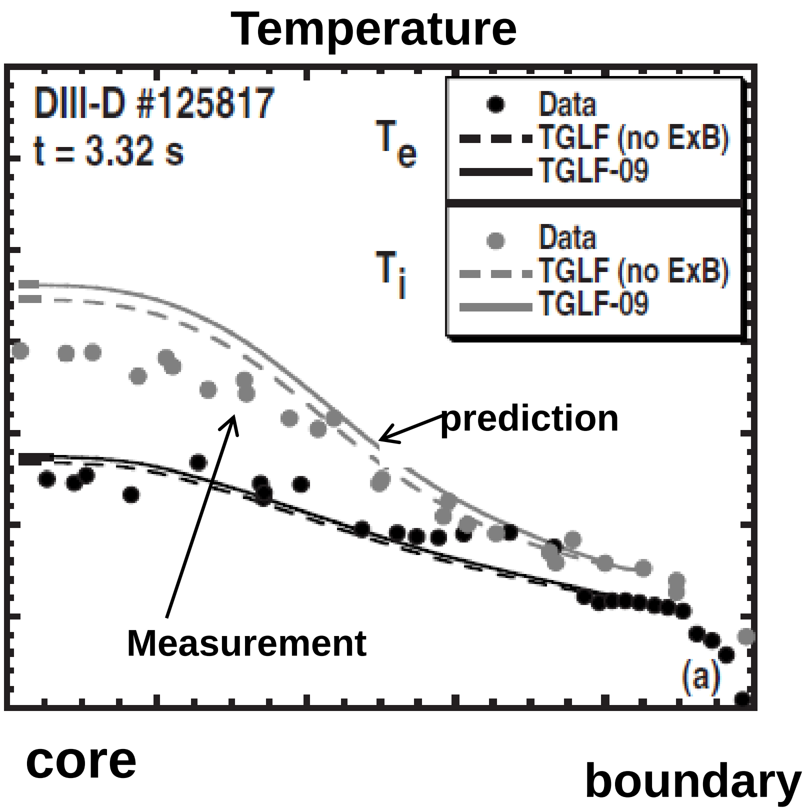
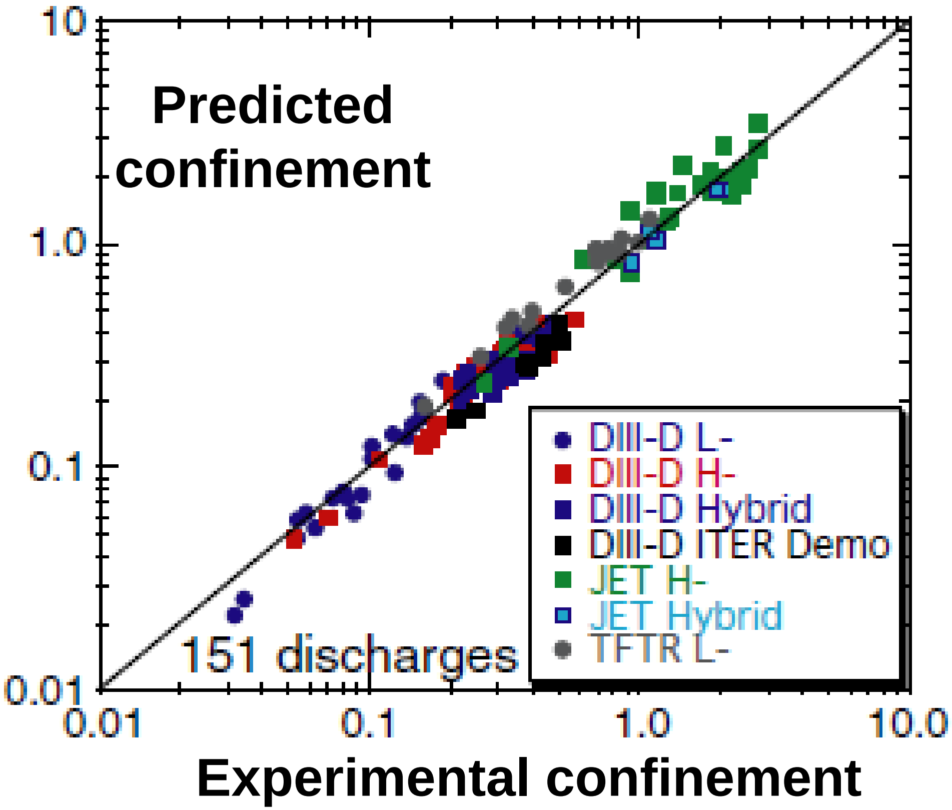
Turbulence Lecture #5:
Modeling turbulence & transport





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Lecture #5 outline

- Simple Illustration of building a turbulent transport model

- Few examples of modern turbulent transport models

Have learned a lot from validating first-principles gyrokinetic simulations with experiment (Lecture 4)

- But the simulations are expensive (1 local multi-scale simulation ~ 20M cpu-hrs)
- Desire a model capable of reproducing flux-gradient relationship that is far quicker, so we can do integrated predictive modeling ("flight simulator")
- All physics based models are local & gradient-driven, i.e. given gradients *from a single flux surface* they predict fluxes:

$$\begin{bmatrix} \Gamma \\ \Pi_\varphi \\ Q_i \\ Q_e \end{bmatrix} = -\begin{bmatrix} \text{flux} - \text{gradient} \\ \text{relationship} \\ \text{matrix} \end{bmatrix} \cdot \begin{bmatrix} \nabla n \\ R\nabla\Omega \\ \nabla T_i \\ \nabla T_e \end{bmatrix}$$

that can be used in solving the 1D transport equation predictively

$$\frac{3}{2} n(\rho,t) \frac{\partial T(\rho,t)}{\partial t} + \nabla \cdot Q(\rho,t) = \dot{P}_{source}(\rho,t) - \dot{P}_{sink}(\rho,t)$$

Is local assumption appropriate?

- If $\rho_* = \rho_i / L$ is small enough (<~1/300), local is good → OK for ITER and most reactor designs (at least in the core, *not the edge*)

- Challenges: In the edge, additional effects may change how we model transport / gradient relationship
  - Large, intermittent edge fluctuations with strong non-local effects may demand full-F gyrokinetic simulations (XGC-1, Gkeyll)
  - Local transport time scale, i.e. evolution of $T(\rho,t)$, is increasingly fast relative to turbulence
  - Related -- edge turbulence should perhaps more realistically be thought of as source driven vs. gradient driven (think external forcing vs. linear instability)
    - We're heating the plasma and watching the temperature respond, not experimentally prescribing a temperature gradient
  - **Unclear how to incorporate these effects in reduced models**

TRANSPORT MODEL DEVELOPMENT

Illustration of how to develop a simple plasma turbulence drift wave transport model

- Decompose flux expressions into wavenumber, amplitude spectra, and cross-phases

$$\Gamma_{k_\theta} = \frac{nT_e}{B} k_\theta \left| \frac{N^*(k_\theta)}{n} \right| \left| \frac{\Phi_r(k_\theta)}{T_e} \right| \sin\left\{ \alpha_{n\varphi}(k_\theta) \right\}$$

- Amplitude could be estimated using mixing-length hypothesis:

$$\frac{\tilde{n}}{n} = \frac{1}{k_r L_n} \sim \frac{\rho_s}{L_n}$$

Would like a representation for cross-phase based on linear stability characteristics

- Greg (Lecture 3) derived for you the ion response for an electron drift wave ($\sim\nabla$n)

$$\frac{\tilde{n}_i}{n} = \frac{k_y V_{*e}}{\omega} \frac{1}{\left(1 + k_\perp^2 \rho_s^2\right)} \frac{e\tilde{\varphi}}{T_e} = \frac{\omega_{*e}}{\omega} \frac{1}{\left(1 + k_\perp^2 \rho_s^2\right)} \frac{e\tilde{\varphi}}{T_e}$$

where I've added the effect of polarization $(1 + k_\perp^2 \rho_s^2)^{-1}$

- For simplicity and illustration, assume electrons nearly-Boltzmann with a small "iδ" imaginary component (<<1) representing instability drive (e.g. TEM)

$$\frac{\tilde{n}_e}{n} = \frac{e\tilde{\varphi}}{T_e}(1 - i\delta)$$

Resulting dispersion relation depends on ω_{*e}, δ, and $k_\perp \rho_s$

$$\omega = \omega_{*e}(1+k_\perp^2\rho_s^2)^{-1}$$

$$\gamma \approx \delta \frac{\omega_{*e}}{1+k_\perp^2\rho_s^2} = \delta \frac{c_s}{L_n} \frac{k_\theta \rho_s}{1+k_\perp^2\rho_s^2}$$

- Expecting growth rates to peak around $k_\theta \rho_s \leq 1$

Linear stability (iδ) also gives us cross-phase information

$$\frac{\tilde{n}_e}{n} = \frac{e\tilde{\varphi}}{T_e}(1 - i\delta)$$

$$\alpha_{n\varphi} = \tan^{-1}(\tilde{n}_e^*\tilde{\varphi}) = \tan^{-1}(1+i\delta) \approx \delta$$

$$\alpha_{n\varphi} \sim \delta \sim \gamma/\omega$$

- In this case, cross phase very simply related to growth rate

Evaluate transport expression using linear stability and mixing-length estimate

$$\Gamma_{k_\theta} = \frac{nT_e}{B} k_\theta \left|\frac{N^*(k_\theta)}{n}\right| \left|\frac{\Phi_r(k_\theta)}{T_e}\right| \sin\{\alpha_{n\varphi}(k_\theta)\}$$

$$\boxed{\Gamma \approx n \frac{T_e}{B} k_\theta \left|\frac{1}{k_r L_n}\right|^2 \frac{\gamma}{\omega_{*e}}}$$

- Model transport determined by (1) mixing length amplitude, and (2) linear stability characteristics

Written as a diffusivity, we recover a mixing-length eddy-diffusivity

$$D_{turb} = -\Gamma/\nabla n = \Gamma \cdot L_n/n$$

$$D_{turb} = \frac{\gamma}{k_r^2}$$

- This is a very common form for "quasi-linear" turbulence diffusion coefficient (often more generally $\gamma/k_\perp^2$)
- Essentially a mixing-length eddy-diffusivity:
  - Radial step size $\langle \Delta x \rangle = k_r^{-1}$, typically evaluated at a single $k_\perp \rho_s$~0.1-0.3, representative of strongest fluctuations in experiment (& eventually sim.)
  - Time step $\langle \Delta t \rangle = \gamma^{-1}$ determined by relevant linear stability dynamics (in this case, iδ)

Using dispersion relation, we recover gyroBohm scaling factor

$$\gamma \approx \delta\omega_{*e} = \delta \cdot k_\theta T_e / B L_n$$

$$k_\theta \rho_s \sim k_r \rho_s \sim 1$$

- $k_\theta \rho_s$ for expected peak γ
- Assuming isotropic

$$D_{turb} = \frac{\gamma}{k_r^2} = \delta \frac{\rho_s}{L_n} \frac{T_e}{B}$$

$$\boxed{D_{turb} \approx \delta \cdot \chi_{GB}}$$

- *In the local (small ρ_*) limit, all transport quantities have leading order gyroBohm scaling*
- **But linear stability (δ) still matters (e.g. thresholds & stiffness)**

Early models (60's-80's) used analytic fluid or gyrokinetic theory to evaluate linear stability

- Fancy non-linear theories also used to refine model for saturated fluctuation amplitudes

- A turning point in model sophistication was the advent of gyrofluid equations & increased computational power
  - Hammett, Perkins, Dorland, Beer, Waltz, ….

➢ Take fluid moments of gyrokinetic equation

➢ Pick suitable kinetic closures

➢ Tweak closure free parameters to best match linear gyrokinetic simulations
  - Linear GK simulations became routine in mid-90's, but expensive and slow relative to gyrofluid

MODERN TURBULENT TRANSPORT MODELS

Breakthrough in understanding was recognition of threshold and stiffness

$$Q_{model} = Q_{GB} \cdot F(s, q, \ldots) \cdot \left(\frac{R}{L_T} - \frac{R}{L_{T,crit}}\right)^{\alpha}$$

- All local models have gyroBohm prefactor (Q_{GB})
- First modern model approaches fit coefficients in above equation to large numbers of GF and/or GK simulations
  - $R/L_{T,crit}$ from linear simulations
  - Additional scaling coefficient F(s,q,…) from nonlinear simulations

➢ *A bunch of fit coefficients, but entirely from first principles*

IFS-PPPL able to reproduce a large database of TFTR discharges

- Recovers a number of important scalings, e.g. stabilization of ITG (larger $R/L_{Ti,crit}$) at increasing T_i/T_e (see Lecture 3)

Model ITG transport (not all shown)

$$\chi_i = C_0 \max(\chi_i^{(1)}, \chi_i^{(2)}) \rho_i^2 v_{ti}/R,$$

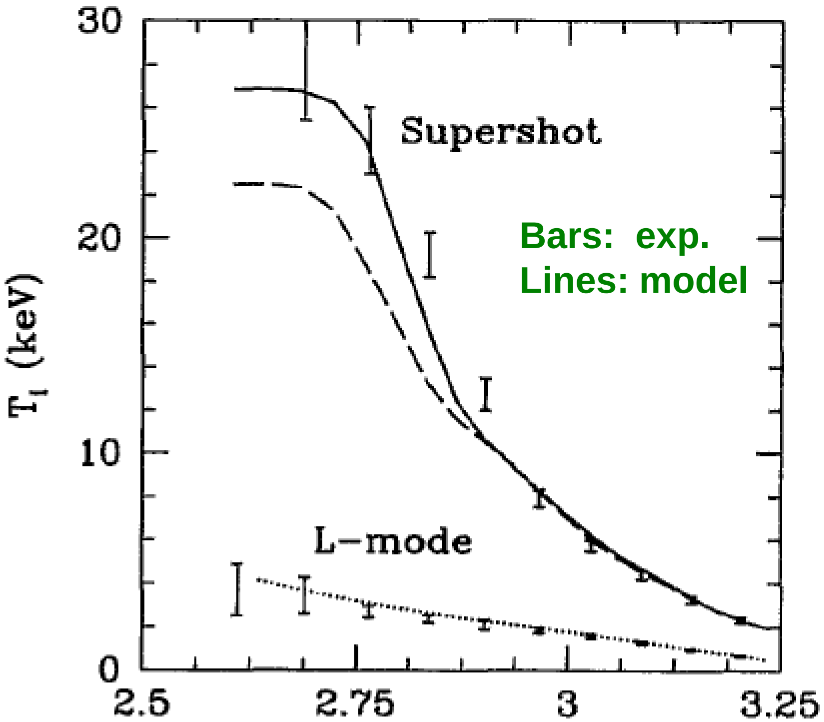
$$\chi_i^{(1)} = \frac{(q/\tau_b)^{1.1}}{1+\hat{s}^{0.84}} \left(1 + \frac{6.7\epsilon}{q\nu^{0.26}}\right) \mathscr{Z}(Z_{\rm eff}^*) \mathscr{G}^{(1)}\left(\frac{R}{L_T}\right)$$

Model ITG threshold (not all shown)

$$R/L_{T\,\rm crit}^{(1)} = f(\{p_j\}) g(\{p_j\}) h(\{p_j\}), \quad (2)$$

where $f = 1 - 0.2 Z_{\rm eff}^{*0.5} \hat{s}^{-0.7}(14\epsilon^{1.3}\nu^{-0.2} - 1)$, $g = (0.7 + 0.6\hat{s} - 0.2R/L_n^*)^2 + 0.4 + 0.3R/L_n^* - 0.8\hat{s} + 0.2\hat{s}^2)$, and $h = 1.5(1 + 2.8/q^2)^{0.26} Z_{\rm eff}^{*0.7} \tau_b^{0.5}$. Here, $R/L_n^* \equiv$ max

TFTR



ITG
Kotschenreuther (1995)

Equivalent parameters found for TEM-dominant conditions ($T_e >> T_i$)

Model TEM transport

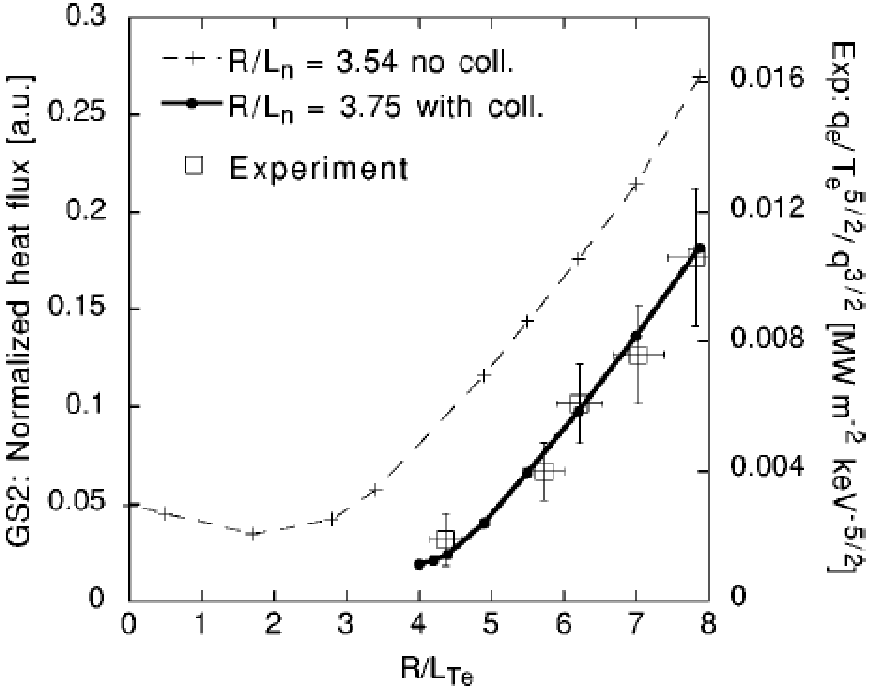
$$q_e = \lambda^* \frac{n_e q^{3/2} \rho_s^2 c_s T_e}{R^2}\left(\frac{R}{L_{Te}} - \left.\frac{R}{L_{Te}}\right|_{\mathrm{crit}}\right) \times H\left(\frac{R}{L_{Te}} - \left.\frac{R}{L_{Te}}\right|_{\mathrm{crit}}\right).$$

Model TEM threshold

$$\frac{R}{L_{Tcrit}} = \frac{0.357\sqrt{\epsilon}+0.271}{\sqrt{\epsilon}}\left[4.90 - 1.31\frac{R}{L_N} + 2.68\hat{s} + \ln(1+20\nu_{\mathrm{eff}})\right],$$

$$\lambda^* = 9\sqrt{\epsilon}(1.0 - 0.39\hat{s} - 0.1\nu_{\mathrm{eff}}),$$

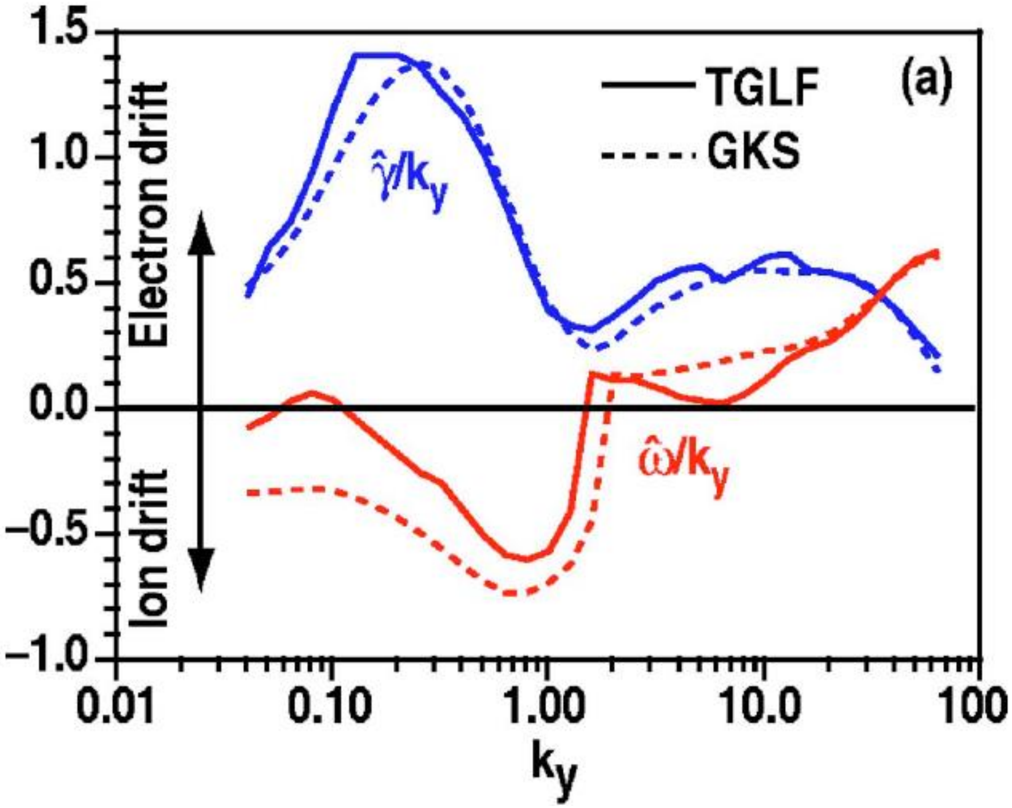
ASDEX Upgrade
Flux-gradient relationship



TEM (T_e>>T_i)
Peeters (2005)

Modern GF models use moment equations to solve for linear equations over entire k space

- Closure models calibrated to ~1800 linear GK simulations
  - Original GLF23 with 8 fluid equations (1997)
  - Updated TGLF now uses 15 fluid equations / species (2005)
- Example shows multiscale growth rates agreeing with gyrokinetics (GKS)



Staebler (2007)

Write transport expressions in terms of cross-phases and amplitudes

$$Q_{i,\mathrm{ql}} = \frac{3}{2}\frac{n_i T_i}{n_e T_e}\,\mathrm{Re}[ik_y\tilde{\Phi}^*\tilde{p}_i]/|\tilde{\Phi}|^2,$$

$$Q_{e,\mathrm{ql}} = \frac{3}{2}\,\mathrm{Re}[ik_y\tilde{\Phi}^*\tilde{p}_e]/|\tilde{\Phi}|^2,$$

$$\Gamma_{\mathrm{ql}} = \mathrm{Re}[ik_y\tilde{\Phi}^*\tilde{n}_e]/|\tilde{\Phi}|^2,$$

- Linear analysis gives distinct cross-phases for each transport channel (far more information than isolated ITG or TEM models above)

Staebler (2007)

Rather complicated saturation rule for the amplitude spectra

- Also keeps a spectrum of saturated mode amplitudes

$$\bar{\Phi}^2 = \Delta_{ky} \frac{\hat{\omega}_{d0}^2}{k_y^4} \Lambda, \quad \Lambda = \frac{\bar{\gamma}^{\beta_\gamma}(\alpha_{d0} + [\alpha_d \mathrm{Max}(\bar{\omega}_d, 0)]^{\beta_d})}{[1 + (\alpha_\gamma \bar{\gamma})^{\beta_\gamma}][1 + (\alpha_d |\bar{\omega}_d|)^{\beta_d}] k_y^{\beta_k}},$$

$$\hat{\omega}_{d0} = k_y (a/R),$$

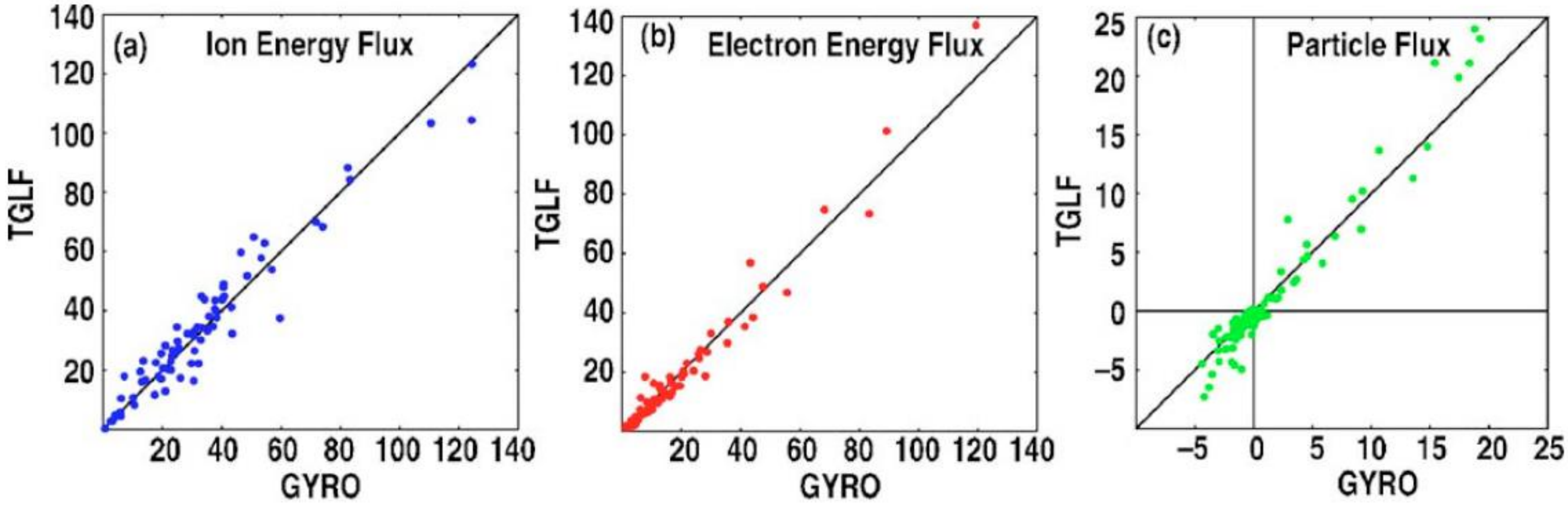
$$\bar{\omega}_d = \langle \hat{\omega}_d \rangle / \hat{\omega}_{d0}, \quad \bar{\gamma} = \mathrm{Max}[(\hat{\gamma} - \alpha_{\mathrm{ZF}} \hat{\gamma}_{\mathrm{ZF}} - \alpha_{\mathrm{E}} \hat{\gamma}_{\mathrm{ExB}}) / \hat{\omega}_{d0}, 0], \tag{3}$$

$$\hat{\gamma}_{\mathrm{ZF}} = \hat{\omega}_{d0} [\mathrm{Max}(\hat{\gamma} - \alpha_{\mathrm{E}} \hat{\gamma}_{\mathrm{ExB}}, 0) / \hat{\omega}_{d0}]^{\beta_{z\gamma}} / (k_y^{\beta_{zk}} q^{\beta_{zq}}).$$

Staebler (2007)

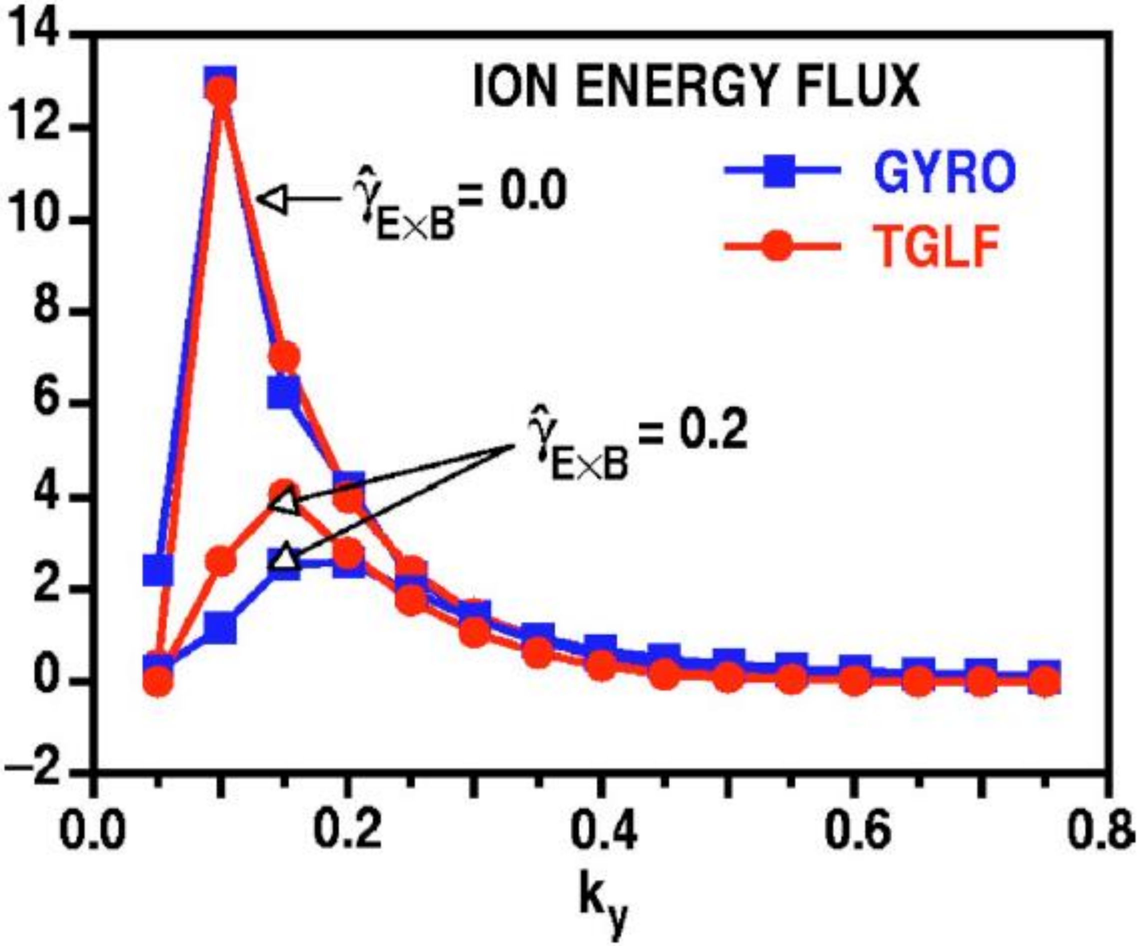
Saturation coefficients chosen to best match transport fluxes from ~100 nonlinear gyrokinetic simulations

- Many fit coefficients in the reduced model, but all determined from first-principles simulations (*no calibrating to experiment*)



Staebler (2007)

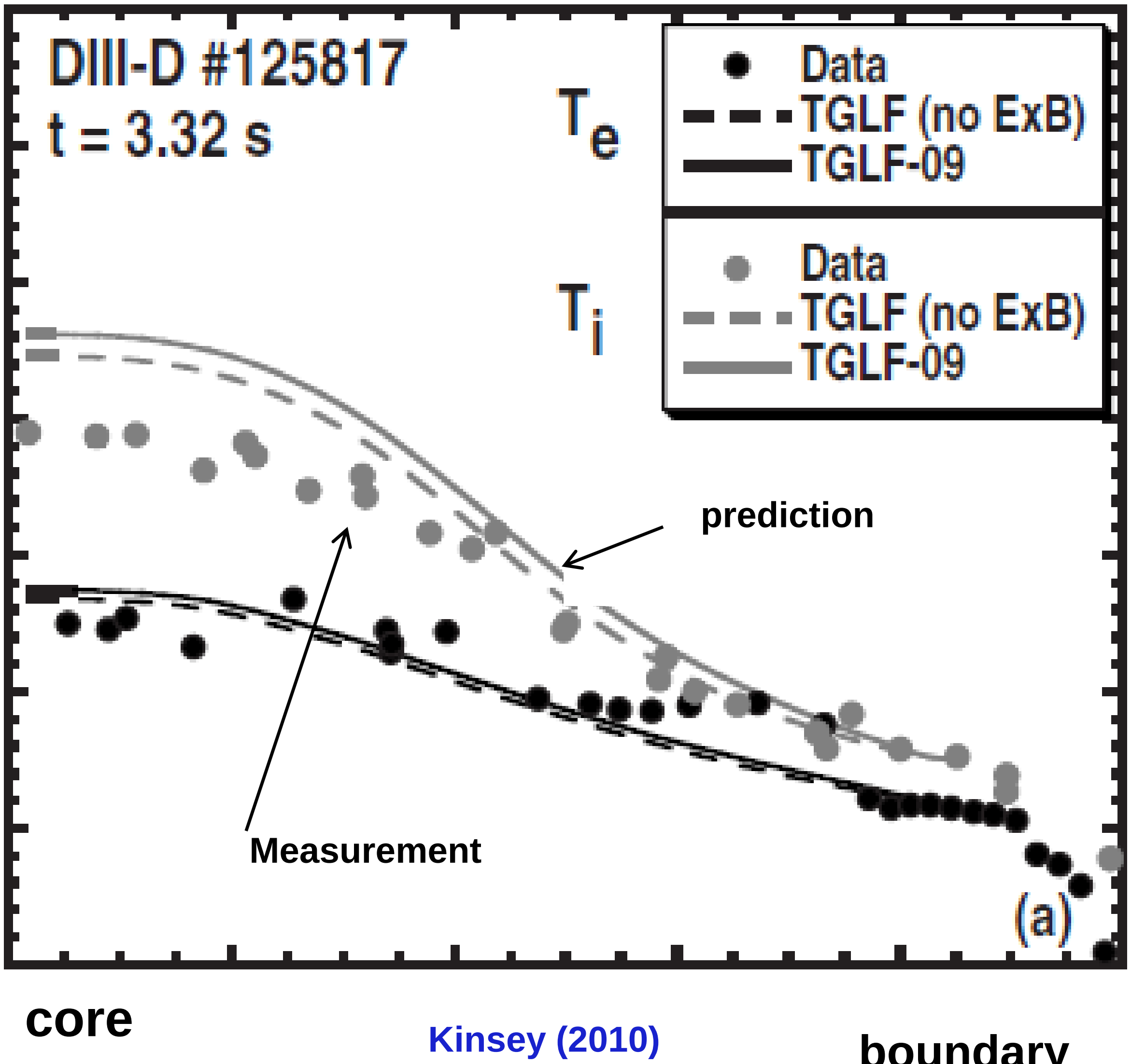
Able to match first-principles (gyrokinetic) transport spectrum



Staebler (2007)

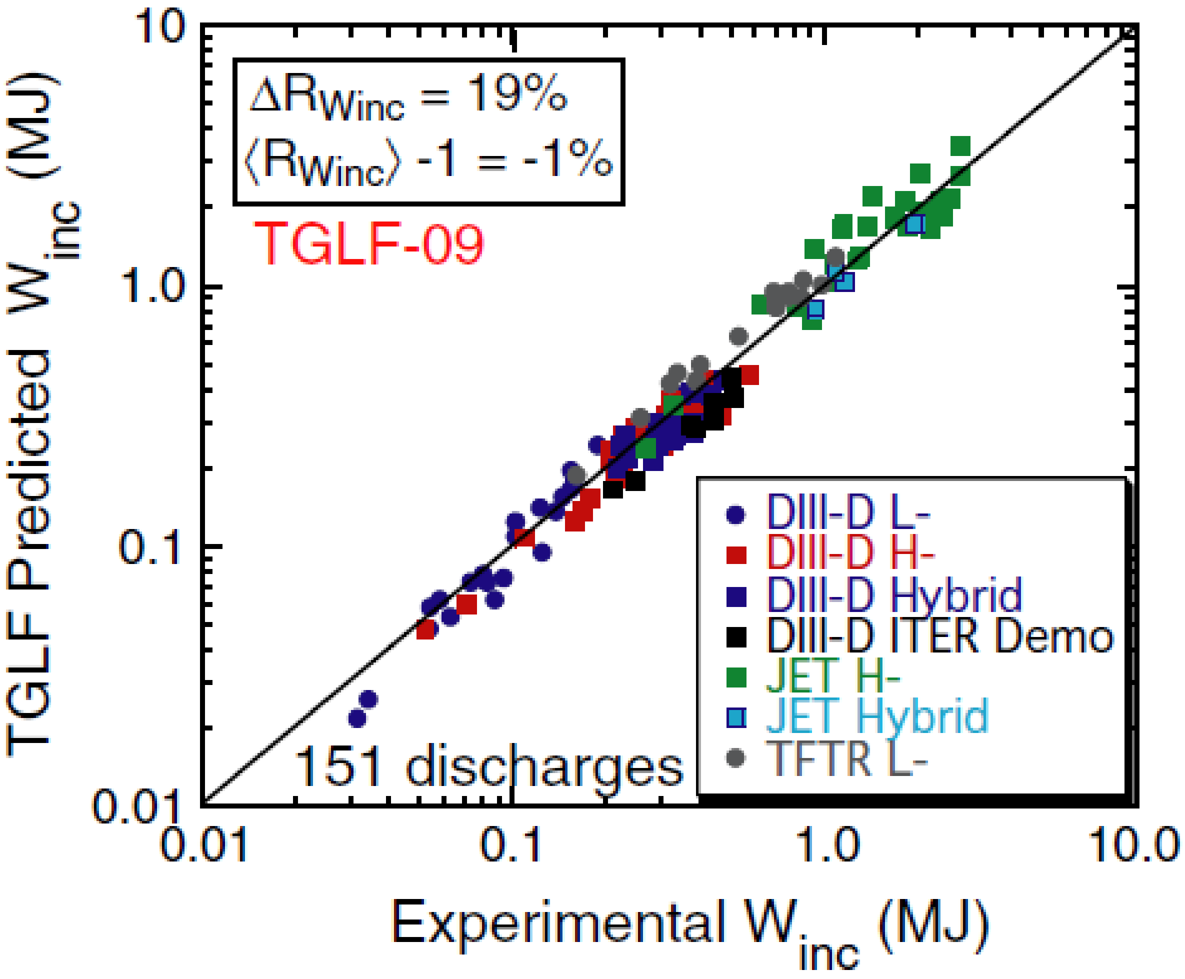
Some success in profile predictions

Temperature



Kinsey (2010)

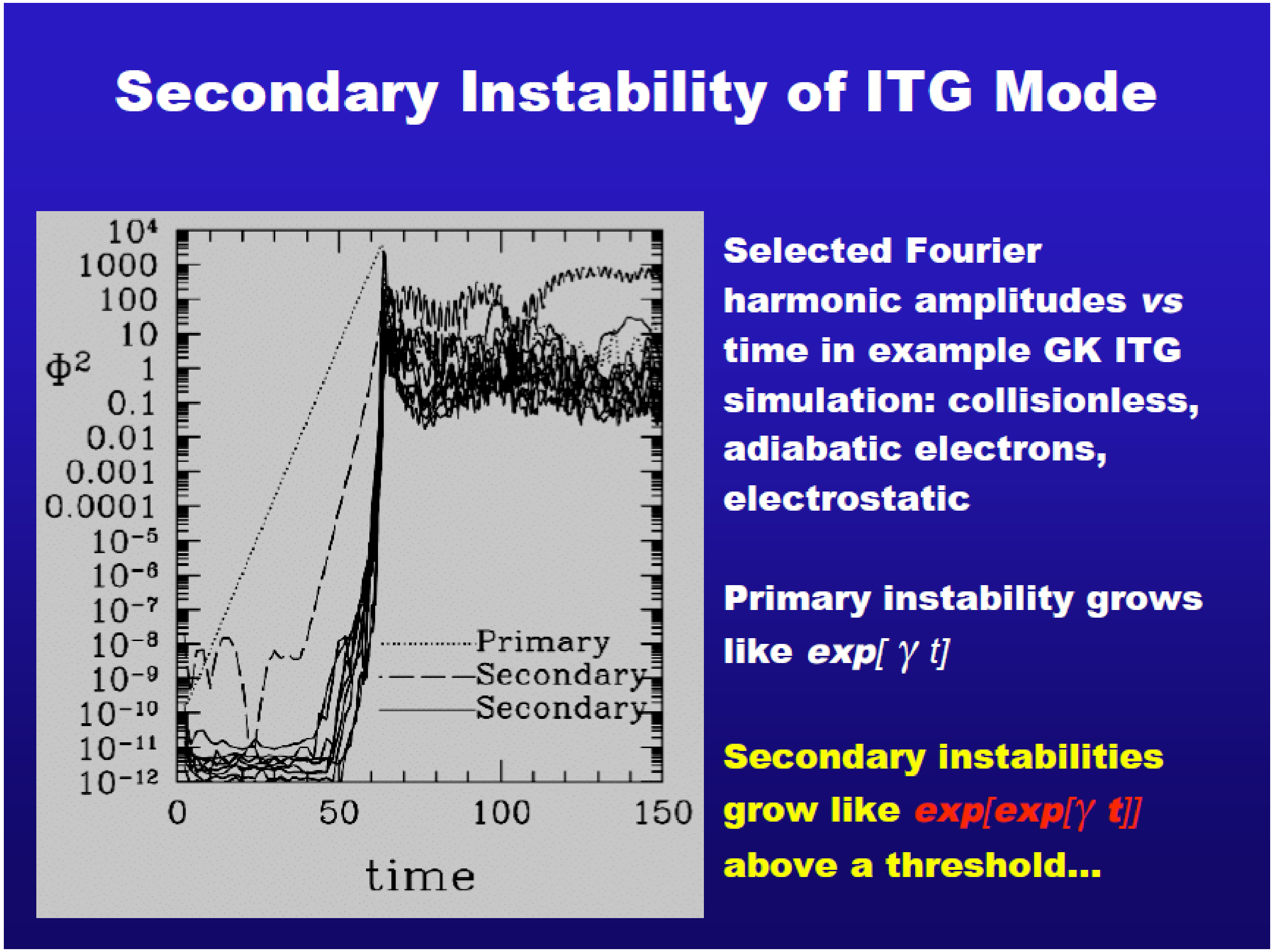
Good agreement in predicted energy confinement over database of discharges



Kinsey (2011)

Zonal flows play a key role in saturating ITG turbulence

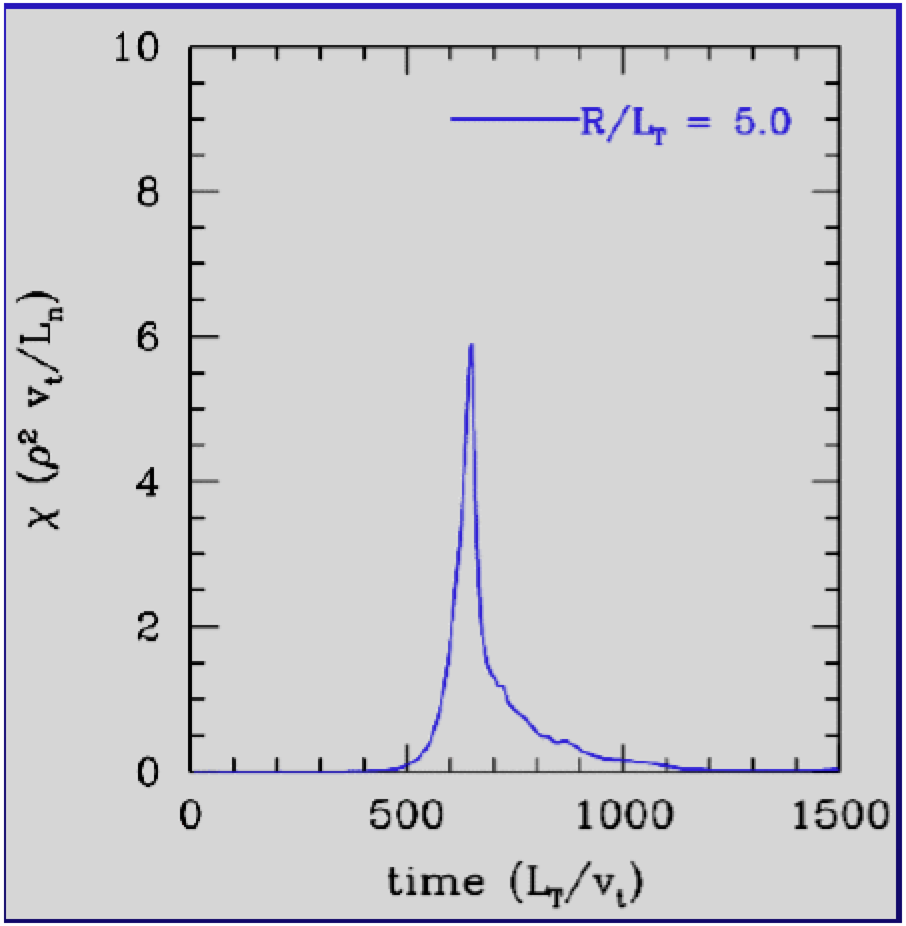
- Driven unstable by linearly growing primary modes ~ exp(exp(γt))
- Large amplitude helps saturate turbulence

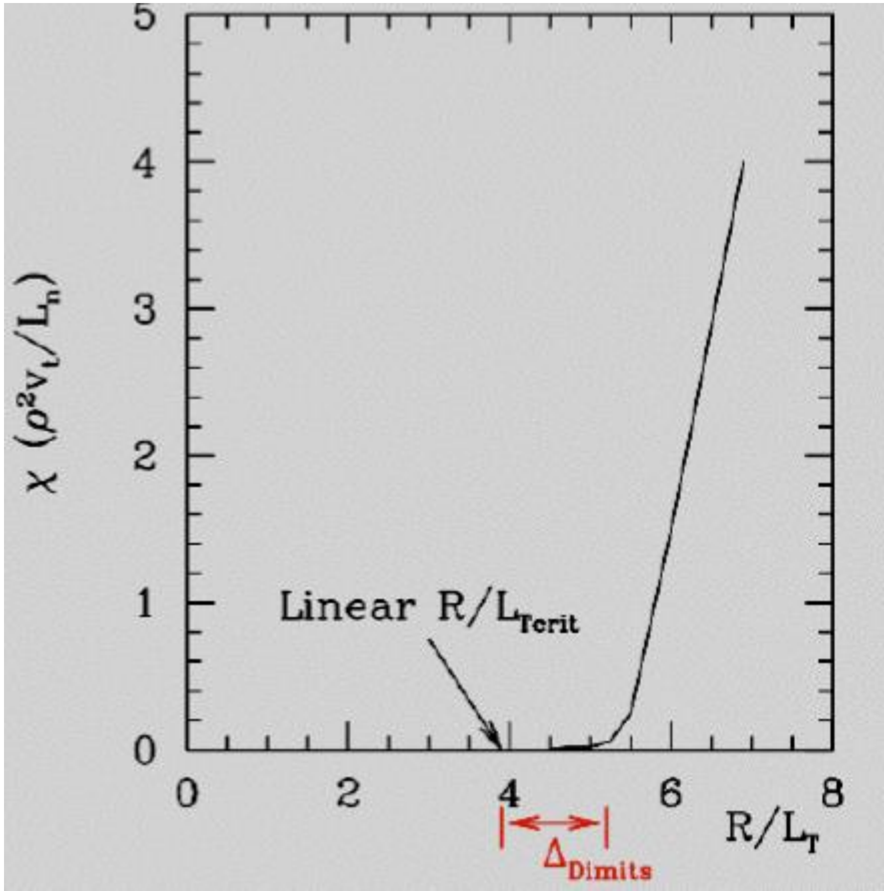


Dorland (2000)

Near linear threshold, strong zonal flows can suppress primary instability

- Leads to nonlinear upshift of effective threshold





Dorland (2000)

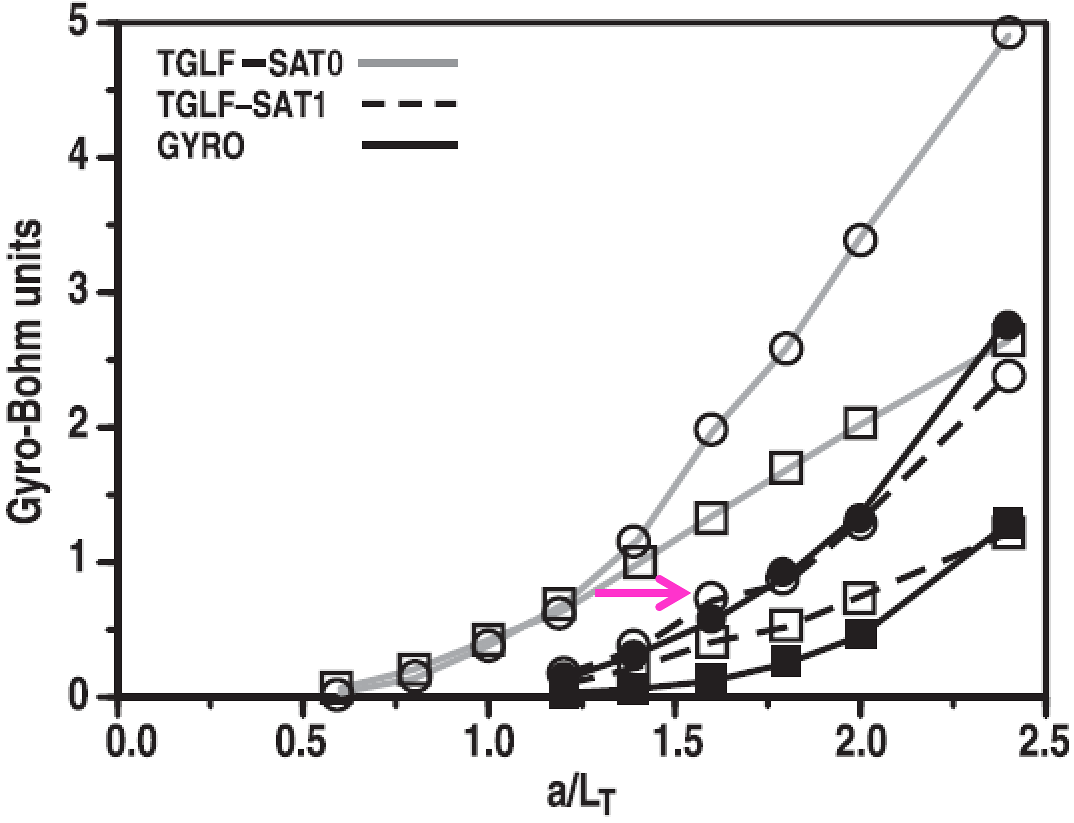
Capturing ZF effects in gyrofluid models

- Balance between linear drive, nonlinear k_x-mixing due to ZF, & nonlinear drift wave mixing → able to model energy redistribution in (k_x, k_y) space

$$\frac{\partial \tilde{f}(k_x,k_y)}{\partial t} = \gamma_{lin}\tilde{f}(k_x,k_y) + \sum_{k'_x} k'_x \tilde{\Phi}(k'_x,0) k_y \tilde{f}(k_x - k'_x, k_y) + \sum_{k'_x} \sum_{k'_y}^{k'_y \neq 0} \left(k'_x k_y - k'_y k_x\right) \tilde{\Phi}\left(k'_x, k'_y\right) \tilde{f}(k_x - k'_x, k_y - k'_y).$$

Staebler (2016)

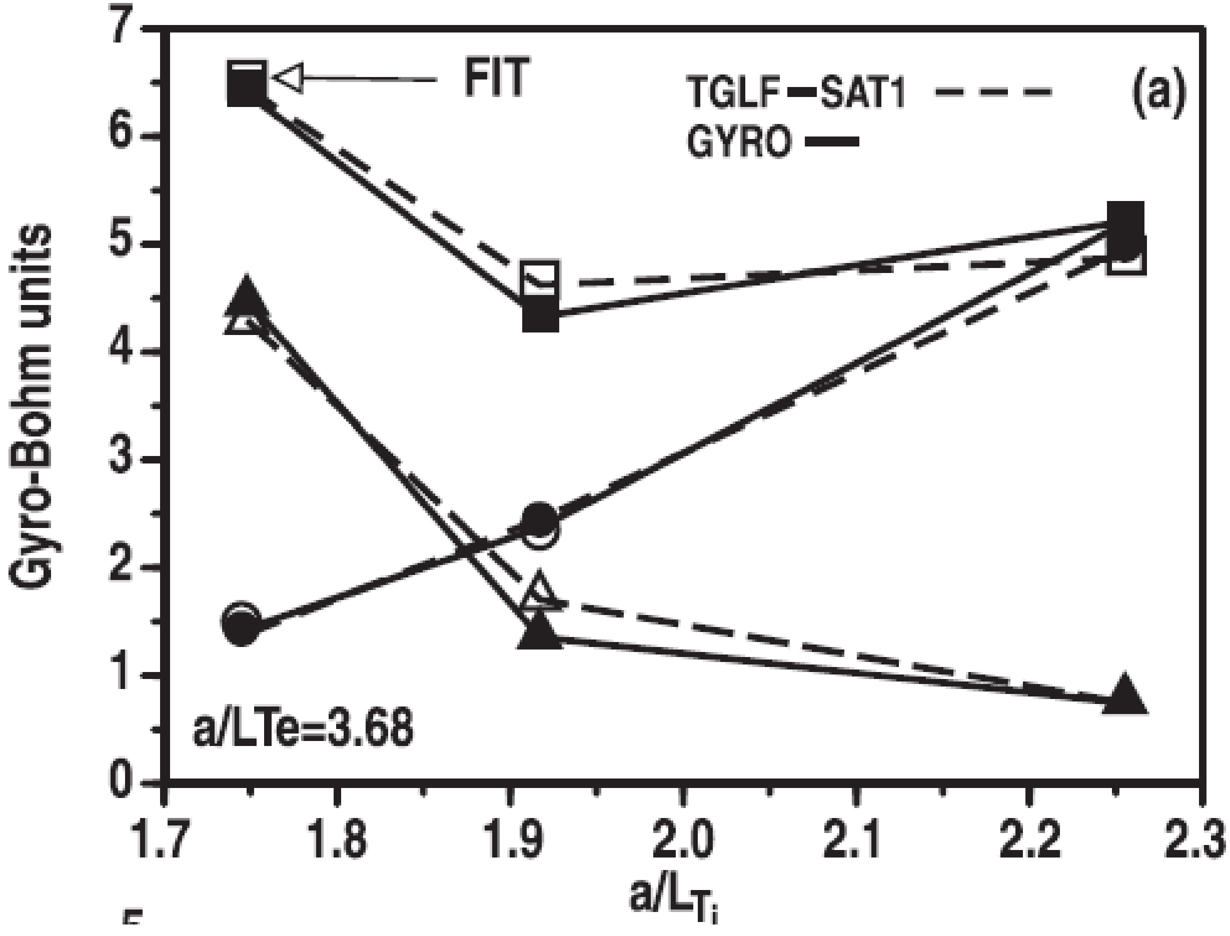
Captures nonlinear upshift predicted in turbulence (GYRO) simulations



Staebler (2016)

Also reproduces cross-scale coupling (discussed in Lecture 4)

- Due to ZF-catalyzed NL energy transfer



Staebler (2016)

More exotic effects may eventually be included in modeling turbulence saturation & dissipation

- Coupling to damped eigenmodes (that exist at all $k_\perp$ scales, with different cross-phases) can influence spectral saturation and partitioning of transport, e.g. Q_e vs. Q_i vs. Γ, … (Terry, Hatch)

- Different routes to dissipation have been addressed theoretically:
- Critical balance (Goldreich-Sridhar, Schekochihin, M. Barnes): balance nonlinear $\perp$ dynamics with linear || dynamics
  - 2D perpendicular nonlinearity at different parallel locations creates fine parallel structure ($k_{||}\uparrow$) → through Landau damping generates fine $v_{||}$ structure → dissipation through collisions
  - Can happen at all $k_\perp$ scales
  - Simple scaling argument reproduces transport scaling
- Nonlinear phase mixing (Hammett, Dorland, Schekochihin, Tatsuno):
  - At sufficient amplitude, gyroaveraged nonlinear term $\langle \delta v_E \rangle \cdot \nabla \delta f \sim J_0\left(\frac{k_\perp v_\perp}{\Omega}\right) \delta v_E \cdot \nabla \delta f$ generates structure in $\mu \sim v_\perp^2$ → dissipation through collisions

While orders faster than GK, reduced GF models still slow for predictive modeling

- Current research → train neural nets on expansive database of GF model predictions to use in predictive models (Citrin)

Multi-scale gyrokinetics	Gyrofluid transport model	NN evaluation of GF model
~10^{10} cpu-sec	**~10^{0} cpu-sec**	**~10^{-5} cpu-sec**

- Sufficient speed up enables real-time control or faster-than-real-time forecasting

Summary

- Magnetized turbulent transport models fundamentally use quasi-linear calculation of cross-phases plus mixing-length like saturation estimates

- Have shown a number of successes in reproducing first-principles simulations

- As always, discrepancies (and failures) increase moving towards the edge → a frontier of turbulence and transport modeling