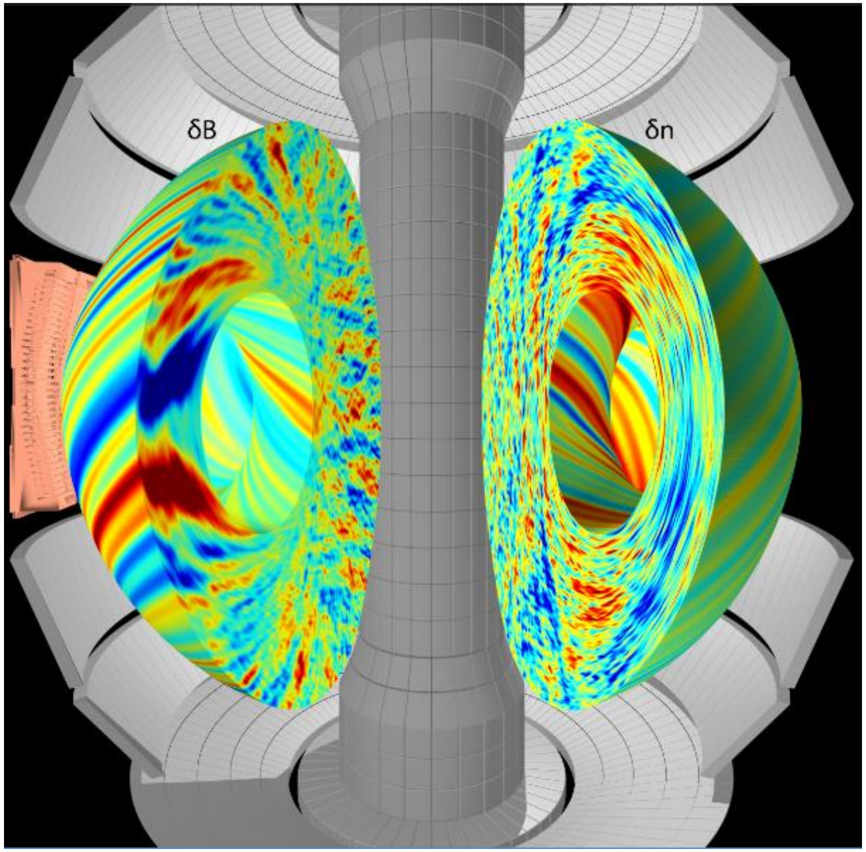
Toroidal magnetized plasma turbulence & transport



Walter Guttenfelder

Graduate Summer School 2020

Categorization of turbulence lectures this week

- 3D hydrodynamic turbulence (Lectures #1/2)
  - Neutral fluid, incompressible Euler equations (continuity + Navier-Stokes)
  - Nonlinear energy cascade in inertial range (between forcing and dissipation scales) $\rightarrow$ Kolmogorov spectrum

- 3D MHD turbulence (Lectures #2/4)
  - Alfven waves in presence of guiding B field $\rightarrow$ anisotropy, additional linear term
  - Derived in single-fluid MHD limit
  - Assumed uniform field and plasma (no background gradients ∇B, ∇n, ∇T)

- 2D drift wave turbulence (Lecture #3, today)
  - Strong anisotropy due to strong background magnetic field
  - Driven by cross-field background thermal gradients ($\nabla_\perp F_M \rightarrow \nabla_\perp n$, $\nabla_\perp T$) $\rightarrow$ additional linear term, source of instability turbulence to relax gradients
  - Inhomogeneous B gives rise to ∇B & curvature drifts and particle trapping $\rightarrow$ additional dynamics for instability
  - Derived in two-fluid (or two-species kinetic) limit

Concepts of turbulence to remember

- Turbulence is deterministic yet unpredictable (chaotic), appears random
  - Turbulence is not a property of the *fluid / plasma*, it's a feature of the *flow*
  - We often treat & diagnose statistically, but also rely on first-principles direct numerical simulation (DNS)

- Turbulence spans a wide range of spatial & temporal
  - Re >>> 1 for neutral fluids, $L_{forcing} / \ell_{dissipation} \sim (Re)^{3/4}$
  - Or in the case of hot, low-collisionality plasma, a wide range of scales in 6D phase-space ($\mathbf{x}$,$\mathbf{v}$)

- Turbulence causes increased mixing, transport larger than collisional transport
  - **Transport** is the key application of why we care about turbulence (e.g. fusion gain ~ $nT\tau_E$, energy confinement time τ_E set by turbulence)

- It's cool! "Turbulence is the most important unsolved problem in classical physics" (~Feynman)

Turbulent transport is an advective process

- Transport a result of finite average 2nd order correlation between perturbed drift velocity (δv) and perturbed fluid moments (δn, δT, δv)
  - Particle flux, $\Gamma = \langle \delta v \delta n \rangle$
  - Heat flux, $Q = 3/2 n_0 \langle \delta v \delta T \rangle + 3/2 T_0 \langle \delta v \delta n \rangle$
  - Momentum flux, $\Pi \sim \langle \delta v \delta v \rangle$ ("Reynolds stress")

- Electrostatic turbulence often most relevant in tokamaks $\rightarrow$ E$\times$B drift from potential perturbations: $\delta v_E = B \times \nabla(\delta\varphi)/B^2 \sim k_\theta(\delta\varphi)/B$

- Can also have magnetic contributions at high beta, $\delta v_B \sim v_{||}(\delta B_r/B)$ (magnetic "flutter" transport)

Implications of a strong toroidal magnetic field

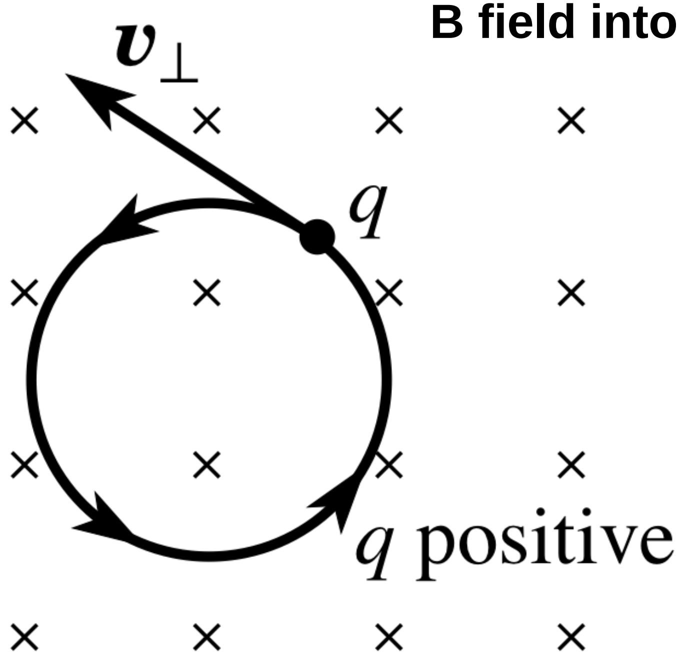
Charged particles experience Lorentz force in a magnetic field → gyro-orbits

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

- Magnetic force acts perpendicular to direction of particle

→Particles follow circular gyro-orbits

B field into plane



$$\Omega_c = \frac{eB}{m}$$

$f_c \sim 10^7$ / 10^{10} Hz

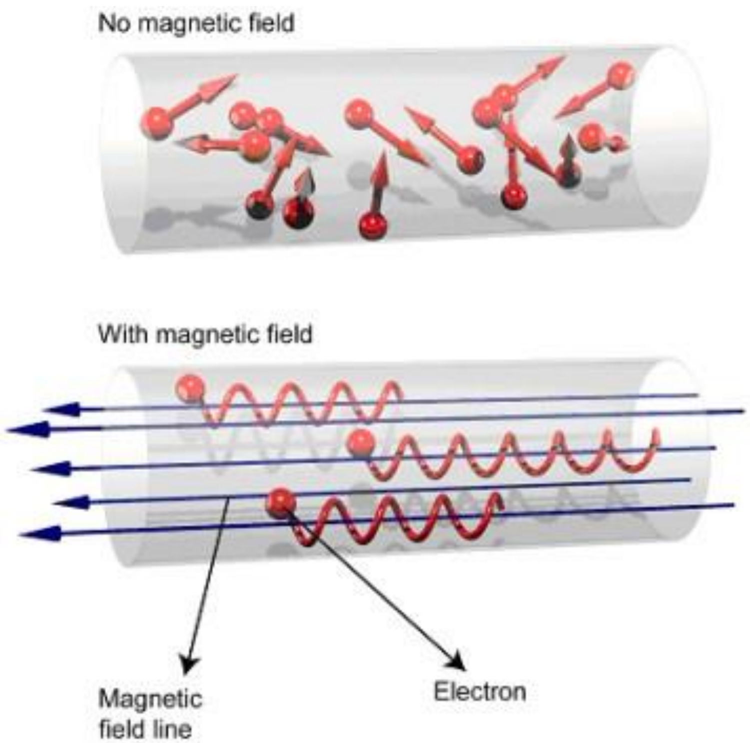
(for deuteron / electron, B=5 T)

Magnetic field confines particles away from boundaries, leads to strong anisotropy

gyroradius: $\boldsymbol{\rho = \frac{v_T}{\Omega_c}}$

$B \approx 5$ T
$T \approx 10$ keV

$\rho_i \sim$ **3 mm** $\rho_e \sim$ **0.05 mm**	<<	**1-2 meter device size**



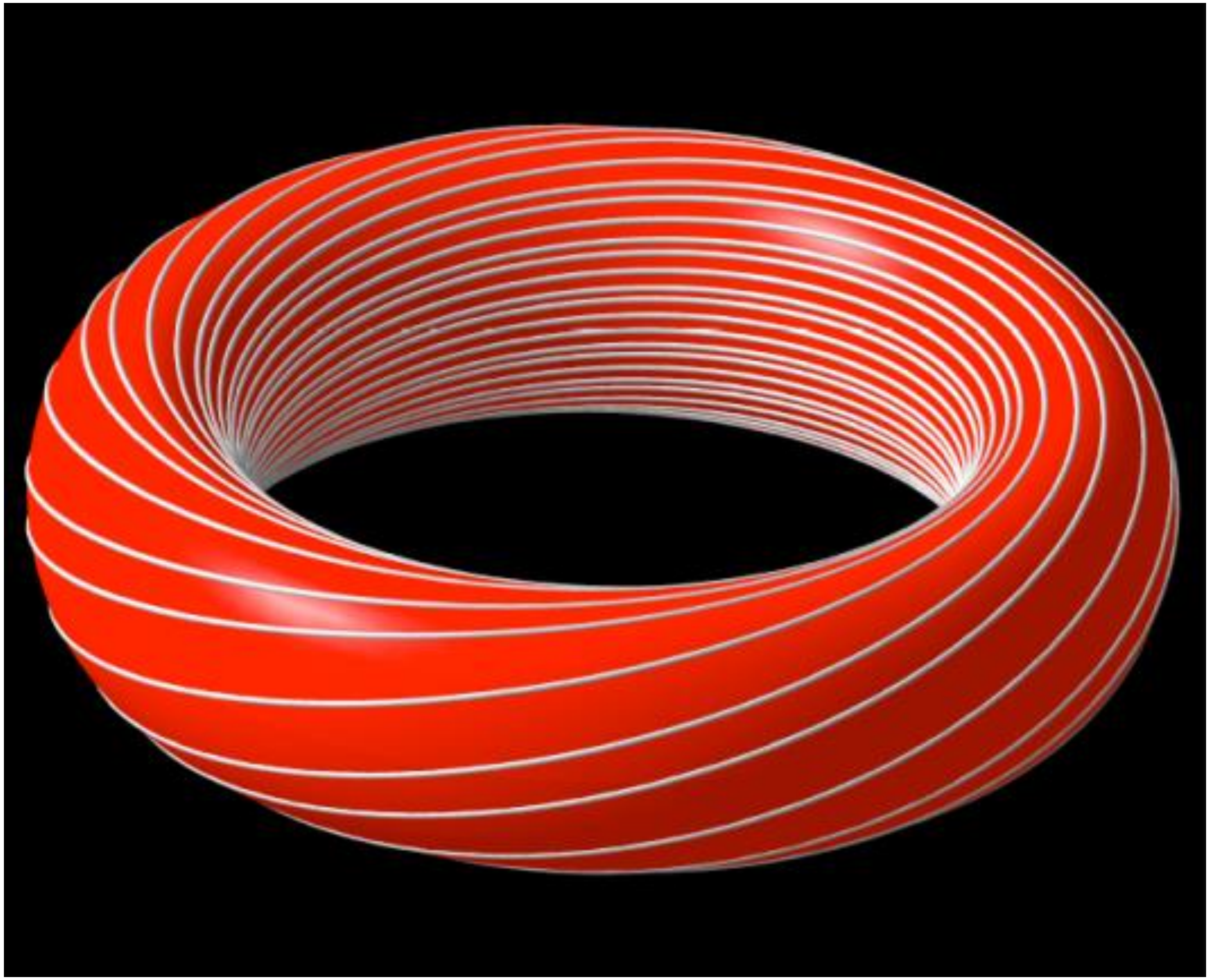
<u>Low collision frequency $\nu \sim n/T^{3/2}$</u>

$\lambda_{MFP} \sim$ km's >> device size

$\lambda_{MFP} / \rho_i \sim 10^6$

$\chi_{||}/\chi_{\perp} \sim (\lambda_{mfp}/\rho)^2 \sim 10^{12} \rightarrow$ *strong anisotropy*

Particles easily lost from ends → bend into a torus

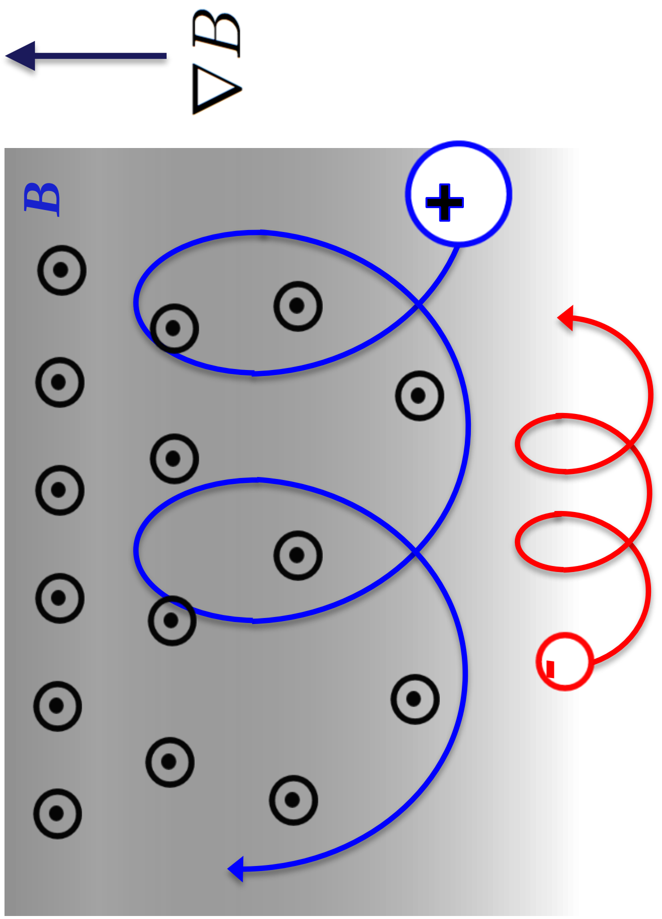


But toroidicity leads to vertical drifts from ∇B & curvature

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \qquad B \sim \frac{1}{R}$$

$$v_{\nabla B} \approx \left(\frac{\rho}{R}\right) v_T \approx \frac{T}{qBR}$$

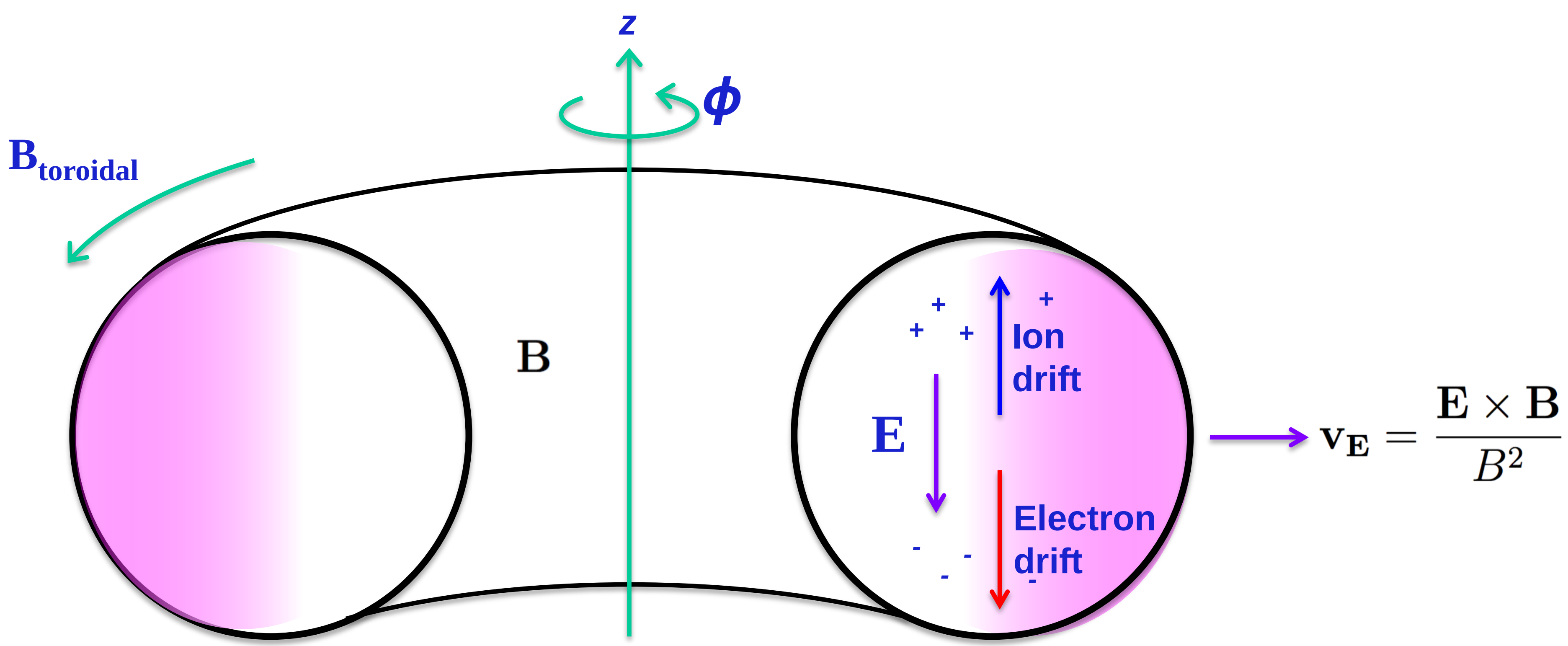
τ_{loss} ~ 5 ms from vertical drifts (B~5 T, R~5 m, T~15 keV)



$$\boxed{\rho_* = \frac{\rho}{R}}$$

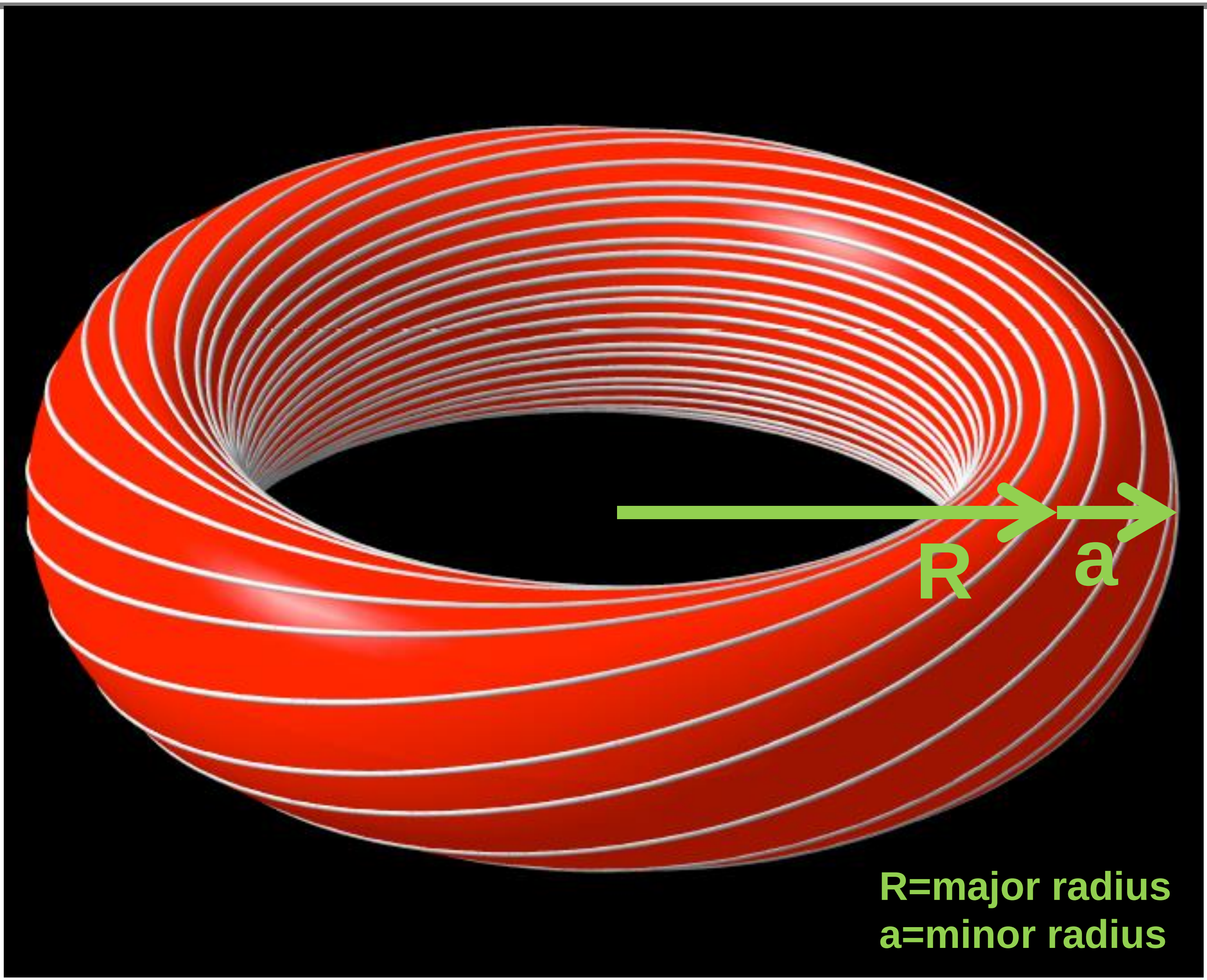
Key parameter in magnetized confinement

Even worse, charge separation leads to faster E×B drifts out to the walls

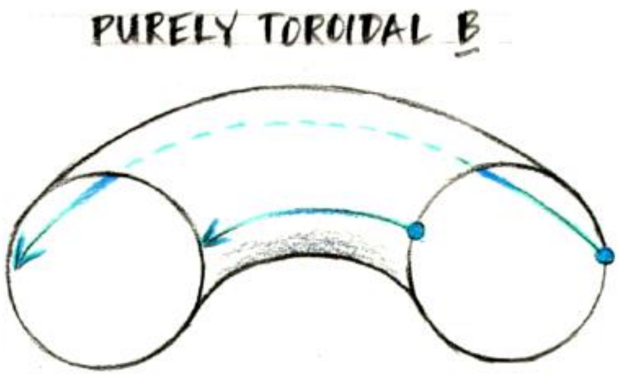


τ_{loss} ~ μs from E×B drifts (due to charge separation from vertical drifts)

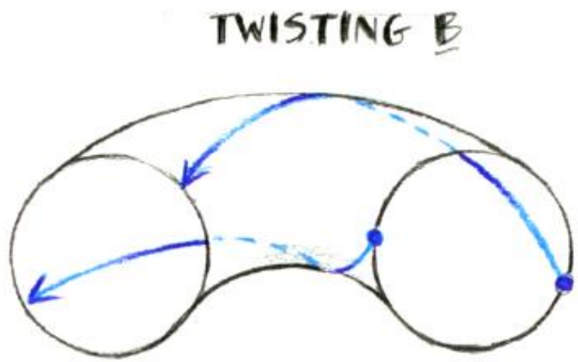
Solution: need a helical magnetic field for confined (closed) particle orbits



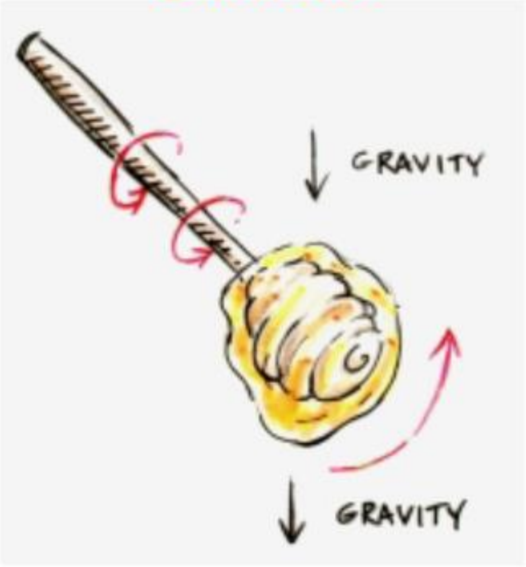
Helical B field carries plasma from “bad curvature” region to “good curvature” region



Unstable



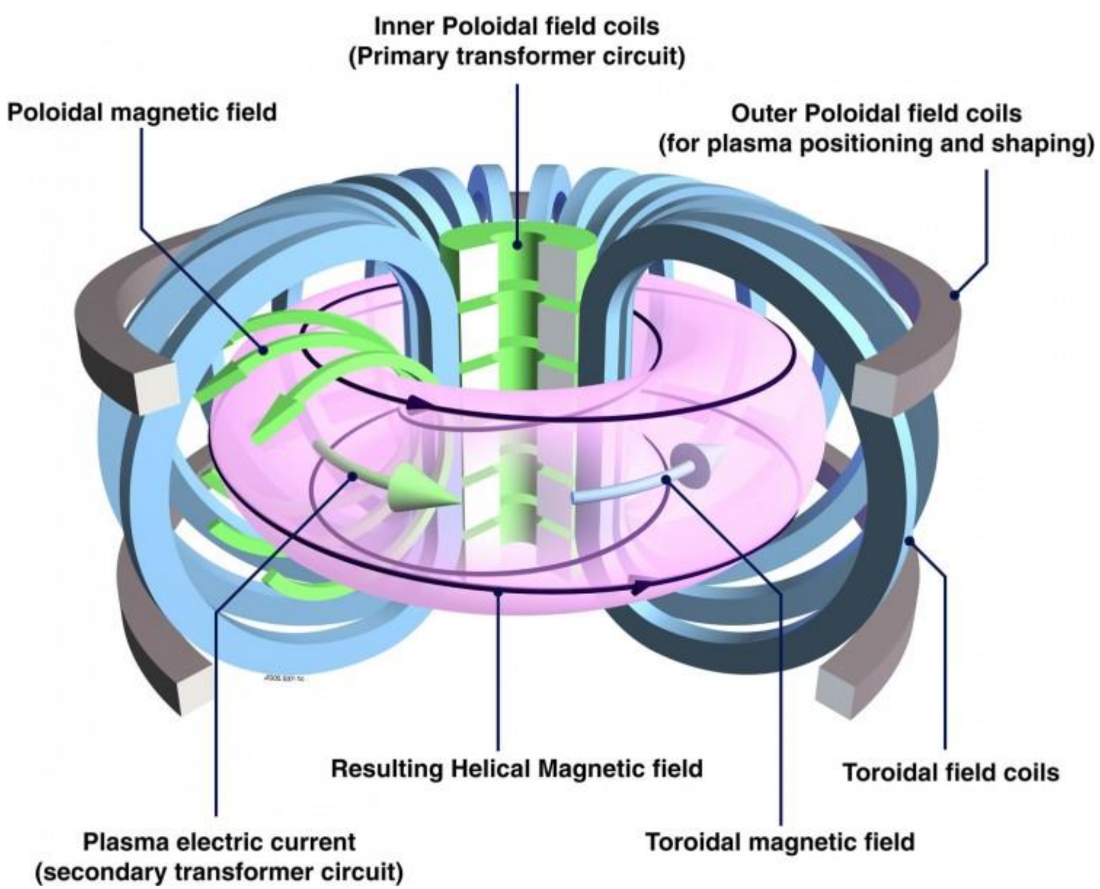
Stable



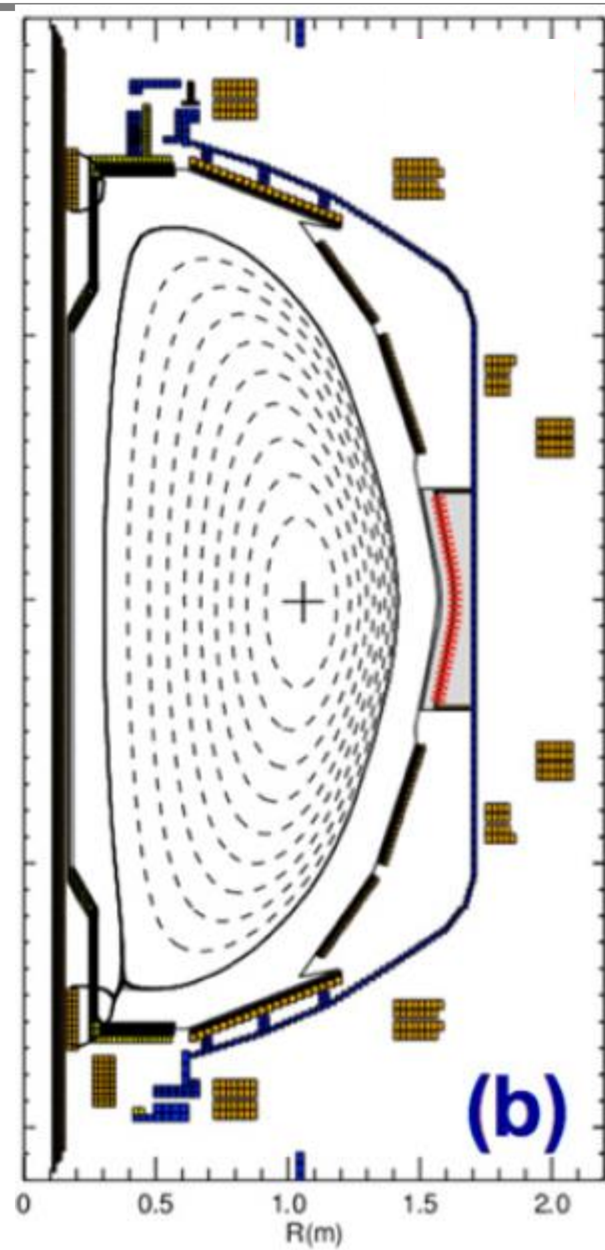
Similar to how honey dipper prevents honey from dripping

Tokamaks

- Toroidal, axisymmetric
- Helical field lines confine plasma
- Closed, nested flux surfaces in force balance:
  $J \times B = \nabla p$



NSTX



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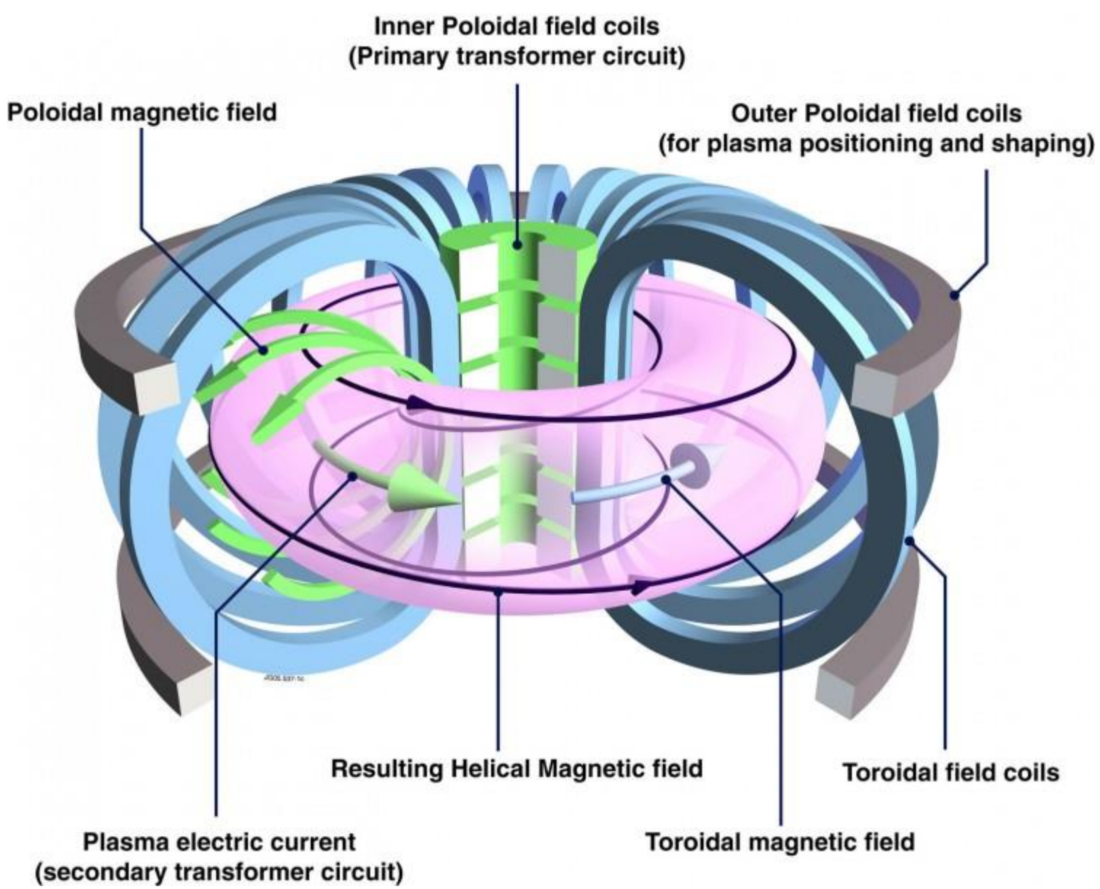

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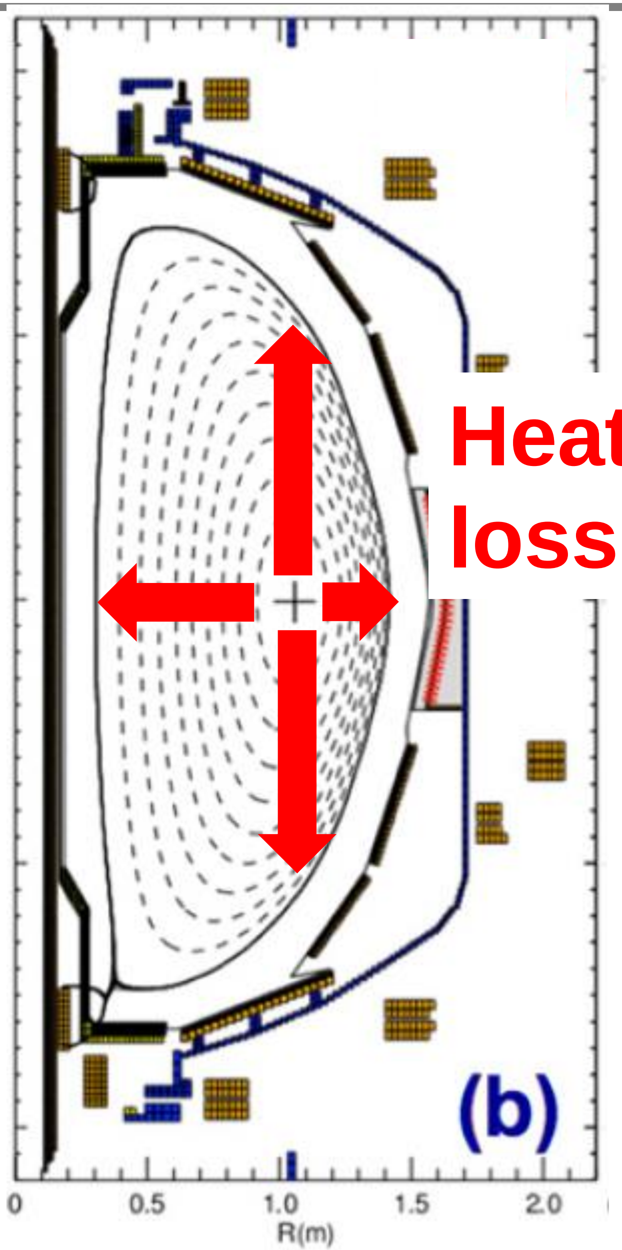
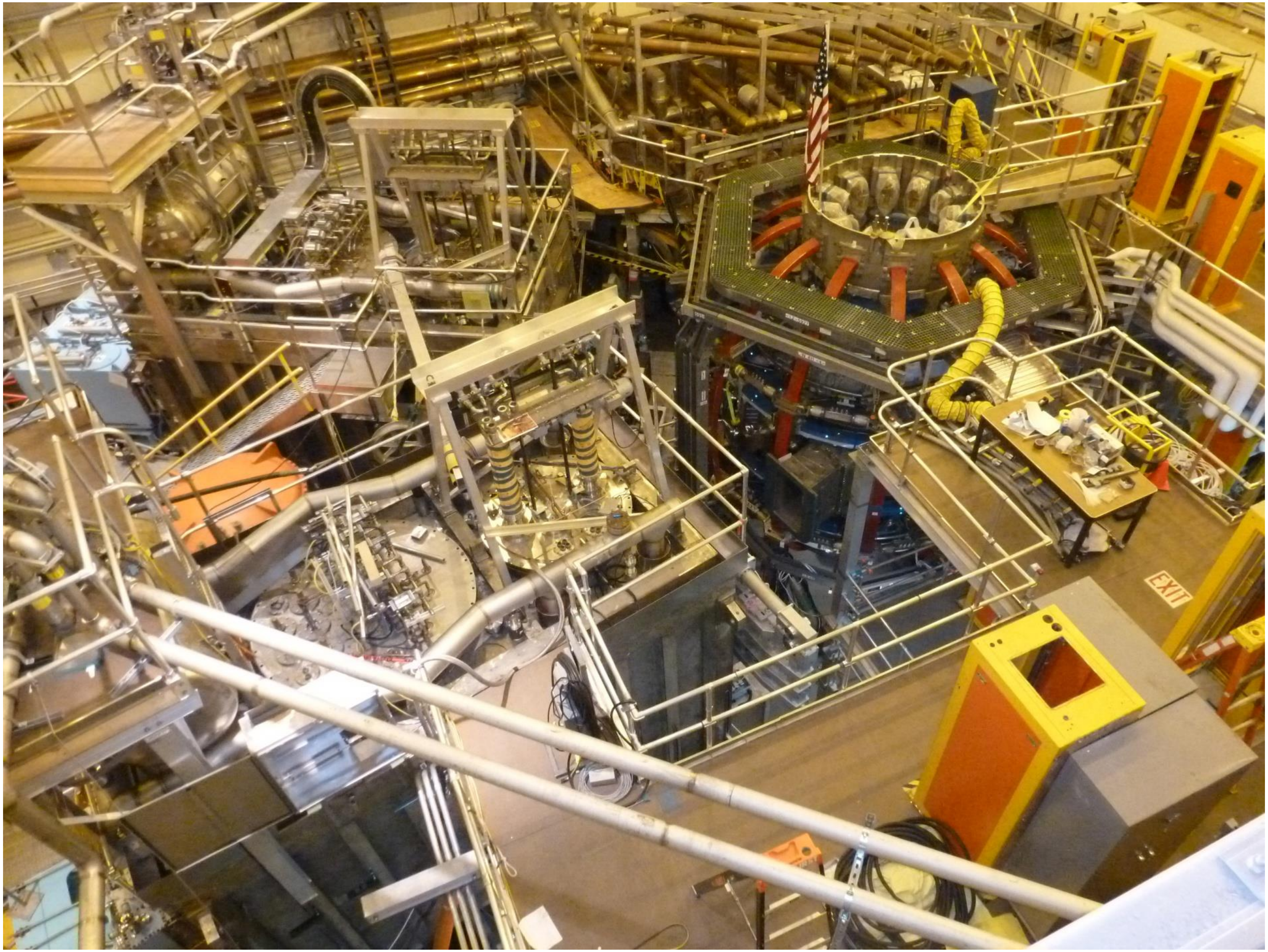


At Princeton Plasma Physics Lab (PPPL): National Spherical Torus Experiment-Upgrade (NSTX-U)

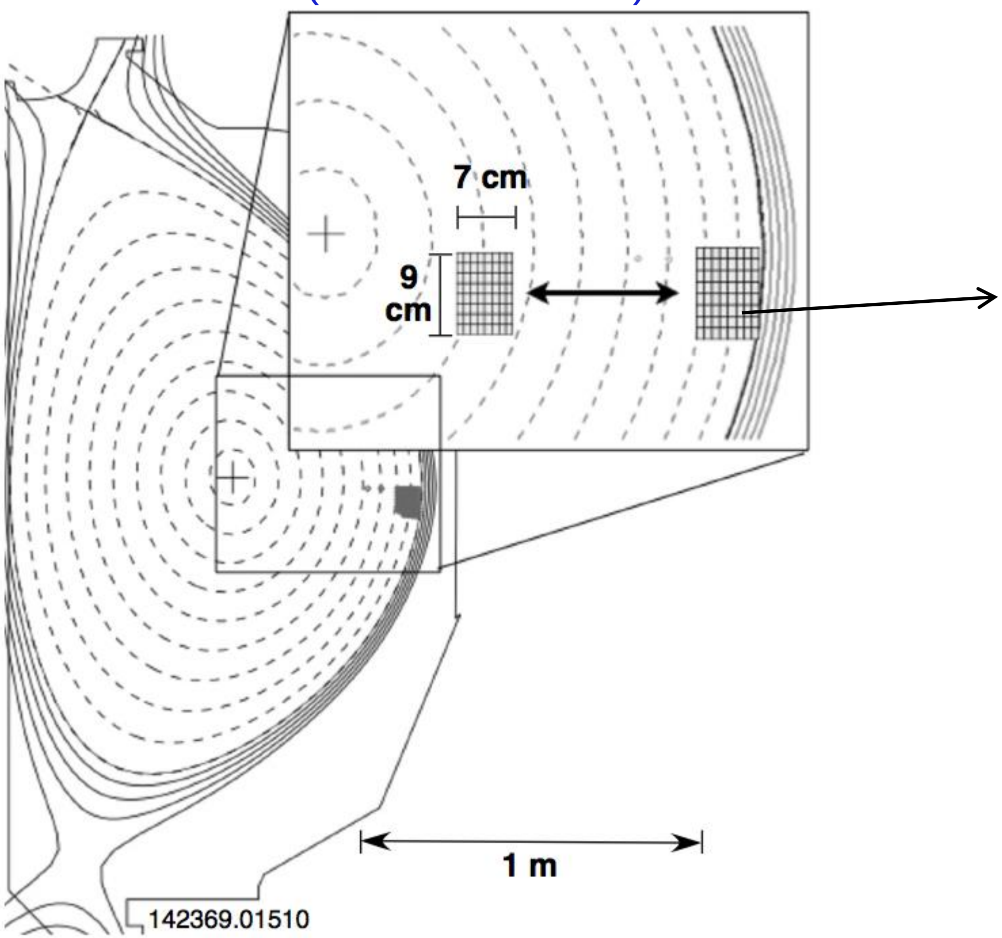


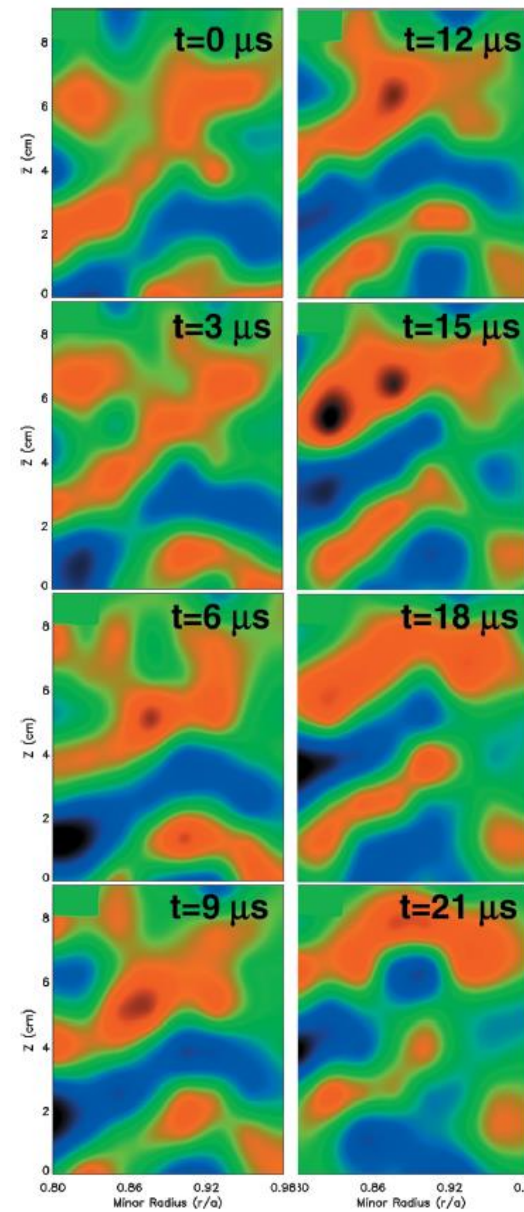
Turbulence characteristics in tokamaks

Spectroscopic imaging provides a 2D picture of turbulence in tokamaks: cm spatial scales, μs time scales, <1% amplitude

- Beam Emission Spectroscopy (UW-Madison) measures Doppler shifted D_α from neutral beam heating to infer plasma density

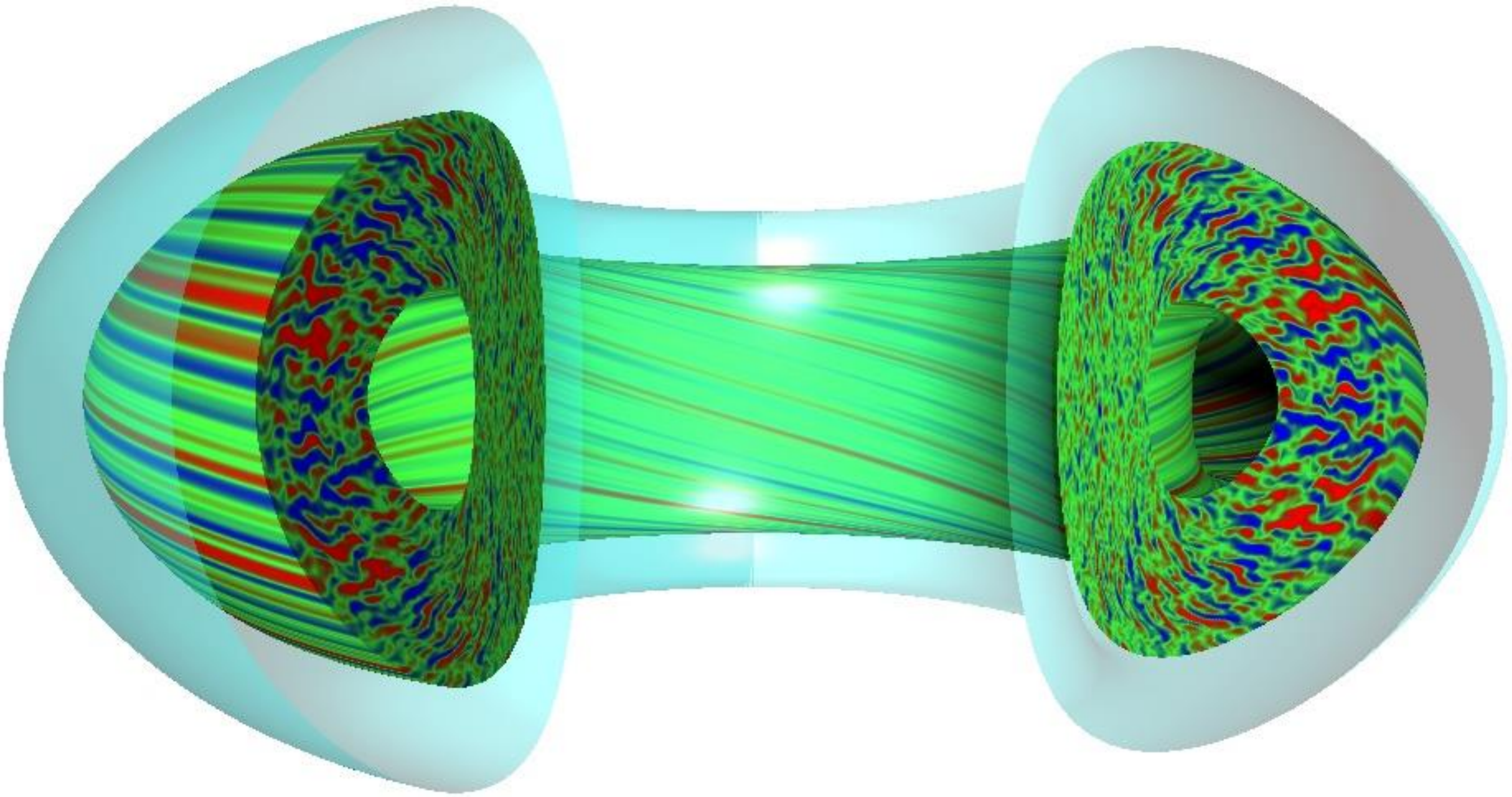
DIII-D tokamak (General Atomics)





Movies at: https://fusion.gat.com/global/BESMovies

Simulations provide detailed prediction of expected turbulence characteristics



GYRO simulation (Candy, Waltz – General Atomics)

Transport is of order the Gyrobohm diffusivity

- Although turbulence is advective, can estimate order of transport due to drift waves as a diffusive process, $\chi_{turb} \sim \langle \Delta x^2 \rangle / \langle \Delta t \rangle \sim (L_{\perp,corr})^2 / \tau_{corr}$

$L_{\perp,corr} \sim$ few ρ_s (~ cm's)

$\tau_{corr}^{-1} \sim c_s/R$ (~10^5 1/s)

$$\rho_s = c_s/\Omega_{ci}$$
$$c_s = \sqrt{T_e/m_d}$$

gyroBohm diffusivity

$$\boxed{\chi_{turb} \sim \chi_{GB} = \frac{L_\perp^2}{\tau_{corr}} = \frac{\rho_s^2 c_s}{R} = \frac{\rho_s}{R}\rho_s c_s = \frac{\rho_s}{R}\frac{T_e}{B}}$$

ρ_*

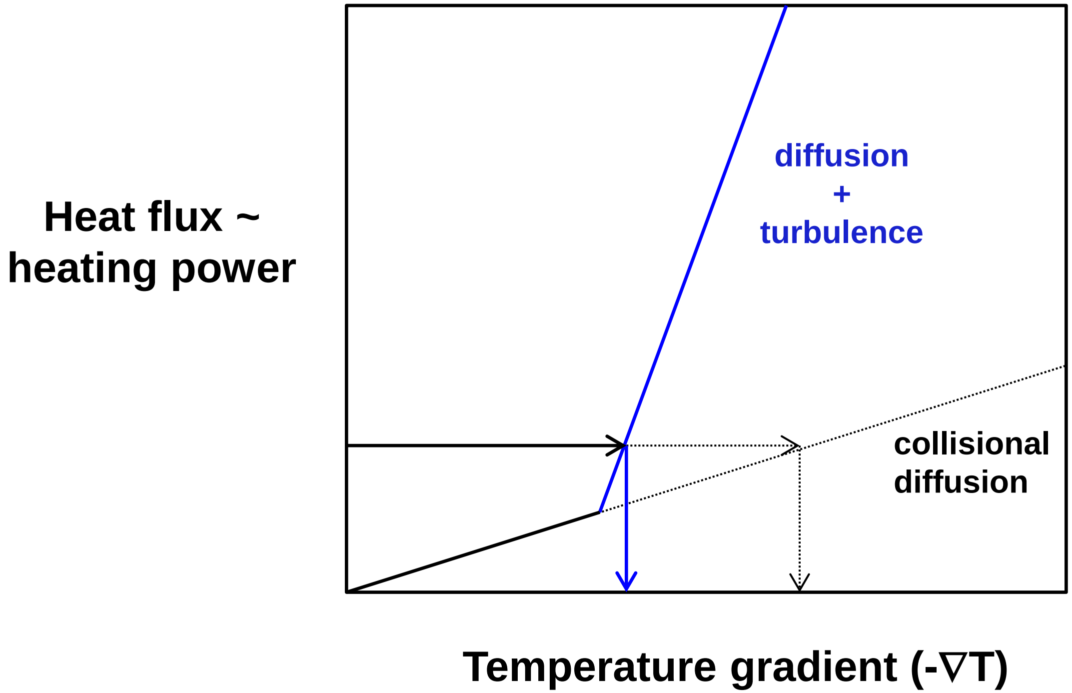
Bohm diffusivity $\approx \frac{1}{16}\frac{T_e}{B}$

$$\tau_E \sim \frac{a^2}{\chi} \sim \frac{R^3 B^2}{T^{3/2}}$$

$\tau_E \sim$ (0.1) sec for current devices
$\tau_E \sim$ (1+) sec for fusion gain (ITER)

- τ_E improves with field strength (B) and machine size (R)

Tokamak turbulence has a threshold gradient for onset, transport tied to linear stability and nonlinear saturation



$$q_{turb} = -n\chi_{GB}[\nabla T - \nabla T_{crit}]F(\cdots)$$

$$q_{col} = -n\chi_{col}\nabla T$$

- GyroBohm scaling important, but liner threshold and scaling also matters

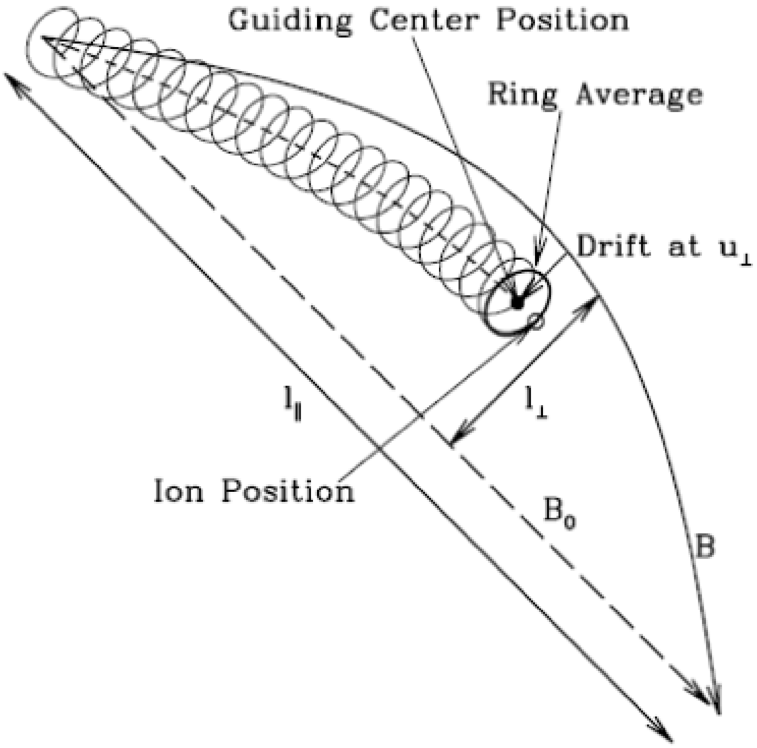
$\Rightarrow$ We must discuss linear drift wave and micro-stability in tokamaks as part of the turbulent transport problem (enter gyrokinetic theory)

Gyrokinetics in brief – evolving 5D gyro-averaged distribution function

$$\boxed{\frac{\omega}{\Omega} << 1}$$

$\sim \frac{10^5}{10^7}$ ***for ions***

$$f(\vec{x}, \vec{v}, t) \xrightarrow{\text{gyroaverage}} f(\vec{R}, v_\parallel, v_\perp, t)$$



- Average over fast gyro-motion → evolve a distribution of gyro-rings

(for each species)

Howes et al., Astro. J. (2006)

Gyrokinetics in brief – evolving 5D gyro-averaged distribution function

$$\frac{\omega}{\Omega}, \frac{\rho}{L}, \frac{\delta f}{f_0}, \frac{k_{||}}{k_\perp} << 1 \qquad f(\vec{x}, \vec{v}, t) \xrightarrow{\text{gyroaverage}} f(\vec{R}, v_\parallel, v_\perp, t) \qquad f = F_M + \delta f$$

$$\frac{\partial(\delta f)}{\partial t} + \underbrace{v_\parallel \hat{b}\cdot\nabla\delta f}_{\text{Fast parallel motion}} + \underbrace{\vec{v}_d\cdot\nabla\delta f}_{\text{Slow perpendicular toroidal drifts}} + \underbrace{\delta\vec{v}\cdot\nabla F_M}_{\text{Advection across equilibrium gradients } (\nabla T_0,\ \nabla n_0,\ \nabla V_0)} + \underbrace{\vec{v}_{E0}(r)\cdot\nabla\delta f}_{\text{Dopper shift due to sheared equilibrium } E_r(r)} + \underbrace{\delta\vec{v}\cdot\nabla\delta f}_{\text{Perpendicular non-linearity}} = \underbrace{C(\delta f)}_{\text{Collisions}}$$

$$\vec{v}_\kappa = m v_{||}^2 \frac{\hat{b}\times\vec{\kappa}}{qB}$$

$$\vec{v}_{\nabla B} = \frac{m v_\perp^2}{2}\frac{\hat{b}\times\nabla B/B}{qB}$$

$$\delta \mathbf{v}_a \doteq \frac{c}{B}\mathbf{b}\times\nabla\Psi_a$$

$$\Psi_a(\mathbf{R}) \doteq \left\langle \delta\phi(\mathbf{R}+\rho) - \frac{1}{c}(\mathbf{V}_0+\mathbf{v})\cdot\delta\mathbf{A}(\mathbf{R}+\rho)\right\rangle_{\mathbf{R}}$$

- Must also solve gyrokinetic Maxwell equations self-consistently to obtain $\delta\varphi$, δB

Drift waves

40+ years of theory predicts turbulence in magnetized plasma should often be drift wave in nature

<u>General predicted drift wave characteristics:</u>

- Finite-frequency drifting waves, $\omega(k_\theta)\sim\omega_*\sim k_\theta V_*\sim(k_\theta\rho)v_T/L_n$
  - Driven by ∇n, ∇T ($1/L_n = -1/n\cdot\nabla n$)

- Quasi-2D, elongated along the field lines ($L_{||}>>L_\perp$, $k_{||} << k_\perp$)
  - Particles can rapidly move along field lines to smooth out perturbations
  - Perpendicular sizes linked to local gyroradius, $L_\perp\sim\rho_{i,e}$ or $k_\perp\rho_{i,e}\sim1$

- In a tokamak expected to be "ballooning", i.e. stronger on outboard side
  - Due to "bad curvature"/"effective gravity" pointing outwards from symmetry axis

- Transport has gyrobohm scaling, $\chi_{GB}=\rho_i^2 v_{Ti}/R$
  - But other factors important like threshold and stiffness: $\chi_{turb} \sim \chi_{GB}\cdot F(\cdots)\cdot[R/L_T-R/L_{T,crit}]$

Can identify key terms in “gyrofluid” equations responsible for drift wave dynamics

- Start with toroidal GK equation in the δf limit ($\delta f/F_M << 1$)
- Take fluid moments ($\int d^3v\, \delta f\, [1, v, \frac{1}{2}v^2]$)
- Apply clever closures that “best” reproduce linear toroidal gyrokinetics (Hammett, Perkins, Beer, Dorland, Waltz, …), e.g.:

ion continuity and energy (M. Beer thesis, 1995):

$$\frac{\partial \tilde{n}}{\partial t} + \mathbf{v}_E \cdot \nabla n_0 + \mathbf{v}_E \cdot \nabla \tilde{n} + n_0 \nabla \cdot \mathbf{v}_E + \frac{2}{m\Omega B^2}\mathbf{B} \times \nabla B \cdot \nabla \tilde{p} = 0. \tag{1.5}$$

$$\frac{\partial \tilde{p}}{\partial t} + \mathbf{v}_E \cdot \nabla p_0 + \mathbf{v}_E \cdot \nabla \tilde{p} + p_0 \nabla \cdot \mathbf{v}_E + p_0 B \mathbf{v}_E \cdot \nabla \frac{1}{B} + \frac{p_0}{n_0 m \Omega B^2}\mathbf{B} \times \nabla B \cdot \nabla (6\tilde{p} - 3\tilde{n} T_0) = 0. \tag{1.12}$$

- **Perturbed E×B drift + background gradients ($\delta v_E \cdot \nabla n_0$, $\delta v_E \cdot \nabla T_0$) are fundamental to drift wave dynamics**

Simple classic electron drift wave in a magnetic slab ($B=B_Z$)

- Assume cool ions ($v_{Ti} << \omega/k_{||}$), isothermal electrons, no temperature gradients, no toroidicity, electrostatic ($\beta \rightarrow 0$), no nonlinear term

$$\frac{\partial}{\partial t}\delta \mathbf{n_i} + \delta \mathbf{v_E} \cdot \nabla \mathbf{n_0}(\mathbf{x}) = \mathbf{0} \qquad \textit{ion continuity}$$

$$\delta v_E = \frac{\hat{b} \times \nabla \delta\phi}{B} = \frac{-ik_y \delta\phi}{B}\widehat{e_x}$$

$$\delta\phi \sim \exp\left(i\vec{k} \cdot \vec{x} - i\omega t\right)$$

Gradient scale length (L_n)

$$\frac{dn_0}{dx} = -\frac{n_0}{L_n}$$

$$\delta v_E \cdot \nabla n_0(x) = \frac{-ik_y \delta\phi}{B}\frac{dn_0}{dx} = in_0\frac{k_y \delta\phi}{BL_n}$$

$$\delta v_E \cdot \nabla n_0(x) = in_0 k_y \frac{T_e}{BL_n}\frac{\delta\phi}{T_e}$$

With some algebra we obtain a diamagnetic drift velocity & frequency

$$\delta v_E \cdot \nabla n_0(x) = i n_0 k_y \frac{T_e}{B L_n} \frac{\delta\phi}{T_e}$$

$$\frac{T_e}{B} = \rho_s c_s$$

$$\delta v_E \cdot \nabla n_0(x) = i n_0 k_y \frac{\rho_s}{L_n} c_s \frac{\delta\phi}{T_e} = i n_0 \omega_{*e} \frac{\delta\phi}{T_e}$$

$$\omega_{*e} = k_y V_{*e} \qquad V_{*e} = \frac{\rho_s}{L_n} c_s$$

ρ_* like parameter

Electron diamagnetic drift velocity & frequency (a fluid drift, not a particle drift)

Simplified ion continuity equation

$$\frac{\partial}{\partial t}\delta n_i + \delta v_E \cdot \nabla n_0(x) = 0$$

$$-i\omega\frac{\delta n_i}{n_0} + i\omega_{*e}\frac{\delta\phi}{T_e} = 0$$

- Expect characteristic frequency ~ ω_{*e} ~ $(k_y\rho_s)\cdot c_s/L_n$

Dynamics Must Satisfy Quasi-neutrality; Rapid parallel electron motion gives Boltzmann distribution

- Quasi-neutrality (Poisson equation, $k_\perp^2\lambda_D^2 << 1$) requires

$$-\nabla^2\tilde{\varphi} = \frac{1}{\varepsilon_0}\sum_s eZ_s \int d^3v f_s$$

$$\left(k_\perp^2\lambda_D^2\right)\frac{\tilde{\varphi}}{T} = \frac{\tilde{n}_i - \tilde{n}_e}{n_0}$$

$$\boxed{\tilde{n}_i = \tilde{n}_e}$$

- For characteristic drift wave frequency, parallel electron motion is very rapid -- from parallel electron momentum eq, assuming isothermal T_e:

$$\omega < k_{||}v_{Te} \ \rightarrow \ 0 = -T_e\nabla_{||}\tilde{n}_e + n_e e\nabla_{||}\tilde{\phi}$$

$\Rightarrow$ Electrons (approximately) maintain a Boltzmann distribution

$$\left(n_0 + \tilde{n}_e\right) = n_0 \exp\left(e\tilde{\varphi}/T_e\right)$$

$$\tilde{n}_e \approx n_0 e\tilde{\varphi}/T_e \Rightarrow \underline{\tilde{n}_e \approx \tilde{\varphi}}$$

Ion continuity + quasi-neutrality + Boltzmann electron = electron drift wave (linear, slab, cold ions)

$$-i\omega\frac{\delta n_i}{n_0}+i\omega_{*e}\frac{\delta\phi}{T_e}=0$$

$$\frac{\delta n_i}{n_0}=\frac{\delta n_e}{n_0}=\frac{\delta\phi}{T_e}$$

$$\boxed{\omega=\omega_{*e}=k_y V_{*e}}$$

- Density and potential wave perturbations propagating perpendicular to B_Z and ∇n_0
  - $\delta v_E \cdot \nabla n_0$ gives δn 90° out-of-phase with initial δn perturbation
- Simple linear dispersion relation (will change with polarization drift / finite Larmor radius effects, toroidicity, other gradients)
- **No mechanism to drive instability (collisions, temperature gradient, toroidicity / trapped particles, …)**

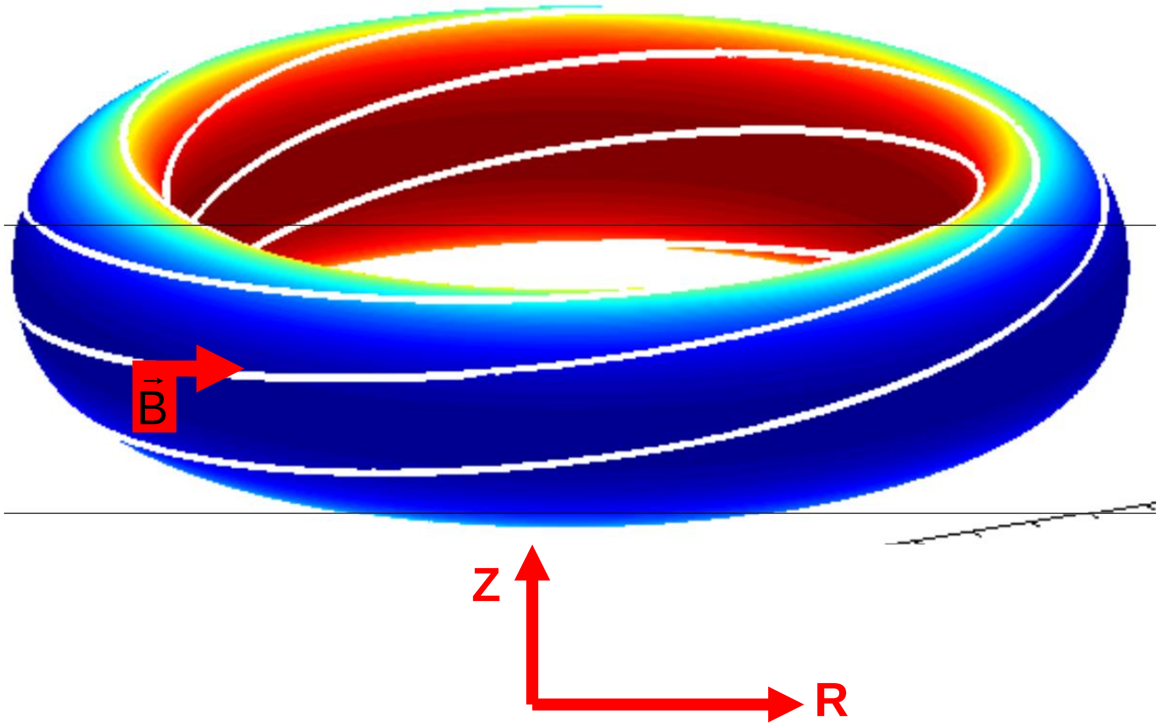
Linear stability analysis of toroidal Ion Temperature Gradient (ITG) micro-instability (expected to dominate in ITER)

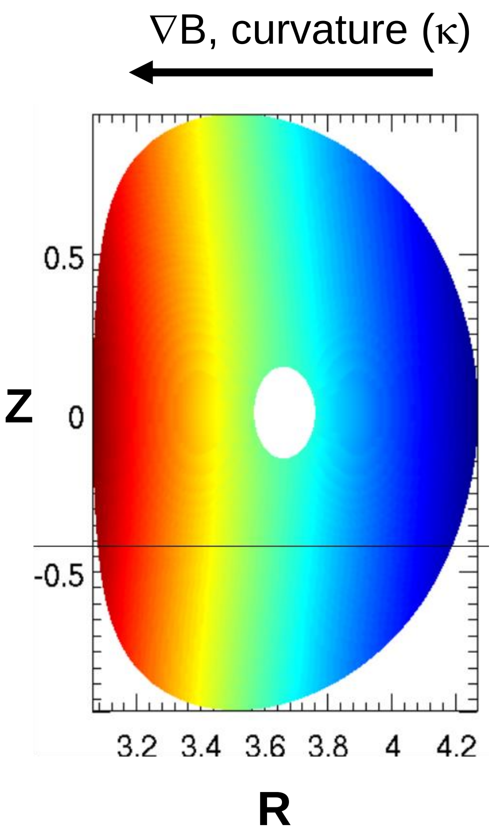
Toroidicity Leads To Inhomogeneity in |B|, gives ∇B and curvature (κ) drifts

$$\vec{v}_\kappa = m v_\parallel^2 \frac{\hat{b}\times\vec{\kappa}}{qB} \sim T_\parallel$$

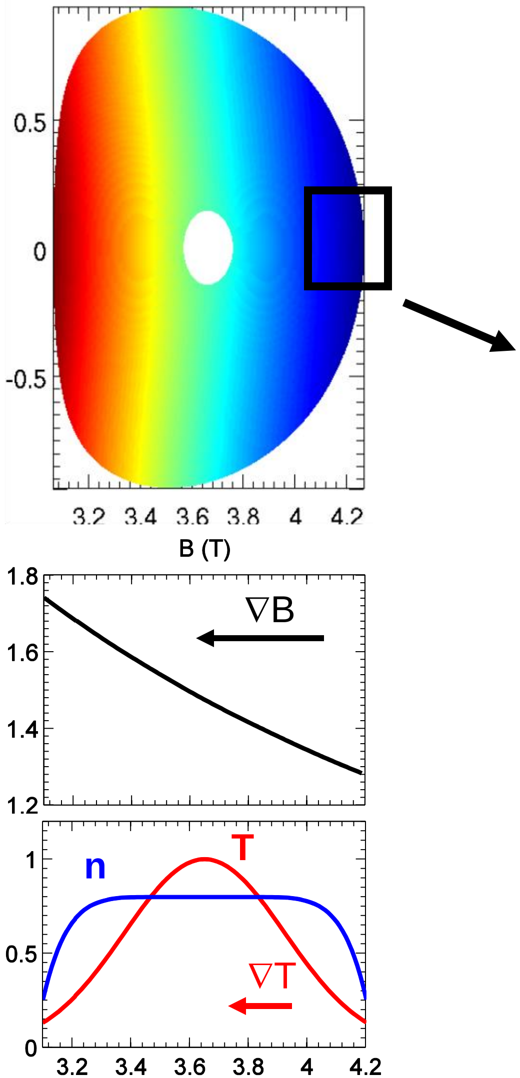
$$\vec{v}_{\nabla B} = \frac{m v_\perp^2}{2}\frac{\hat{b}\times\nabla B/B}{qB} \sim T_\perp$$

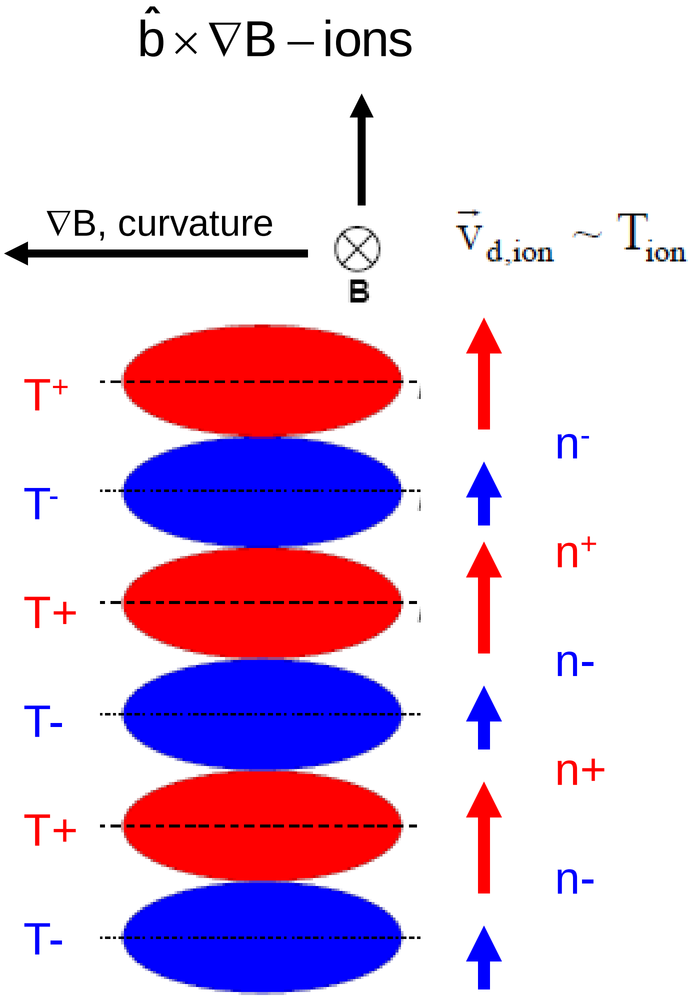
- What happens when there are small perturbations in $T_\parallel$, $T_\perp$? $\Rightarrow$ Linear stability analysis…





Temperature perturbation (δT) leads to compression ($\nabla \cdot v_{di}$), density perturbation – 90° out-of-phase with δT





- Fourier decompose perturbations in space ($k_\theta \rho_i \leq 1$)
- Assume small δT perturbation

Dynamics Must Satisfy Quasi-neutrality; Rapid parallel electron motion gives Boltzmann distribution

- Quasi-neutrality (Poisson equation, $k_\perp^2\lambda_D^2 \ll 1$) requires

$$-\nabla^2\tilde{\varphi} = \frac{1}{\varepsilon_0}\sum_s eZ_s \int d^3v f_s$$

$$\left(k_\perp^2\lambda_D^2\right)\frac{\tilde{\varphi}}{T} = \frac{\tilde{n}_i - \tilde{n}_e}{n_0}$$

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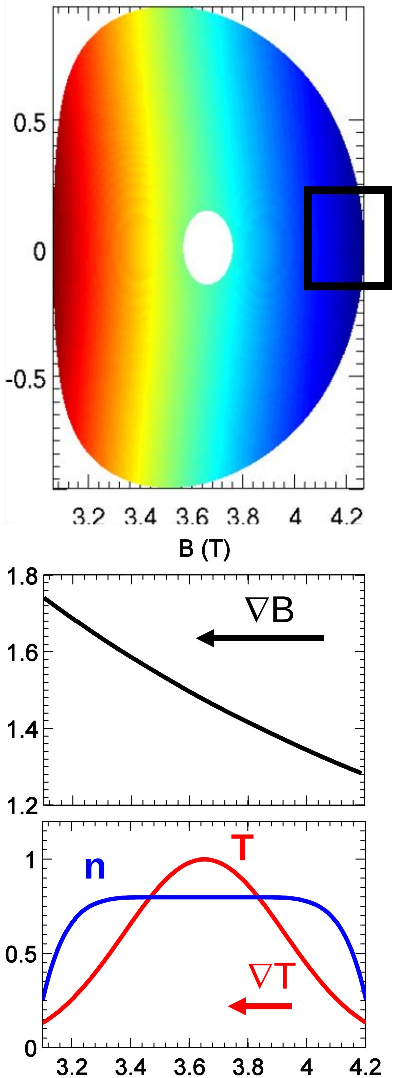
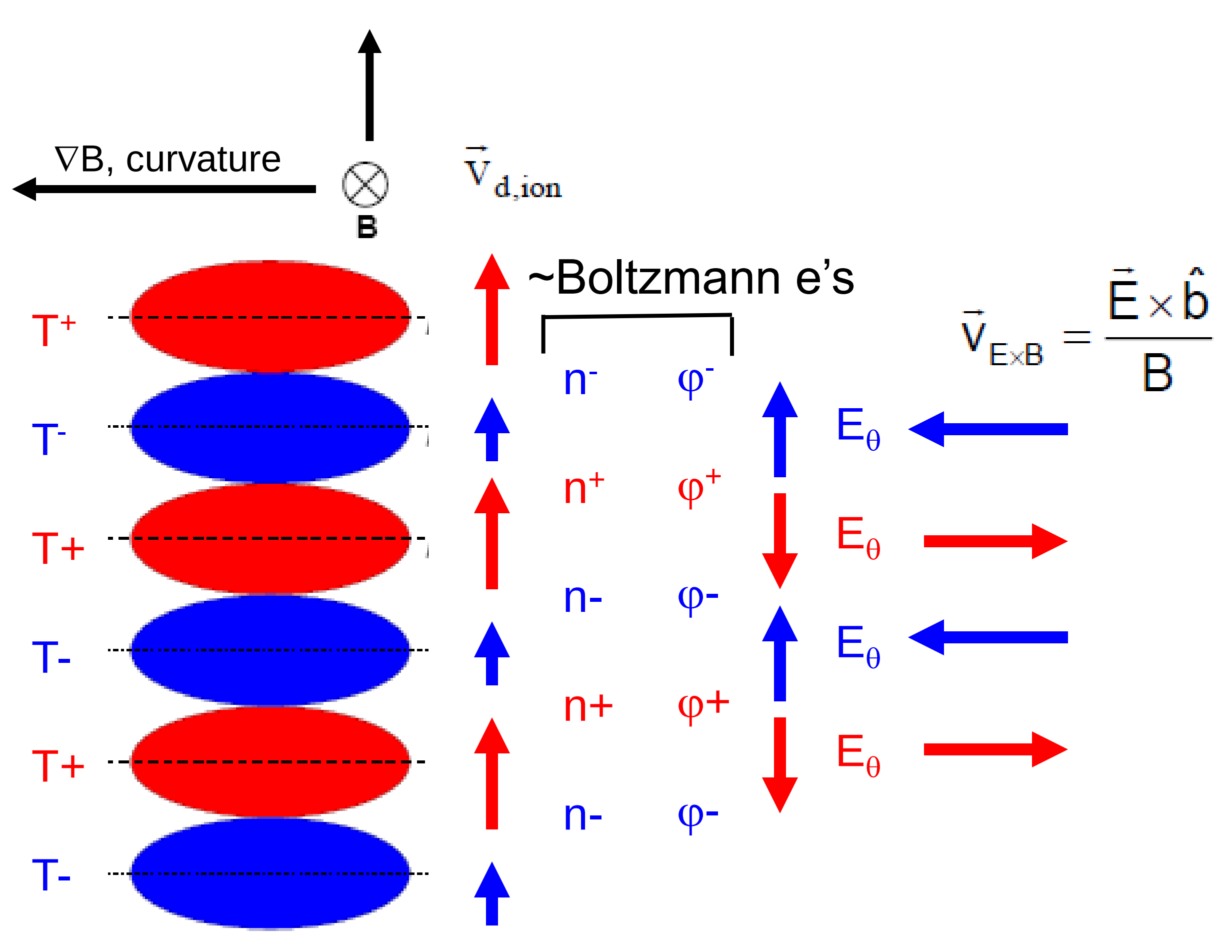
$$\left(n_0 + \tilde{n}_e\right) = n_0 \exp\left(e\tilde{\varphi}/T_e\right)$$

$$\tilde{n}_e \approx n_0 e\tilde{\varphi}/T_e \Rightarrow \underline{\tilde{n}_e \approx \tilde{\varphi}}$$

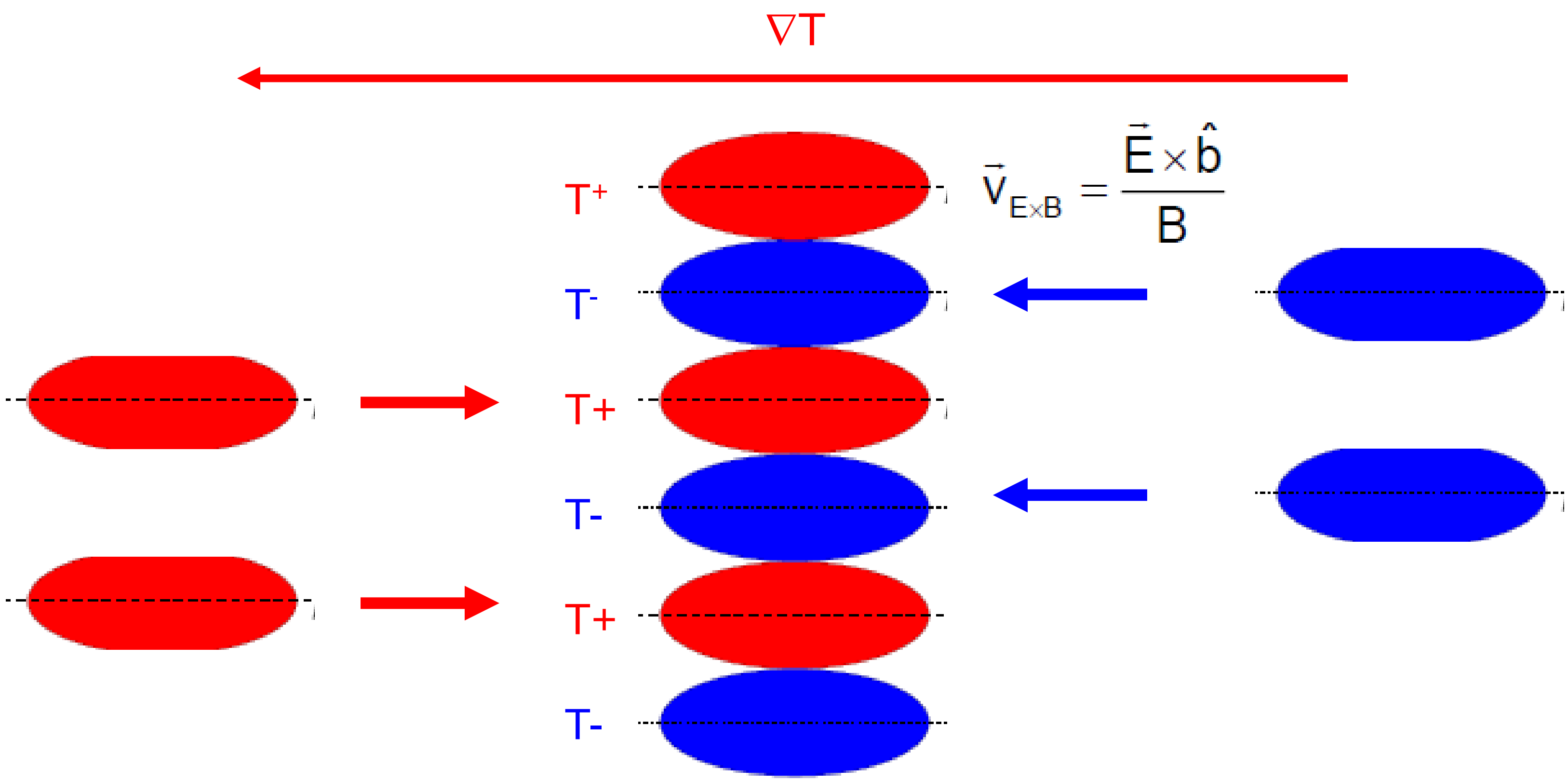
Perturbed Potential Creates E×B Advection

- Advection occurs in the radial direction





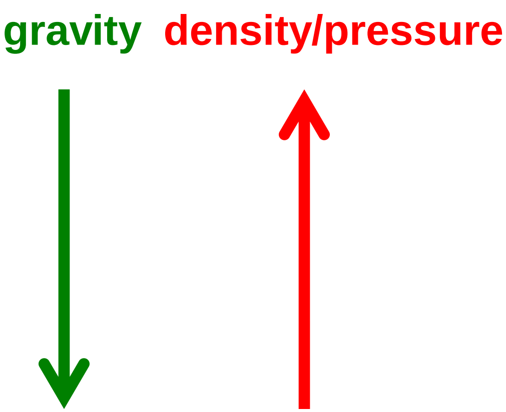
Background Temperature Gradient Reinforces Perturbation ⇒ Instability



This simple cartoon gives a purely growing "interchange" like mode (coarse derivation in backup slides). The complete derivation (all drifts, gradients) will give a real frequency dispersion, i.e. $\omega_r=\omega_r(k_\theta)$

Analogy for turbulence in tokamaks – Rayleigh-Taylor instability

- Higher density on top of lower density, with gravity acting downwards



Inertial force in toroidal field acts like an effective gravity

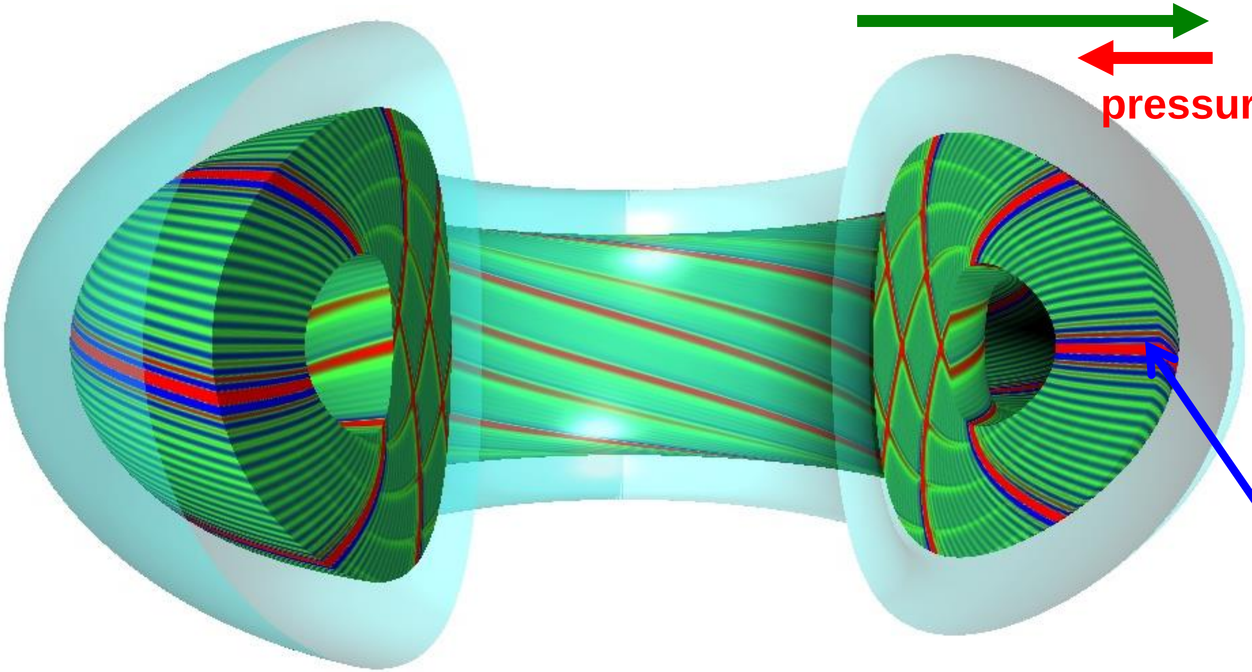
centrifugal force

→effective gravity

pressure

gravity

pressure

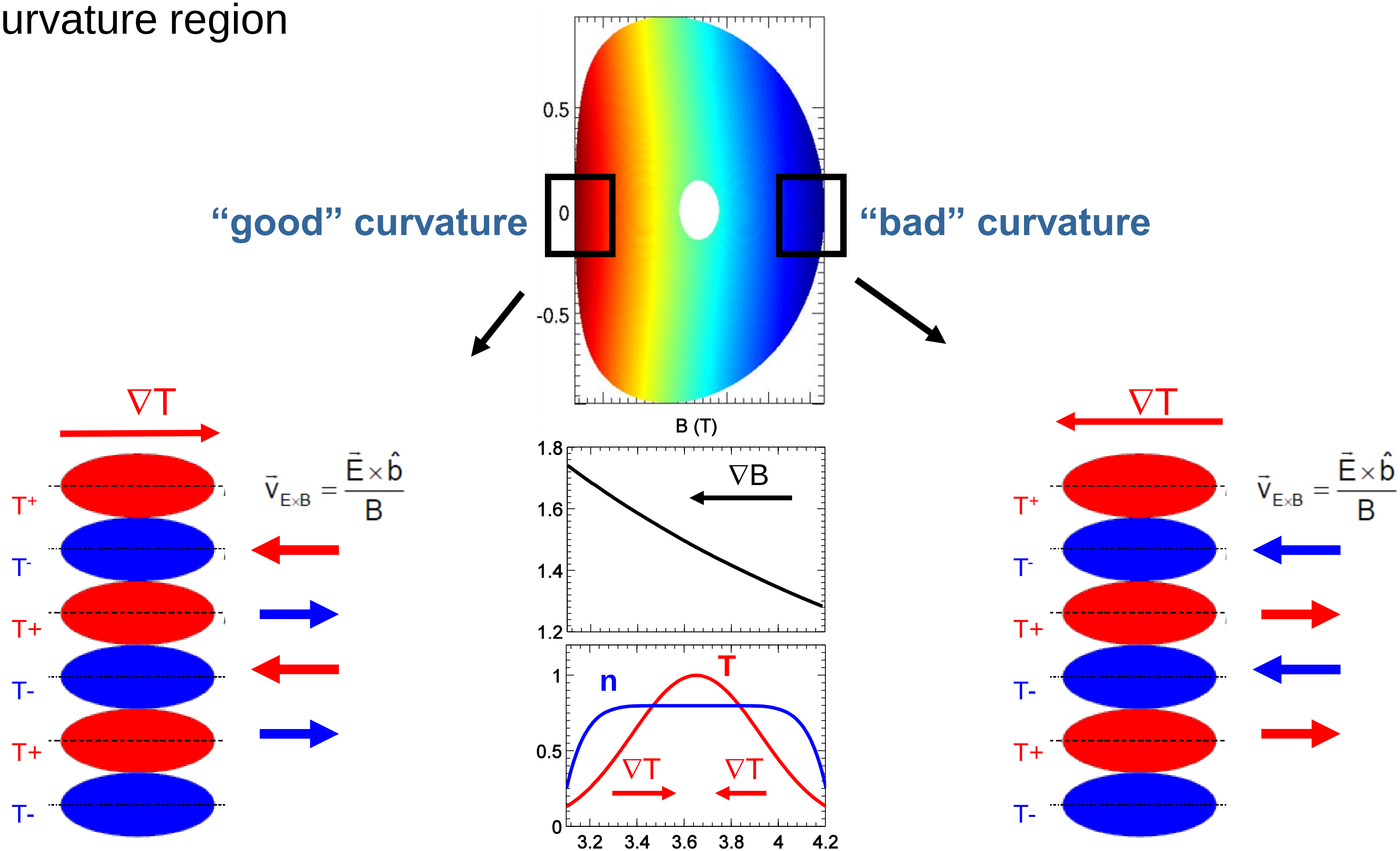


Unstable in the outer region

GYRO code
https://fusion.gat.com/theory/Gyro

Same Dynamics Occur On Inboard Side But Now Temperature Gradient Is Stabilizing

- Advection with ∇T counteracts perturbations on inboard side – “good” curvature region



Fast Parallel Motion Along Helical Field Line Connects Good & Bad Curvature Regions

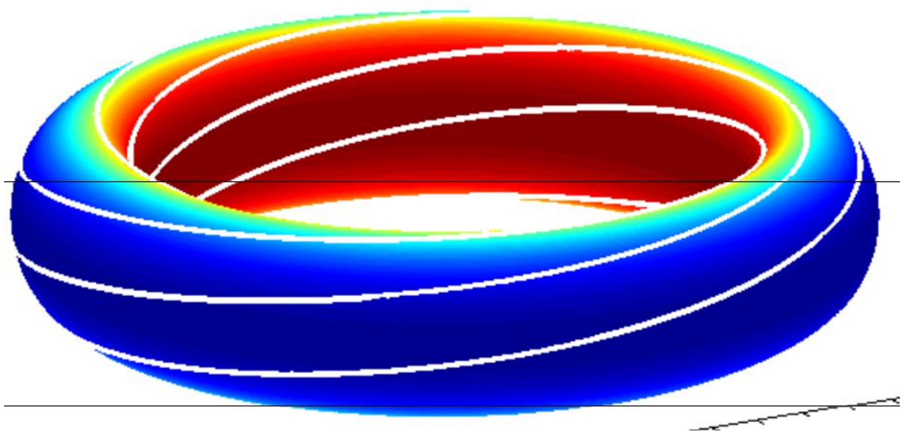
- Approximate growth rate on outboard side

  effective gravity: $g_{eff} = v_{th}^2/R$

  gradient scale length: $1/L_T = -1/T \cdot \nabla T$

$$\gamma_{instability} \sim \left(\frac{g_{eff}}{L}\right)^{1/2} \sim \frac{v_{th}}{\sqrt{RL_T}}$$

- Parallel transit time along helical field line with “safety factor” q



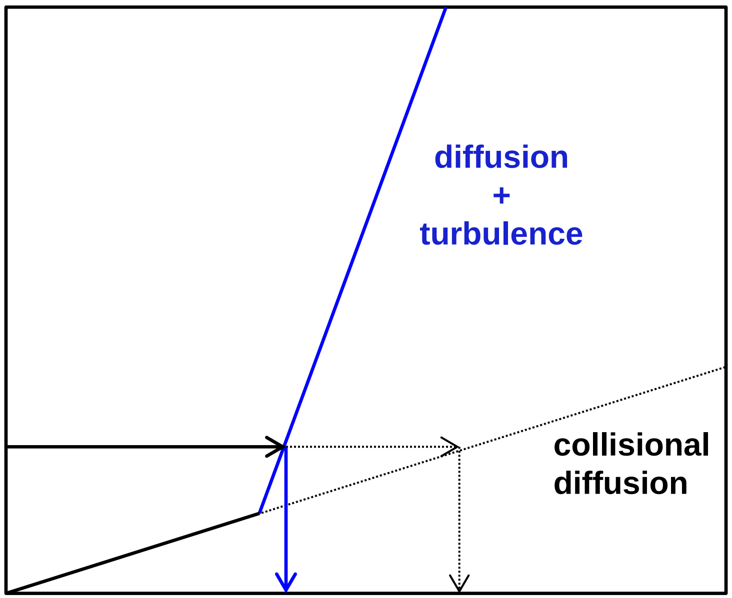
$$q = \frac{\#\ \text{toroidal transits}}{\#\ \text{poloidal transits}}$$

$$\gamma_{parallel} \sim \frac{v_{th}}{qR}$$

- Expect instability if $\gamma_{instability} > \gamma_{parallel}$, or $\left(\frac{R}{L_T}\right)_{threshold} \approx \frac{1}{q^2}$

Threshold-like behavior analogous to Rayleigh-Benard instability

Heat flux ~ heating power



Temperature gradient
$(T_{hot} - T_{cold})$

Analogous to convective transport when heating a fluid from below … boiling water (before the boiling)

Rayleigh, Benard, early 1900's

Critical gradient for ITG determined from theory + linear gyrokinetic simulations

$$\left(\frac{R}{L_T}\right)_{crit}^{ITG} = \mathbf{Max}\left[\left(\mathbf{1}+\frac{\mathbf{T_i}}{\mathbf{T_e}}\right)\left(\mathbf{1.3}+\mathbf{1.9}\frac{\mathbf{s}}{\mathbf{q}}\right)(\ldots)\right]$$

Jenko (2001)
Hahm (1989)
Romanelli (1989)

- $R/L_T = -R/T \cdot \nabla T$ is the normalized temperature gradient
- Natural way to normalize gradients for toroidal drift waves, i.e. ratio of diamagnetic-to-toroidal drift frequencies:

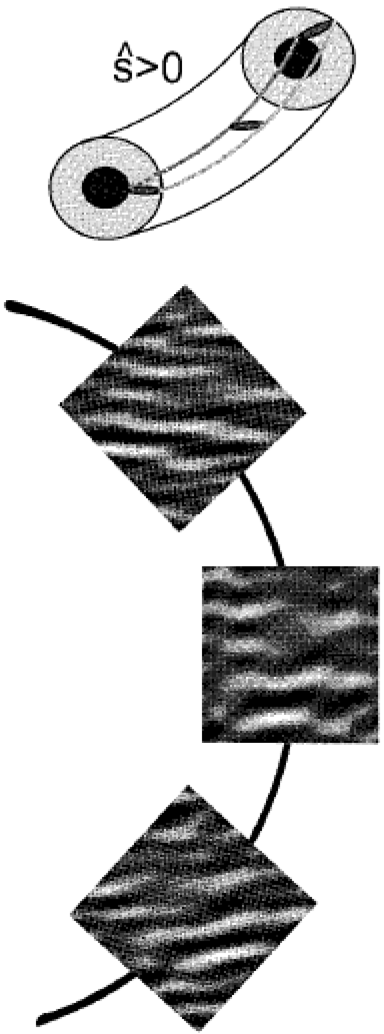
$\omega_{*T} = k_y(B\times\nabla p) / nqB^2 \rightarrow (k_\theta\rho_i)v_T/L_T$

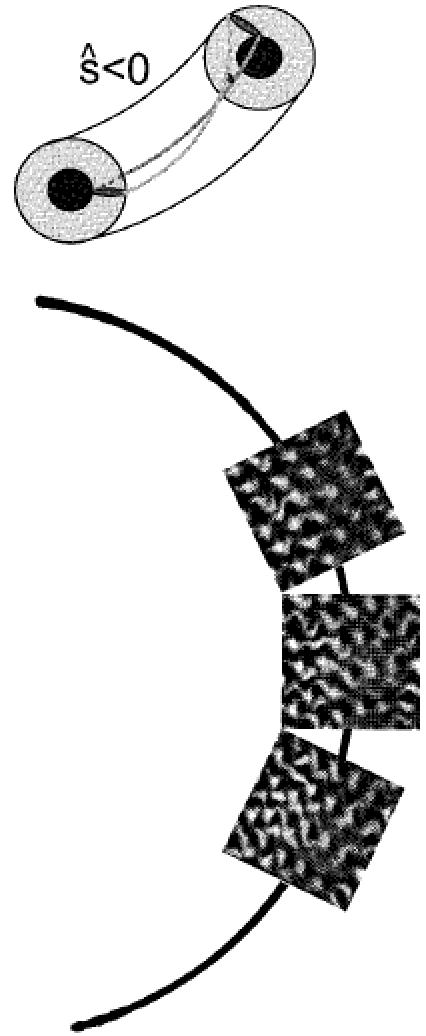
$\omega_D = k_y(B\times m v_\perp^2 \nabla B/2B) / qB^2 \rightarrow (k_\theta\rho_i)v_T/R$

$\rightarrow \omega_{*T}/\omega_D = R/L_T$

With physical understanding, can try to manipulate/optimize microstability

- E.g., magnetic shear influences stability by twisting radially-elongated instability to better align (or misalign) with bad curvature drive





Antoneson

How does magnetized turbulence saturate?

What sets spatial scales (drive vs. dissipation)?

Spectrum shape / distribution governed by nonlinear (2D perpendicular) three-wave interactions

- Linearly unstable modes grow: $\delta\phi(k) \sim \exp\,[ik \cdot x + i\omega(k)t + \gamma(k)t]$

- At large amplitude, interact via nonlinear advection, $\boldsymbol{\delta v_E \cdot \nabla \delta f}$
  i.e. "three-wave" coupling in (2D perpendicular) wavenumber space

$$\frac{\partial}{\partial t}\delta f \sim \delta v_E \cdot \nabla \delta f$$

$$\frac{\partial}{\partial t}\delta f_{k_{\perp 3}} \sim \sum_{\substack{k_{\perp 1}, k_{\perp 2} \\ k_{\perp 3} = k_{\perp 1} + k_{\perp 2}}} \left(b \times k_{\perp 1}\delta\phi_{k_{\perp 1}}\right) \cdot k_{\perp 2}\delta f_{k_{\perp 2}}$$

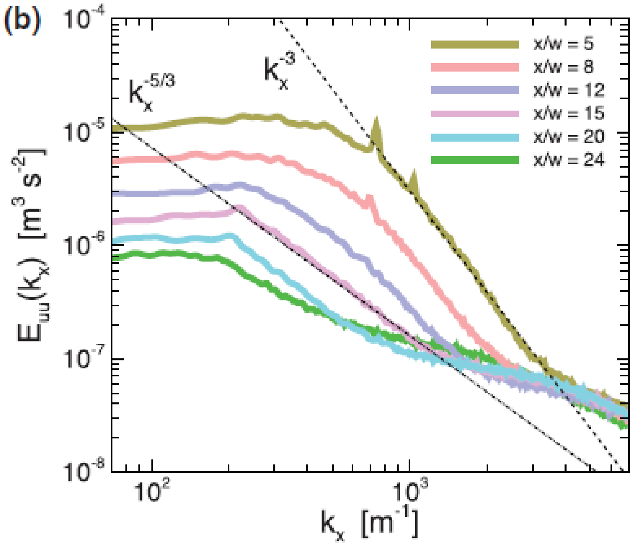
- Energy gets distributed across k space (& velocity space) until damped by stable modes (& collisions) → saturation
  - Local (in k) 2D cascades
  - Non-local (in k) interactions drive "zonal flows" that also mediate turbulence

Energy cascade in 2D turbulence is different than 3D

- Change in non-linear conservation properties → energy and vorticity is conserved
  - ***Inverse*** energy cascade $E(k) \sim k^{-5/3}$
  - Forward enstrophy $[\omega^2 \sim (\nabla \times v)^2]$ cascade $E(k) \sim k^{-3}$
  - Non-local wavenumber interactions can couple over larger range in k-space (e.g. to zonal flows)

<u>Quasi-2D turbulence exists in many places</u>

- Geophysical flows like ocean currents, tropical cyclones, polar vortex, chemical mixing in polar stratosphere (→ ozone hole)
- Soap films →

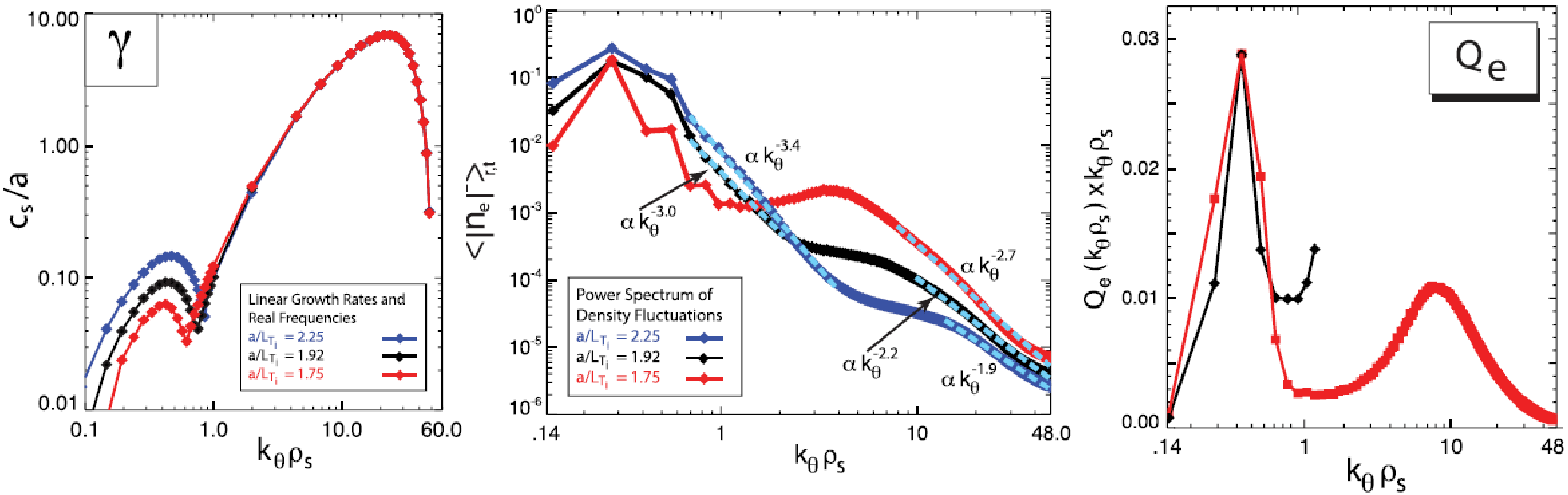


Liu et al., PRL (2016)

Energy drive can occur across large range of scales, but turbulent spectra still exhibit decay

- 2D energy & enstrophy cascades remain important → nonlinear spectra often downshifted in k_θ (w.r.t. linear growth rates)
- Both drive and damping can overlap over wide range of $k_\perp$ (very distinct from neutral fluid turbulence)

Linear growth rates | *Nonlinear density power spectra* | *Nonlinear heat flux spectra*



Howard, PoP & NF (2016)

Additional effects proposed to model turbulence saturation & dissipation

- Coupling to damped eigenmodes (that exist at all $k_\perp$ scales, with different cross-phases) can influence spectral saturation and partitioning of transport, e.g. Q_e vs. Q_i vs. Γ, … (Terry, Hatch)

- Different routes to dissipation have been addressed theoretically:
- Critical balance (Goldreich-Sridhar, Schekochihin, M. Barnes): balance nonlinear $\perp$ dynamics with linear || dynamics
  - 2D perpendicular nonlinearity at different parallel locations creates fine parallel structure ($k_{||}\uparrow$) → through Landau damping generates fine $v_{||}$ structure → dissipation through collisions
  - Can happen at all $k_\perp$ scales
  - Simple scaling argument reproduces transport scaling
- Nonlinear phase mixing (Hammett, Dorland, Schekochihin, Tatsuno):
  - At sufficient amplitude, gyroaveraged nonlinear term $\langle\delta \mathrm{v_E}\rangle\cdot\nabla\delta \mathrm{f}\sim \mathrm{J_0}\left(\frac{\mathrm{k_\perp v_\perp}}{\Omega}\right)\delta\mathrm{v_E}\cdot\nabla\delta\mathrm{f}$ generates structure in $\mu\sim v_\perp^2$ → dissipation through collisions

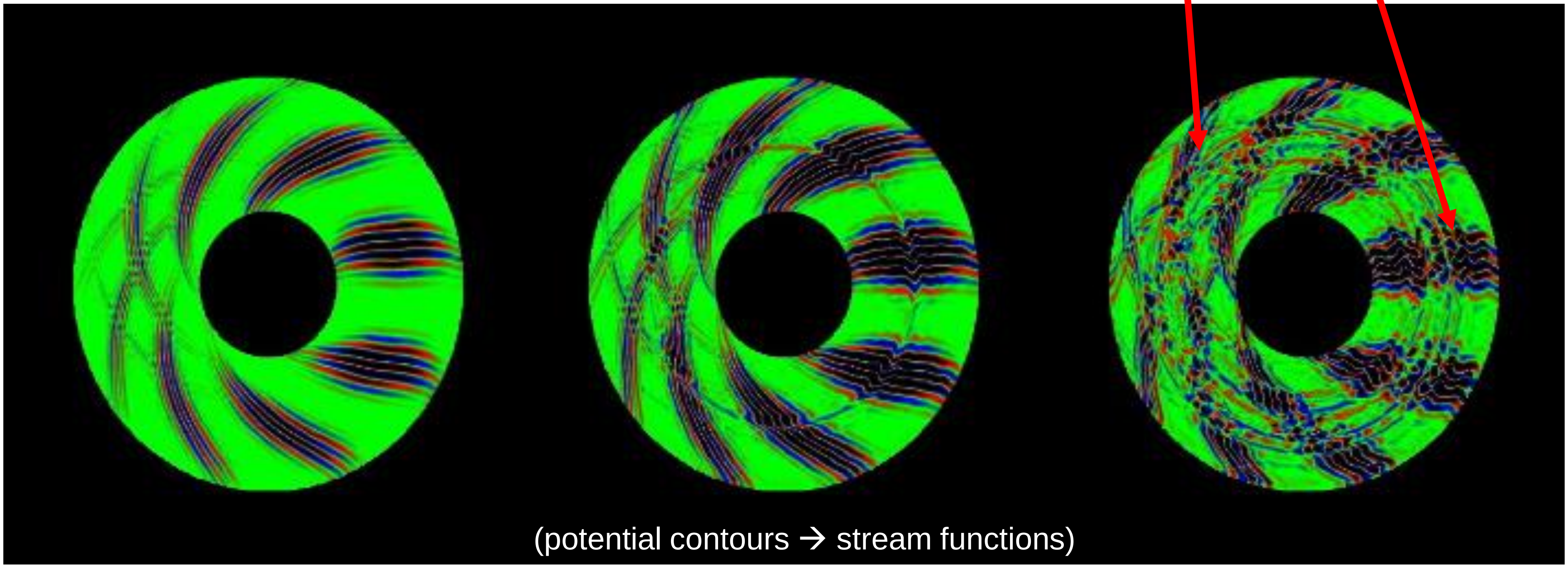
Nonlinearly-generated "zonal flows" also impact saturation

- Potential perturbations uniform on flux surfaces (k_y=0) → marginally stable, do not cause transport
- Turbulence can condense to system size → ZF driven largely by non-local (in k) NL interactions ($k \gg k_{ZF}$)

Linear instability stage demonstrates structure of fastest growing modes

Large flow shear from instability cause perpendicular "zonal flows"

Zonal flows help moderate the turbulence



(potential contours → stream functions)

Rayleigh-Taylor like instability driving Kelvin-Helmholtz-like instability

Code: **GYRO**

Authors: Jeff Candy and Ron Waltz

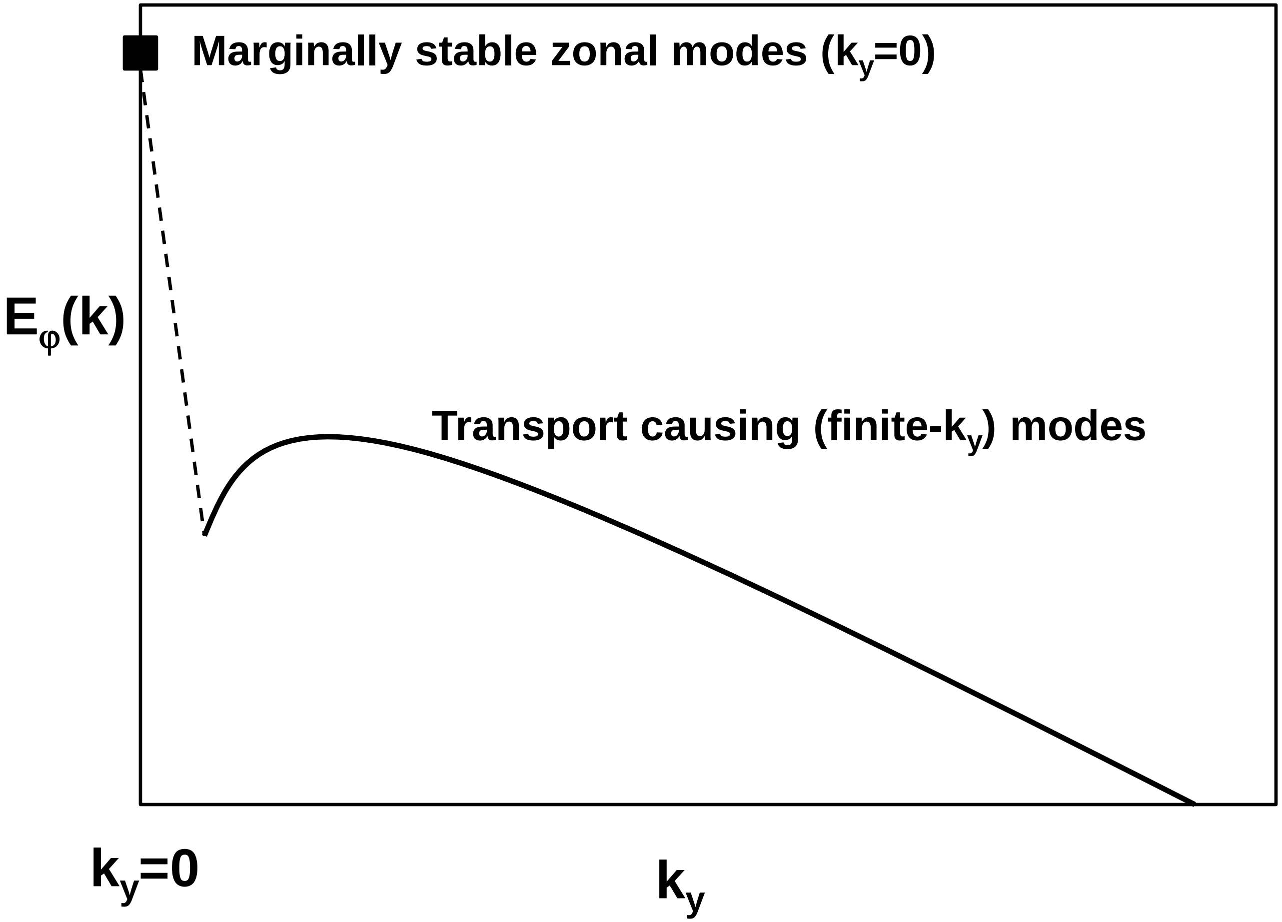
Generation of zonal flows in tokamaks similar to "Kelvin-Helmotz" instability found throughout nature

Variation of flows in one direction…

leads to instability, flows in another direction

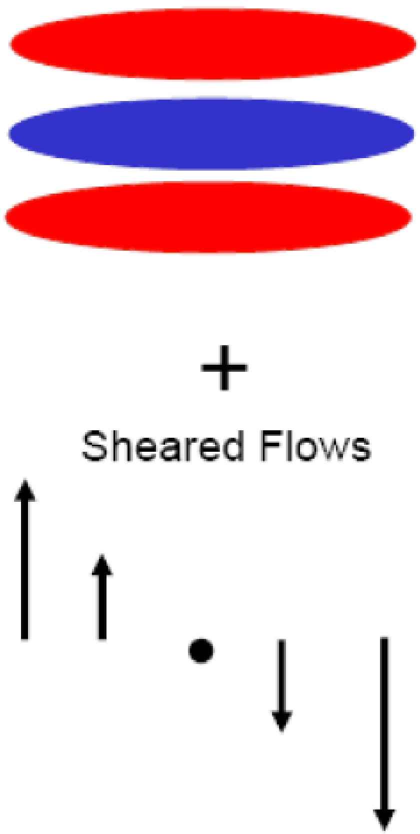
Zonal flows can saturate at relatively large amplitude for toroidal ITG turbulence

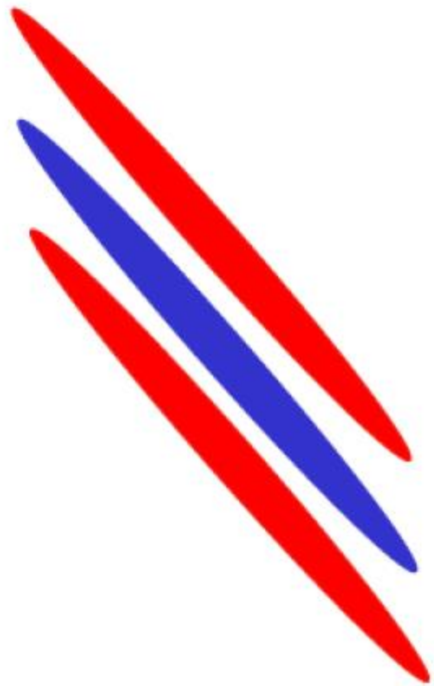
- Regulates saturation via (i) shear decorrelation of eddies, (ii) energy sink into marginal (non-transport-causing) modes
- Typically have distinct k_x spectra (overall 2D spectra anisotropic in k_x,k_y)



Large scale equilibrium sheared flows also influence saturation

- Large scale background flow shear distorts eddies → reduces radial correlation length, fluctuation strength, cross-phases and transport





Turbulent transport expected to be reduced as the mean flow shear rate (ω_s~dU_0/dy) approaches the turbulence decorrelation rate ($\Delta\omega_D$)

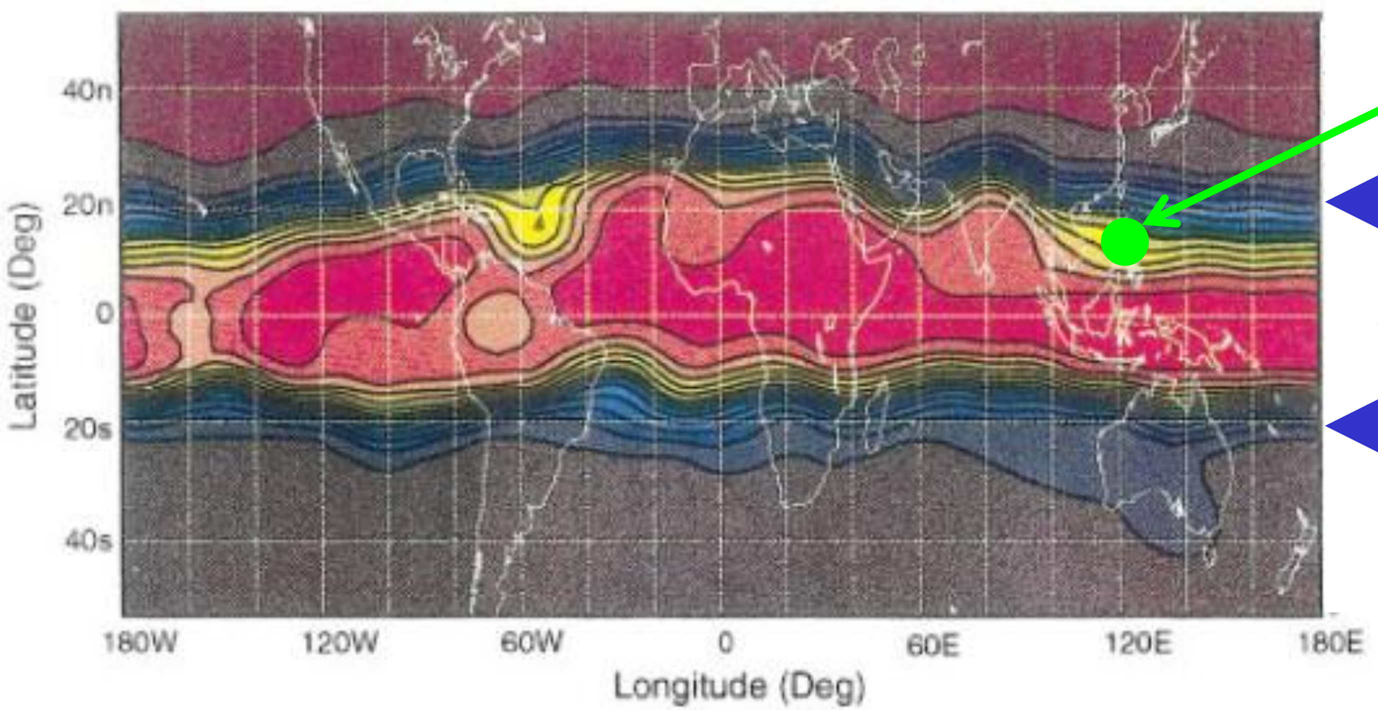
Biglari, Diamond, Terry 1990

In neutral fluids, sheared flows are often a source of free energy to drive turbulence

- Thin (quasi-2D) atmosphere in axisymmetric geometry of rotating planets similar to tokamak plasma turbulence → can also suppress transport

- Stratospheric ash from Mt. Pinatubo eruption (1991) spread rapidly around equator, **but confined in latitude by flow shear**

Aerosol concentration



Large shear in stratospheric equatorial jet

(Trepte, 1993)

Beyond general characteristics, there are many theoretical "flavors" of drift waves possible in tokamak core & edge

- Usually think of drift waves as gradient driven (∇T_i, ∇T_e, ∇n)
  - Often exhibit threshold in one or more of these parameters

- Different theoretical "flavors" exhibit different parametric dependencies, predicted in various limits, depending on gradients, T_e/T_i, ν, β, geometry, location in plasma…
  - Electrostatic, ion scale ($k_\theta \rho_i \leq 1$)
    - Ion temperature gradient (ITG) – driven by ∇T_i, weakened by ∇n
    - Trapped electron mode (TEM) – driven by ∇T_e & ∇n_e, weakened by ν_e
    - Parallel velocity gradient (PVG) – driven by $R\nabla\Omega$ (like Kelvin-Helmholtz)
  - Electrostatic, electron scale ($k_\theta \rho_e \leq 1$)
    - Electron temperature gradient (ETG) - driven by ∇T_e, weakened by ∇n
  - Electromagnetic, ion scale ($k_\theta \rho_i \leq 1$)
    - Kinetic ballooning mode (KBM) - driven by $\nabla \beta_{pol} \sim \alpha_{MHD}$
    - Microtearing mode (MTM) – driven by ∇T_e, at sufficient β_e

Some additional sources & references

- Greg Hammett has a lot of great introductory material to fusion, tokamaks, drift waves, ITG turbulence, gyrokinetics, etc… (*w3.pppl.gov/~hammett*)

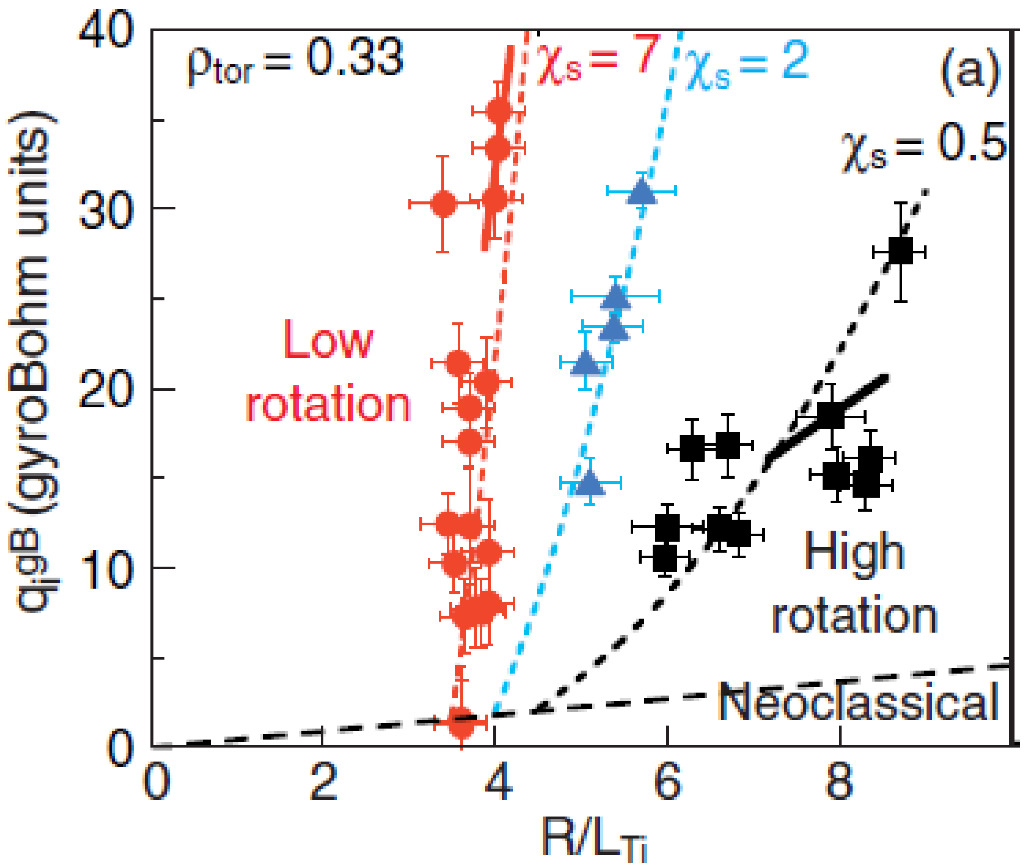
- See the following for broader reviews and thousands of useful references

- Transport & Turbulence reviews:
  - Liewer, Nuclear Fusion (1985)
  - Wootton, Phys. Fluids B (1990)
  - Carreras, IEEE Trans. Plasma Science (1997)
  - Wolf, PPCF (2003)
  - Tynan, PPCF (2009)
  - ITER Physics Basis (IPB), Nuclear Fusion (1999)
  - Progress in ITER Physics Basis (PIPB), Nuclear Fusion (2007)
- Drift wave reviews:
  - Horton, Rev. Modern Physics (1999)
  - Tang, Nuclear Fusion (1978)
- Gyrokinetic simulation review:
  - Garbet, Nuclear Fusion (2010)
- Zonal flow/GAM reviews:
  - Diamond et al., PPCF (2005)
  - Fujisawa, Nuclear Fusion (2009)
- Measurement techniques:
  - Bretz, RSI (1997)

THE END

Threshold-like behavior observed experimentally

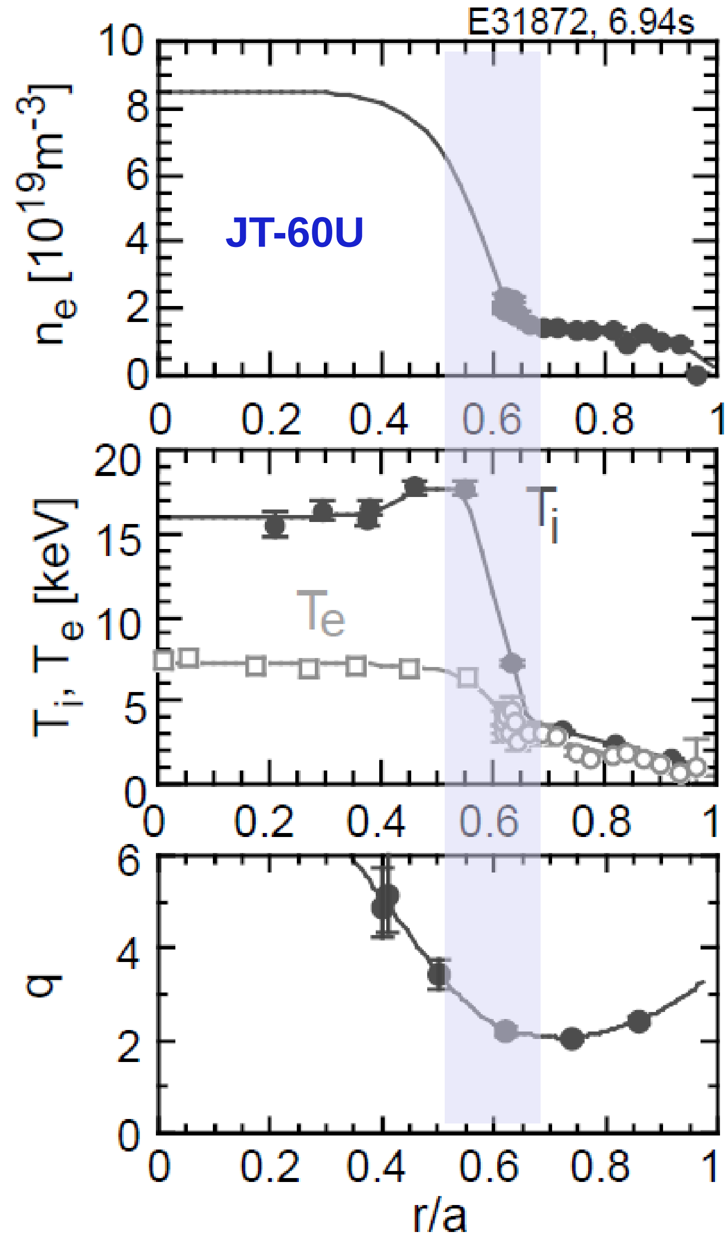
- Experimentally inferred threshold varies with equilibrium, plasma rotation, ...
- Stiffness (~$dQ/d\nabla T$ above threshold) also varies
- $\chi = -Q/n\nabla T$ highly nonlinear (also use perturbative experiments to probe stiffness)



JET
Mantica, PRL (2011)

Reverse magnetic shear can lead to internal transport barriers (ITBs)

- ITBs established on numerous devices
- Used to achieve “equivalent” $Q_{DT,eq}$~1.25 in JT-60U (in D-D plasma)
- $\chi_i \sim \chi_{i,NC}$ in ITB region (complete suppression of ion scale turbulence)



Ishida, NF (1999)

2 slide summary of some turbulent transport concepts in magnetized fusion plasmas (1)

- For fusion gain $Q \sim nT\tau_E$ (& 100% non-inductive tokamak operation) we need excellent energy confinement, τ_E
- Energy confinement depends on turbulence ($\tau_E \sim a^2/\chi_{turb}$)
  - As does particle, impurity & momentum transport
- Core turbulence generally accepted to be drift wave in nature
  - Quasi-2D ($L_\perp \sim \rho_i, \rho_e << L_{||} \sim qR$)
  - Driven by ∇T & ∇n
  - Frequencies ~ diamagnetic drift frequency ($\omega \sim \omega_* \sim k_\theta \rho_i \cdot c_s/L_{n,T}$)
  - Drift wave transport generally follows gyroBohm scaling $\chi_{turb} \sim \chi_{GB} \sim \rho_i^2 v_{Ti}/a$, *however…*
  - Thresholds and stiffness are critical, i.e. $\chi_{turb} \sim \chi_{GB} \cdot F(\ldots) \cdot (\nabla T - \nabla T_{crit})$
- Toroidal ion temperature gradient (ITG) drift wave is a key instability for controlling confinement in current tokamaks
  - Unstable due to interchange-like toroidal drifts, analogous to Rayleigh-Taylor instability
  - Threshold influenced by magnetic equilibrium (q, s) and other parameters
  - Nonlinear saturated transport depends on zonal flows & perpendicular E×B sheared flow

2 slide summary of some turbulent transport concepts in magnetized fusion plasmas (2)

- Reduced models are constructed by quasi-linear calculations + “mixing-length” estimates for nonlinear saturation
  - We rely heavily on direct numerical simulation using gyrokinetic codes to guide model development
  - Reasonably predict confinement scaling and core profiles
- Many other flavors of turbulence exist (TEM, ETG, PVG, MTM, KBM)
  - ρ_i or ρ_e scale
  - Electrostatic or electromagnetic (at increasing beta)
  - Different physical drives, parametric dependencies, & influence on transport channels (Γ vs. Q vs. Π)
- Things get more complicated for edge / boundary turbulence
  - Changing topology (closed flux surfaces → X-point (poloidal field null) → open field lines & sheaths at physical boundary)
  - Larger gyroradius / banana widths, $\rho_{banana}/\Delta_{ped}$~1 → orbit losses & non-local effects
  - Large amplitude fluctuations, $\delta n/n_0$~1 (delta-f → full-F simulations)
  - Neutral particles, radiation, other atomic physics…

Very simple growth rate derivation of previous toroidal ITG cartoon picture

Can identify key terms in “gyrofluid” equations responsible for toroidal ITG instability

- Start with toroidal GK equation in the δf limit ($\delta f/F_M << 1$)
- Take fluid moments
- Apply clever closures that “best” reproduce linear toroidal gyrokinetics (Hammett, Perkins, Beer, Dorland, Waltz, …)

Ion continuity and energy (M. Beer thesis, 1995):

$$\frac{\partial \tilde{n}}{\partial t} + \mathbf{v}_E \cdot \nabla n_0 + \mathbf{v}_E \cdot \nabla \tilde{n} + n_0 \nabla \cdot \mathbf{v}_E + \frac{2}{m\Omega B^2}\mathbf{B} \times \nabla B \cdot \nabla \tilde{p} = 0. \qquad (1.5)$$

$$\frac{\partial \tilde{p}}{\partial t} + \mathbf{v}_E \cdot \nabla p_0 + \mathbf{v}_E \cdot \nabla \tilde{p} + p_0 \nabla \cdot \mathbf{v}_E + p_0 B \mathbf{v}_E \cdot \nabla \frac{1}{B} + \frac{p_0}{n_0 m \Omega B^2}\mathbf{B} \times \nabla B \cdot \nabla (6\tilde{p} - 3\tilde{n}T_0) = 0. \qquad (1.12)$$

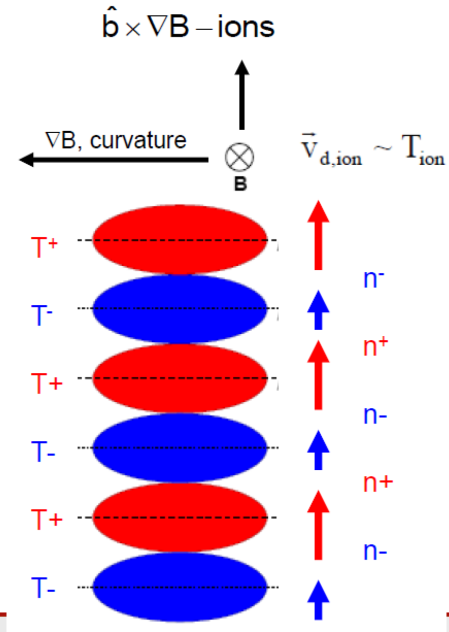
Temperature perturbation (δT) leads to compression ($\nabla\cdot v_{di}$), density perturbation – 90° out-of-phase with δT

$dn/dt + \nabla\cdot(nv)=0$

$\boldsymbol{-i\omega\delta n}$ **from** $\boldsymbol{-n_0\nabla\cdot\delta v_d} \sim -n_0\nabla\cdot(\delta T_\perp\, b\times\nabla B/B)/B \sim -n_0\, ik_y\delta T / BR$

$-i\omega(\delta n/n_0) \sim -ik_y(\delta T/T_0)\ T/BR \sim -i(k_yV_D)\ (\delta T/T_0) \sim -i\omega_D\ (\delta T/T_0)$

$$\boldsymbol{-i(\omega_r+i\gamma)(\delta n/n_0) = -i\omega_D\ (\delta T/T_0)}$$

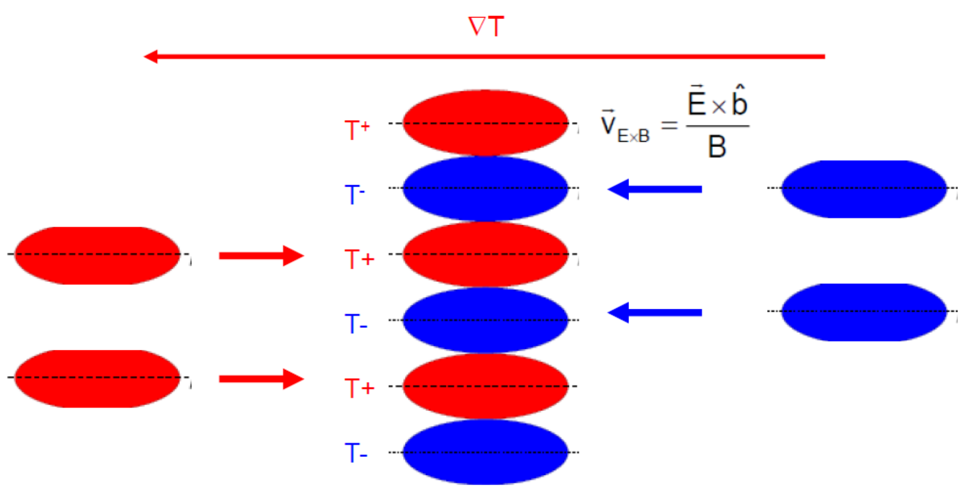


Background Temperature Gradient Reinforces Perturbation $\Rightarrow$ Instability

$-i\omega\delta T$ from $-\delta v_E\cdot\nabla T_0$ $\sim -(b\times\nabla\delta\varphi/B)\nabla T_0 \sim ik_y\delta\varphi/B\cdot\nabla T_0 \sim ik_y\delta\varphi(T/B)/L_T$

$-i\omega(\delta T/T) \sim ik_y(\delta\varphi/T)T/BL_T \sim i(k_yV_{*T})(\delta\varphi/T) \sim i\omega_{*T}(\delta\varphi/T)$

$$\boldsymbol{-i(\omega_r+i\gamma)(\delta T/T) = i\omega_{*T}(\delta\varphi/T)}$$



Simplest dispersion from these 3 terms

(1) Compression from toroidal drifts

$$\omega(\delta n_i/n_0) = \omega_{Di}\,(\delta T_i/T_{i0})$$

(2) Quasi-neutrality + Boltzmann electron response

$$(\delta n_i/n_0) = (\delta n_e/n_0) = (\delta\varphi/T_{e0}) = (\delta\varphi/T_{i0})(T_i/T_e)$$

(3) E×B advection of background gradient

$$-\omega(\delta T_i/T_{i0}) = \omega_{*T}(\delta\varphi/T_i)$$

(1)+(2): $\omega(T_i/T_e)(\delta\varphi/T_{i0}) = \omega_{Di}\,(\delta T_i/T_{i0})$

(+3): $\omega(T_i/T_e) = -\omega_{Di}\,\omega_{*T} / \omega$

$\omega^2 = -(k_y\rho_i)^2 v_{Ti}^2 / RL_T$ (assume T_e=T_i)

$\boldsymbol{\omega = +/- \; i\,(k_y\rho_i)v_{Ti} / (RL_T)^{1/2}}$

Finite gyroradius effects limit characteristic size to ion-gyroradius ($k_\perp\rho_i$~1)

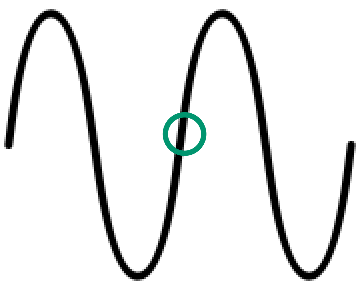
- Drift velocity increases with smaller wavelength (larger $k_\perp\rho_i$)

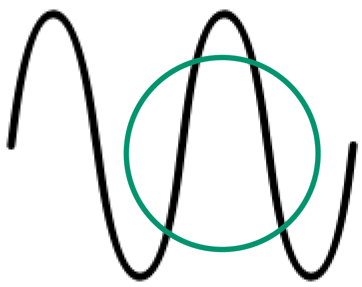
$$\vec{v}_E = \frac{\hat{b}\times\nabla\varphi}{B} = -ik_\perp\frac{\varphi}{B} = -ik_\perp\left(\frac{\varphi}{T_i}\right)\left(\frac{T_i}{B}\right) = -i(k_\perp\rho_i)\left(\frac{\varphi}{T_i}\right)v_{Ti}$$

- If wavelength approaches ion gyroradius $(k_\perp\rho_i)\geq1$, average electric field experienced over fast ion-gyromotion is reduced:

$\langle\nabla\varphi\rangle_{\text{gyro-average}} \sim \nabla\varphi$

$\langle\nabla\varphi\rangle_{\text{gyro-average}} \sim \nabla\varphi[1-(k_\perp\rho_i)^2]$



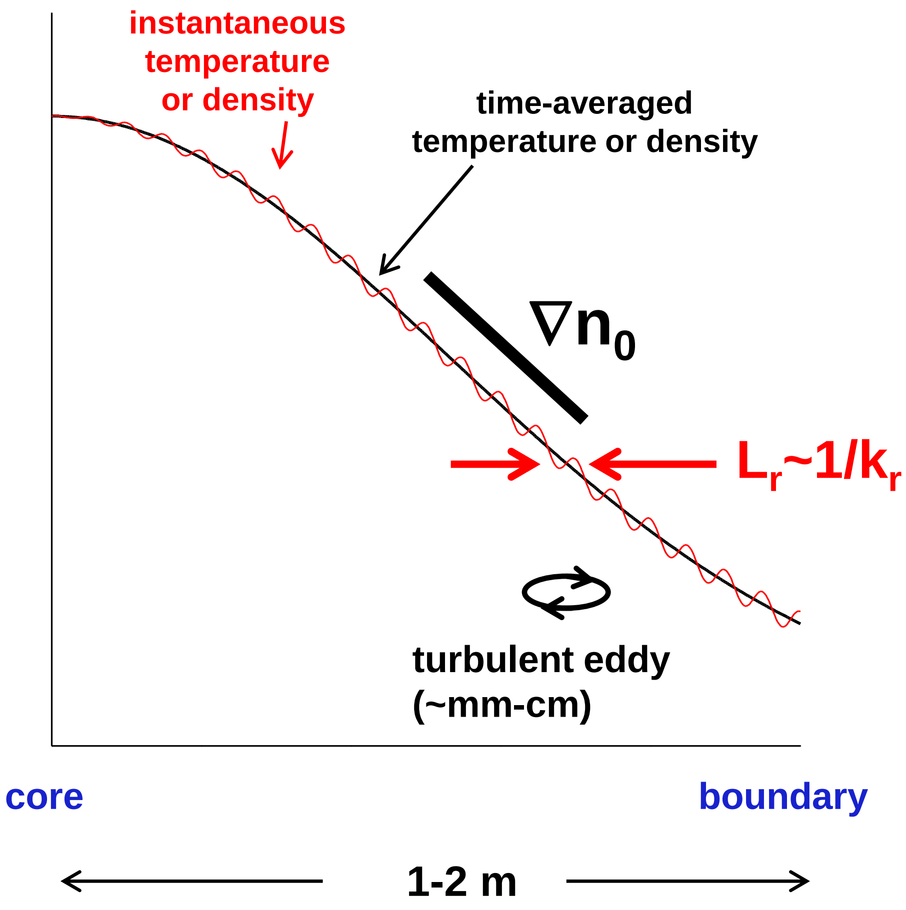


$\Rightarrow$ **Maximum growth rates (and typical turbulence scale sizes) occur for $(k_\perp\rho_i) \lesssim 1$**

Mixing length estimate of fluctuation amplitude

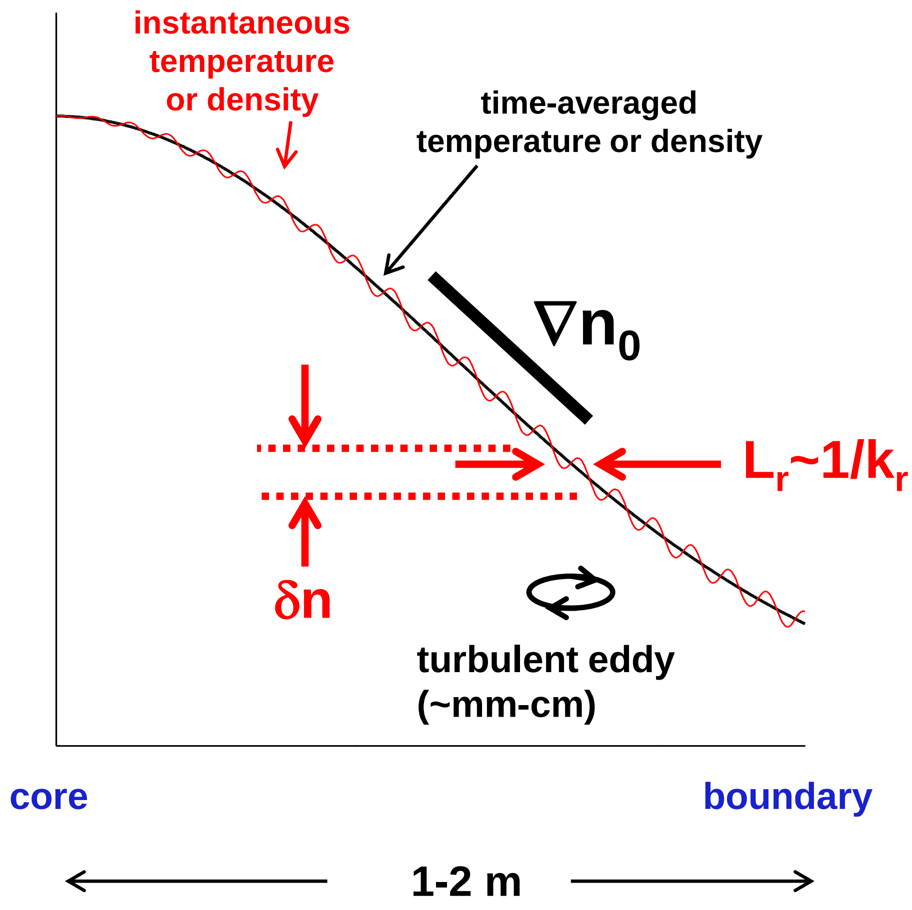
Mixing length estimate for fluctuation amplitude

- In the presence of an equilibrium gradient, ∇n_0, turbulence with radial correlation L_r will mix regions of high and low density



Mixing length estimate for fluctuation amplitude

- In the presence of an equilibrium gradient, ∇n_0, turbulence with radial correlation L_r will mix regions of high and low density
- Leads to fluctuation δn



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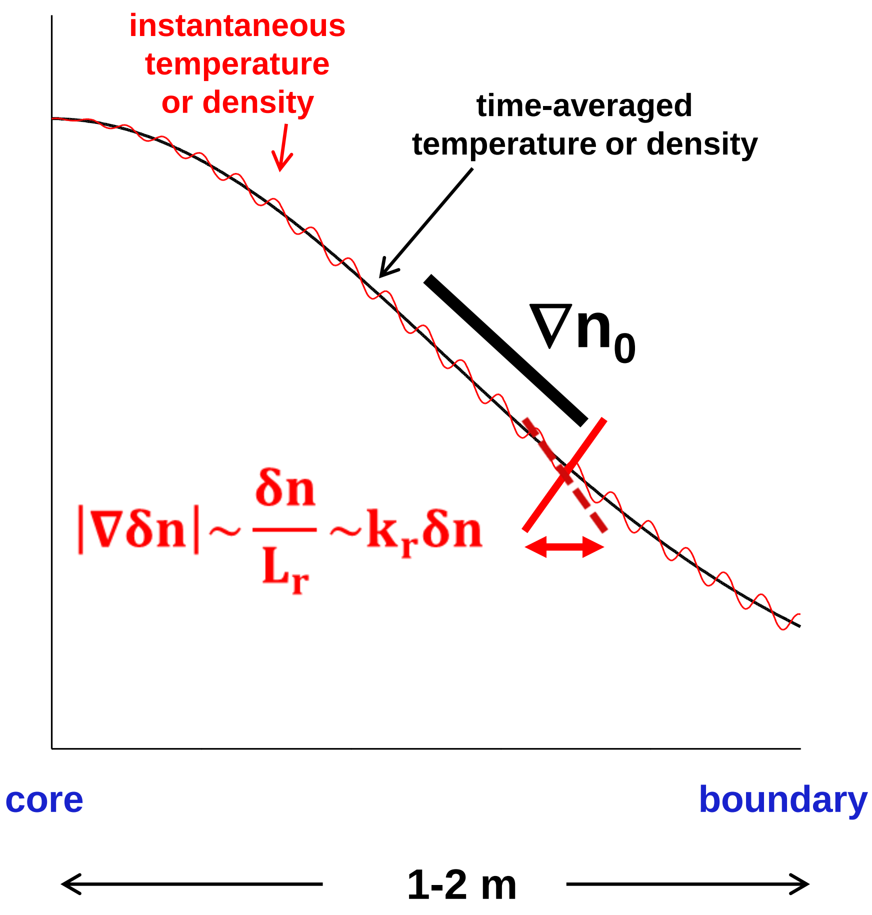


- Another interpretation: local, instantaneous gradient limited to equilibrium gradient

Mixing length estimate for fluctuation amplitude

$$\delta n \approx \nabla n_0 \cdot L_r$$

$$\frac{\delta n}{n_0} \approx \frac{\nabla n_0}{n_0} \cdot L_r \approx \frac{L_r}{L_n} \qquad \left(1/L_n = \nabla n_0 / n_0\right)$$

$$\frac{\delta n}{n_0} \sim \frac{1}{k_\perp L_n} \sim \frac{\rho_s}{L_n} \qquad \left(k_\perp^{-1} \sim L_r ; k_\perp \rho_s \sim \text{constant}\right)$$

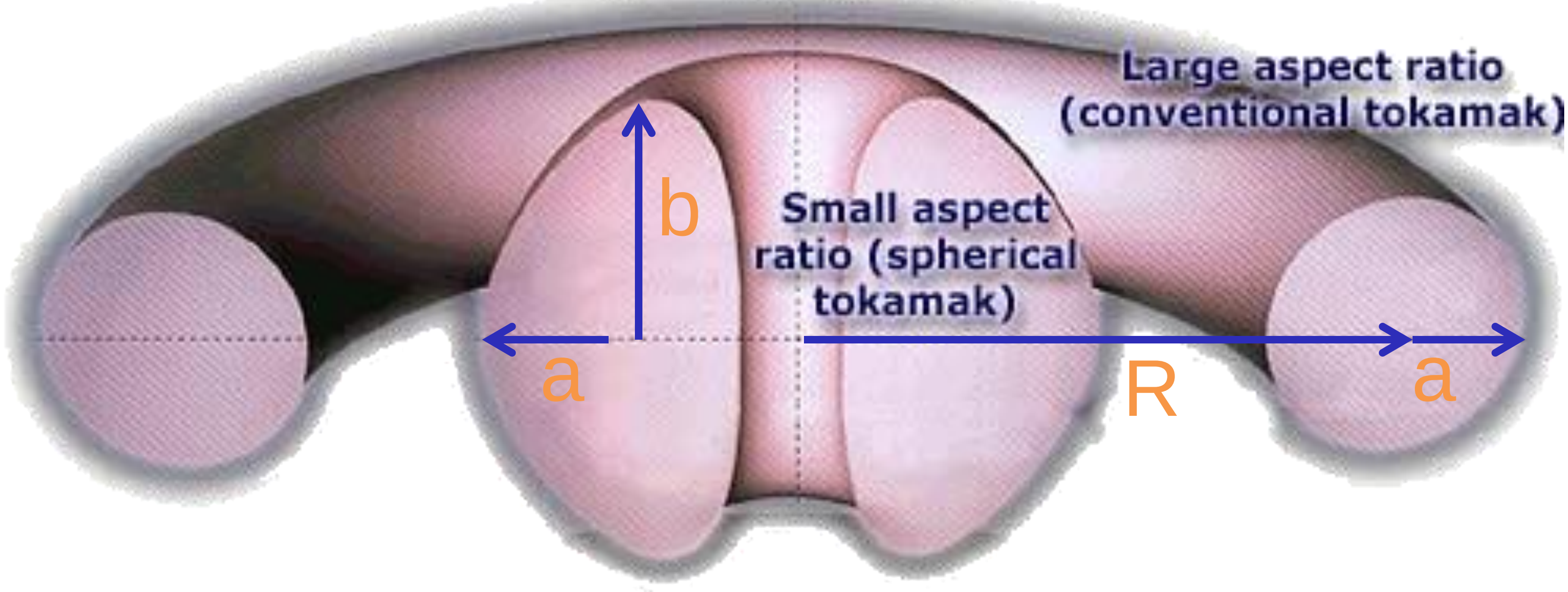
Expect $\delta n/n_0 \sim \rho_s/L \sim \rho_*$

Low aspect ratio equilibrium can stabilize electrostatic drift waves, minimize transport (i.e. one motivation for NSTX-U at PPPL)

Aspect ratio is an important free parameter, can try to make more compact devices (i.e. hopefully cheaper)

Aspect ratio A = R / a
Elongation κ = b / a

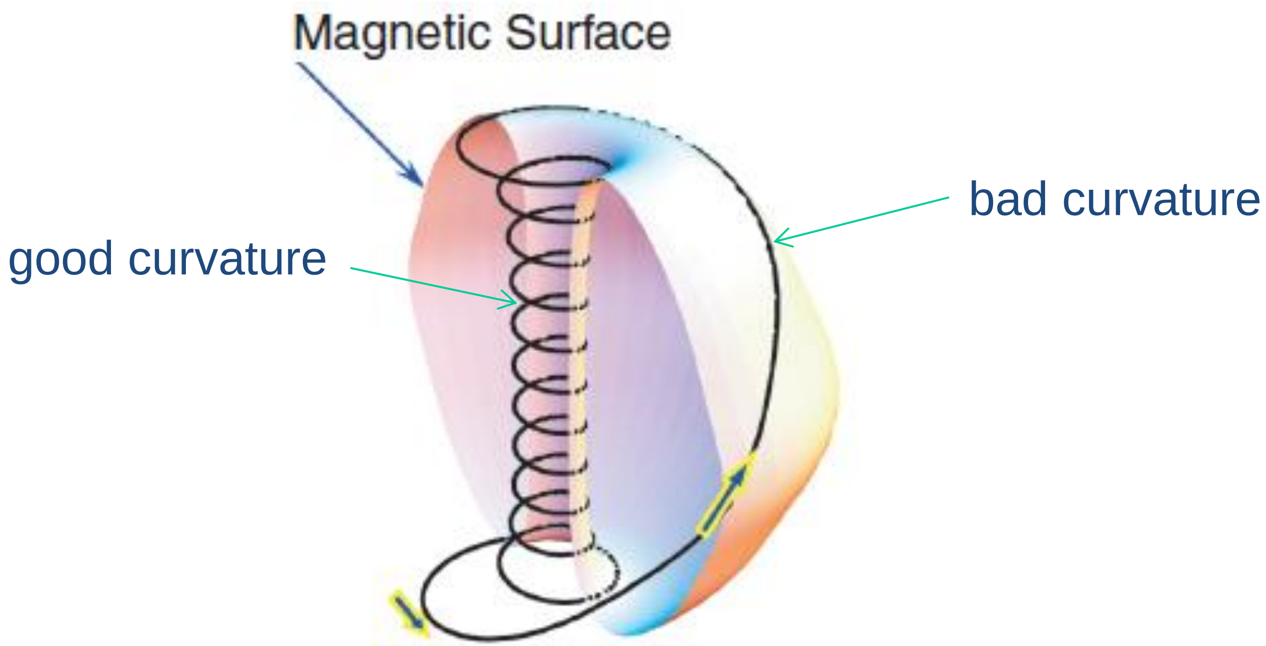
R = major radius, a = minor radius, b = vertical ½ height



But smaller R = larger curvature, ∇B (~1/R) -- isn't this terrible for "bad curvature" driven instabilities?!?!?!

Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

- Short connection length $\rightarrow$ **smaller average bad curvature**



Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

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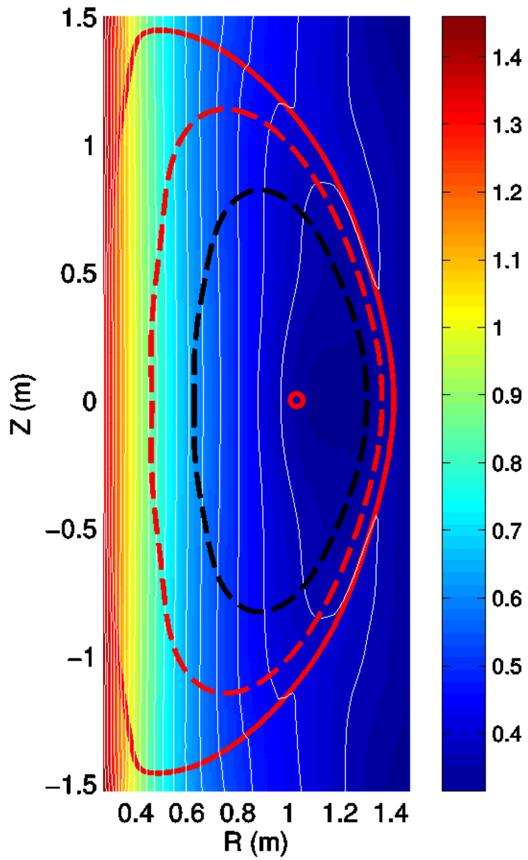
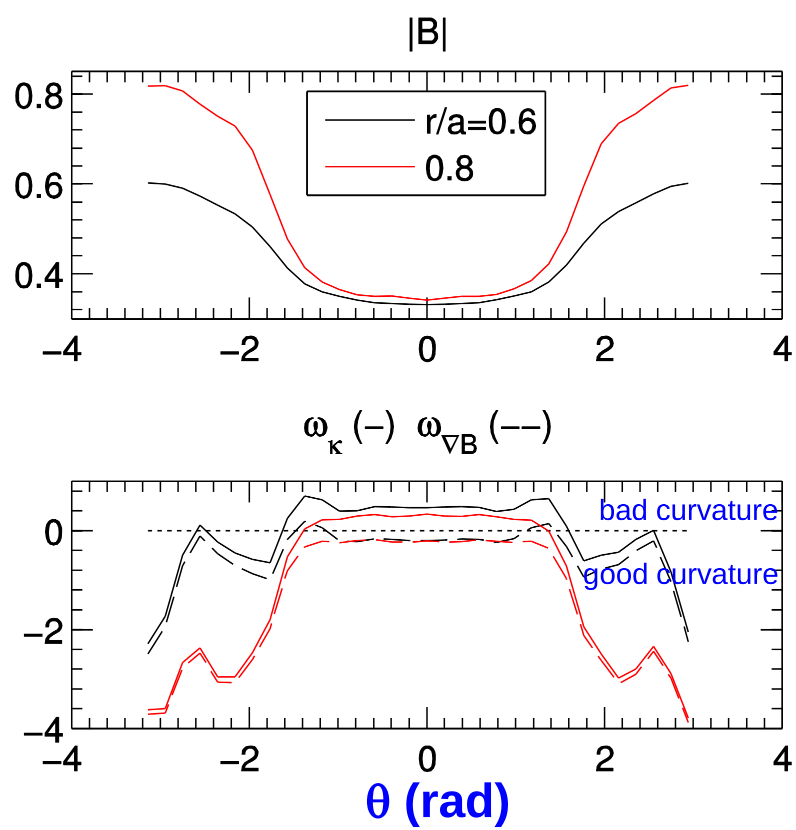
- Quasi-isodynamic (~constant B) at high β → **grad-B drifts stabilizing [Peng & Strickler, NF 1986]**





$$\vec{v}_\kappa = m v_{||}^2 \frac{\hat{b}\times\vec{\kappa}}{qB}$$

$$\vec{v}_{\nabla B} = \frac{m v_\perp^2}{2}\,\frac{\hat{b}\times\nabla B/B}{qB}$$

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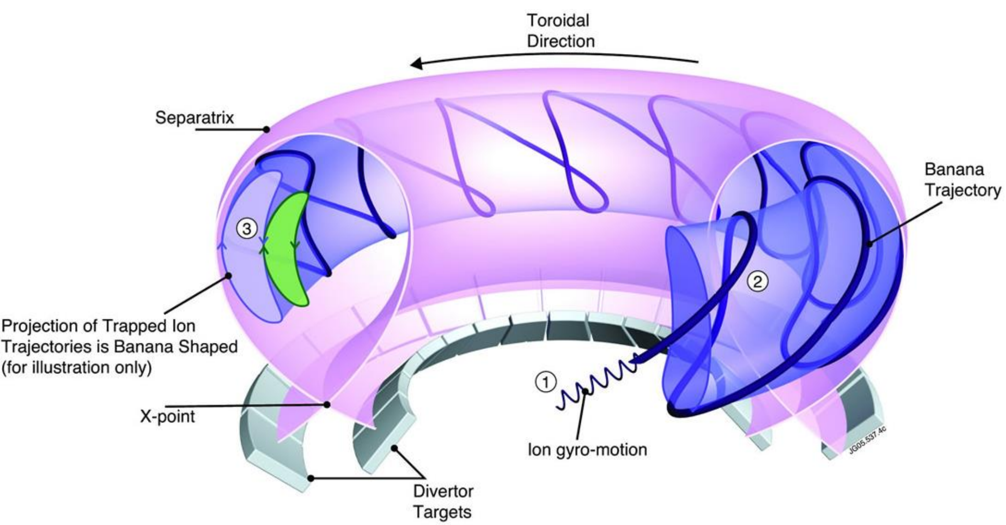
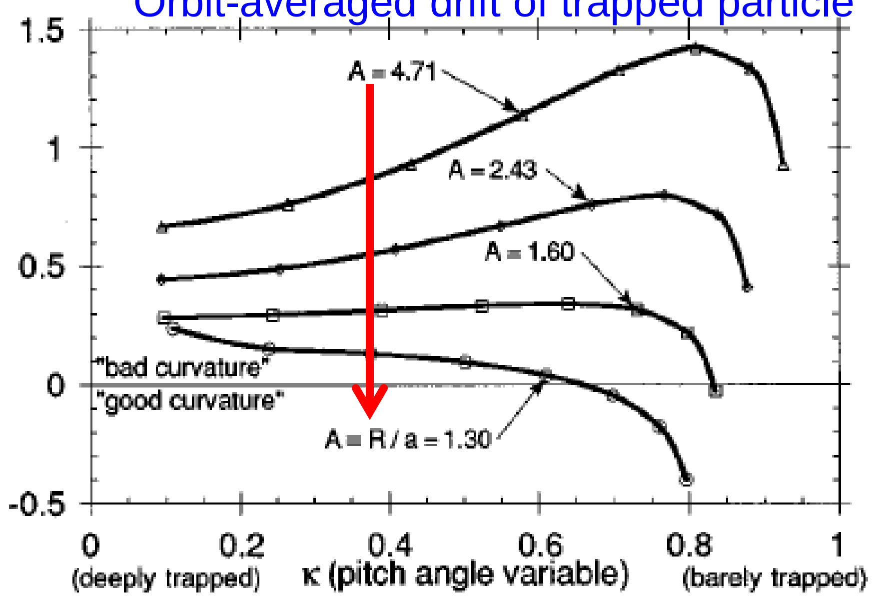


Orbit-averaged drift of trapped particle



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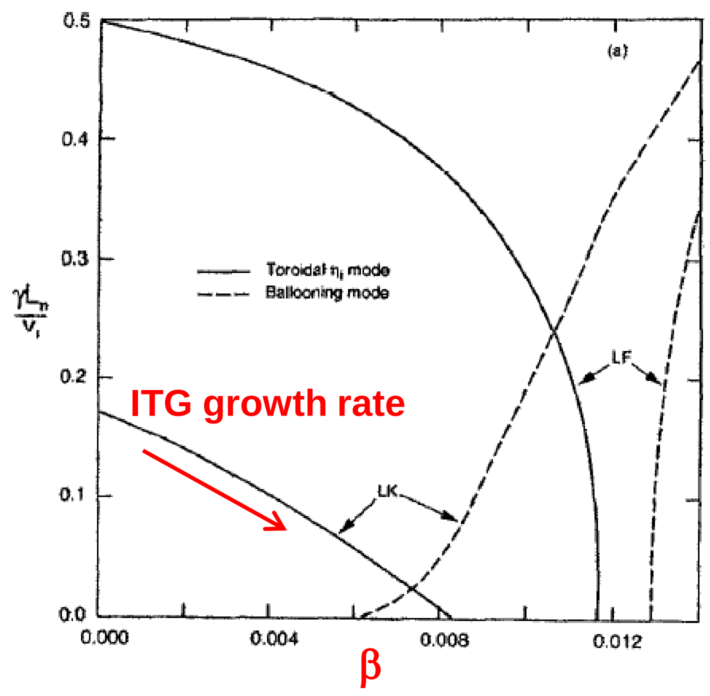

Kim, Horton, Dong, PoFB (1993)

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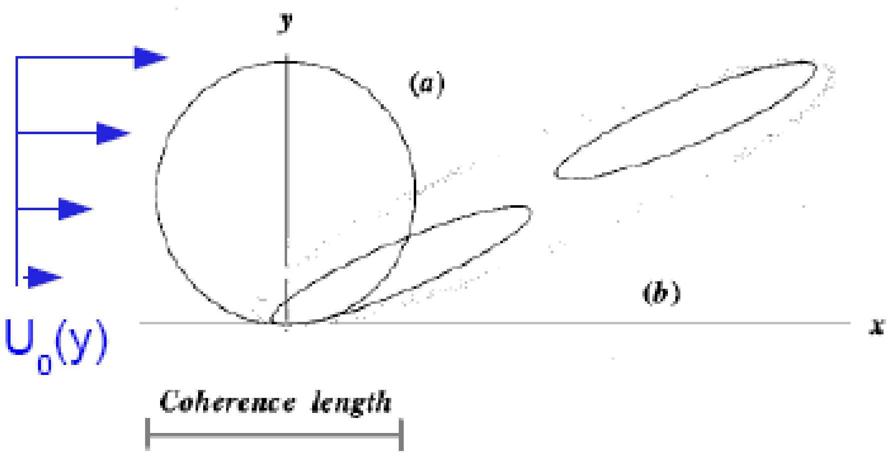

Biglari, Diamond, Terry, PoFB (1990)

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$\Rightarrow$ **Not expecting strong ES ITG/TEM instability (much higher thresholds)**

- BUT
- High beta drives EM instabilities: microtearing modes (**MTM**) ~ $\beta_e \cdot \nabla T_e$, kinetic ballooning modes (**KBM**) ~ $\alpha_{MHD} \sim q^2 \nabla P/B^2$
- Large shear in parallel velocity can drive Kelvin-Helmholtz-like instability ~$dv_{||}/dr$