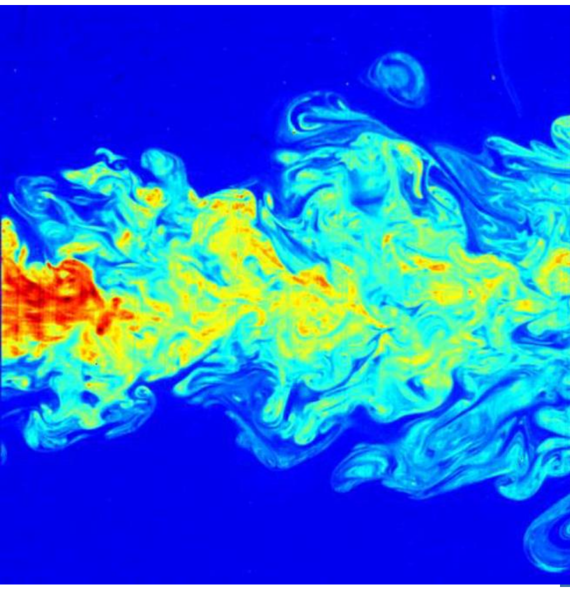
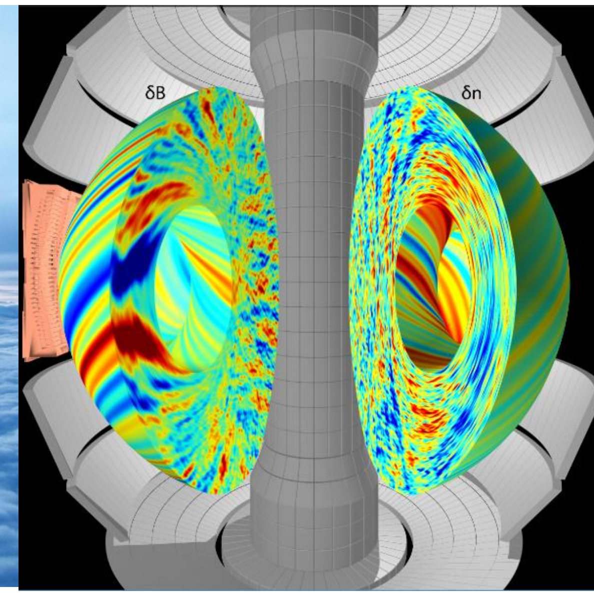
Intro to neutral & magnetized plasma turbulence





Walter Guttenfelder

PPPL Graduate Student Seminar
Feb. 18, 2019

Some additional sources & references

- Greg Hammett has a lot of great introductory material to fusion, tokamaks, drift waves, ITG turbulence, gyrokinetics, etc… (*w3.pppl.gov/~hammett*)

- Greg & I recently gave five 90 minute lectures on turbulence at the 2018 Graduate Summer School (*gss.pppl.gov*)

- See the following for broader reviews and thousands of useful references

- Transport & Turbulence reviews:
  - Liewer, Nuclear Fusion (1985)
  - Wootton, Phys. Fluids B (1990)
  - Carreras, IEEE Trans. Plasma Science (1997)
  - Wolf, PPCF (2003)
  - Tynan, PPCF (2009)
  - ITER Physics Basis (IPB), Nuclear Fusion (1999)
  - Progress in ITER Physics Basis (PIPB), Nuclear Fusion (2007)
- Drift wave reviews:
  - Horton, Rev. Modern Physics (1999)
  - Tang, Nuclear Fusion (1978)
- Gyrokinetic simulation review:
  - Garbet, Nuclear Fusion (2010)
- Zonal flow/GAM reviews:
  - Diamond et al., PPCF (2005)
  - Fujisawa, Nuclear Fusion (2009)
- Measurement techniques:
  - Bretz, RSI (1997)

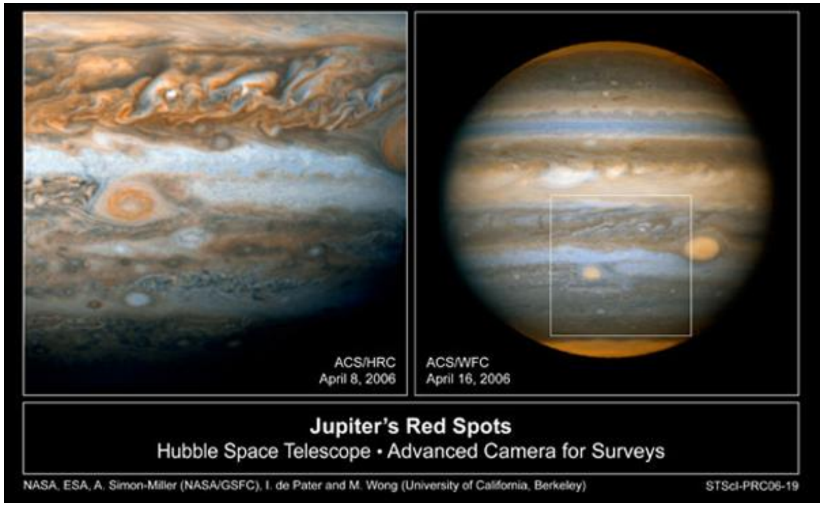
Outline

- Neutral fluid turbulence
  - Examples & concepts
  - Energy cascade

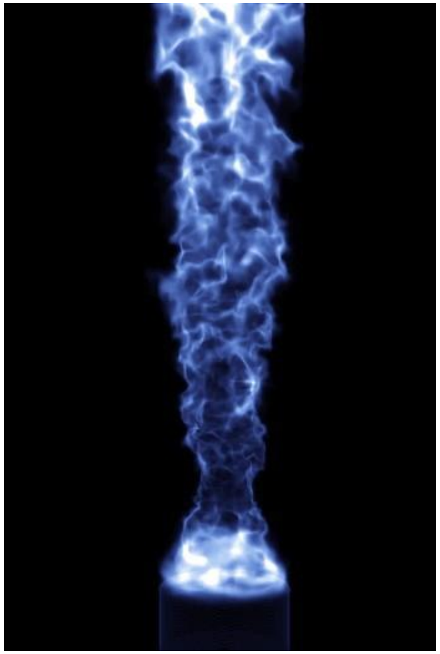
- Magnetized plasma turbulence (e.g. in MFE)
  - Examples (measurements & simulations)
  - Micro-instabilities (ITG dynamics)
  - Saturation (zonal flows)

Examples of turbulence

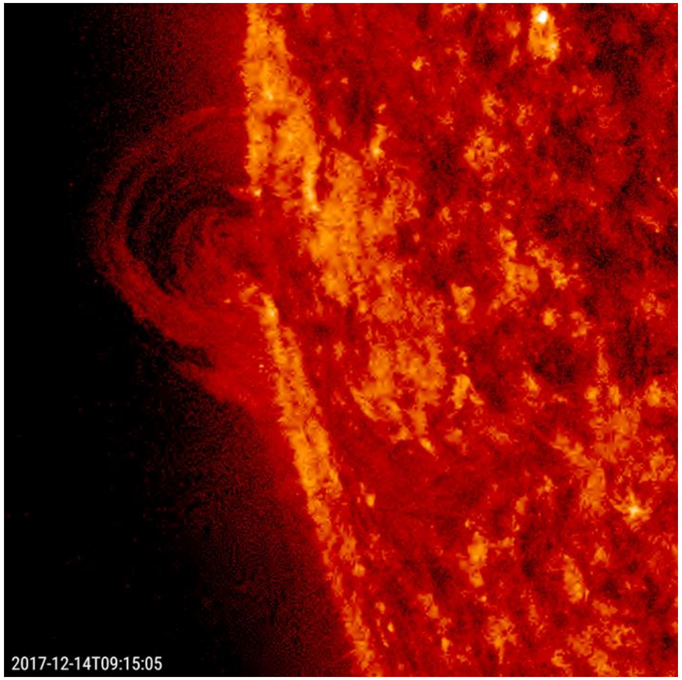
Turbulence found throughout the universe



Steve Morr



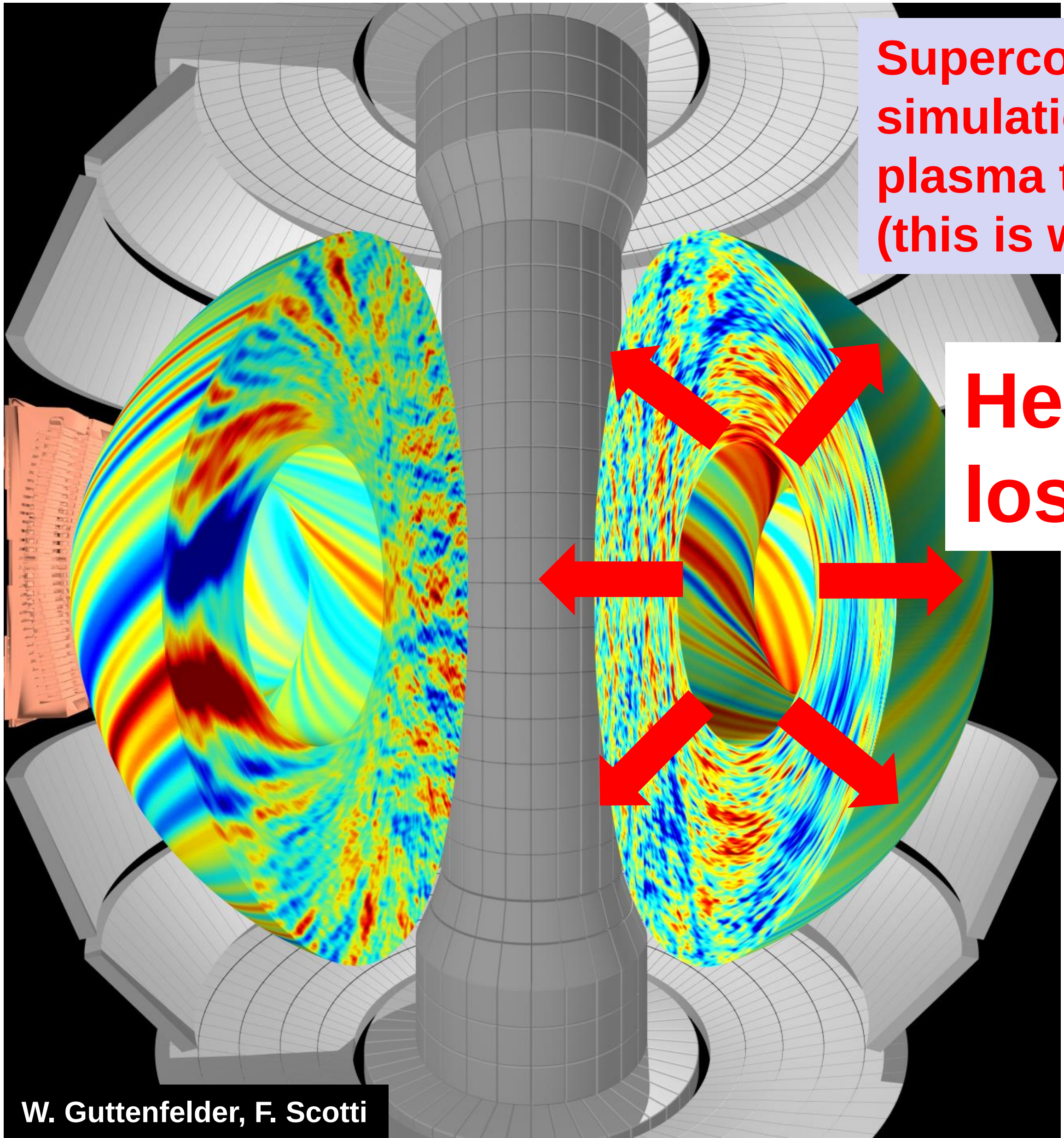
Universität Duisburg-Essen



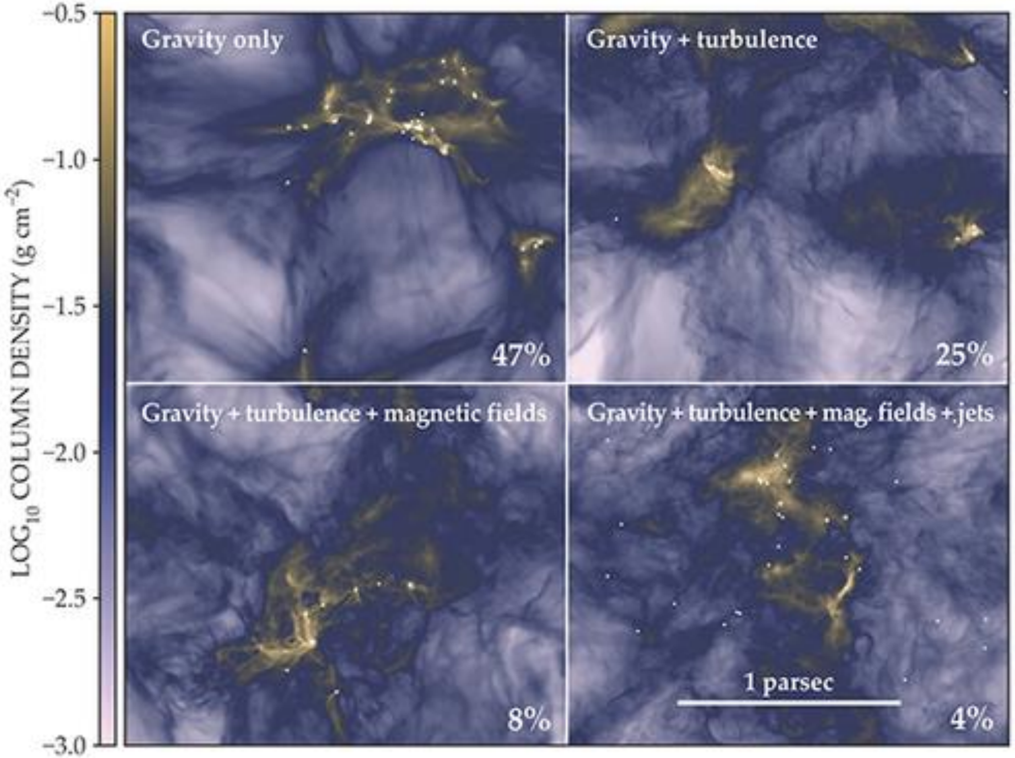
https://sdo.gsfc.nasa.gov/gallery

Turbulence is ubiquitous throughout planetary atmospheres

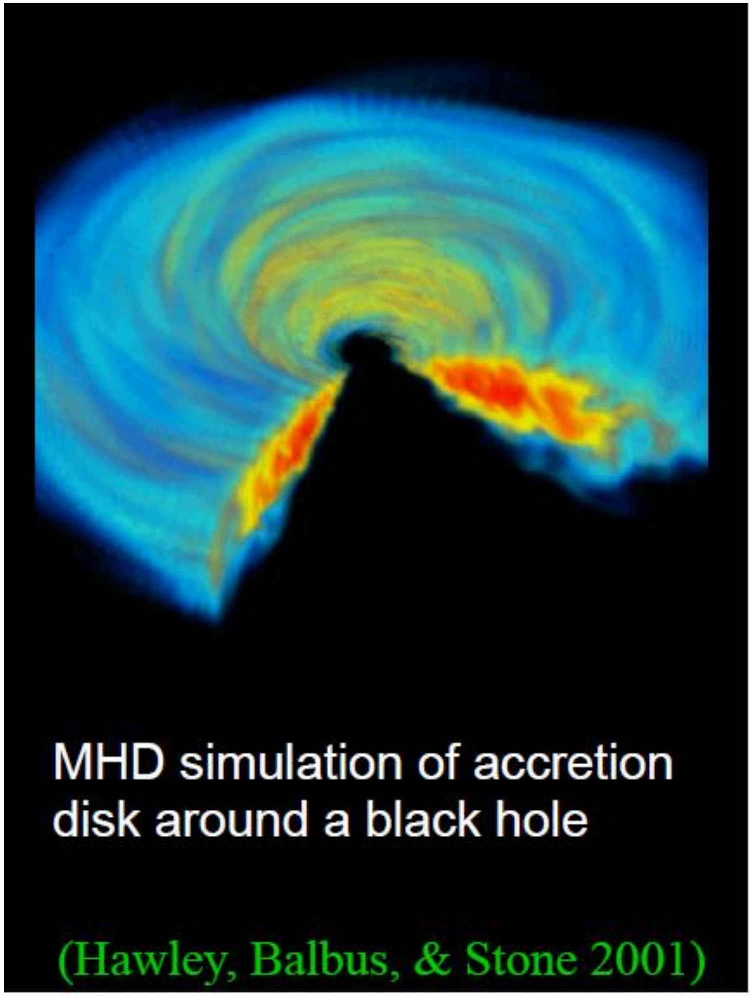
Plasma turbulence degrades energy confinement / insulation in magnetic fusion energy devices



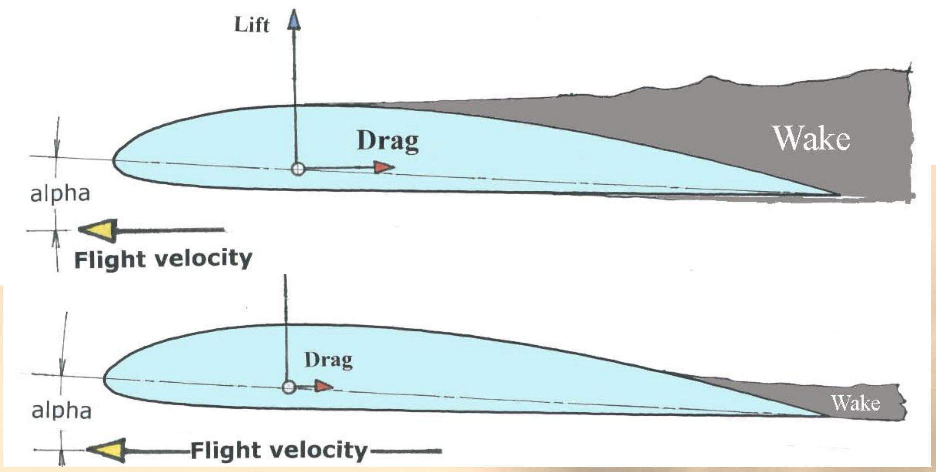
Turbulence is important throughout astrophysics



- Plays a role in star formation (C. Federrath, Physics Today, June 2018)

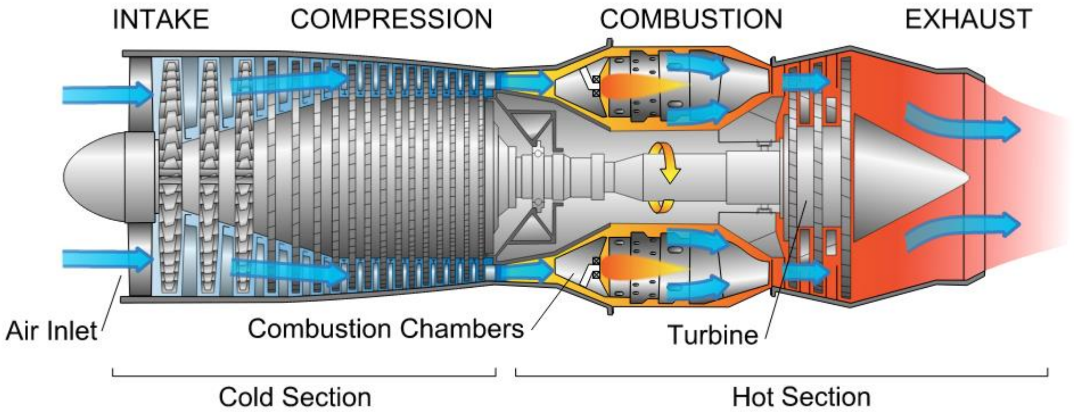


Turbulence is crucial to lift, drag & stall characteristics of airfoils



Increased turbulence on airfoil helps minimize boundary-layer separation and drag from adverse pressure gradient

Turbulence generators



Turbulent mixing of fuel and air is critical for efficient & economical jet engines

What is turbulence?

Concepts of turbulence to remember

- Turbulence is deterministic yet unpredictable (chaotic), appears random
  - We often treat & diagnose statistically, but also rely on first-principles direct numerical simulation (DNS)
- Turbulence causes increased mixing, transport larger than collisional transport
  - **Transport** is the key application of why we care about turbulence
- Turbulence spans a wide range of spatial and temporal scales
  - Or in the case of hot, low-collisionality plasma, a wide range of scales in 6D phase-space (**x**,**v**)
- Turbulence is not a property of the *fluid*, it's a feature of the *flow*

Concepts of turbulence to remember

- Turbulence is deterministic yet unpredictable (chaotic), appears random
  - We often treat & diagnose statistically, but also rely on first-principles direct numerical simulation (DNS)
- Turbulence causes increased mixing, transport larger than collisional transport
  - **Transport** is the key application of why we care about turbulence
- Turbulence spans a wide range of spatial and temporal scales
  - Or in the case of hot, low-collisionality plasma, a wide range of scales in 6D phase-space (**x**,**v**)
- Turbulence is not a property of the *fluid*, it's a feature of the *flow*
- It's cool! "Turbulence is the most important unsolved problem in classical physics" (~Feynman)

Turbulence is an advective process

- Transport a result of finite average correlation between perturbed drift velocity (δv) and perturbed fluid moments (δn, δT, δv)
  - Particle flux, $\Gamma = \langle \delta v \delta n \rangle$
  - Heat flux, $Q = 3/2 n_0 \langle \delta v \delta T \rangle + 3/2 T_0 \langle \delta v \delta n \rangle$
  - Momentum flux, $\Pi \sim \langle \delta v \delta v \rangle$ ("Reynolds stress")

- Electrostatic turbulence often most relevant in tokamaks $\rightarrow$ $E \times B$ drift from potential perturbations: $\delta v_E = B \times \nabla(\delta\varphi)/B^2 \sim k_\theta(\delta\varphi)/B$

- Can also have magnetic contributions at high beta, $\delta v_B \sim v_{||}(\delta B_r/B)$ (magnetic "flutter" transport)

Why such a broad range of scale lengths?
(Enter the Reynolds number)

Incompressible Navier-Stokes (neutral fluids)

- Momentum conservation law:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{f}_B$$

Unsteady flow	Convective acceleration	Pressure force	Viscosity	Body forces (g, $J \times B$, qE)

- Assuming incompressible, from mass conservation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) \rightarrow \nabla \cdot \mathbf{v} = \mathbf{0}$$

Consider externally forced flow, no body forces or pressure drop

- Momentum conservation law:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho}\nabla P + \nu \nabla^2 \mathbf{v} + \mathbf{f}_B$$

Unsteady flow	Convective acceleration	Pressure force	Viscosity	Body forces (g, J×B, qE)

- Assuming incompressible, from mass conservation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) \rightarrow \nabla \cdot \mathbf{v} = \mathbf{0}$$

Use dimensionless ratios to estimate dominant dynamics

- Reynolds number gives order-of-magnitude estimate of inertial force to viscous force

$$\frac{\mathbf{v}\cdot\nabla\mathbf{v}}{\nu\nabla^2\mathbf{v}} \rightarrow \frac{V^2/L}{\nu V/L^2}$$

$$\boxed{\mathrm{Re} = \frac{VL}{\nu}}$$

Viscosities (m^2/s)

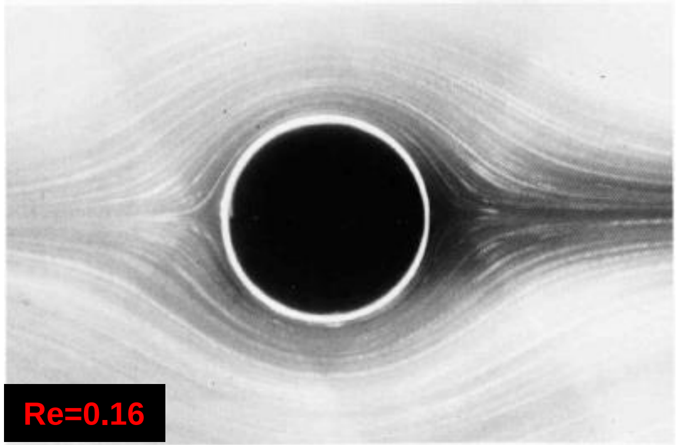
Air	**~1.5×10^{-5}**
Water	**~1.0×10^{-6}**

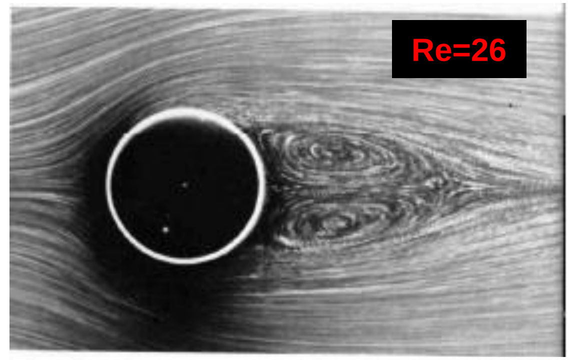
For L~1 m scale sizes and V~10 m/s, Re~10^6-10^7

  – Analogous to magnetic Reynolds number, $S=VL/(\eta/\mu_0)$ (reconnection)

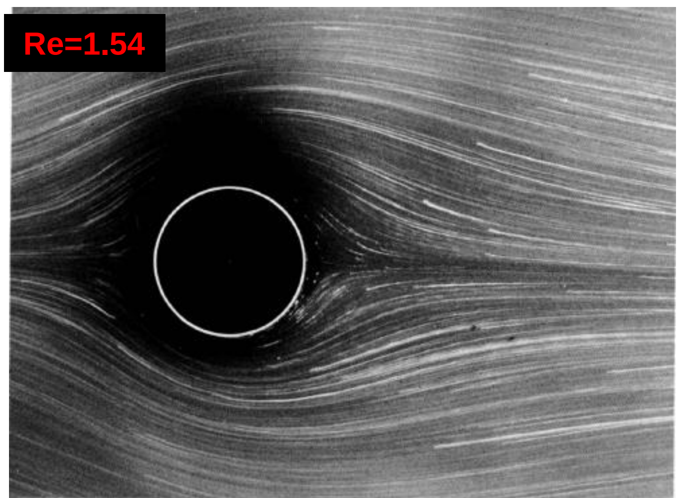
- For similar Reynolds numbers, we expect similar behavior, regardless of fluid type, viscosity or magnitude of V & L (as long as we are at low Mach #)

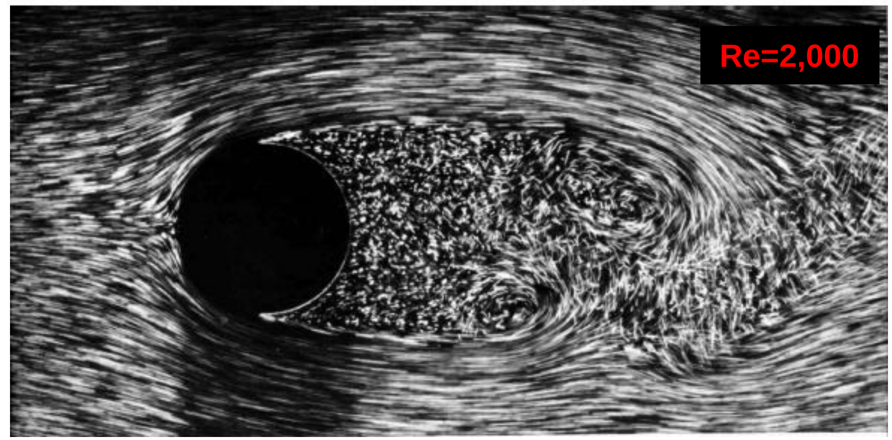
Transition from laminar to turbulent flow with increasing Re

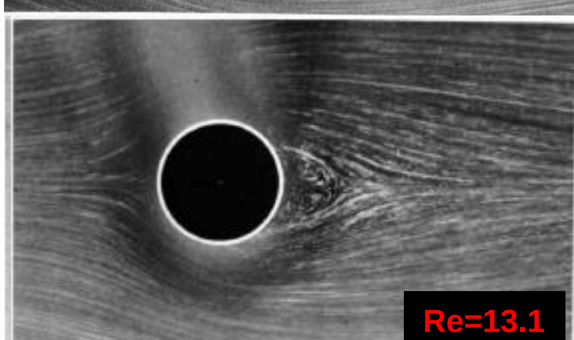


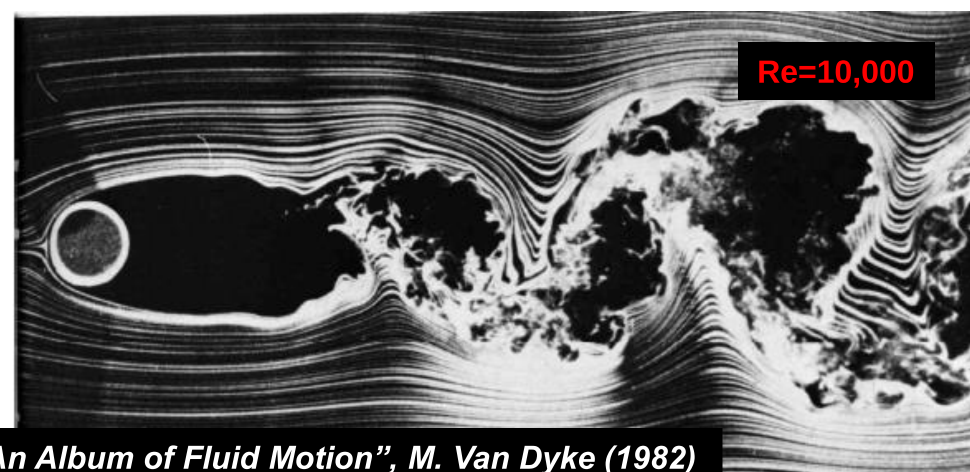


Cylinders or spheres



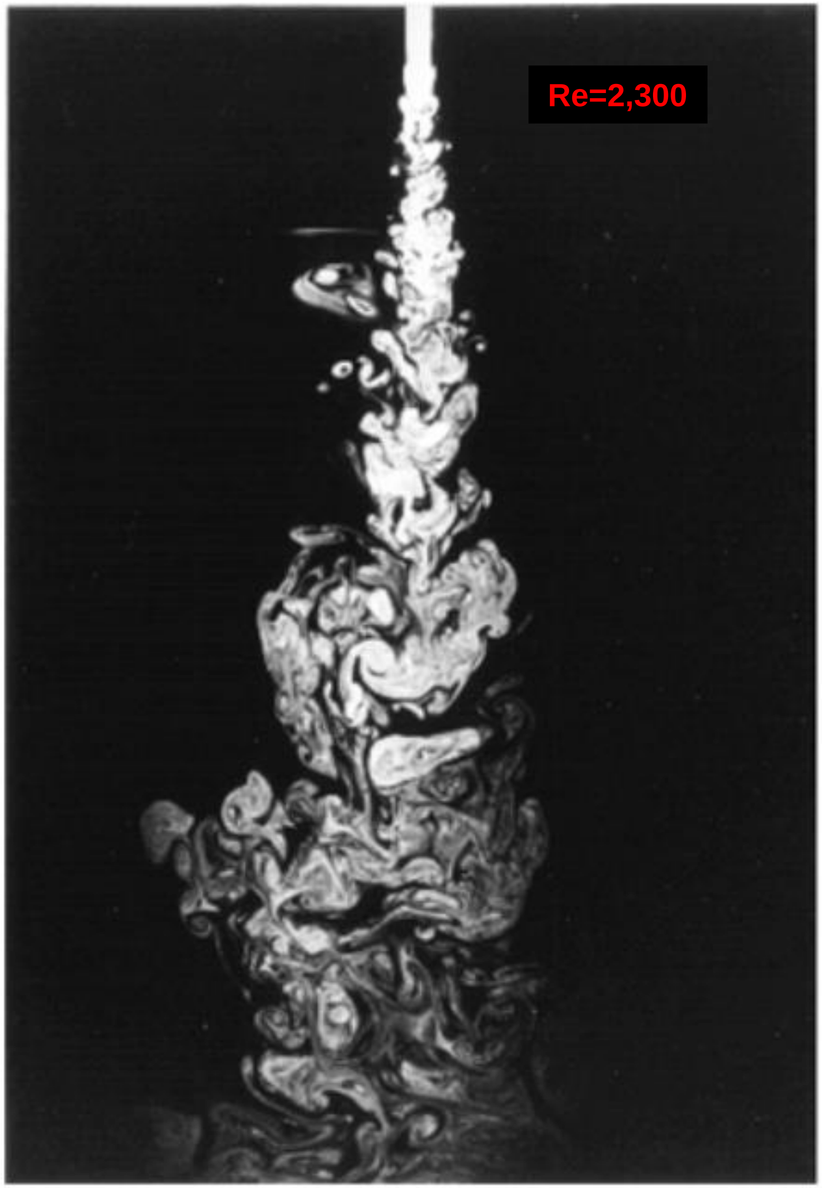


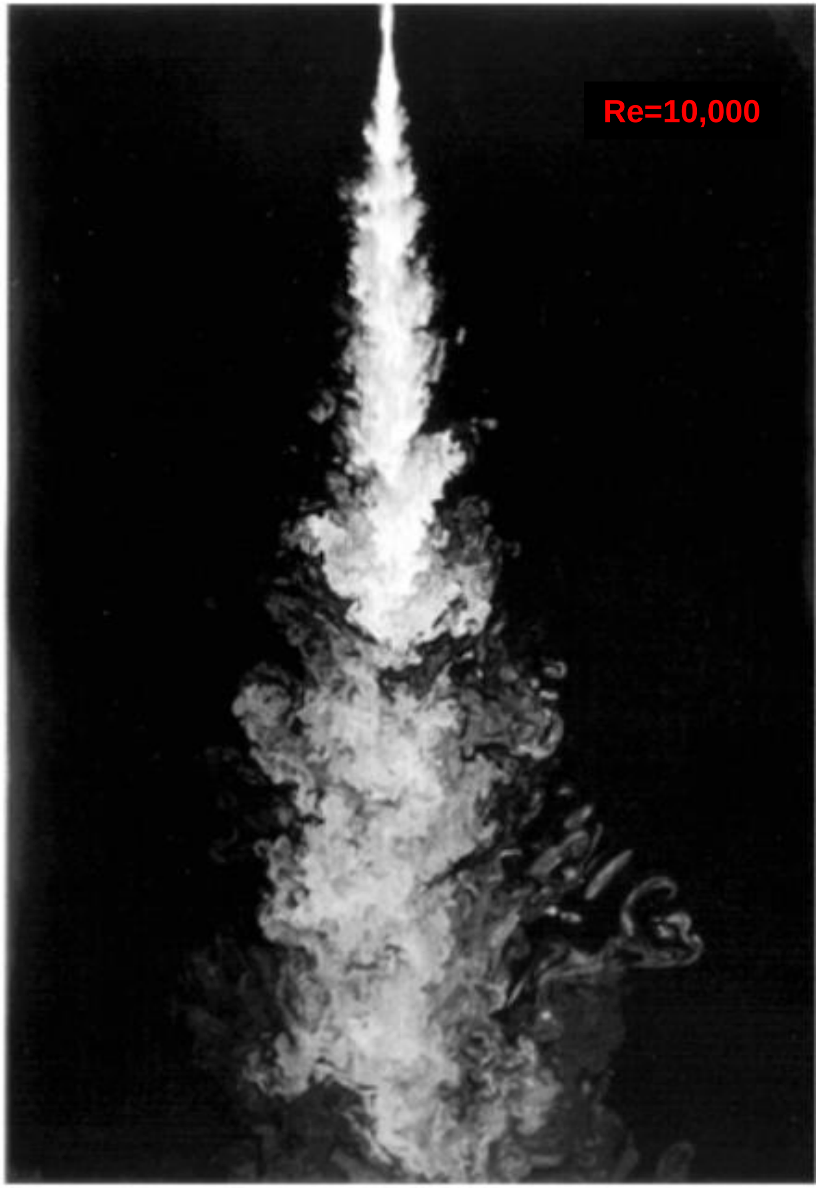




"An Album of Fluid Motion", M. Van Dyke (1982)

Increasing Re # in jet flow (what is changing?)





P. Dimotakis, J. Fluid Mech (2000)

For large Reynolds #, we expect a large range of scale lengths

- Viscosity works via shear stress, $\nu\nabla^2\mathbf{v} \sim \nu v/\ell^2$

- For the energy injection scales (L_0, V_0), viscosity dissipation is tiny compared to nonlinear dynamics, ~1/Re

- Effects of viscosity will become comparable to rate of energy injection at increasing smaller scales $\ell \ll L_0$

  $\ell/L_0 \sim Re^{-1/2}$ (for laminar boundary layer)

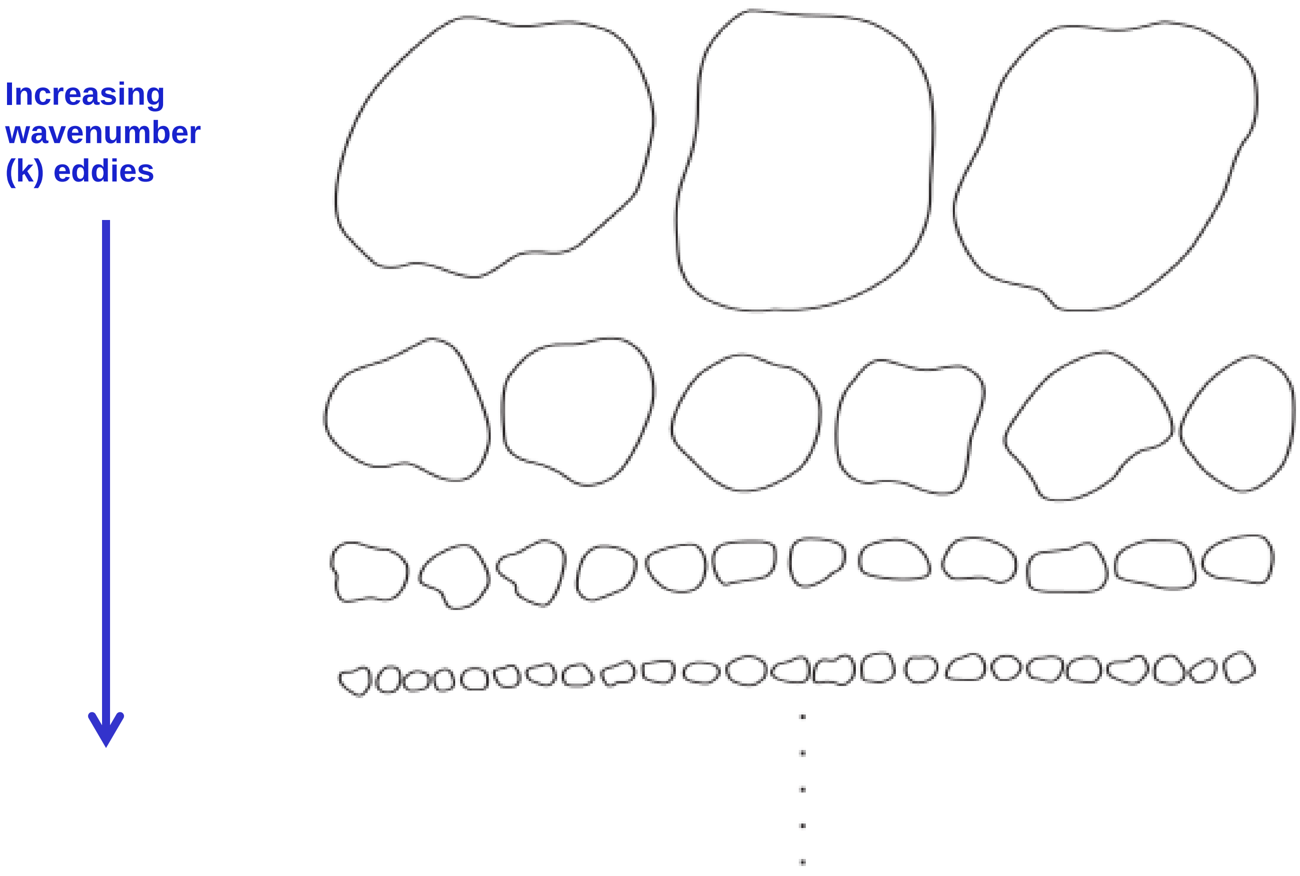
  $\ell/L_0 \sim Re^{-3/4}$ (turbulent flow)

$\Rightarrow$ **What sets the distribution of fluctuations?**

Kolmogorov scaling
(energy cascade through the inertial range)

E.g., imagine there are eddies distributed at various scale lengths

- Of course these different wavenumber eddies are not spatially separated but co-exist in space



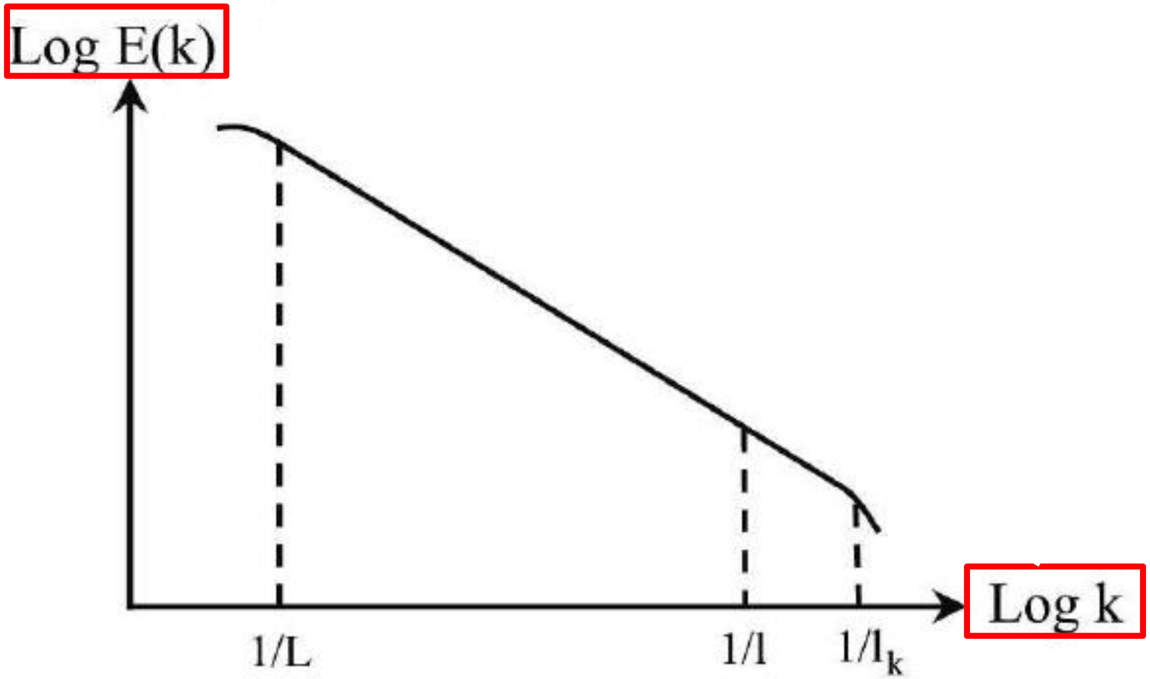
Want to predict distribution of energy with scale length (or wavenumber)

Turbulent energy spectrum

$$\langle \frac{v^2}{2} \rangle = \int E(k)dk$$

$$E(k) \sim v_k^2 / \Delta k$$

$$v_k^2 \sim \Delta k \, E(k)$$



A.N. Kolmogorov (1941) provides a well known derivation of turbulent energy spectrum

Assumptions

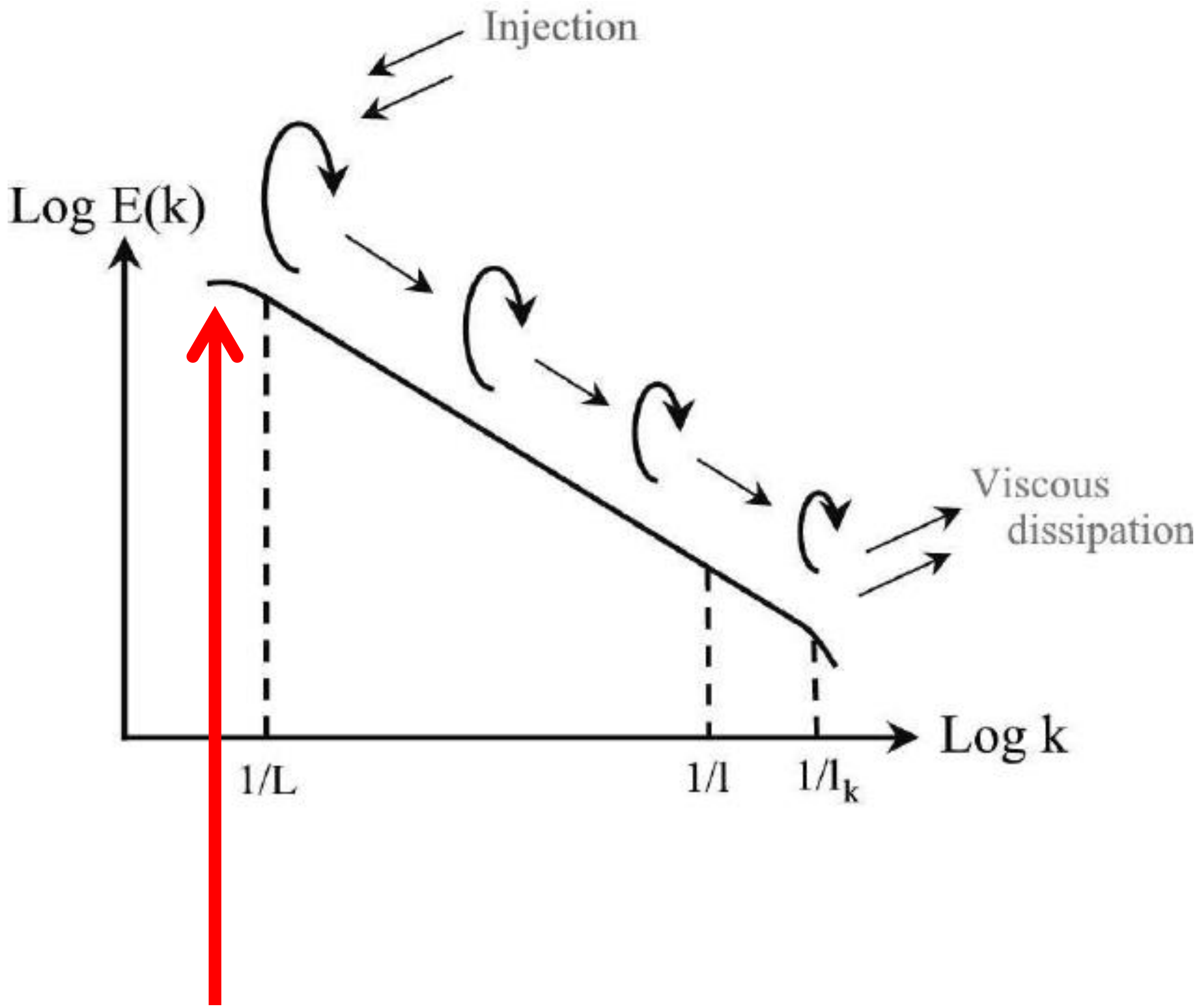
- For sufficient separation of scales ($L >> l >> l_\nu$, i.e. Re >>>> 1), assume non-linear interactions independent of energy injection or dissipation (so called "inertial range")
  - Energy injected at large scales ~ L_0 ($k_{forcing} \sim 1/L_0$)
  - Viscosity only matters at very small scales ~ l_ν ($k_\nu \sim 1/l_\nu$)
- Turbulence assumed to be homogeneous and isotropic in the inertial range
- Assume that interactions occur locally in wavenumber space (for interacting triads $\mathbf{k}_1+\mathbf{k}_2=\mathbf{k}_3$, $|\mathbf{k}_1|\sim|\mathbf{k}_2|\sim|\mathbf{k}_3|$)

$$\mathrm{v}(\mathrm{x},\mathrm{t}) \rightarrow \mathrm{v}_{\mathrm{k}}(\mathrm{t})\exp(i\mathrm{k}\cdot\mathrm{x})$$

$$\mathrm{v}\cdot\nabla\mathrm{v} \rightarrow \mathrm{v}_{\mathrm{k}_1}(\mathrm{t})\cdot\mathrm{k}_2\mathrm{v}_{\mathrm{k}_2}(\mathrm{t})\exp[i(\mathrm{k}_1+\mathrm{k}_2)\cdot\mathrm{x}]$$

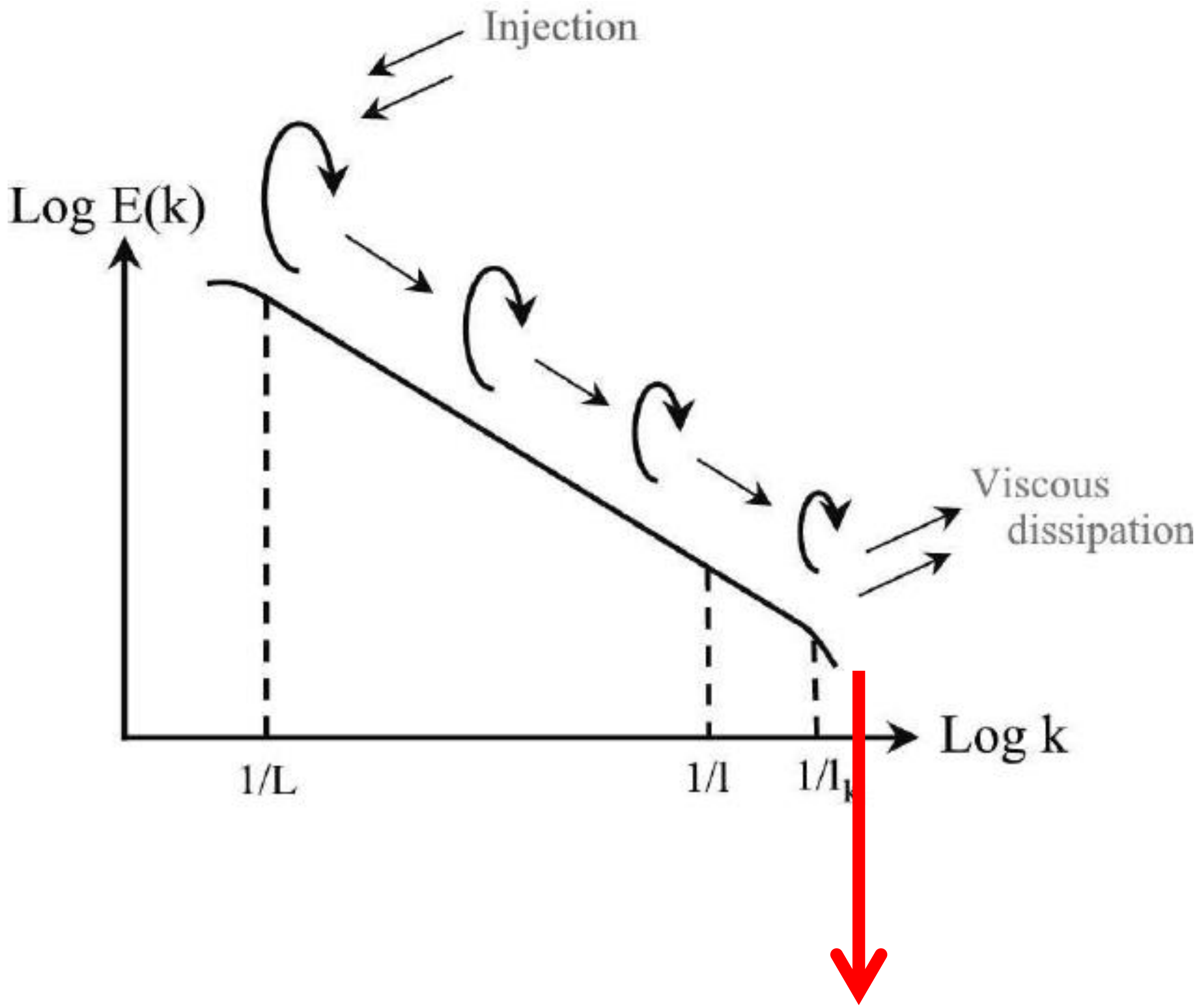
$$\Rightarrow \frac{\partial \mathrm{v}_{k_3}}{\partial \mathrm{t}} = -\mathrm{v}_{\mathrm{k}_1}\cdot\mathrm{k}_2\mathrm{v}_{\mathrm{k}_2} \qquad \mathrm{k}_3=\mathrm{k}_1+\mathrm{k}_2$$

Energy injection occurs at large scales (low k)



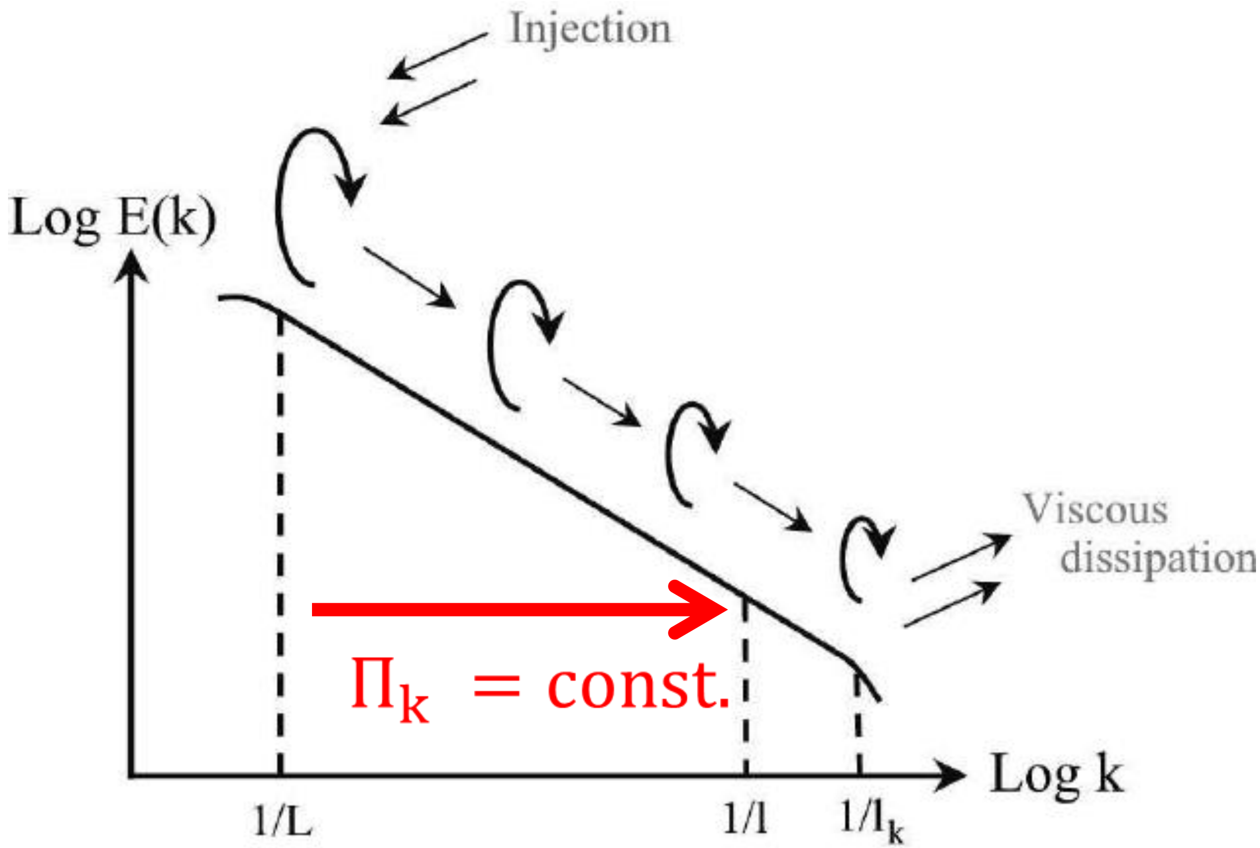
$$\epsilon_{inj} = \frac{d}{dt}(U_0^2) \sim \frac{U_0^3}{L_0}$$

Viscous dissipation strong at small scales (high k)



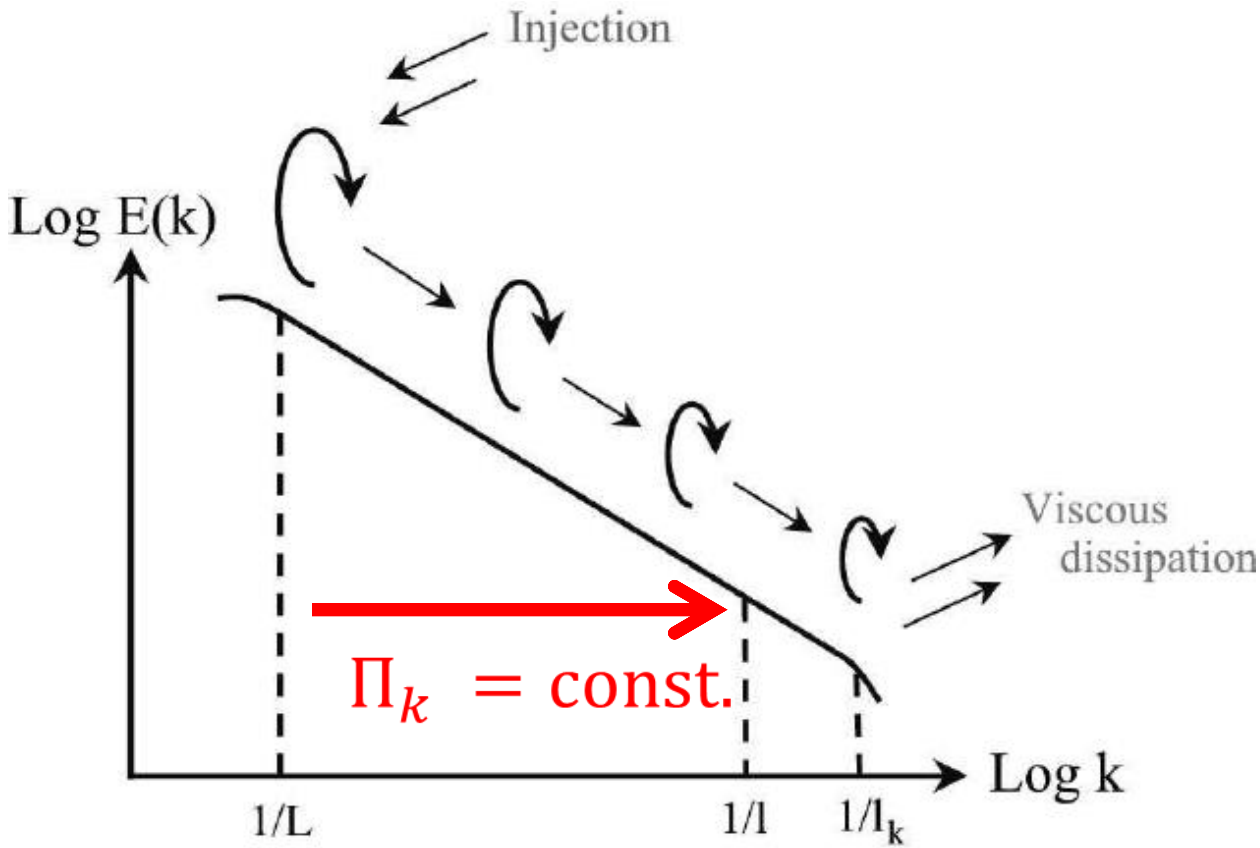
$$\epsilon_{diss} = -\nu k_\nu^2 E(k)$$

Constant forward energy cascade (from large eddies to small eddies) through the "inertial range"



- NL v·∇v interactions occur <u>locally in wavenumber space</u> (e.g. between ~k/2 < ~2*k)
  - Very large eddy will not distort smaller eddy very much (~rigid translation/rotation)
  - Smaller eddies will not distort much larger eddies as they don't act coherently

Consider a Fokker-Plank / advection equation for energy transfer through k-space

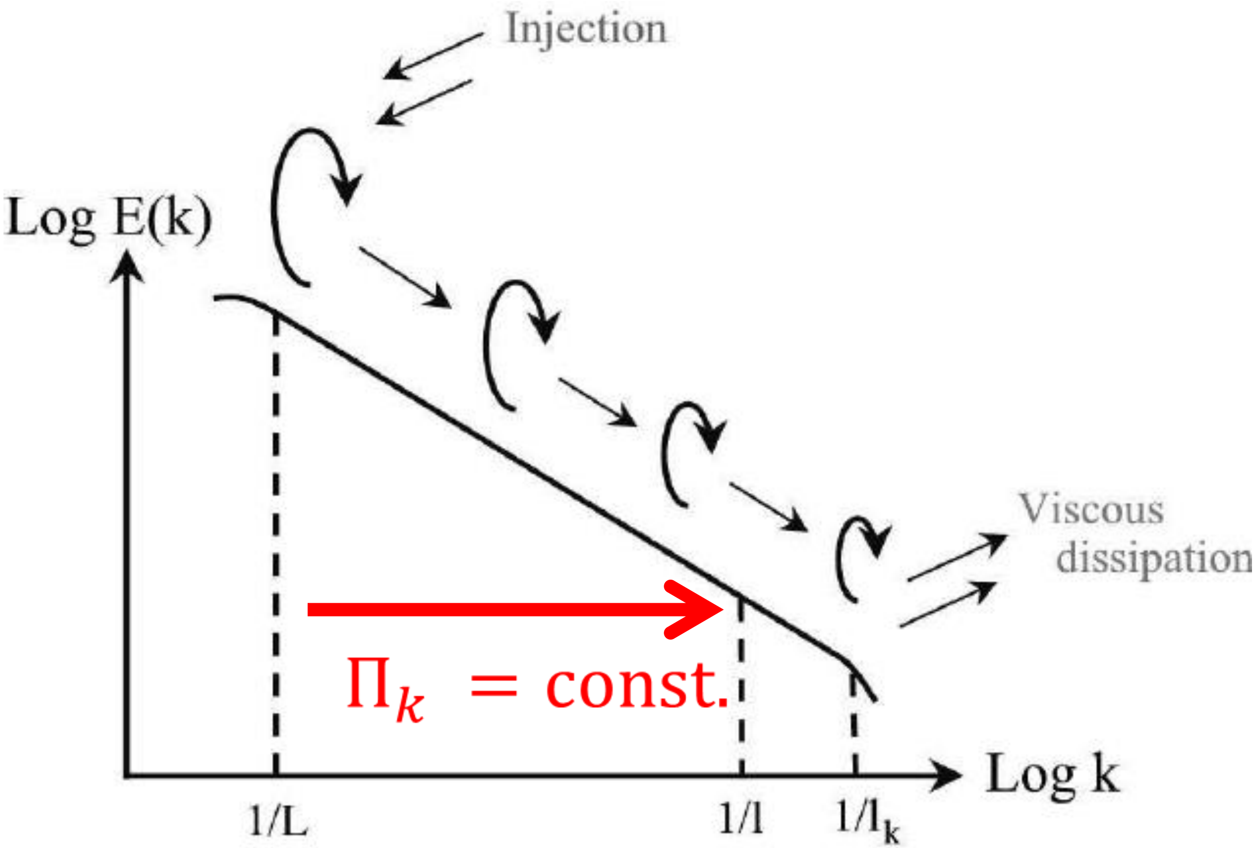


$$\Pi_k = \frac{\langle \Delta k \rangle}{\Delta t} E(k)$$

$$\frac{d}{dt} E(k,t) = \epsilon_{inj} \delta(k - k_f) - \frac{\partial}{\partial k} [\Pi_k] - \nu k^2 E(k) = 0$$

(Hammett class notes)

Consider a Fokker-Plank / advection equation for energy transfer through k-space



$$\Pi_k = \text{const.}$$

$$\Pi_k = \frac{\langle \Delta k \rangle}{\Delta t} E(k)$$

$$\epsilon_{inj} = \epsilon_{diss} = \epsilon \approx \Pi_k = \frac{\langle \Delta k \rangle}{\Delta t} E(k) = \text{const}$$

Constant cascade of energy through the inertial range gives Kolmogorov spectrum $E(k) \sim \varepsilon^{2/3} k^{-5/3}$

$\Delta t \sim \frac{\ell_k}{v_k} \sim \frac{1}{k v_k}$ eddy turn-over time for scale ℓ_k

$v_k^2 \sim \Delta k \, E(k) \sim k \, E(k)$ ($k \sim \Delta k$ for "local-k" interactions)

$$\Delta t \sim \frac{1}{k\sqrt{kE(k)}} \sim \frac{1}{k^{3/2} E^{1/2}}$$

$$\epsilon \sim \frac{\langle \Delta k \rangle}{\Delta t} E(k) \sim k^{5/2} E^{3/2}$$

$$\mathbf{E(k) \sim \epsilon^{2/3} k^{-5/3}}$$

Energy cascades also important in plasma turbulence, but driving and dissipation can co-exist at similar scales

Significant experimental evidence supports inertial cascade at large Reynolds

Kolmogorov scales

$l_K = (\nu^3/\varepsilon)^{1/4}$

$\tau_K = (\nu/\varepsilon)^{1/2}$

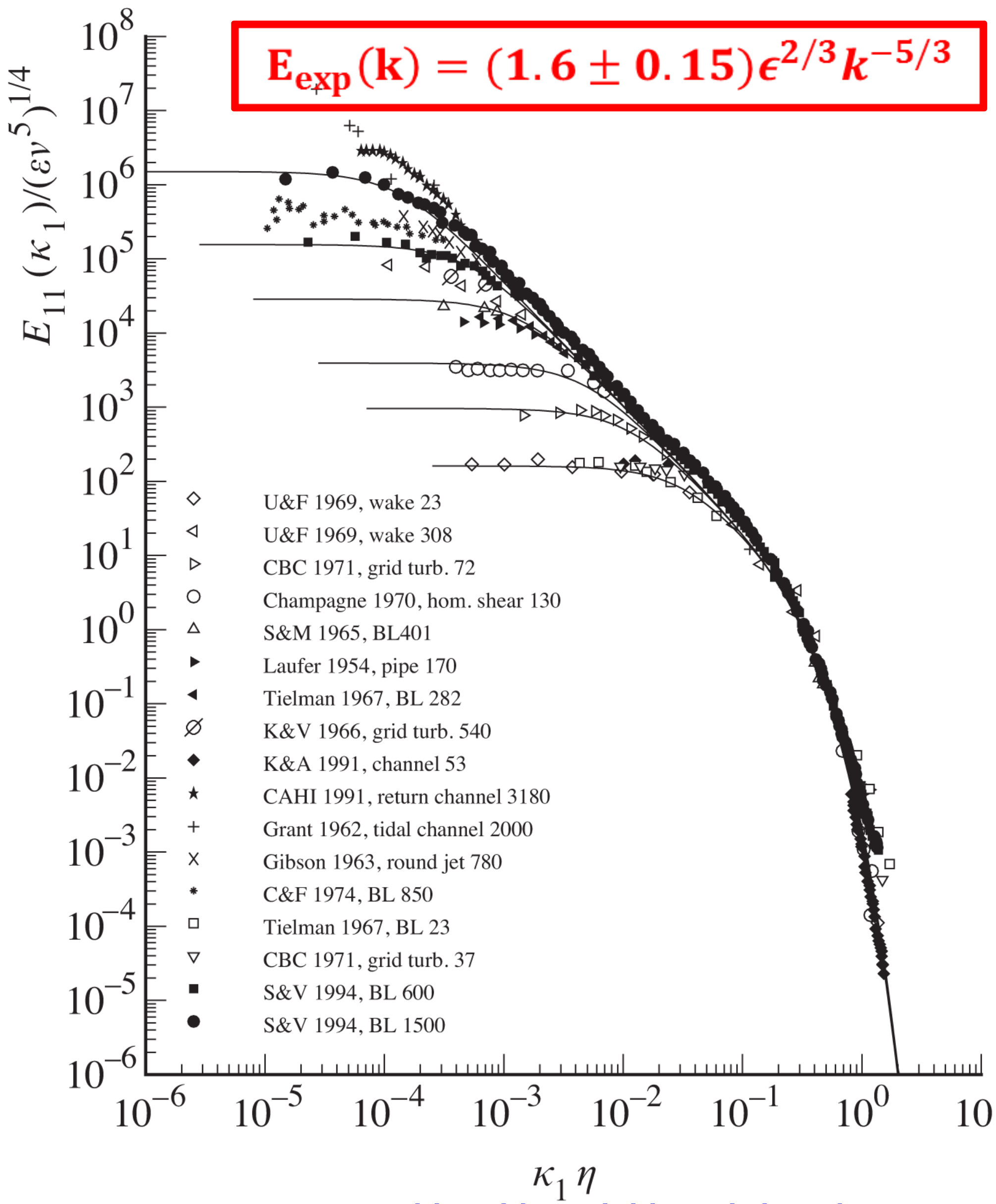
$v_K = (\nu\varepsilon)^{1/4}$

Ratio of Kolmogorov / integral scales

$l_K/l_{int} \sim Re^{-3/4}$

$\tau_K u/l_{int} \sim Re^{-1/2}$

$v_K/u \sim Re^{-1/4}$



S.G. Saddoughi, J. Fluid Mech (1994)

Significant experimental evidence supports inertial cascade at large Reynolds

Kolmogorov scales

$l_K = (\nu^3/\varepsilon)^{1/4}$

$\tau_K = (\nu/\varepsilon)^{1/2}$

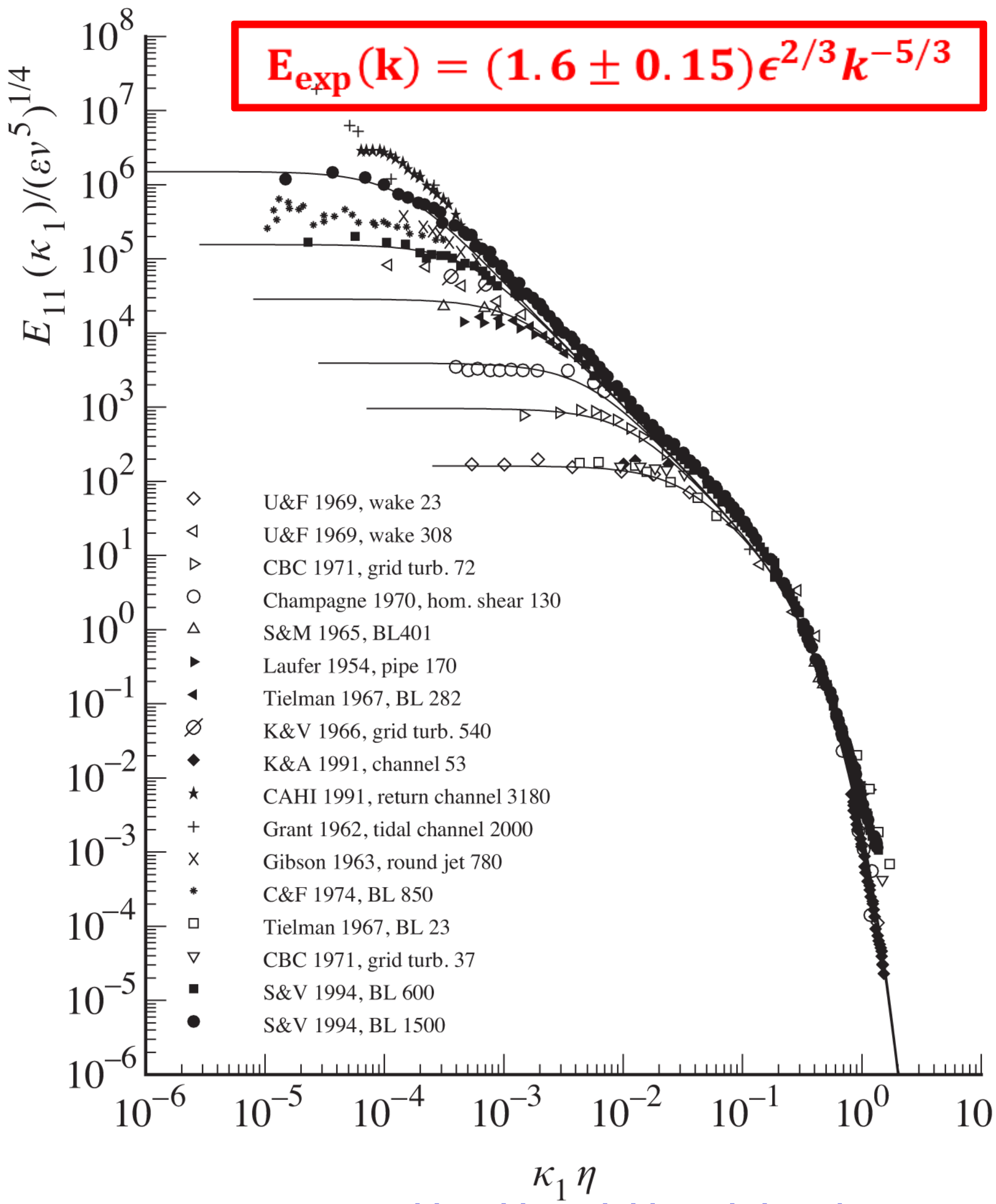
$v_K = (\nu\varepsilon)^{1/4}$

Ratio of Kolmogorov / integral scales

$l_K/l_{int} \sim Re^{-3/4}$

$\tau_K u/l_{int} \sim Re^{-1/2}$

Too expensive to do direction numerical simulation (DNS) of N-S for realistic applications → look to modeling



S.G. Saddoughi, J. Fluid Mech (1994)

How does plasma dynamics change turbulence?

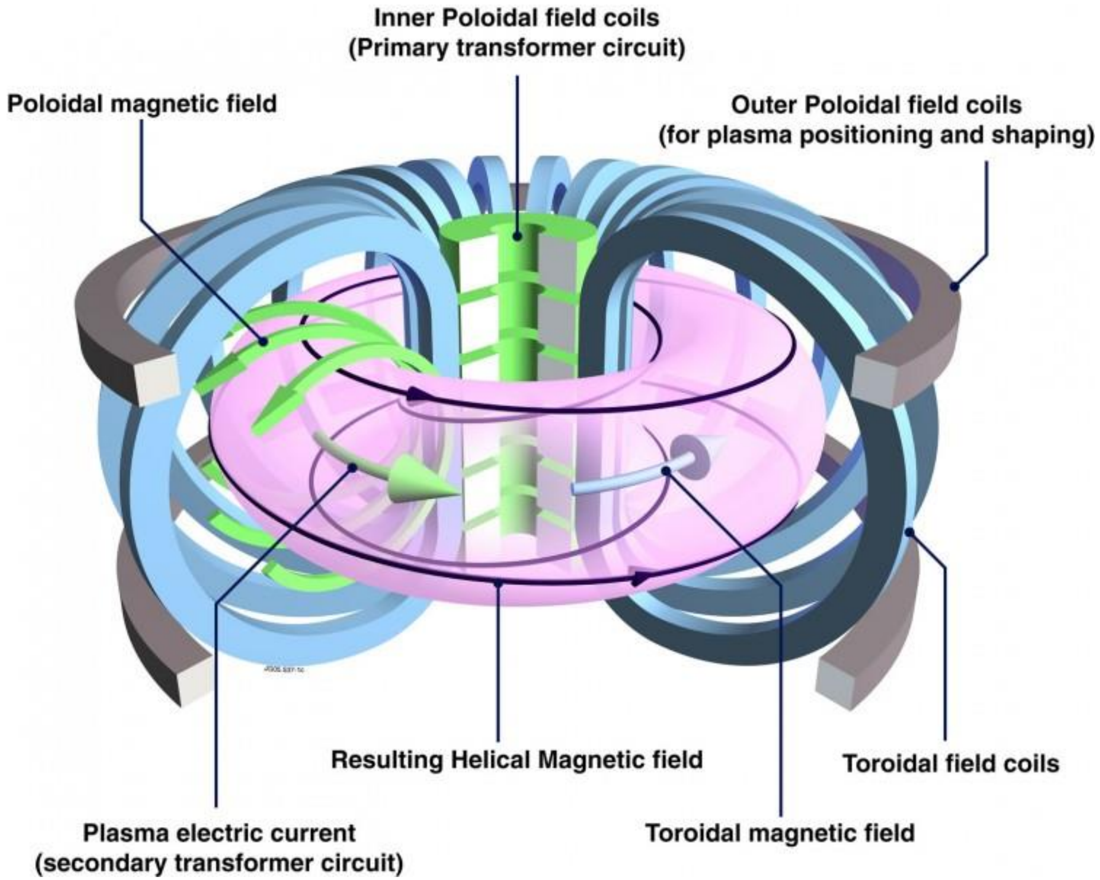
New dynamics arise in plasma turbulence

- New forces & interactions through charged particle motion
  - $\delta n_{e,i}$ & $q\delta v_{e,i}$ → δE, δj, δB → $q[E+\delta E + (v+\delta v)\times(B+\delta B)]$
  - Turbulent dynamos $\delta j\times\delta B$ in conductive plasma gas
  - Additional body forces (neutral beam injection, RF heating, …)
- Manipulated by externally applied E & B fields
  - Strong guide B-field → quasi-2D dynamics, changes inertial scaling
  - Variation in equilibrium E field → can suppress turbulence through sheared V_{ExB} flows (in 2D)
- Introduces additional scale lengths & times
  - $\rho_{i,e}$, c/ω_{pe}, λ_{mfp}, $(\rho/L)v_T$, ν_{coll}
- High temperature plasma → low collisionality → kinetic effects, additional degrees of freedom
  - New sources of instability drive / energy injection (can occur over broad range of spatial scales)
  - Different interpretation of spatial scale separation / Reynolds # → phase-space (**x**,**v**) scale separation / Dorland #
  - Different cascade dynamics & routes to dissipation (that still occurs through collisions / thermalization, but can occur at all spatial scales)

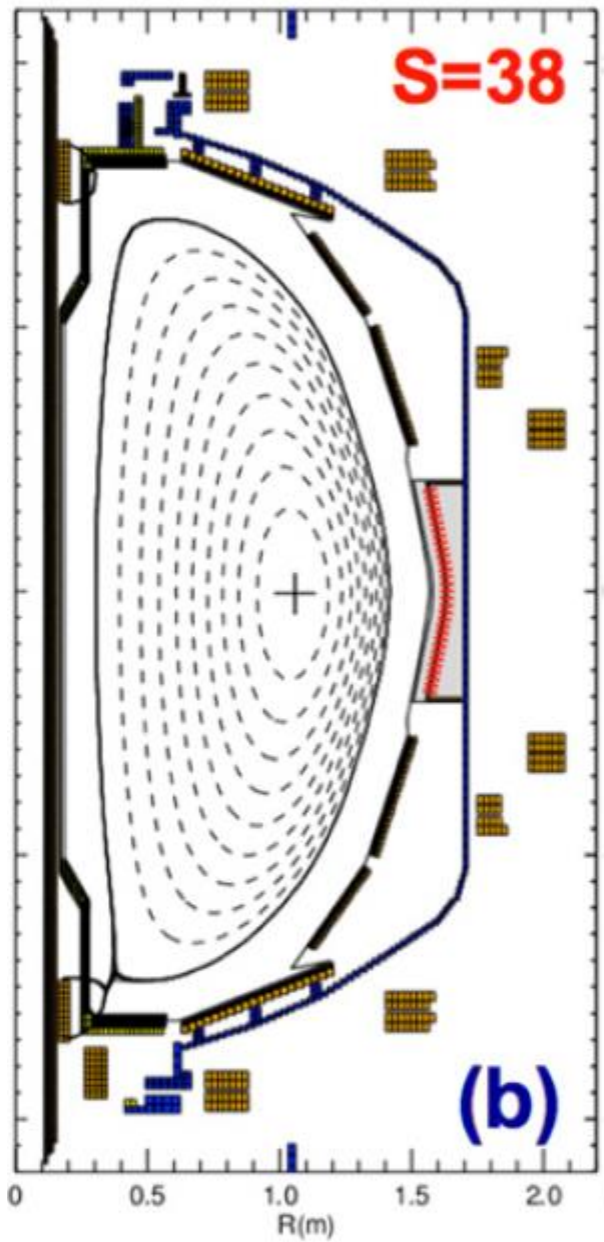
Magnetized plasma turbulence (e.g. for magnetic confinement fusion)

Tokamaks

- Toroidal, axisymmetric
- Helical field lines confine plasma
- Closed, nested flux surfaces

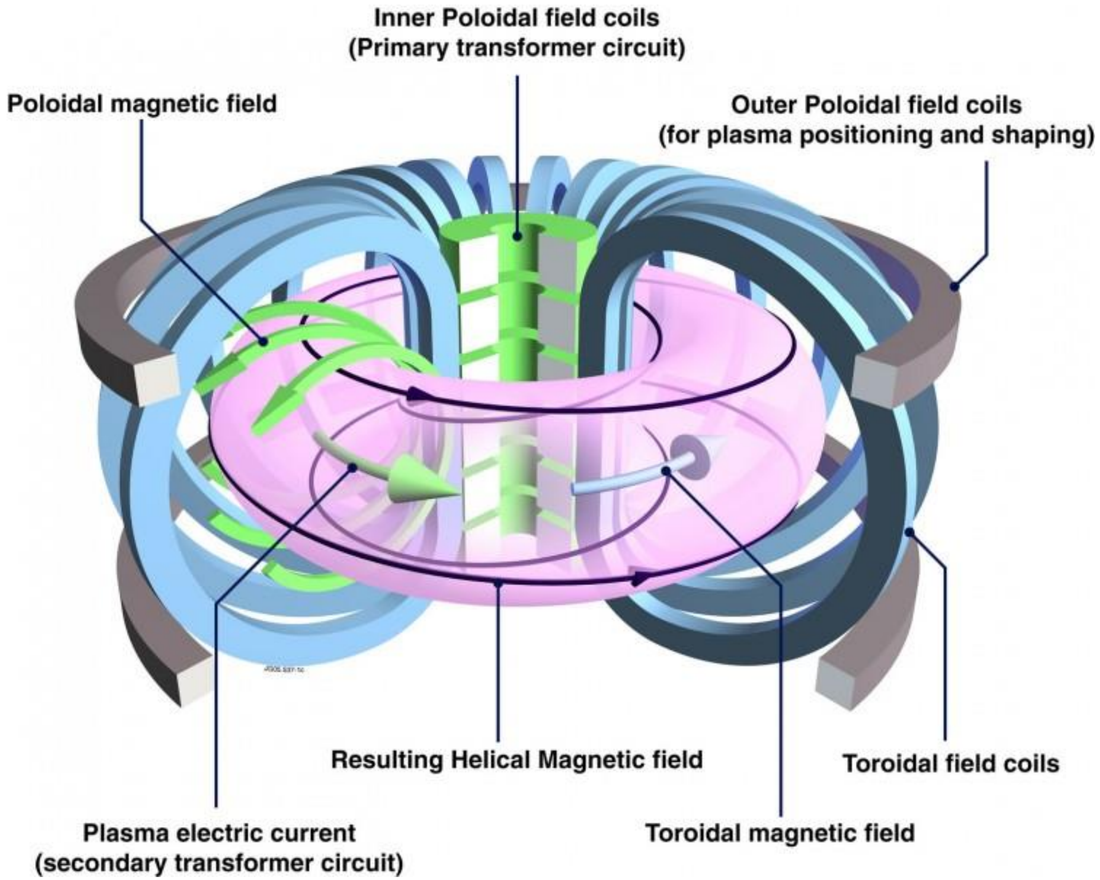


NSTX

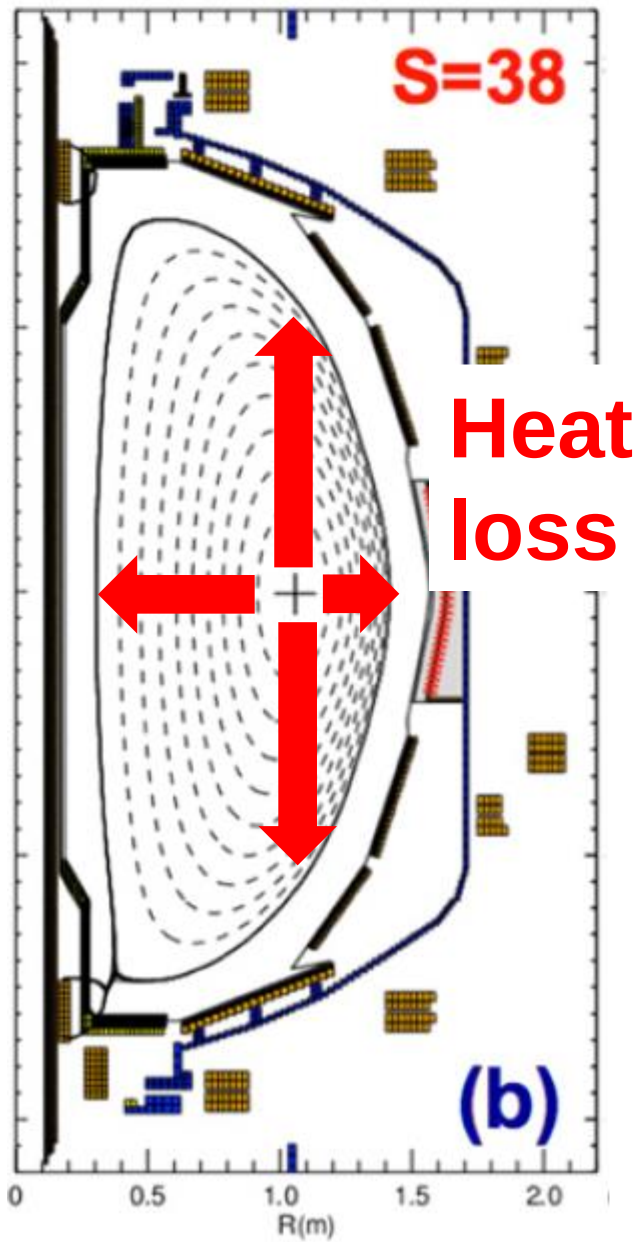


Tokamaks

- Toroidal, axisymmetric
- Helical field lines confine plasma
- Closed, nested flux surfaces



NSTX



We use 1D transport equations to interpret experiments

- Take moments of kinetic equation
- Flux surface average, i.e. everything depends only on flux surface label (ρ)

$$\frac{3}{2}n(\rho,t)\frac{\partial T(\rho,t)}{\partial t} + \nabla \cdot Q(\rho,t) = \dot{P}_{source}(\rho,t) - \dot{P}_{sink}(\rho,t)$$

We use 1D transport equations to interpret experiments

- Take moments of kinetic equation
- Flux surface average, i.e. everything depends only on flux surface label (ρ)
- Average over short space and time scales of turbulence (assume sufficient scale separation, e.g $\tau_{turb} << \tau_{transport}$, $L_{turb} << L_{machine}$) → macroscopic transport equation for evolution of equilibrium (non-turbulent) plasma state

$$\frac{3}{2} n(\rho, t) \frac{\partial T(\rho, t)}{\partial t} + \nabla \cdot Q(\rho, t) = \dot{P}_{source}(\rho, t) - \dot{P}_{sink}(\rho, t)$$

- To infer experimental transport, Q_{exp}:
  - Measure profiles (Thomson Scattering, CHERS)
  - Measure / calculate sources (NBI, RF)
  - Measure / calculate losses (P_{rad})

Inferred experimental transport larger than collisional (neoclassical) theory – extra “anomalous” contribution

$$D = -\frac{\Gamma}{\nabla n}$$

$$\chi = -\frac{Q}{n\nabla T}$$

- Reporting transport as diffusivities – does not mean the transport processes are collisionally diffusive!

TFTR
Hawryluk, Phys. Plasmas (1998)

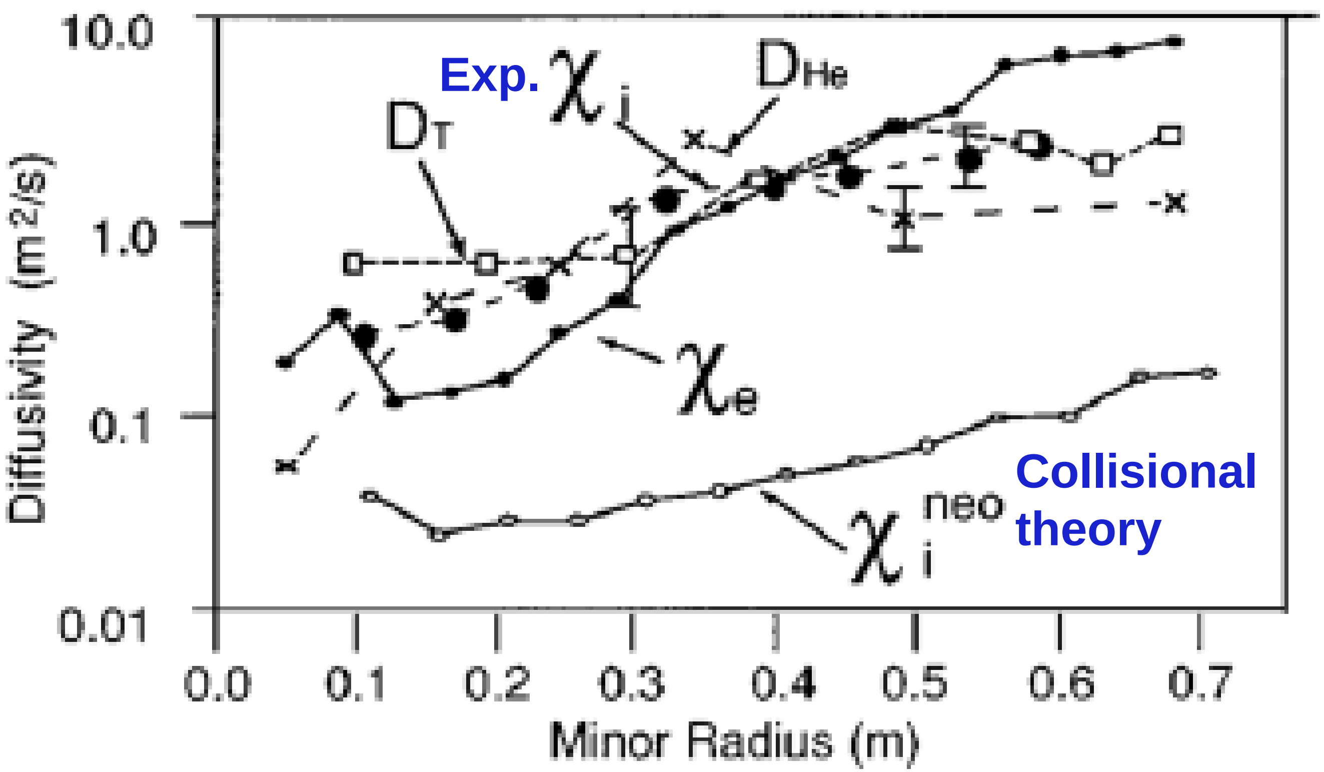
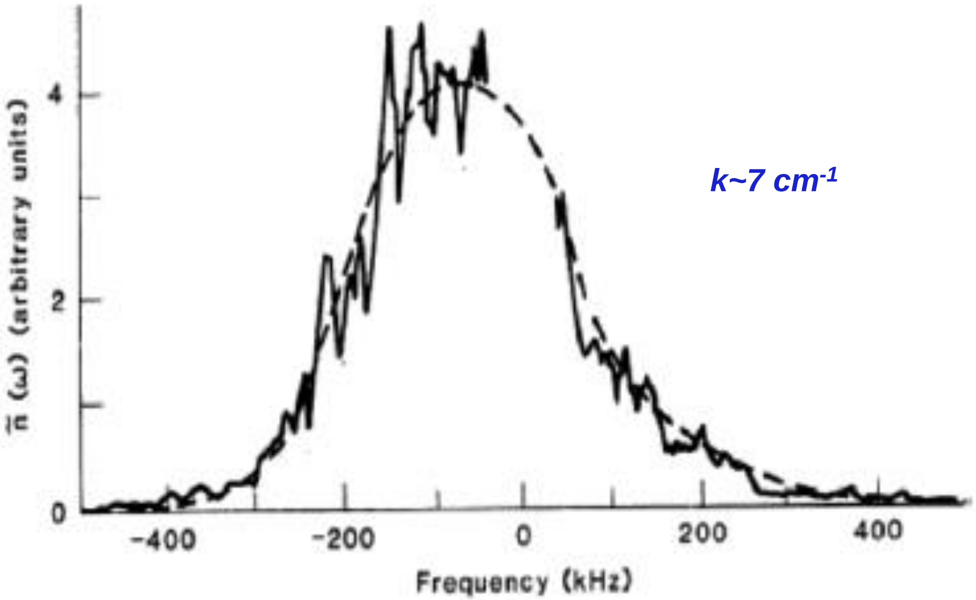


Figure 1. Results from TFTR showing ion thermal, momentum, ‹ diffusivities in an L-mode discharge; reprinted with permission fı American Institute of Physics.

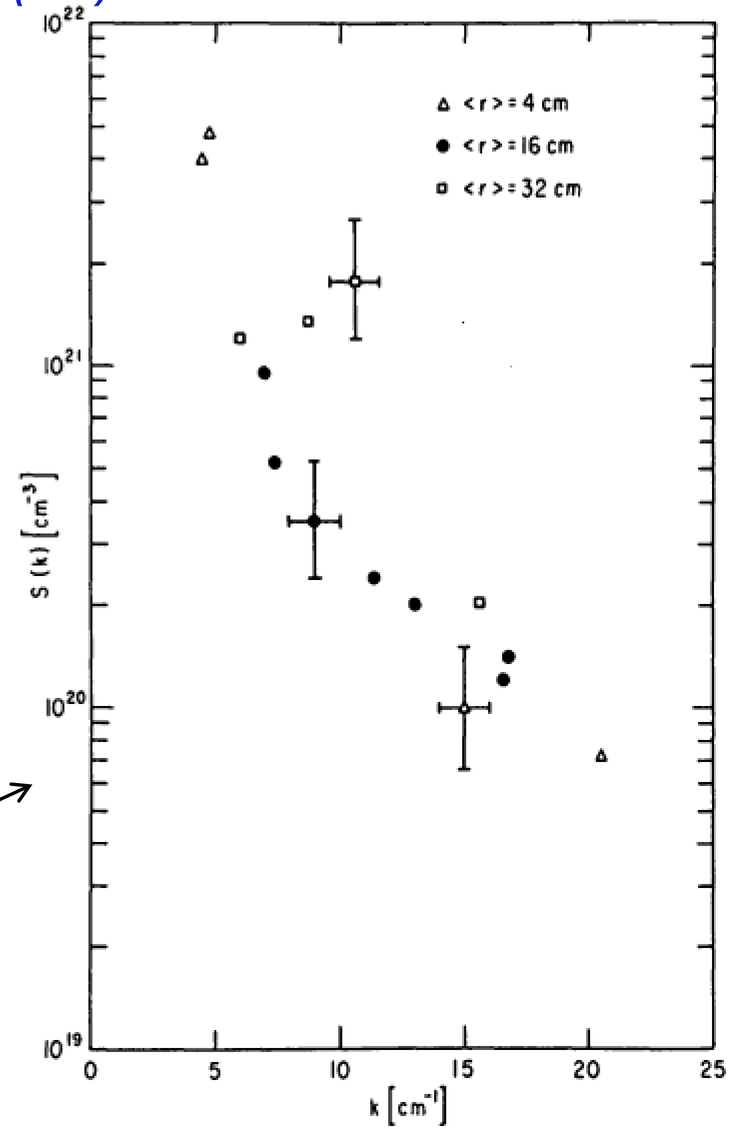
Broad frequency and wavenumber spectra measured, e.g. from microwave scattering

Mazzucato, PRL (1982)
Surko & Slusher, Science (1983)

Princeton Large Torus (PLT)



k~7 cm^{-1}

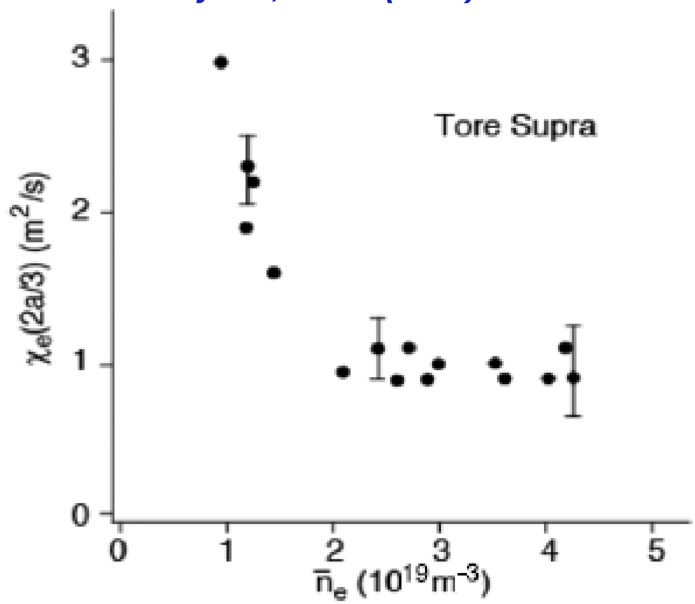


- Different scattering angles measure different k, observe spectral decay in wavenumber

Correlation between local transport and density fluctuations hints at turbulence as source of anomalous transport

Garbet, Nuclear Fusion (1992)
Tynan, PPCF (2009)

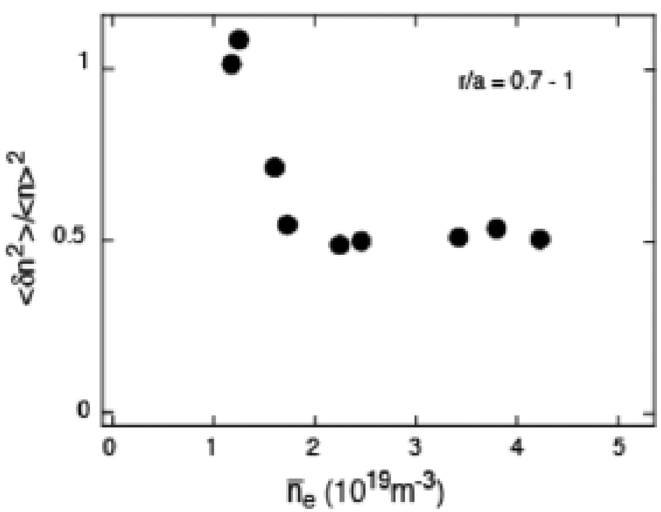
$$\chi = -\frac{Q}{n\nabla T}$$



$$Q_{exp} = Q_{collisions} + Q_{turbulence}$$

Our goal is to understand this

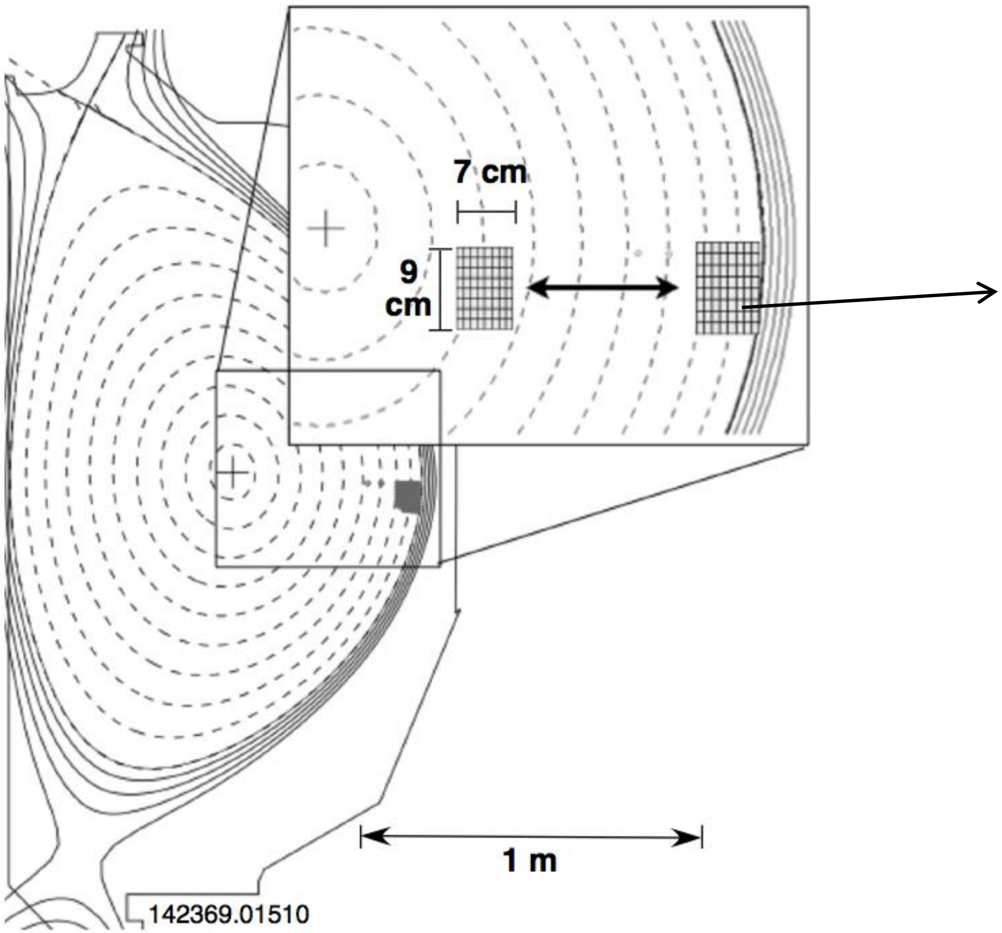
$$\frac{\langle \delta n^2 \rangle}{\langle n \rangle^2}$$

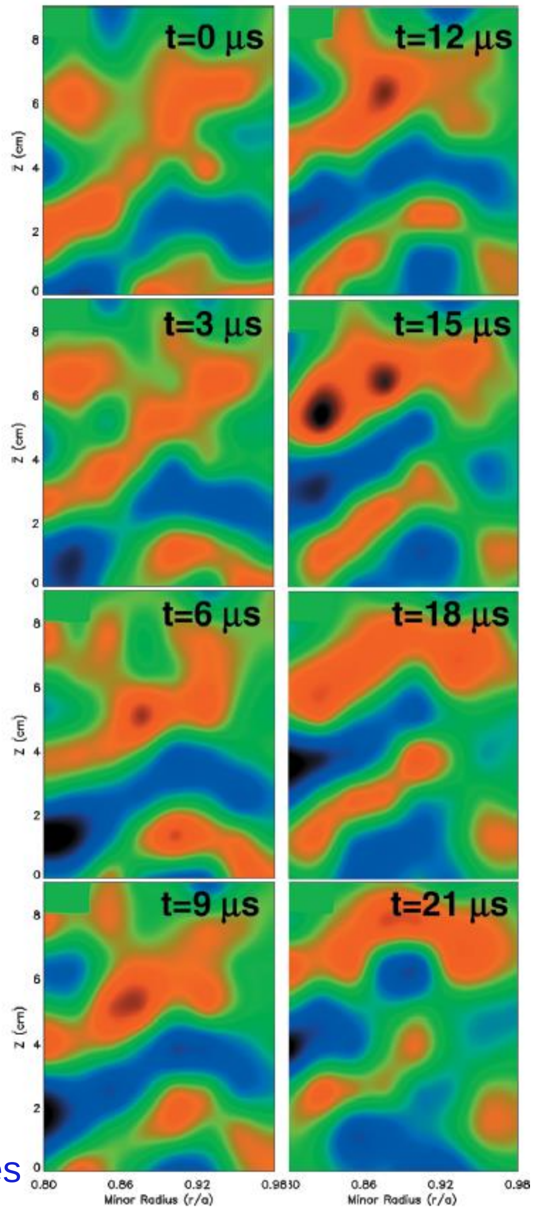


Spectroscopic imaging provides a 2D picture of turbulence in tokamaks: cm spatial scales, μs time scales, <1% amplitude

- Utilize interaction of neutral atoms with charged particles to measure density

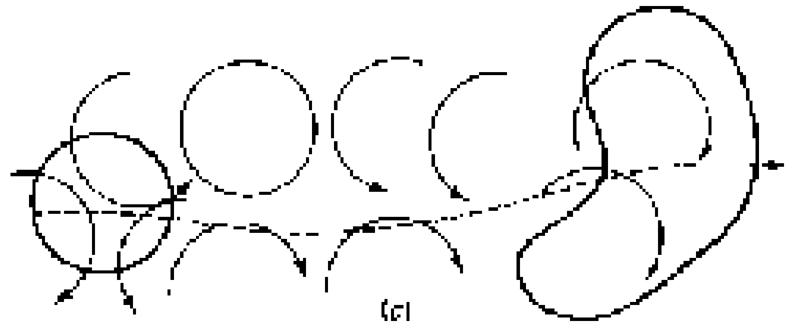
DIII-D tokamak (General Atomics)

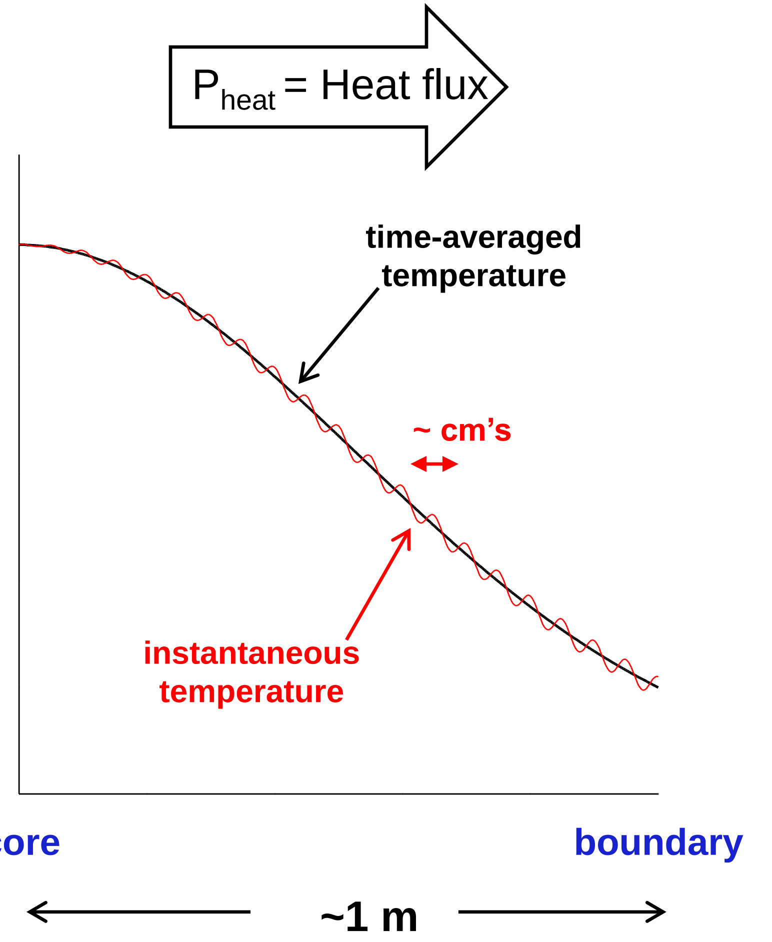




Movies at: https://fusion.gat.com/global/BESMovies

Rough estimate of turbulent diffusivity indicates it's a plausible explanation for confinement





- $D_{turbulence} \sim (\text{step size})^2 \times$ decorrelation rate

  step size ~ 5-10 gyroradii ~ few cm's
  decorrelation rate ~ 100 kHz

$$\text{confinement time} \sim \frac{1}{D_{turbulence}}$$

Turbulence confinement time estimate ~ 0.1 s
Experimental confinement time ~ 0.1 s

Measurements are challenging and limited – also use theory and simulation to help improve understanding

40+ years of theory predicts turbulence in magnetized plasma should often be drift wave in nature

General predicted drift wave characteristics:

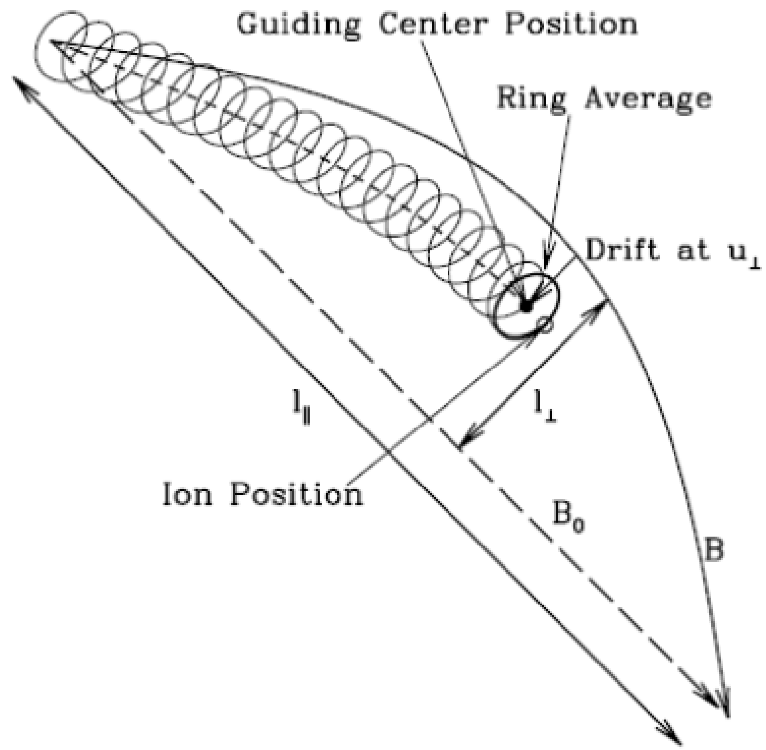
- Finite-frequency drifting waves, $\omega(k_\theta)\sim\omega_*\sim k_\theta V_*\sim(k_\theta\rho)v_T/L_n$
- Driven by ∇n, ∇T ($1/L_n = -1/n\cdot\nabla n$)
  - Can propagate in ion or electron diamagnetic direction, depending on conditions/dominant gradients
- Quasi-2D, elongated along the field lines ($L_{||}>>L_\perp$, $k_{||} << k_\perp$)
  - Particles can rapidly move along field lines to smooth out perturbations
- Perpendicular sizes linked to local gyroradius, $L_\perp\sim\rho_{i,e}$ or $k_\perp\rho_{i,e}\sim1$
- Correlation times linked to acoustic velocity, $\tau_{cor}\sim c_s/R$
- In a tokamak expected to be "ballooning", i.e. stronger on outboard side
  - Due to "bad curvature"/"effective gravity" pointing outwards from symmetry axis
  - Often only measured at one location (e.g. outboard midplane)
- Fluctuation strength loosely follows mixing length scaling ($\delta n/n_0\sim\rho_s/L_n$)
- Transport has gyrobohm scaling, $\chi_{GB}=\rho_i^2 v_{Ti}/R$
  - But other factors important! I.e. $\chi_{turb} \sim \chi_{GB}\cdot F(\cdots)\cdot[R/L_T-R/L_{T,crit}]$

Gyrokinetics in brief – evolving 5D gyro-averaged distribution function

$$\frac{\omega}{\Omega} << 1$$

$$f(\vec{x}, \vec{v}, t) \xrightarrow{\text{gyroaverage}} f(\vec{R}, v_{\parallel}, v_{\perp}, t)$$

- Average over fast gyro-motion → evolve a distribution of gyro-rings



Howes et al., Astro. J. (2006)

Gyrokinetics in brief – evolving 5D gyro-averaged distribution function

$$\frac{\omega}{\Omega}, \frac{\rho}{L}, \frac{\delta f}{f_0}, \frac{k_{||}}{k_\perp} << 1$$

$$f(\vec{x}, \vec{v}, t) \xrightarrow{\text{gyroaverage}} f(\vec{R}, v_\parallel, v_\perp, t)$$

$$f = F_M + \delta f$$

$$\frac{\partial(\delta f)}{\partial t} + v_\parallel \hat{b} \cdot \nabla \delta f + \vec{v}_d \cdot \nabla \delta f + \delta \vec{v} \cdot \nabla F_M + \vec{v}_{E0}(r) \cdot \nabla \delta f + \delta \vec{v} \cdot \nabla \delta f = C(\delta f)$$

Fast parallel motion

Slow perpendicular toroidal drifts

Advection across equilibrium gradients (∇T_0, ∇n_0, ∇V_0)

Dopper shift due to sheared equilibrium $E_r(r)$

Perpendicular non-linearity

Collisions

$$\vec{v}_\kappa = m v_{||}^2 \frac{\hat{b} \times \vec{\kappa}}{qB}$$

$$\vec{v}_{\nabla B} = \frac{m v_\perp^2}{2} \frac{\hat{b} \times \nabla B / B}{qB}$$

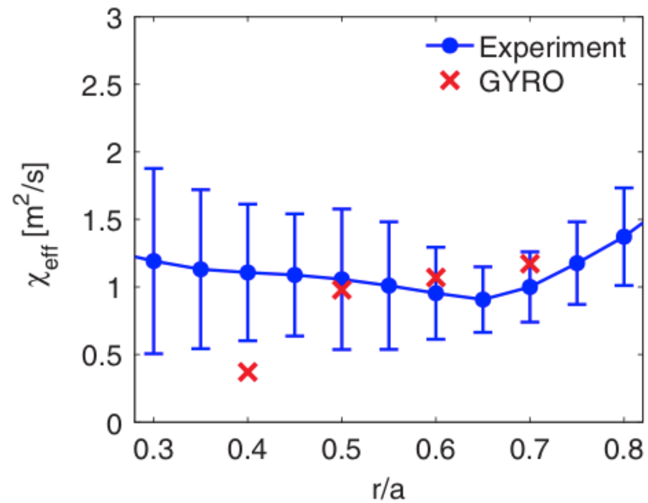
$$\delta \mathbf{v}_a \doteq \frac{c}{B} \mathbf{b} \times \nabla \Psi_a$$

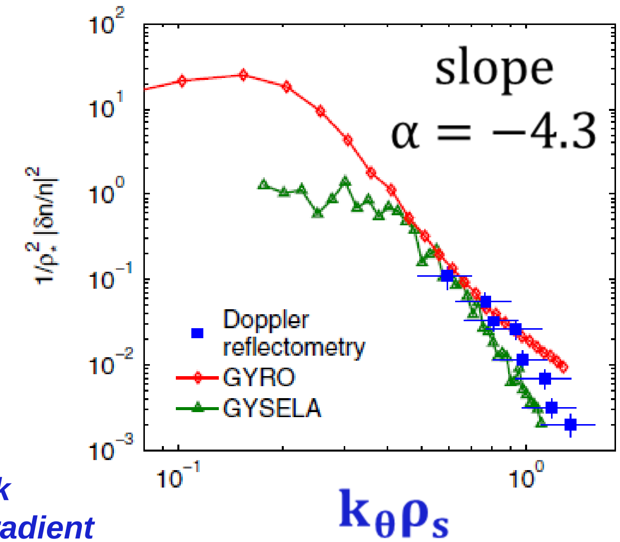
$$\Psi_a(\mathbf{R}) \doteq \left\langle \delta\phi(\mathbf{R} + \rho) - \frac{1}{c}(\mathbf{V}_0 + \mathbf{v}) \cdot \delta\mathbf{A}(\mathbf{R} + \rho) \right\rangle_{\mathbf{R}}$$

- Must also solve gyrokinetic Maxwell equations self-consistently to obtain $\delta\varphi$, δB

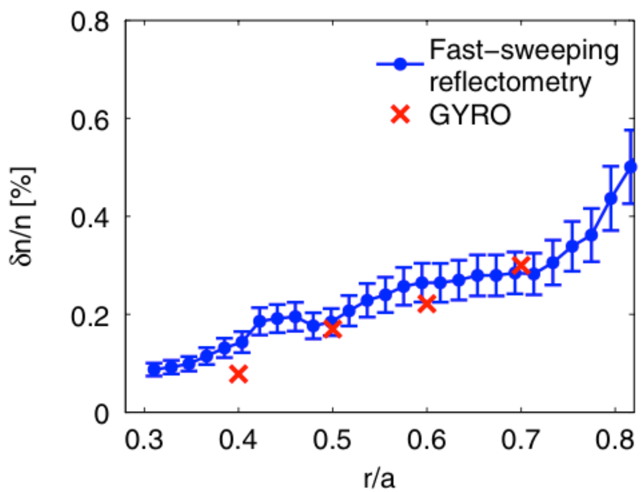
Transport, density fluctuation amplitude (from reflectometry) and spectral characteristics predicted by nonlinear gyrokinetic simulations

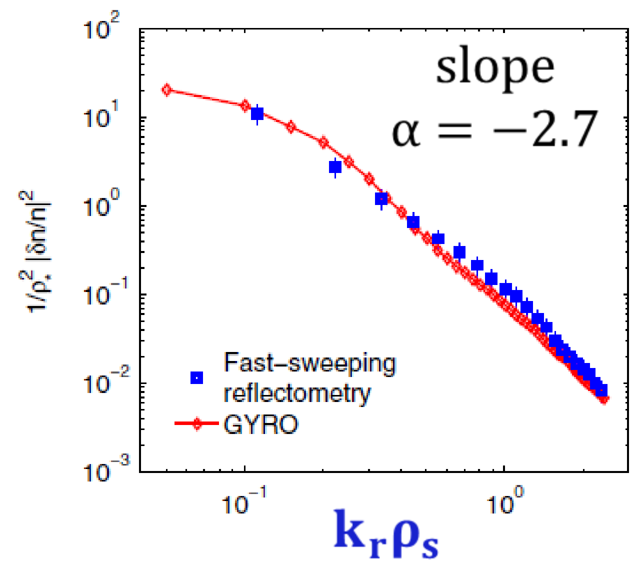
- Provides confidence in interpretation of transport in conditions when ITG instability/turbulence predicted to be most important





Casati, PRL (2009)
Tore Supra tokamak
Ion Temperature Gradient (ITG) turbulence



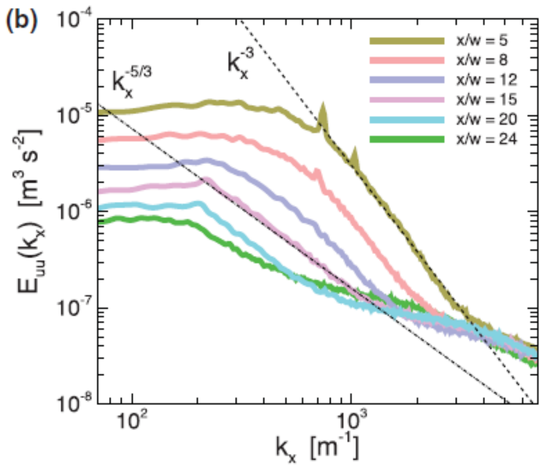


Energy cascade in 2D turbulence is different than 3D

- Loss of vortex stretching, vorticity is conserved → change in non-linear conservation properties
  - ***Inverse*** energy cascade $E(k) \sim k^{-5/3}$
  - Forward enstrophy $[\omega^2 \sim (\nabla \times v)^2]$ cascade $E(k) \sim k^{-3}$ (at larger wavenumbers, smaller scales)
  - Non-local wavenumber interactions can couple over larger range in k-space (e.g. to zonal flows)

<u>Quasi-2D turbulence exists in many places</u>

- Geophysical flows like ocean currents (Charney, 1947), tropical cyclones, polar vortex, chemical mixing in polar stratosphere (→ ozone hole)
- Soap films →



Liu et al., PRL (2016)

Gyrokinetic simulations find that nonlinear transport follows many of the underlying linear instability trends

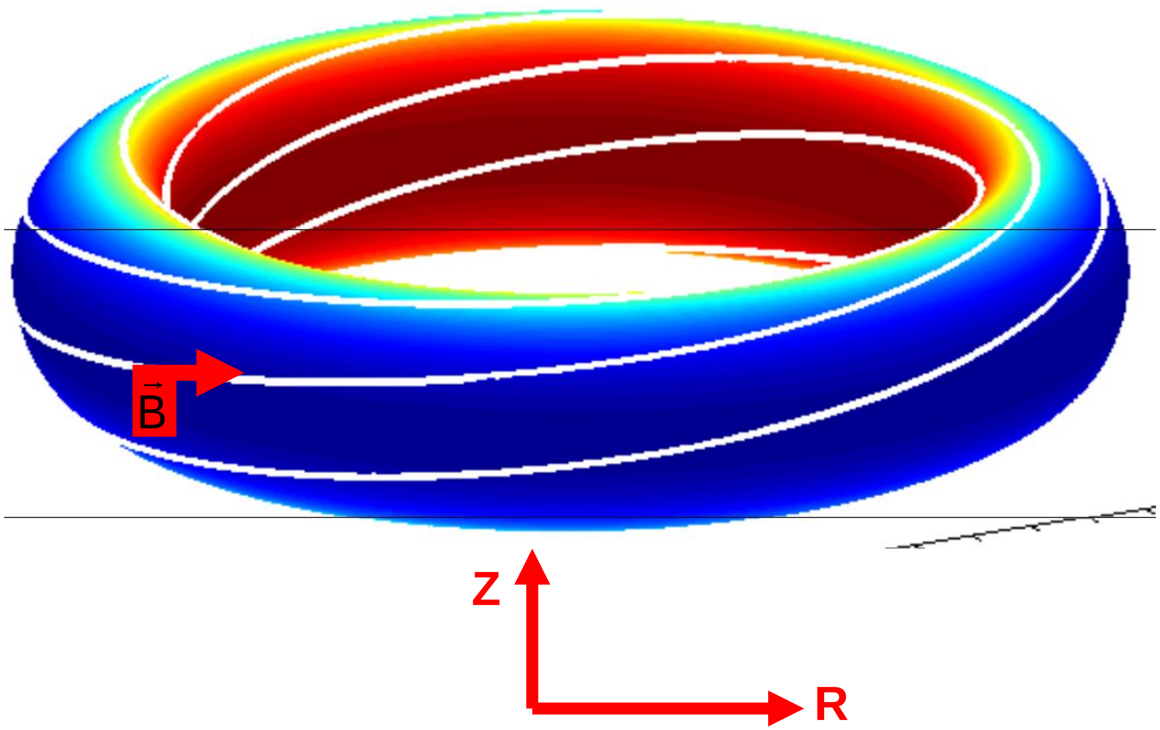
Very valuable to understand linear instabilities → Example:
Linear stability analysis of toroidal Ion Temperature Gradient (ITG) micro-instability (expected to dominate in ITER)

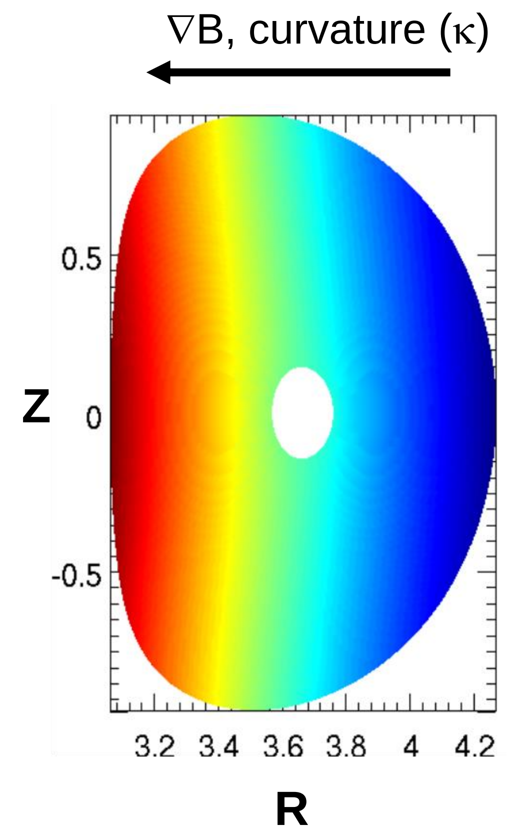
Toroidicity Leads To Inhomogeneity in |B|, gives ∇B and curvature (κ) drifts

$$\vec{v}_\kappa = m v_\parallel^2 \frac{\hat{b}\times\vec{\kappa}}{qB} \sim T_\parallel$$

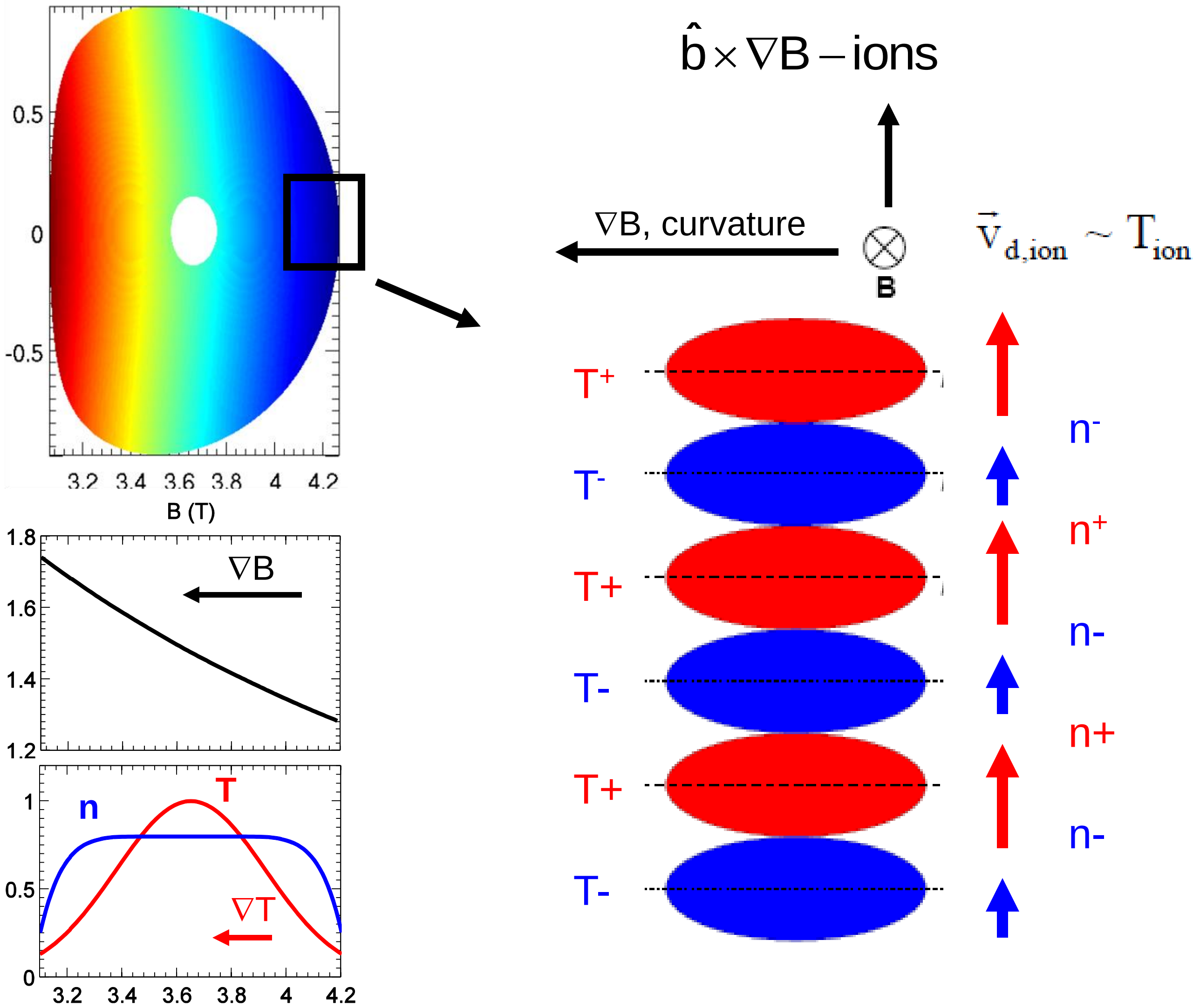
$$\vec{v}_{\nabla B} = \frac{m v_\perp^2}{2}\frac{\hat{b}\times\nabla B / B}{qB} \sim T_\perp$$

- What happens when there are small perturbations in $T_\parallel$, $T_\perp$? $\Rightarrow$ Linear stability analysis…





Temperature perturbation (δT) leads to compression ($\nabla \cdot v_{di}$), density perturbation – 90° out-of-phase with δT



- Fourier decompose perturbations in space ($k_\theta \rho_i \leq 1$)
- Assume small δT perturbation

Dynamics Must Satisfy Quasi-neutrality

- Quasi-neutrality (Poisson equation, $k_\perp{}^2\lambda_D{}^2 << 1$) requires

$$-\nabla^2\tilde{\varphi} = \frac{1}{\varepsilon_0}\sum_s eZ_s \int d^3v f_s$$

$$\left(k_\perp^2\lambda_D^2\right)\frac{\tilde{\varphi}}{T} = \frac{\tilde{n}_i - \tilde{n}_e}{n_0}$$

$$\boxed{\tilde{n}_i = \tilde{n}_e}$$

- For this ion drift wave instability, parallel electron motion is very rapid

$$\omega < k_\parallel v_{Te} \;\rightarrow\; 0 = -T_e\nabla\tilde{n}_e + n_e e\nabla\tilde{\varphi}$$

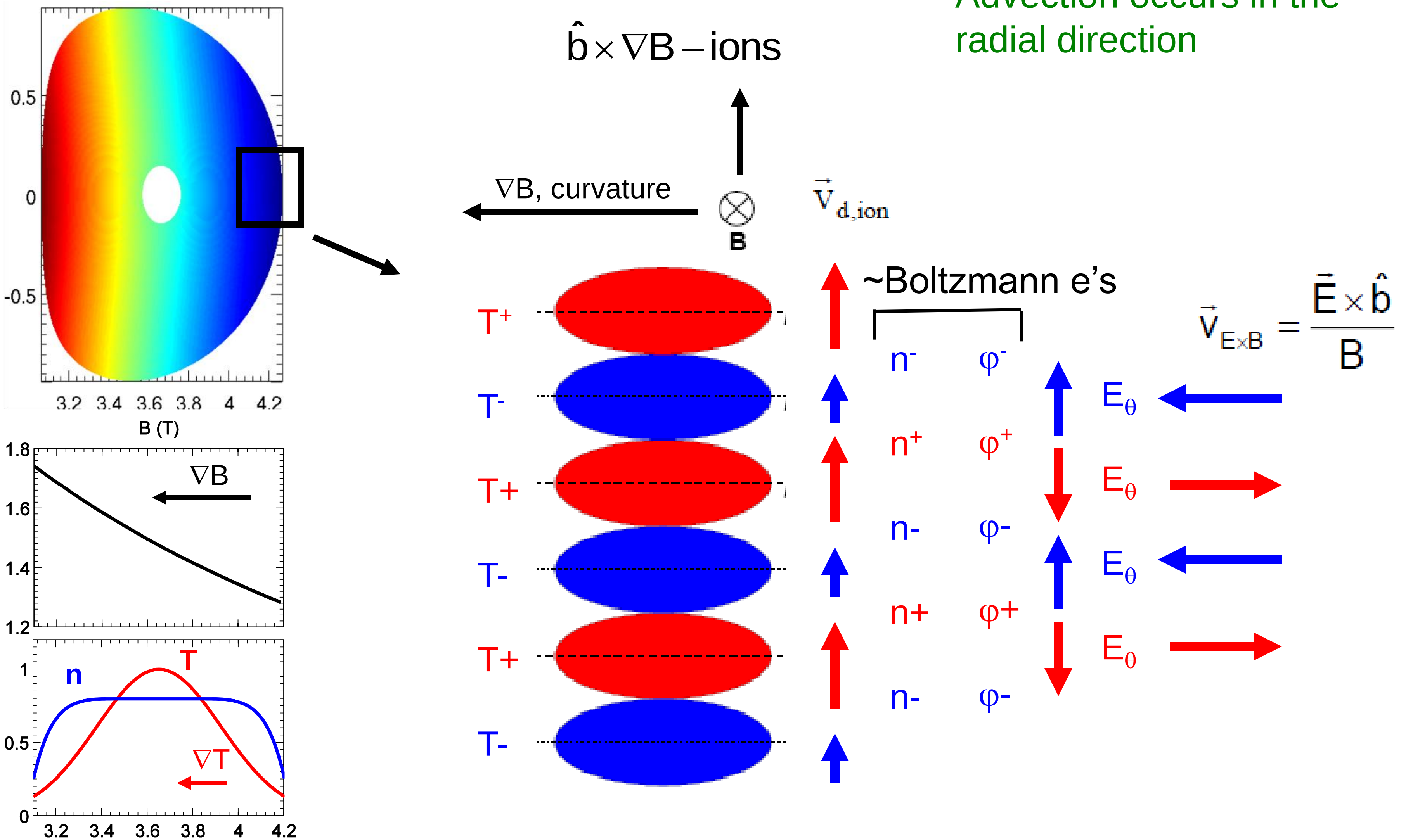
$\Rightarrow$ Electrons (approximately) maintain a Boltzmann distribution

$$\left(n_0 + \tilde{n}_e\right) = n_0 \exp\left(e\tilde{\varphi}/T_e\right)$$

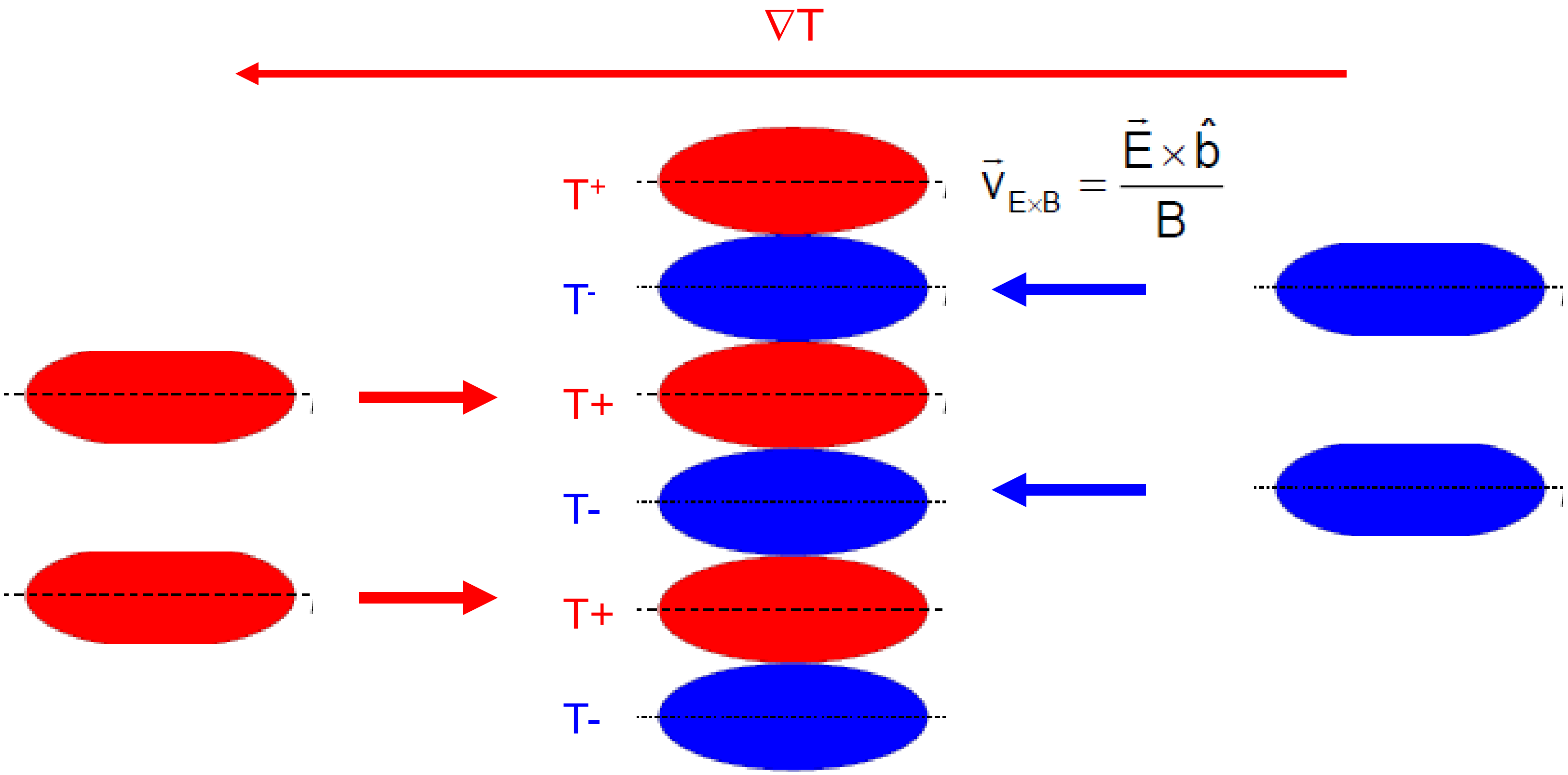
$$\tilde{n}_e \approx n_0 e\tilde{\varphi}/T_e \Rightarrow \underline{\tilde{n}_e \approx \tilde{\varphi}}$$

Perturbed Potential Creates E×B Advection

- Advection occurs in the radial direction

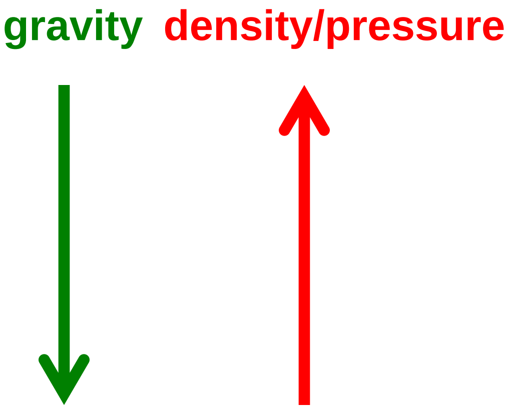


Background Temperature Gradient Reinforces Perturbation ⇒ Instability



Analogy for turbulence in tokamaks – Rayleigh-Taylor instability

- Higher density on top of lower density, with gravity acting downwards



Inertial force in toroidal field acts like an effective gravity

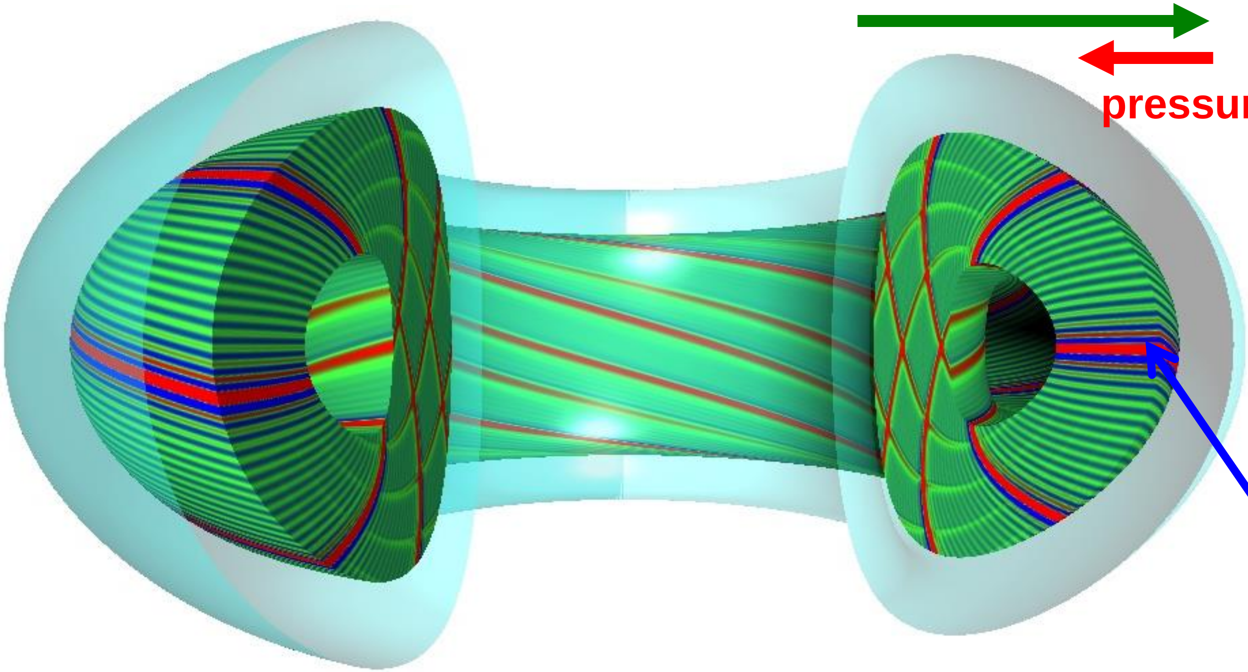
centrifugal force

→effective gravity

pressure

gravity

pressure

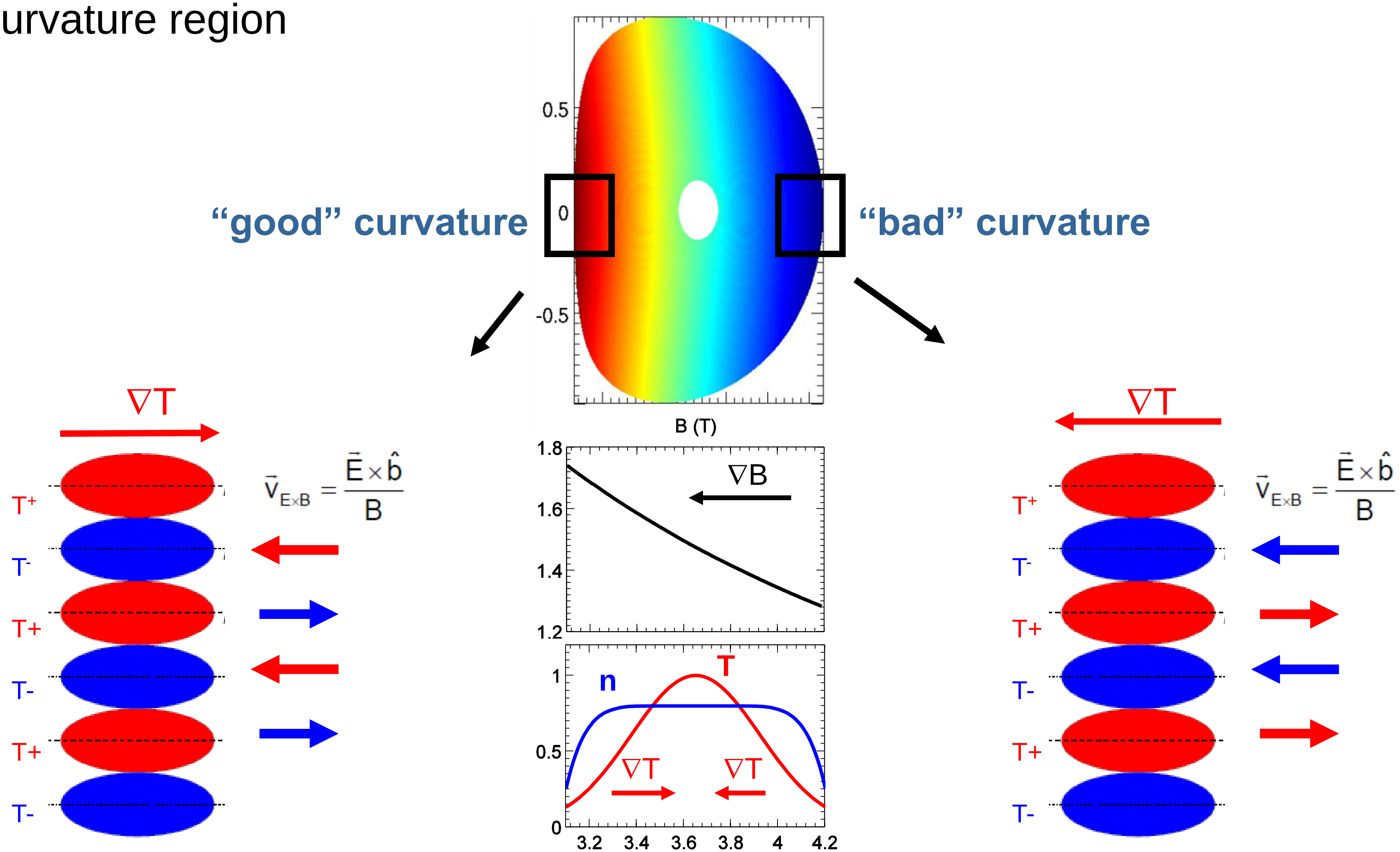


Unstable in the outer region

GYRO code
https://fusion.gat.com/theory/Gyro

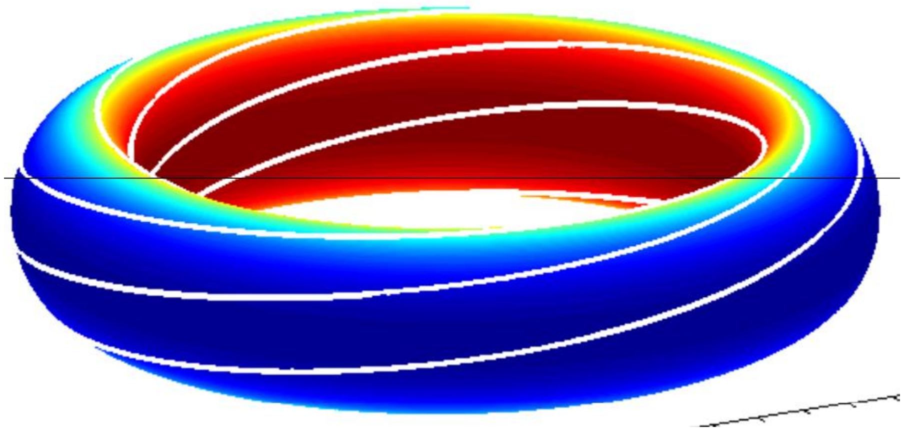
Same Dynamics Occur On Inboard Side But Now Temperature Gradient Is Stabilizing

- Advection with ∇T counteracts perturbations on inboard side – “good” curvature region



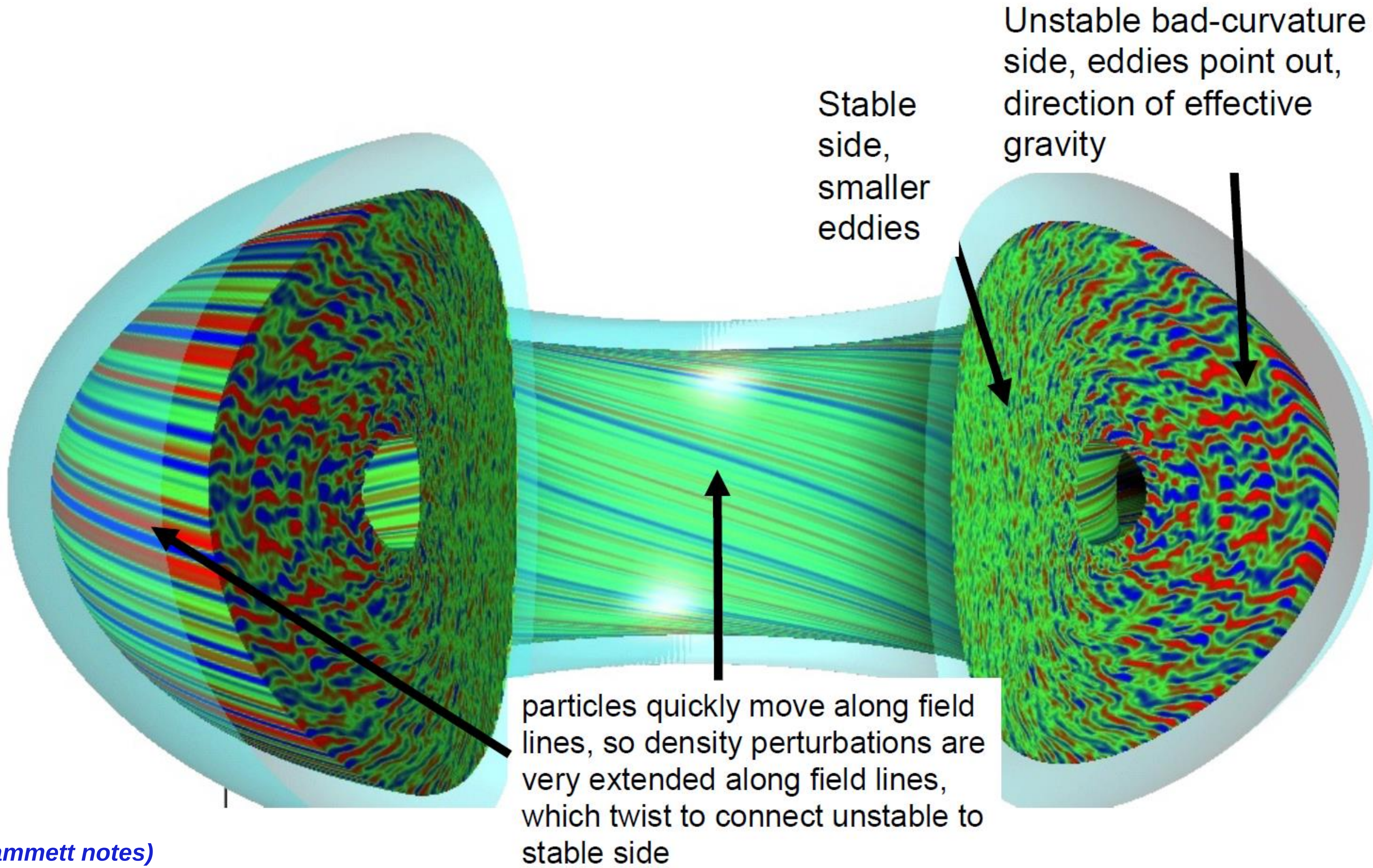
Fast Parallel Motion Along Helical Field Line Connects Good & Bad Curvature Regions

- Approximate growth rate on outboard side $\gamma_{instability} \sim \frac{v_{th}}{\sqrt{RL_T}}$ $1/L_T = -1/T \cdot \nabla T$
- Parallel transit time $\gamma_{parallel} \sim \frac{v_{th}}{qR}$



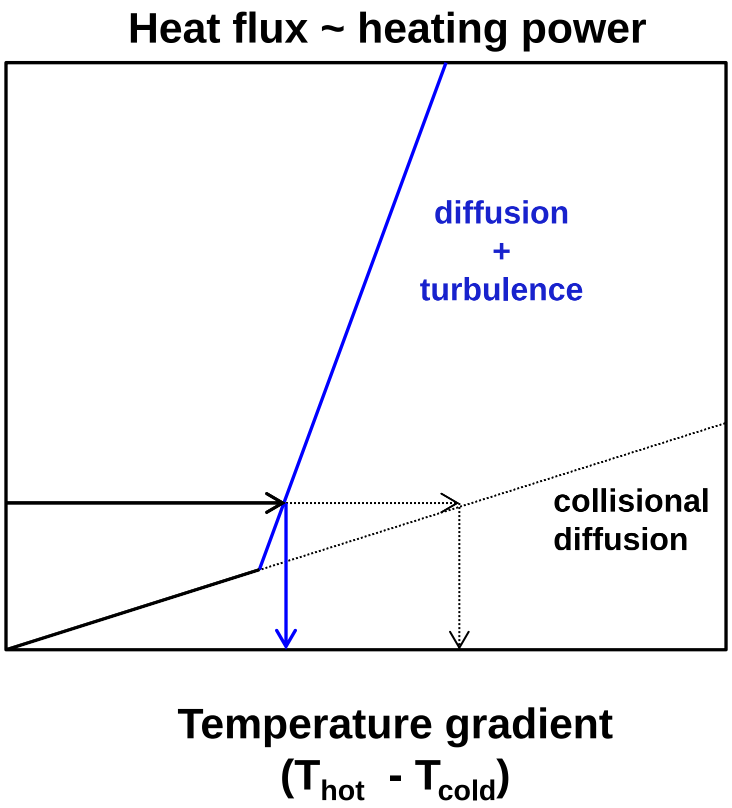
- Expect instability if $\gamma_{instability} > \gamma_{parallel}$, or $\left(\frac{R}{L_T}\right)_{threshold} \approx \frac{1}{q^2}$

Ballooning nature observed in simulations



(Hammett notes)

Threshold-like behavior analogous to Rayleigh-Benard instability



Analogous to convective transport when heating a fluid from below … boiling water (before the boiling)

Rayleigh, Benard, early 1900's

Threshold gradient for temperature gradient driven instabilities have been characterized over parameter space with gyrokinetic simulations

Critical gradient for ITG determined from many linear gyrokinetic simulations (guided by theory)

$$\left(\frac{R}{L_T}\right)_{crit}^{ITG} = Max\left[\left(1+\frac{T_i}{T_e}\right)\left(1.3+1.9\frac{s}{q}\right)(\ldots)\right]$$

Jenko (2001)
Hahm (1989)
Romanelli (1989)

- $R/L_T = -R/T \cdot \nabla T$ is the normalized temperature gradient
- Natural way to normalize gradients for toroidal drift waves, i.e. ratio of diamagnetic-to-toroidal drift frequencies:

$\omega_{*T} = k_y(B\times\nabla p) / nqB^2 \rightarrow (k_\theta\rho_i)v_T/L_T$

$\omega_D = k_y(B\times mv_\perp^2\nabla B/2B) / qB^2 \rightarrow (k_\theta\rho_i)v_T/R$

$\rightarrow \omega_{*T}/\omega_D = R/L_T$

How does magnetized turbulence saturate?

What sets spatial scales (drive vs. dissipation)?

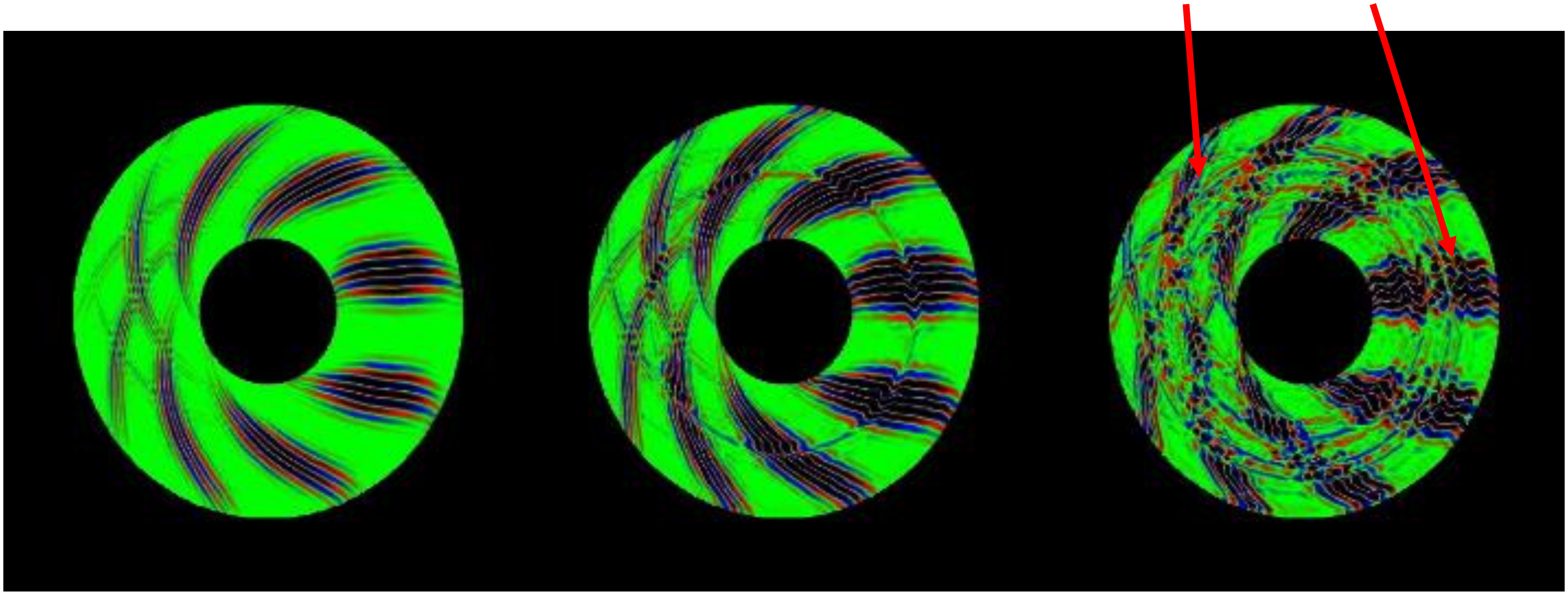
Nonlinearly-generated "zonal flows" impacts saturation of turbulence and overall transport (esp. ITG)

- Potential perturbations uniform on flux surfaces, near zero frequency (f~0)
- Predator-prey like behavior: turbulence drives ZF (linearly stable), which regulates/clamps turbulence; if turbulence drops enough, ZF drive drops, allows turbulence to grow again…

Linear instability stage demonstrates structure of fastest growing modes

Large flow shear from instability cause perpendicular "zonal flows"

Zonal flows help moderate the turbulence!!!



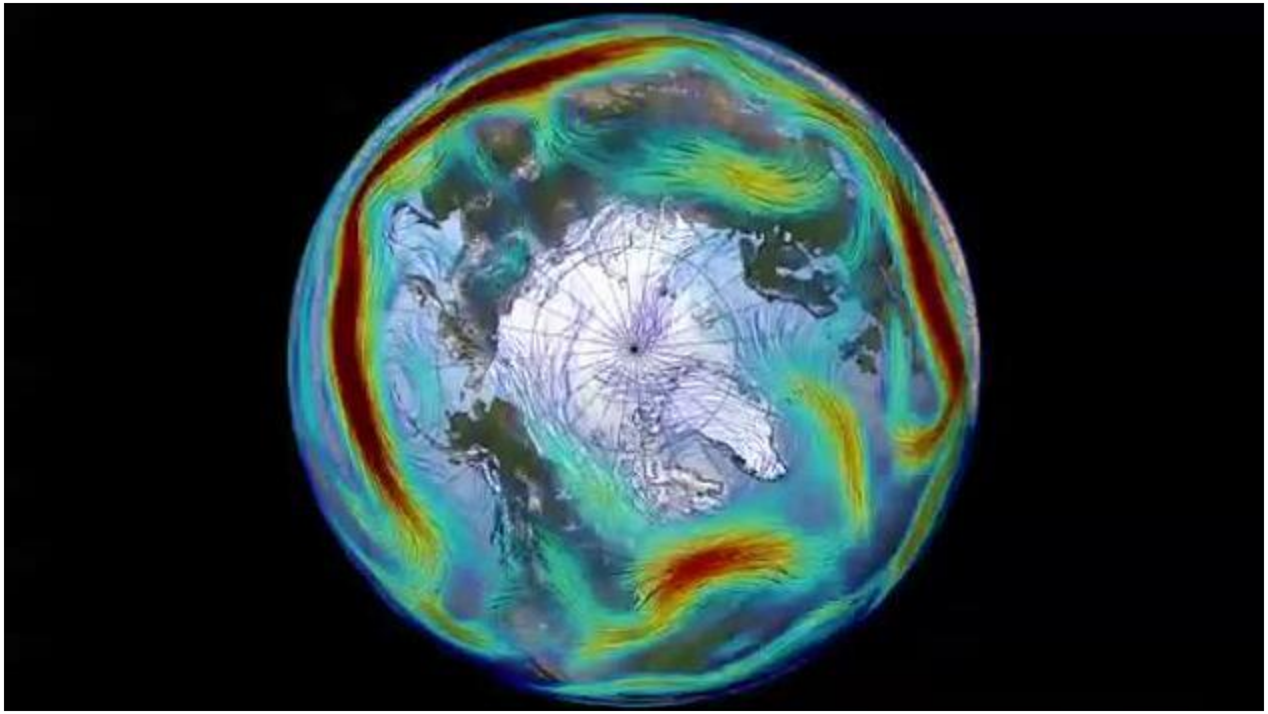
Rayleigh-Taylor like instability ultimately driving Kelvin-Helmholtz-like instability → non-linear saturation

Code: **GYRO**

Authors: Jeff Candy and Ron Waltz

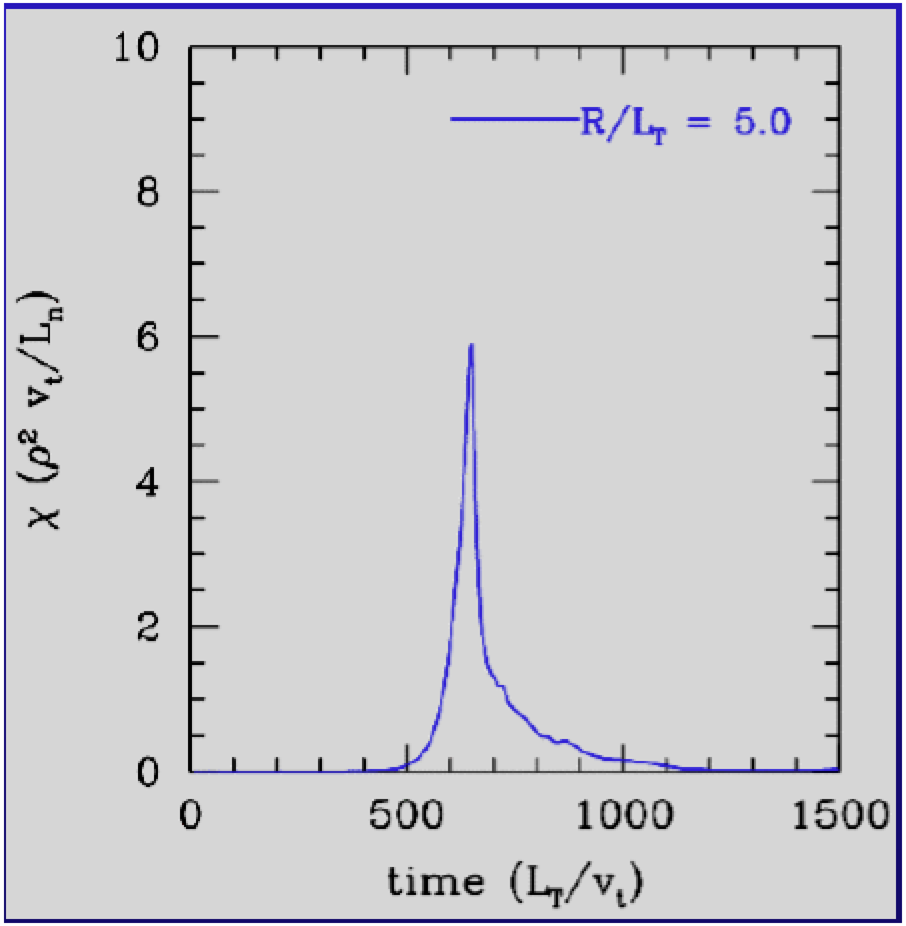
The Jet Stream is a zonal flow (or really, vice-versa)

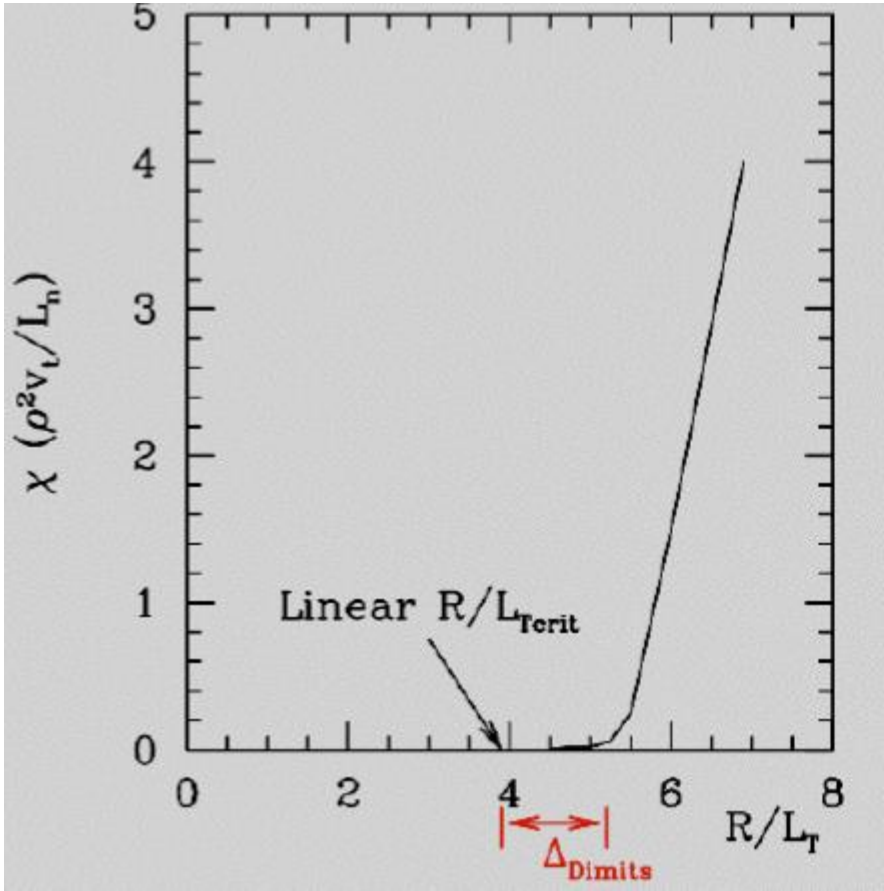
- NASA/Goddard Space Flight Center Scientific Visualization Studio



Near linear threshold, strong zonal flows can suppress primary ITG instability → low time-averaged transport

- Leads to nonlinear upshift of effective threshold
- **Predicting threshold and "stiffness" ~d(Q)/d(∇T) was a key breakthrough in understanding tokamak transport (~90s) – has also been measured**

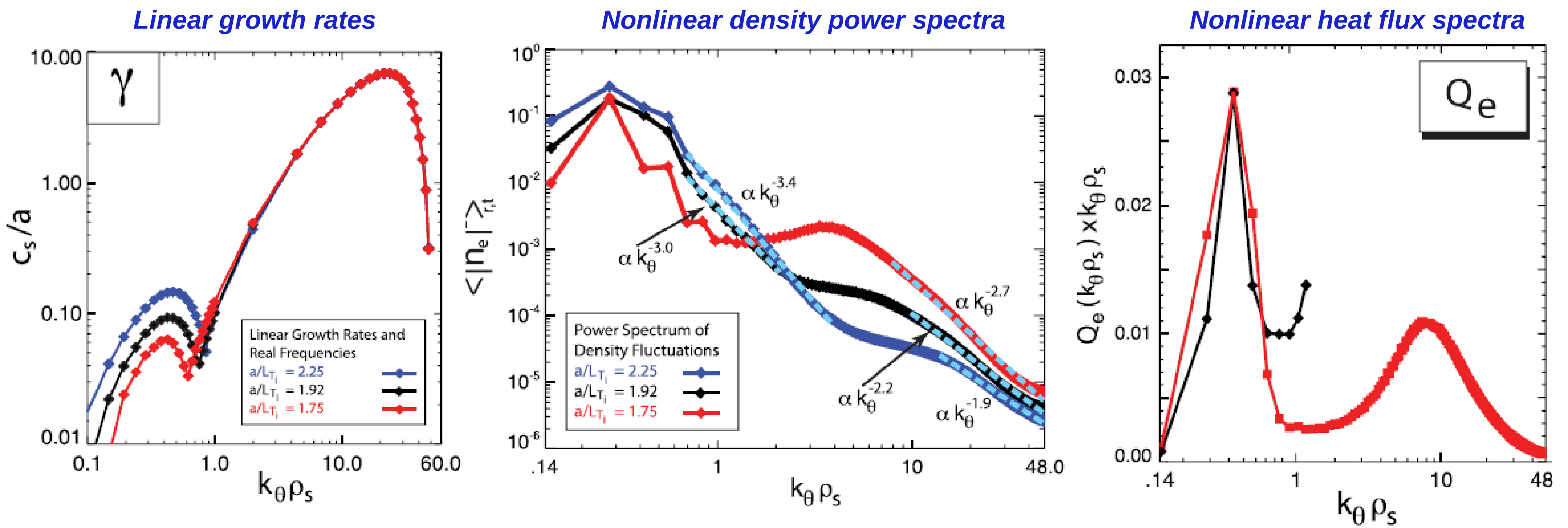




Dorland (2000)

Energy drive can occur across large range of scales, but turbulent spectra still exhibit decay

- 2D energy & enstrophy cascades remain important → nonlinear spectra often downshifted in k_θ (w.r.t. linear growth rates)
- Both drive and damping can overlap over wide range of $k_\perp$ (very distinct from neutral fluid turbulence)



Howard, PoP & NF (2016)

Addition effects proposed to model turbulence saturation & dissipation

- Coupling to damped eigenmodes (that exist at all $k_\perp$ scales, with different cross-phases) can influence spectral saturation and partitioning of transport, e.g. Q_e vs. Q_i vs. Γ, … (Terry, Hatch)

- Different routes to dissipation have been addressed theoretically:
- Critical balance (Goldreich-Sridhar, Schekochihin, M. Barnes): balance nonlinear $\perp$ dynamics with linear || dynamics
  - 2D perpendicular nonlinearity at different parallel locations creates fine parallel structure ($k_{||}$ ↑) → through Landau damping generates fine $v_{||}$ structure → dissipation through collisions
  - Can happen at all $k_\perp$ scales
  - Simple scaling argument reproduces transport scaling
- Nonlinear phase mixing (Hammett, Dorland, Schekochihin, Tatsuno):
  - At sufficient amplitude, gyroaveraged nonlinear term $\langle \delta v_E \rangle \cdot \nabla \delta f \sim J_0\left(\frac{k_\perp v_\perp}{\Omega}\right) \delta v_E \cdot \nabla \delta f$ generates structure in $\mu \sim v_\perp^2$ → dissipation through collisions

Summary

- Turbulence ubiquitous throughout the universe
  - Lots of free energy sources

- Turbulence is deterministic yet unpredictable (chaotic), appears random

- Turbulence causes increased mixing, transport larger than collisional transport
  - Transport is the key application of why we care about turbulence
  - **Understanding and reducing transport critical for fusion reactors**

- Turbulence spans a wide range of spatial and temporal scales
  - Large Reynolds # (3D neutral fluids) / Dorland # (6D kinetic plasmas)
  - 6D kinetic plasmas lead to additional degrees of freedom for driving and dissipation mechanisms

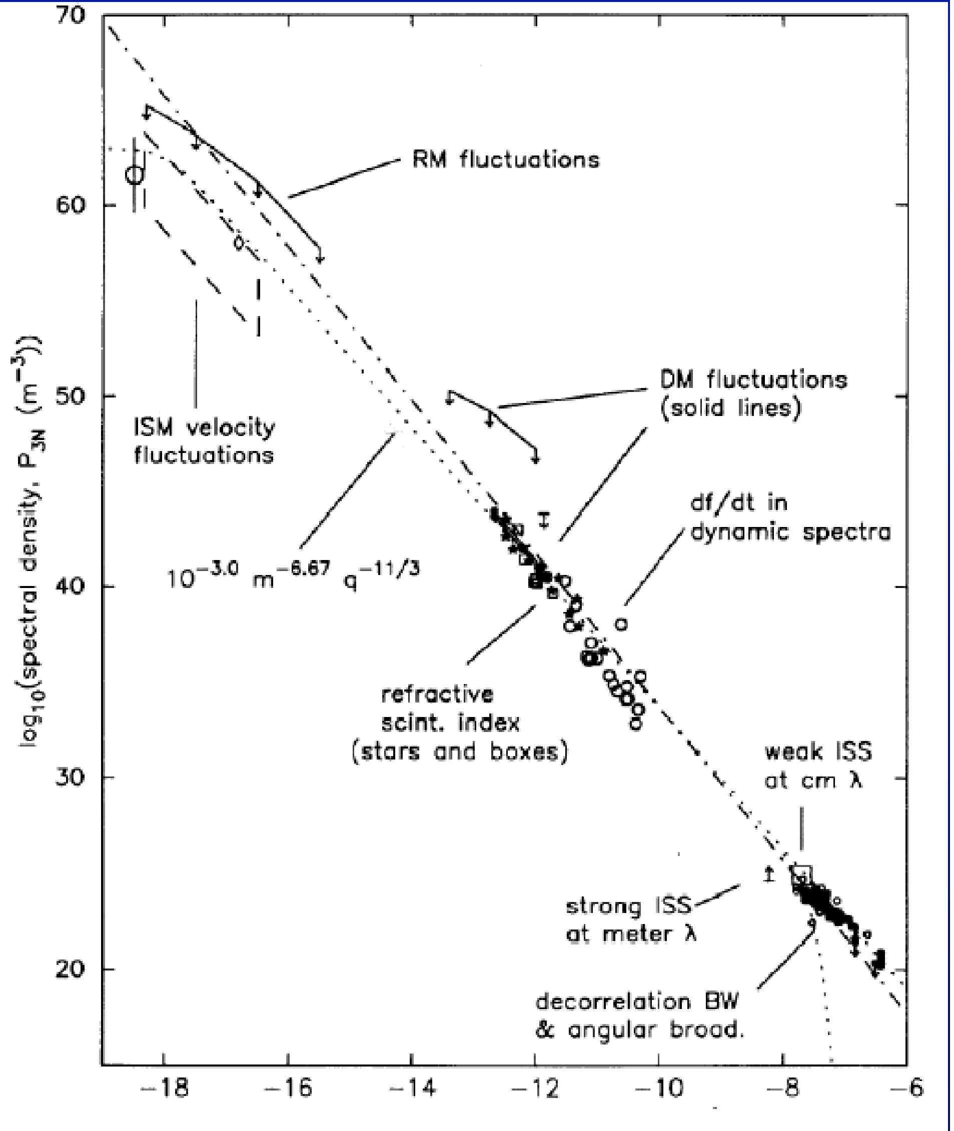
Turbulence in the Interstellar Medium
Power law for 12 orders of magnitude!

Power Spectrum of Electron Density Fluctuations

Density fluctuations change the index of refraction of the plasma & thus modify the propagation of radio waves: "Interstellar scintillation/ scattering"

Consistent with Kolmogorov

$$P_{\rm tot} = \int d^3k\, P_{3N}$$
$$= \int dk\, 4\pi k^2 P_{3N}$$
$$k^2 k^{-11/3} = k^{-5/3}$$



Wavenumber q (m^{-1})

Armstrong, Rickett, Spangler 1995

(Hammett class notes)

Have learned a lot from validating first-principles gyrokinetic simulations with experiment

- But the simulations are expensive (1 local multi-scale simulation ~ 20M cpu-hrs)
- Desire a model capable of reproducing flux-gradient relationship that is far quicker, so we can do integrated predictive modeling ("flight simulator")
- All physics based models are local & gradient-driven, i.e. given gradients *<u>from a single flux surface</u>* they predict fluxes:

$$\begin{bmatrix} \Gamma \\ \Pi_\varphi \\ Q_i \\ Q_e \end{bmatrix} = -\begin{bmatrix} \text{flux} - \text{gradient} \\ \text{relationship} \\ \text{matrix} \end{bmatrix} \cdot \begin{bmatrix} \nabla n \\ R\nabla\Omega \\ \nabla T_i \\ \nabla T_e \end{bmatrix}$$

that can be used in solving the 1D transport equation predictively

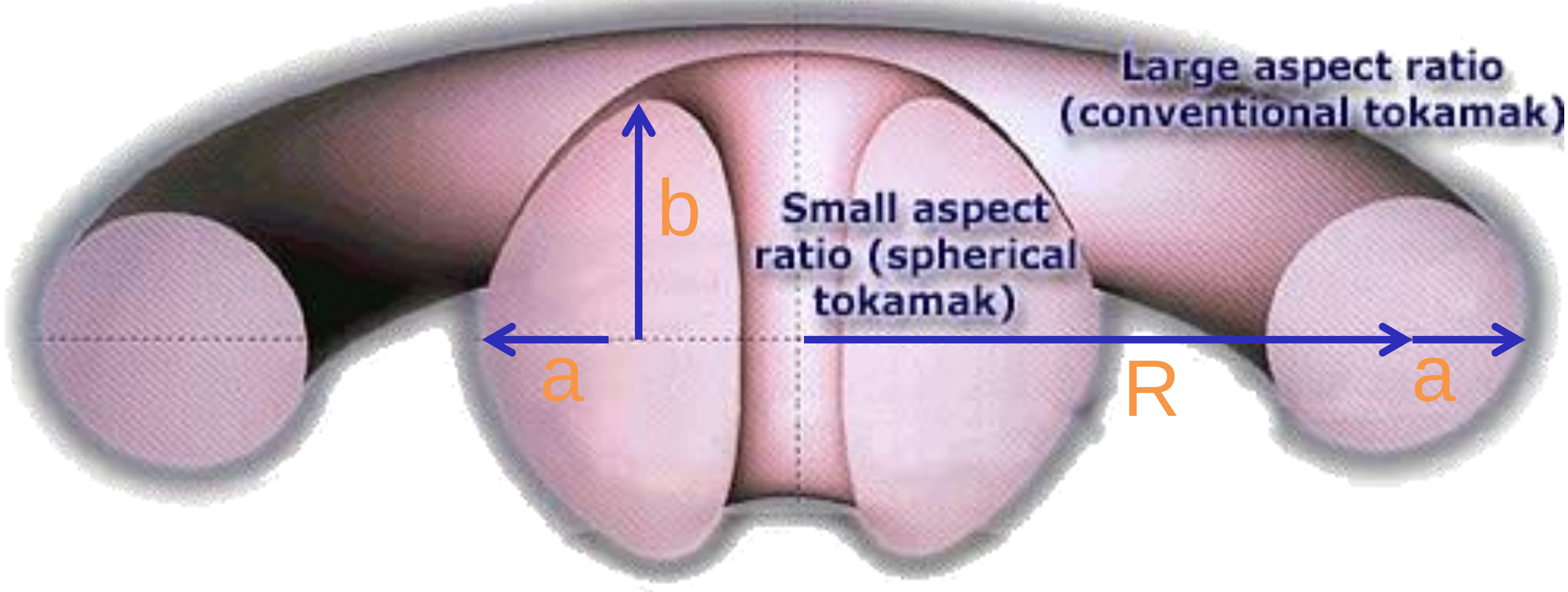
$$\frac{3}{2} n(\rho,t) \frac{\partial T(\rho,t)}{\partial t} + \nabla \cdot Q(\rho,t) = \dot{P}_{source}(\rho,t) - \dot{P}_{sink}(\rho,t)$$

Low aspect ratio equilibrium can stabilize electrostatic drift waves, minimize transport (i.e. one motivation for NSTX-U at PPPL)

Aspect ratio is an important free parameter, can try to make more compact devices (i.e. hopefully cheaper)

Aspect ratio $A = R / a$
Elongation $\kappa = b / a$

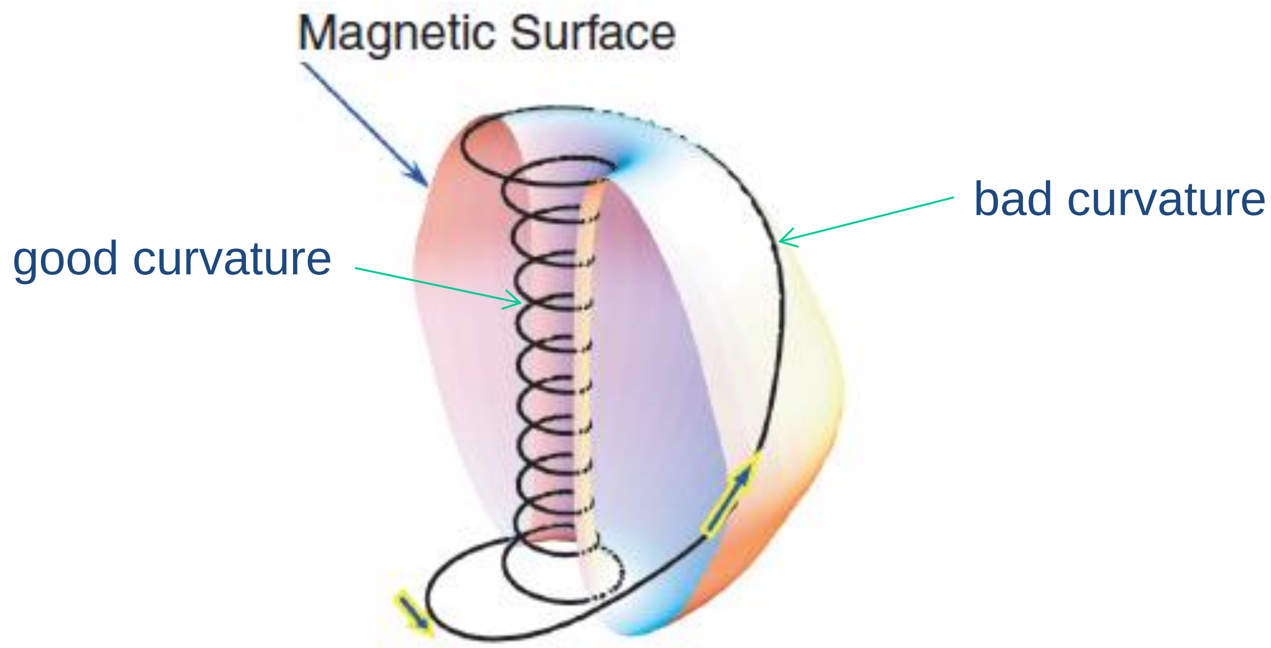
R = major radius, a = minor radius, b = vertical ½ height



But smaller R = larger curvature, ∇B (~1/R) -- isn't this terrible for "bad curvature" driven instabilities?!?!?!

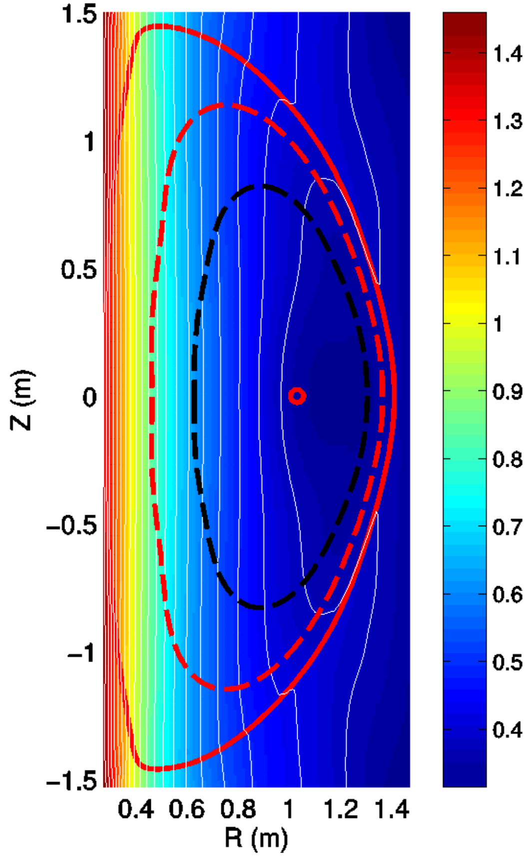
Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

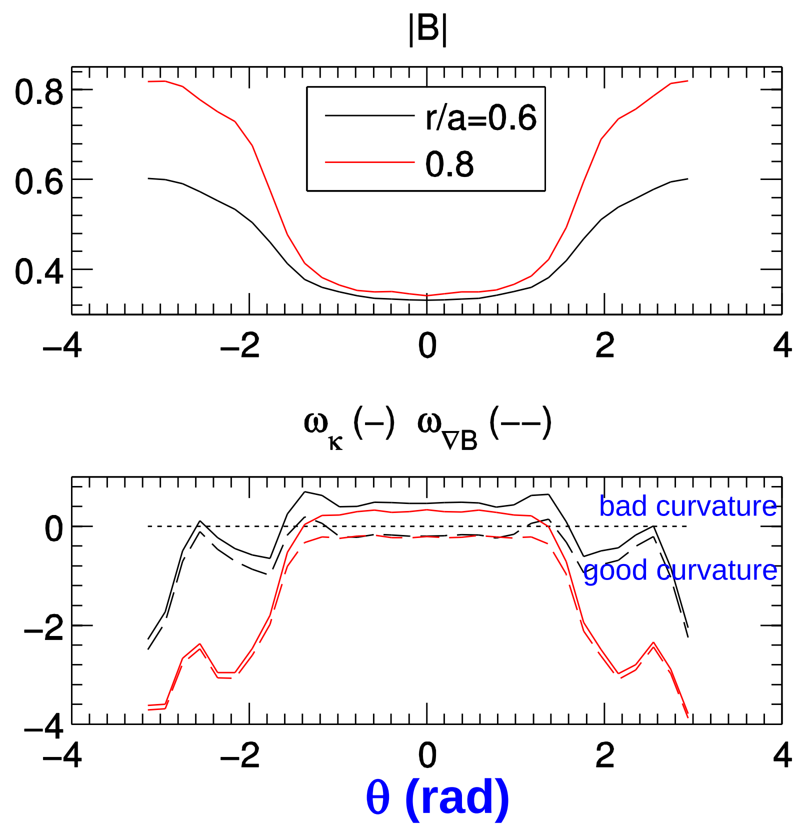
- Short connection length → **smaller average bad curvature**



Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

- Short connection length → **smaller average bad curvature**
- Quasi-isodynamic (~constant B) at high β → **grad-B drifts stabilizing [Peng & Strickler, NF 1986]**



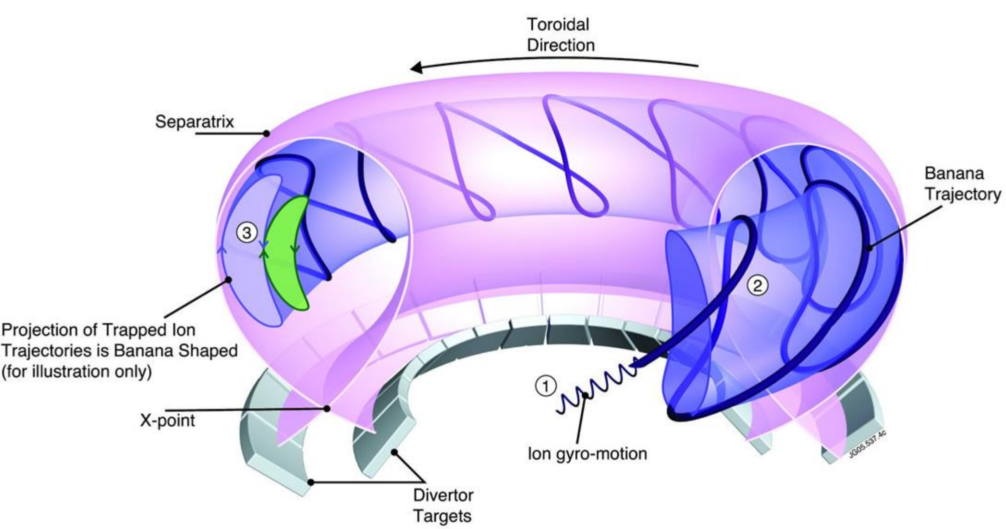


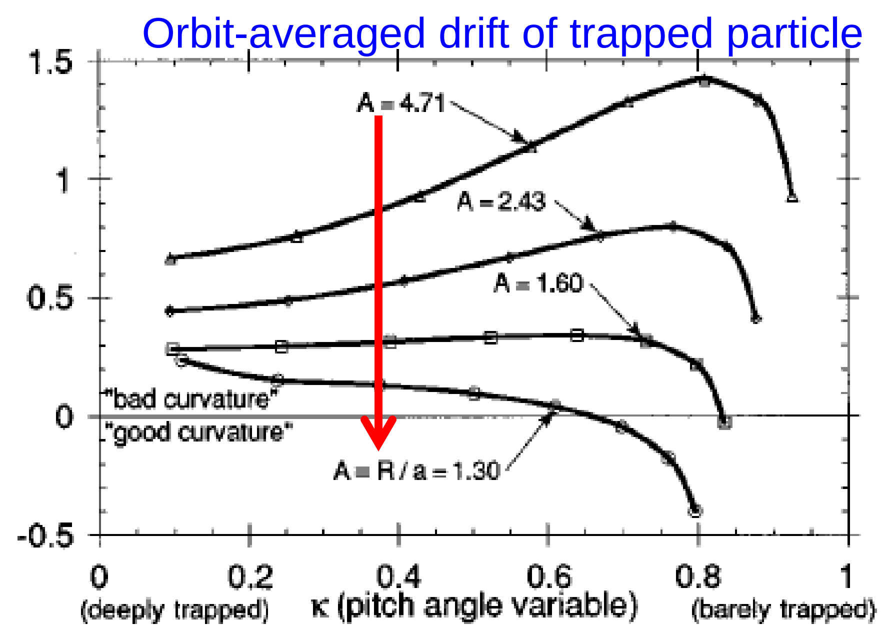
$$\vec{v}_\kappa = m v_{||}^2 \frac{\hat{b}\times\vec{\kappa}}{qB}$$

$$\vec{v}_{\nabla B} = \frac{m v_\perp^2}{2}\,\frac{\hat{b}\times\nabla B/B}{qB}$$

Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

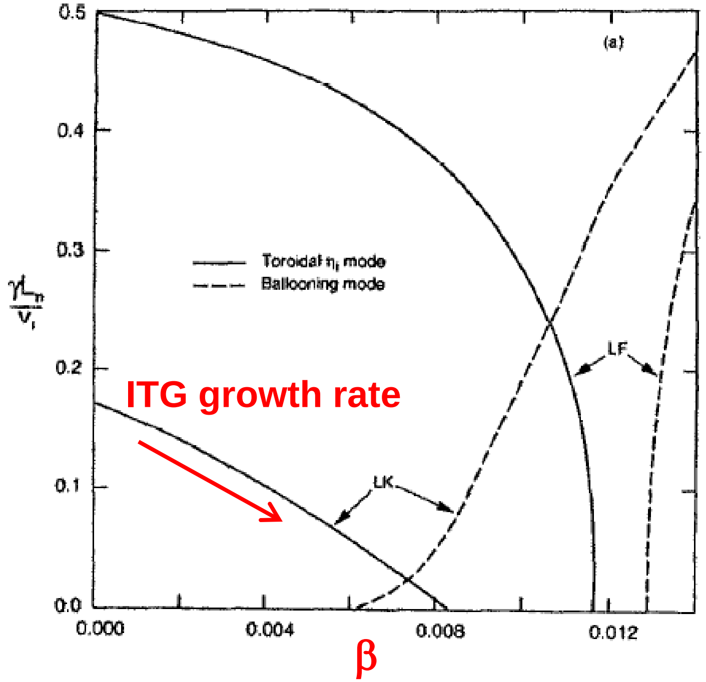
- Short connection length → **smaller average bad curvature**
- Quasi-isodynamic (~constant B) at high β → **grad-B drifts stabilizing [Peng & Strickler, NF 1986]**
- Large fraction of trapped electrons, BUT precession weaker at low A → **reduced TEM drive [Rewoldt, Phys. Plasmas 1996]**





Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

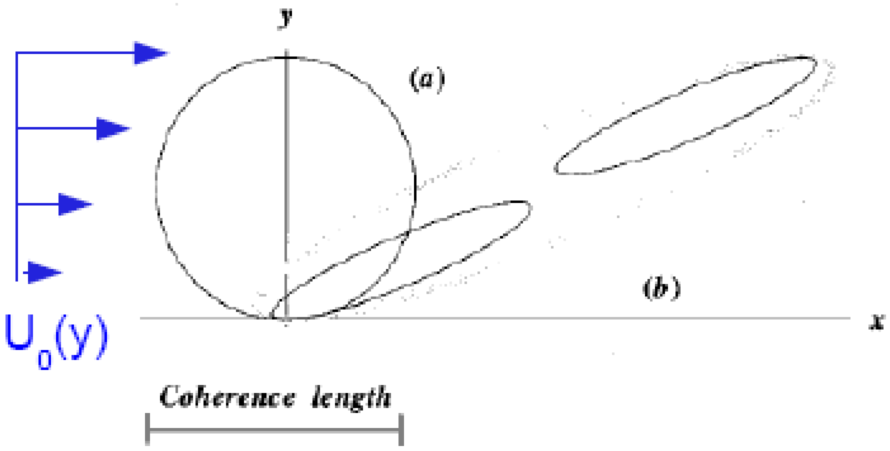
- Short connection length → **smaller average bad curvature**
- Quasi-isodynamic (~constant B) at high β → **grad-B drifts stabilizing [Peng & Strickler, NF 1986]**
- Large fraction of trapped electrons, BUT precession weaker at low A → **reduced TEM drive [Rewoldt, Phys. Plasmas 1996]**
- Strong coupling to $\delta B_\perp \sim \delta A_{||}$ at high β → **stabilizing to ES-ITG**



Kim, Horton, Dong, PoFB (1993)

Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

- Short connection length → **smaller average bad curvature**
- Quasi-isodynamic (~constant B) at high β → **grad-B drifts stabilizing [Peng & Strickler, NF 1986]**
- Large fraction of trapped electrons, BUT precession weaker at low A → **reduced TEM drive [Rewoldt, Phys. Plasmas 1996]**
- Strong coupling to $\delta B_\perp \sim \delta A_{||}$ at high β → **stabilizing to ES-ITG**
- Small inertia ($nm\underline{R}^2$) with uni-directional NBI heating gives strong toroidal flow & flow shear → **E×B shear stabilization ($dv_\perp/dr$)**



Biglari, Diamond, Terry, PoFB (1990)

Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

- Short connection length $\rightarrow$ **smaller average bad curvature**
- Quasi-isodynamic (~constant B) at high $\beta \rightarrow$ **grad-B drifts stabilizing [Peng & Strickler, NF 1986]**
- Large fraction of trapped electrons, BUT precession weaker at low A $\rightarrow$ **reduced TEM drive [Rewoldt, Phys. Plasmas 1996]**
- Strong coupling to $\delta B_\perp \sim \delta A_{||}$ at high $\beta \rightarrow$ **stabilizing to ES-ITG**
- Small inertia (nmR^2) with uni-directional NBI heating gives strong toroidal flow & flow shear $\rightarrow$ **E×B shear stabilization ($dv_\perp/dr$)**

$\Rightarrow$ **Not expecting strong ES ITG/TEM instability (much higher thresholds)**

- BUT
- High beta drives EM instabilities: microtearing modes (**MTM**) ~ $\beta_e \cdot \nabla T_e$, kinetic ballooning modes (**KBM**) ~ $\alpha_{MHD} \sim q^2 \nabla P / B^2$
- Large shear in parallel velocity can drive Kelvin-Helmholtz-like instability $\sim dv_{||}/dr$

Why is turbulence everywhere?

Energy gradients can drive linear instabilities → turbulence

- Free energy drive in fluid gradients (∇V, $\nabla \rho$ ∇T) or kinetic gradients $\nabla F(\mathbf{x},\mathbf{v})$ in the case of plasma → **<u>transport</u>** relaxes gradients
- Multiple analogous instabilities in magnetic plasmas

Kelvin-Helmholtz instability ~ ∇V

Rayleigh-Taylor instability ~ $\nabla \rho$

Rayleigh-Benard instability ~ ∇T

- Generally expect large scale separation remains between linearly unstable wavelengths and viscous damping scale lengths (*often not the case in kinetic plasma turbulence*)

Nonlinear instability important for neutral fluid pipe flow

Pipe flow is believed linearly stable for all Re (not rigorously proven). Nevertheless: Laminar for Re < 2300, turbulent for Re>4000 (both possible in transition region). Due to nonlinear instabilities: with large-enough amplitude becomes self-sustained turbulence (transient growth by non-normal modes (Trefethen)). Turbulent drag depends (weakly) on surface roughness.

From http://en.wikipedia.org/wiki/Reynolds_number From Moody (1944), Princeton Prof. of Hydrolic Eng.

(Hammett notes)

- Nonlinear instability (subcritical turbulence) may be important in some plasma scenarios