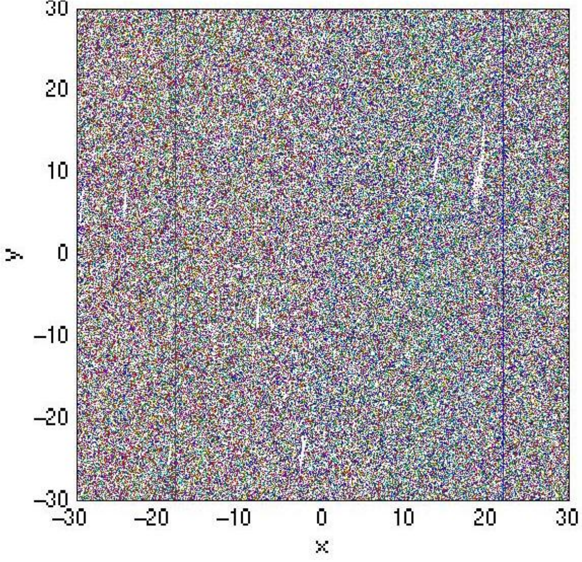
# ~98% of transport due to magnetic “flutter” contribution

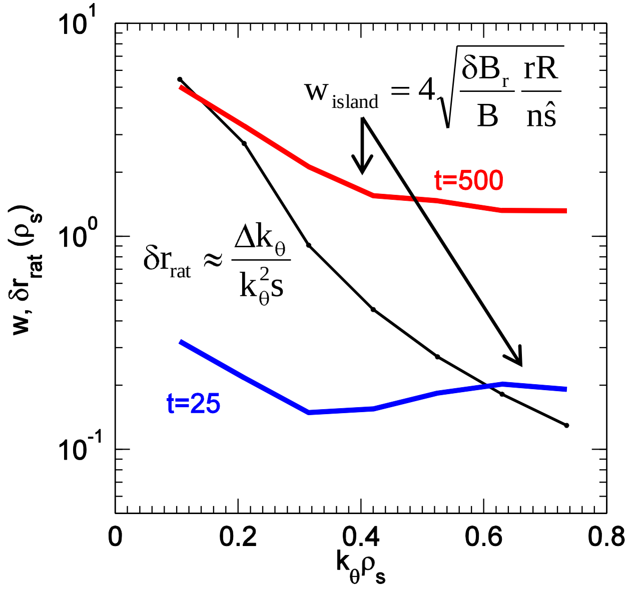
- Flux surfaces become distorted in linear phase (t=25)
- Globally stochastic* in saturated phase, complete island overlap $w_{island}(n) > \delta r_{rat}(n)$

t=25 (linear phase)



t=500 (saturated phase)





- $\chi_{e,EM}$ close to *collisionless* Rechester-Rosenbluth* ($\lambda_{mfp}$=12 m, $L_c \approx$2.5 m)

$$D_{st} = \lim_{s\to\infty} \frac{\left\langle [r_i(s) - r_i(0)]^2 \right\rangle}{2s} \qquad \chi^{RR} \approx 2\left(\frac{2}{\pi}\right)^{1/2} D_{st} v_{Te} f_p \approx 0.9\left(\frac{\rho_s^2 c_s}{a}\right)$$

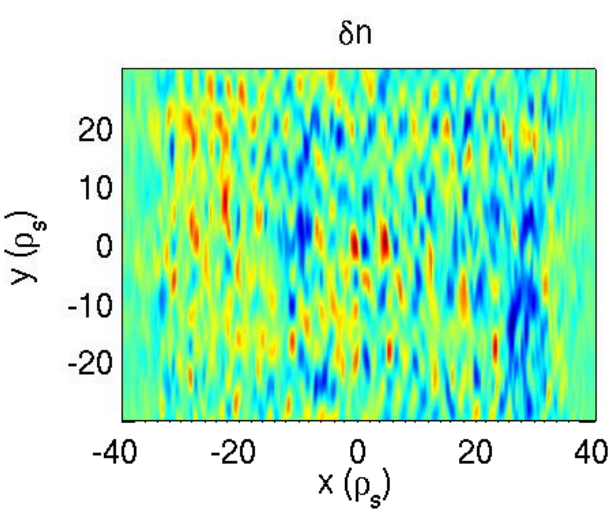
$f_p \approx$63% passing particles

*Wang et al., Phys. Plasmas (2011); Nevins et al., Phys. Rev. Lett. (2011);
Rechester & Rosenbluth, Phys. Rev. Lett. (1978); Harvey et al., Phys. Rev. Lett. (1981)

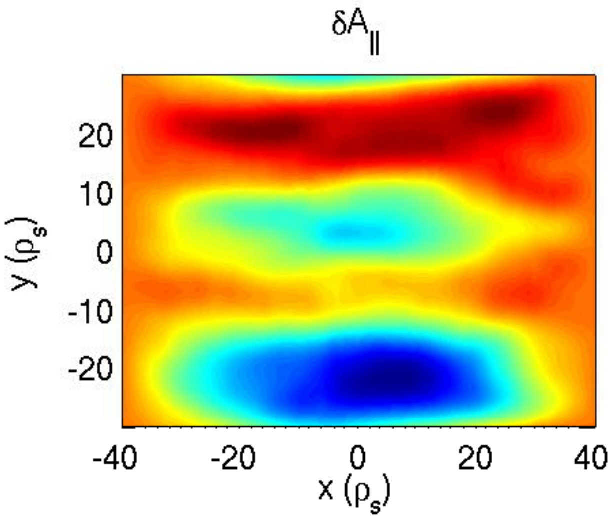
# Narrow density perturbations remain in nonlinear simulations

- Narrow radial n, $\varphi$, $j_{||}$ structures need to be resolved but $A_{||}$ very broad
- $\delta B_r/B \sim 0.15\% \;\sim\; \rho_e/L_{Te} = 0.065\%$
- $\delta B_r/B \sim \rho_e/L_{Te}$ analytic approximation from Drake et al. PRL 1980; used for NSTX in Wong et al. PRL 2007

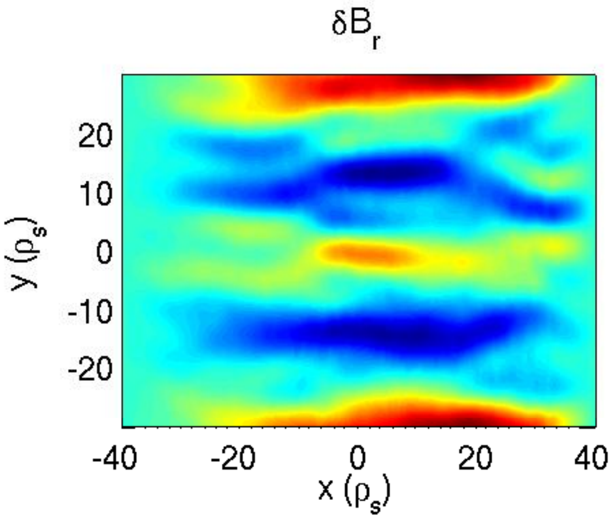
$\delta n/n \approx 0.5\%$



$\delta A_{||}/\rho_s B \approx 0.8\%$



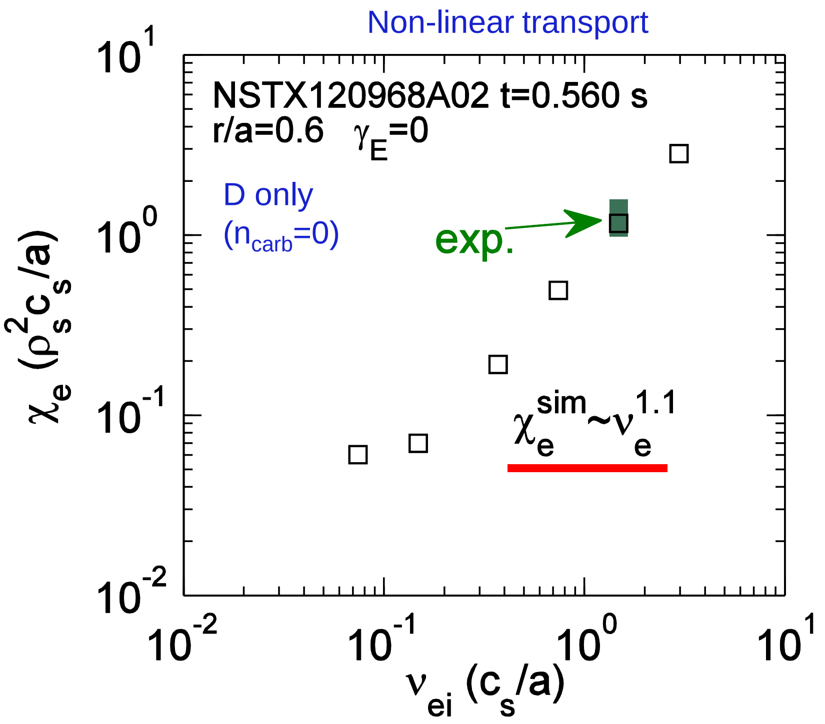
$\delta B_r/B \approx 0.15\%$



$\delta T_e/T_e \approx 2\%$

$\delta v_{e,||}/c_s \approx 6\%$

# Near linear scaling of transport with $\nu_e$ consistent with experimental scaling



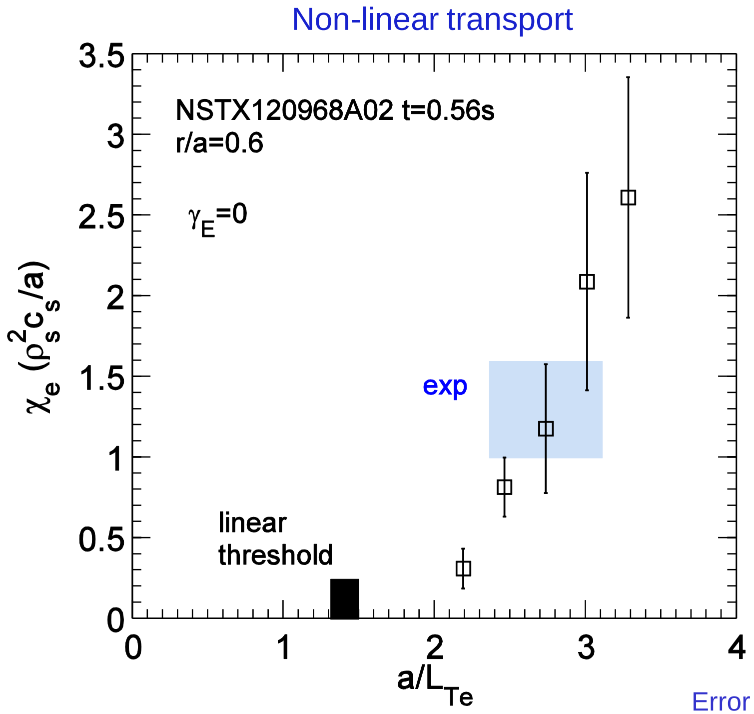
NSTX experimental scaling (Kaye et al., 2007)

$\Omega_i \tau_E \sim \nu_*^{-0.95}$

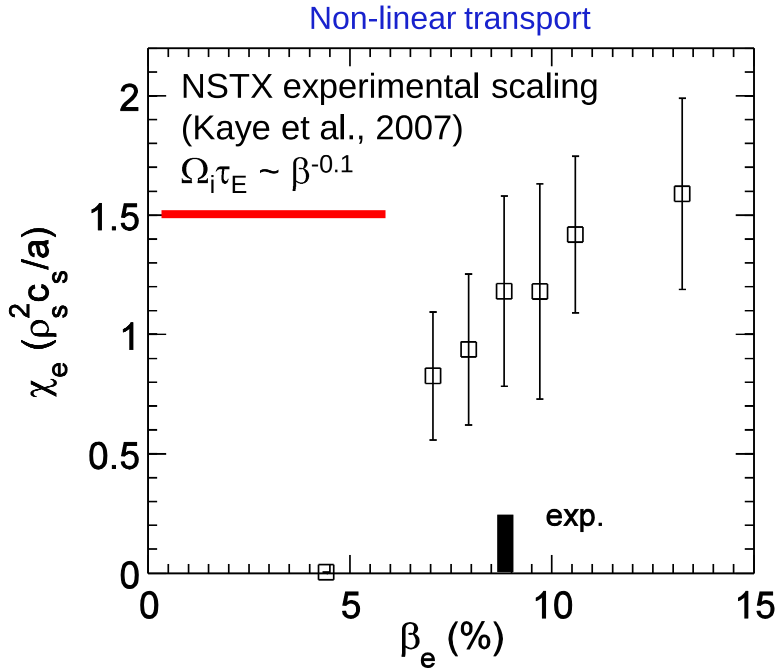
- As transport drops, $a/L_{Te}$ will increase (for fixed heat flux), at some point ETG (TEM?) should become important
- This transition likely to determine limit of “favorable” $\nu_*$ scaling
- Also likely to depend on ionic charge ($Z_{eff}$) – above with D only ($n_c$=0)

# Predicted transport “stiff” with $\nabla T_e$, increases with $\beta_e$

- Complicates simple interpretation from $\chi_{e,sim} \sim \nu_e^{1.1}$ scaling
- Useful to characterize scaling of threshold gradient (work in progress)



Non-linear transport



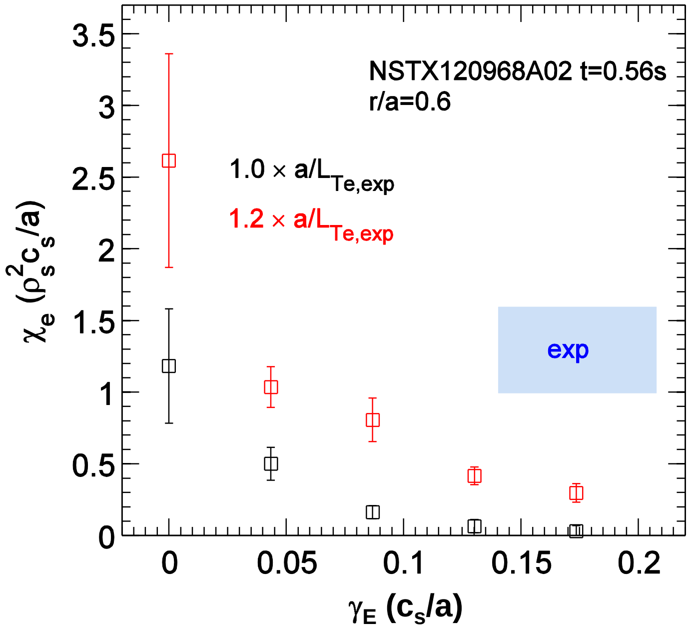
Non-linear transport

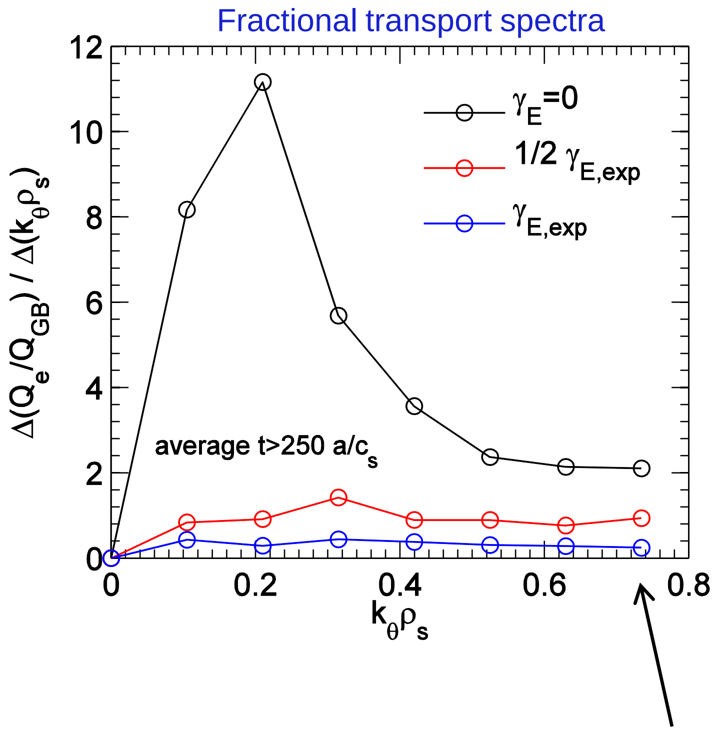
Error bars represent rms variation in transport

- Non-linear threshold ~40% bigger than linear threshold -- possible influence from limited numerical resolution

# Nonlinear microtearing transport sensitive to $\gamma_E/\gamma_{lin}$

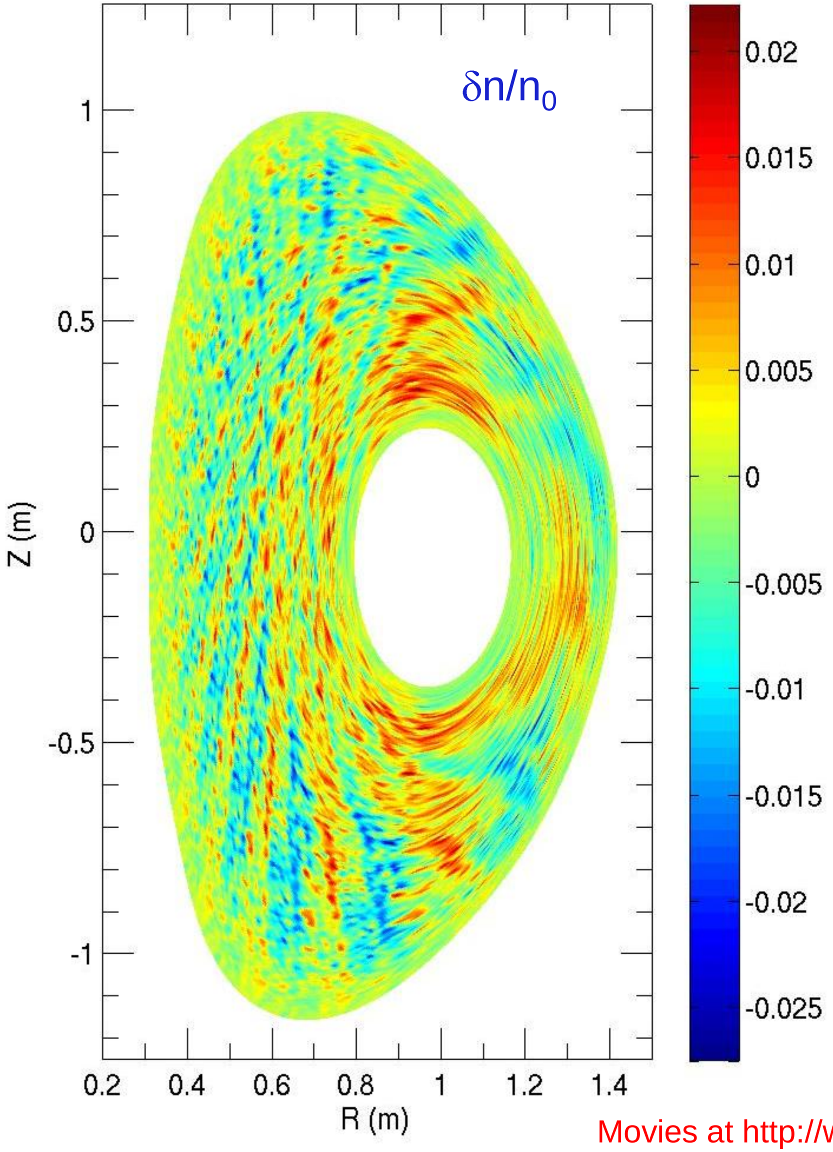
- Transport reduced when increasing $\gamma_E$ to local experimental value ($\gamma_{E,exp}$~$\gamma_{lin,max}$~0.17 $c_s/a$)
- Transport partially recovered with increase in $\nabla T_e$

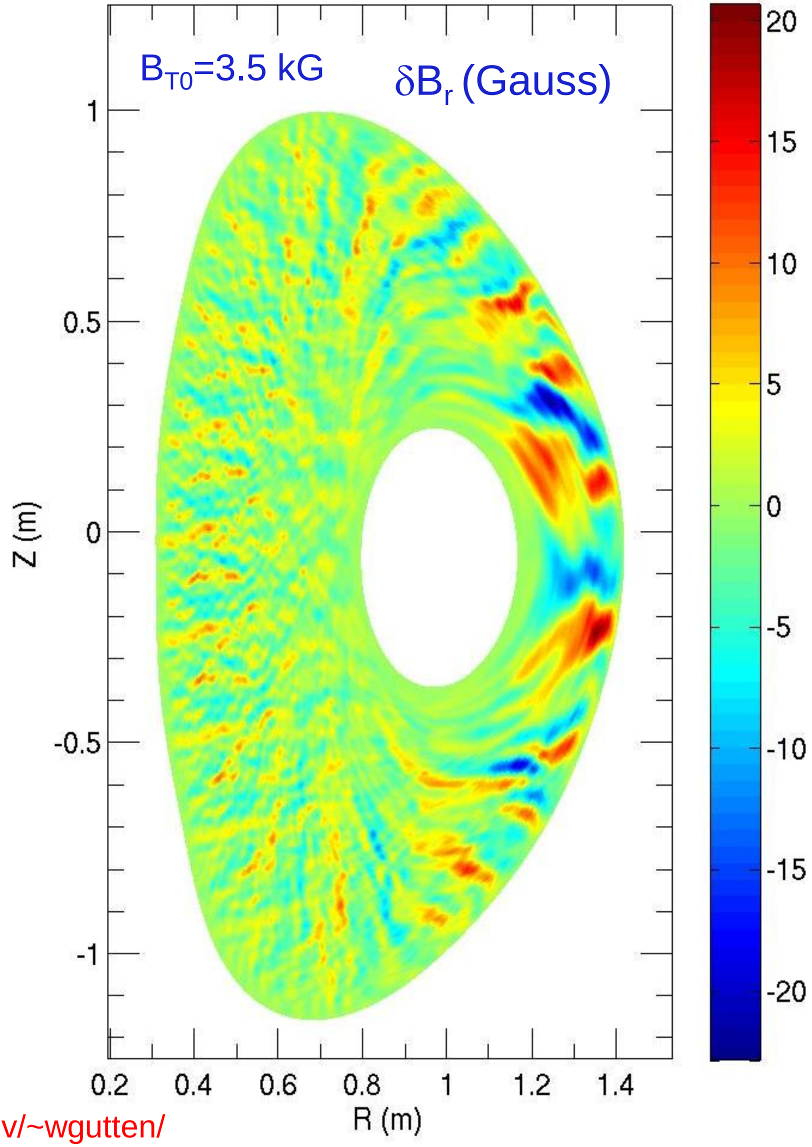




- Higher ionic charge ($Z_{eff}$>1, through adiabatic response) and improved resolution (binormal and radial) could increase transport
- Profile (non-local) effects could also matter - $\rho_s/a \approx 1/100$ & edge more strongly driven

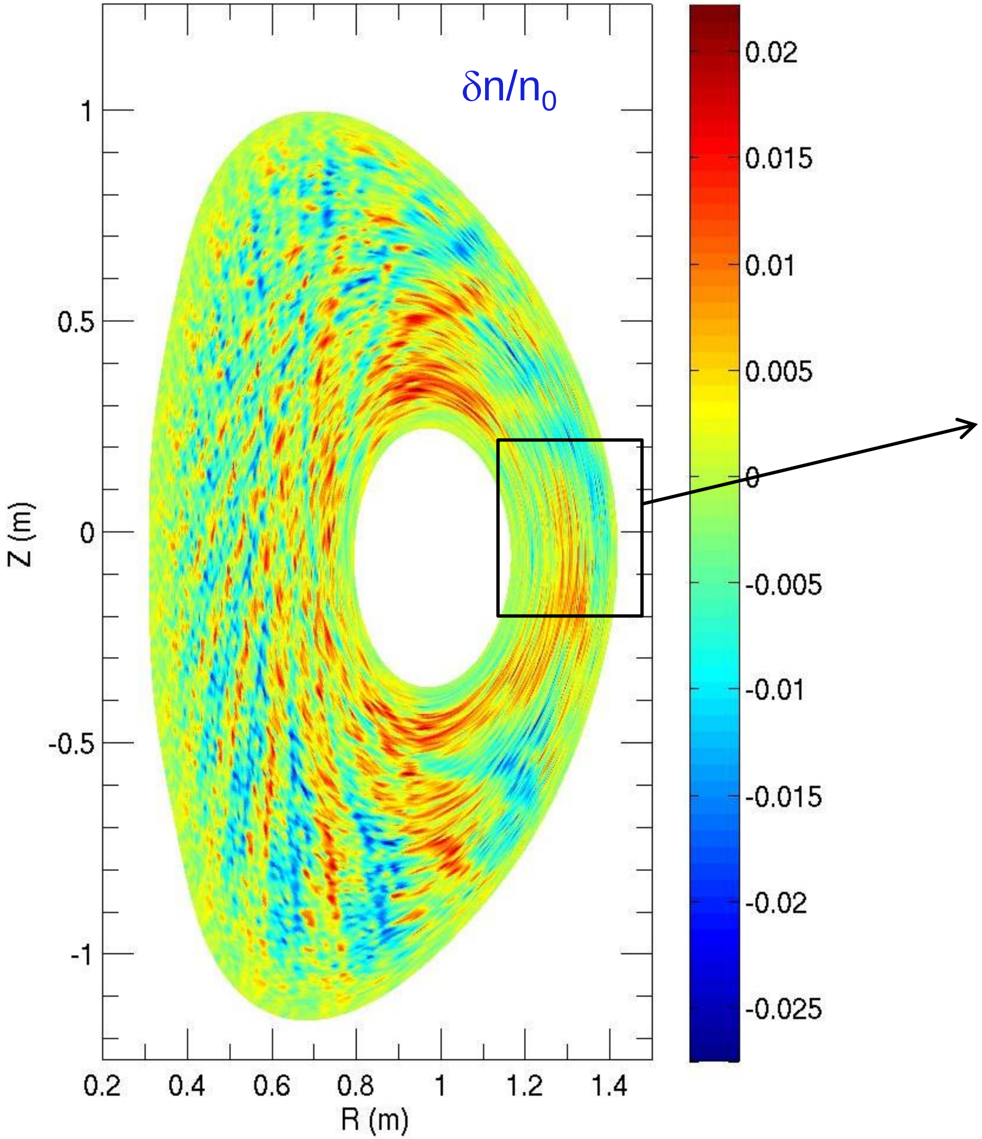
# What hope is there to experimentally identify microtearing modes?

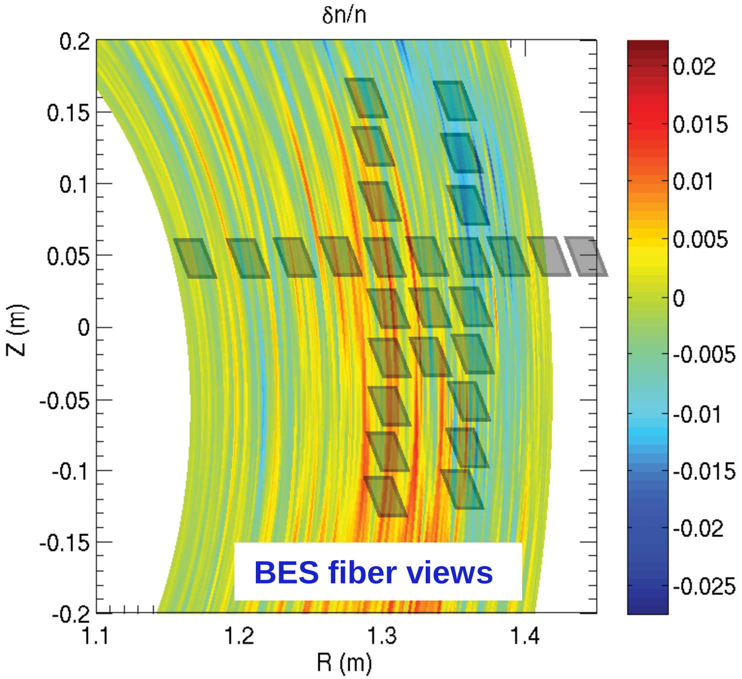




Movies at http://www.pppl.gov/~wgutten/

# BES for density fluctuations

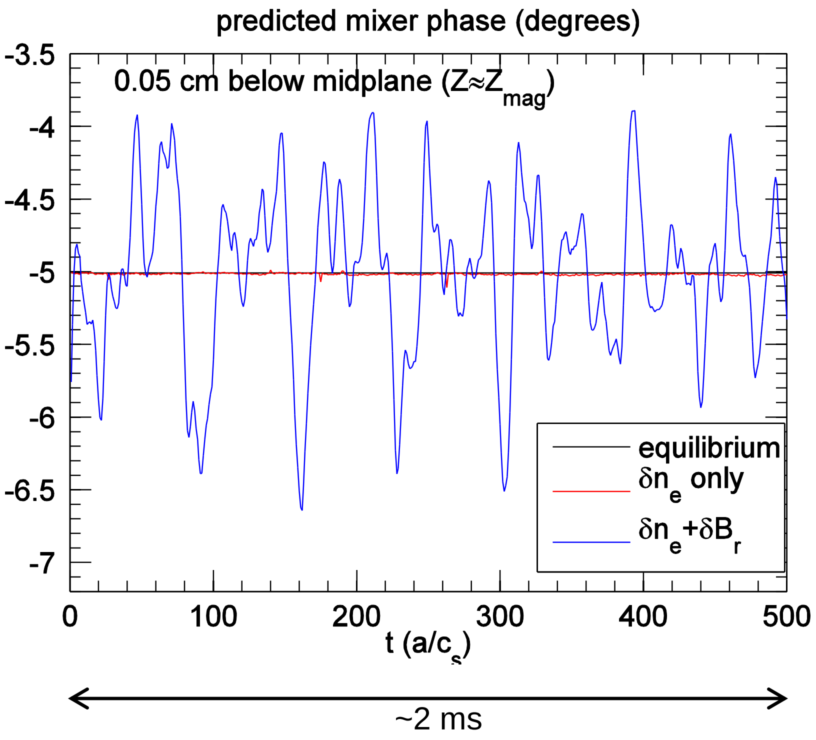


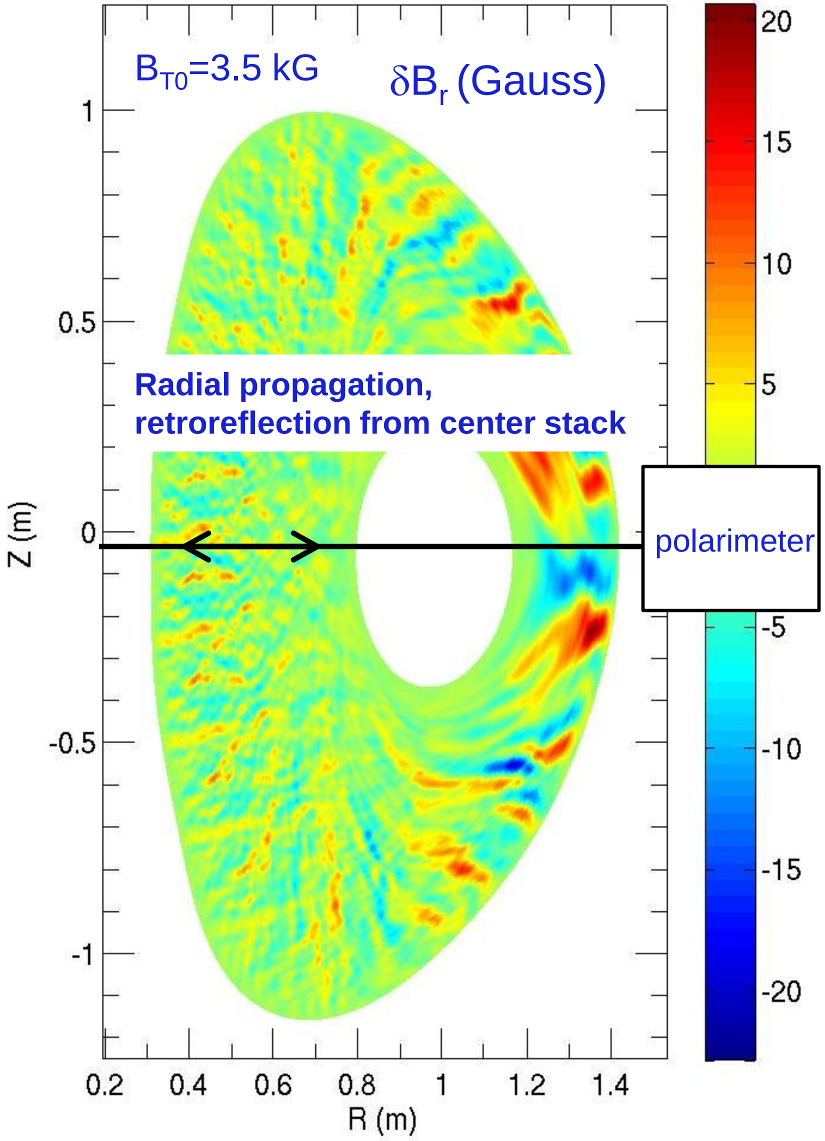


- BES suitable for long poloidal scale (U-Wisconsin, Smith et al., RSI 2010)
- May average over narrow radial scale – requires synthetic diagnostic and instrument function (D. Smith, BO4.2)

# Polarimetry for magnetic field fluctuations

- New UCLA polarimetry system
  (J. Zhang, PP9.71)
- Simulations suggest $(\delta B/B)_{internal} \leq 0.1\%$ may be detectable (1-$2^0$ or ~$0.3^0$ rms mixer phase)





**Radial propagation, retroreflection from center stack**

# Summary

- Microtearing modes found to be unstable in experimental $\nu_*$ scans
  - Scaling of linear growth rates $\gamma_{lin} \sim \nu_e$ – potential candidate to explain experimental confinement trend
  - Linear thresholds exist in $a/L_{Te}$ & $\beta_e$
  - Ionic charge ($Z_{eff}$) can enhance instability (opposite to ETG expectations)

- First non-linear microtearing simulations in NSTX
  - Require relatively fine radial resolution ($\Delta x \approx 0.2\rho_s$, nx=400) to capture physics
  - Transport dominated by electromagnetic contribution ($\delta A_{||}$) → stochastic field lines
  - **Predicted $\chi_{e,sim} \sim \nu_e^{1.1}$ close to experimental scaling**
  - "Stiff" with $\nabla T_e$ but suppressible by experimental levels of E×B shear

## Acknowledgements

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