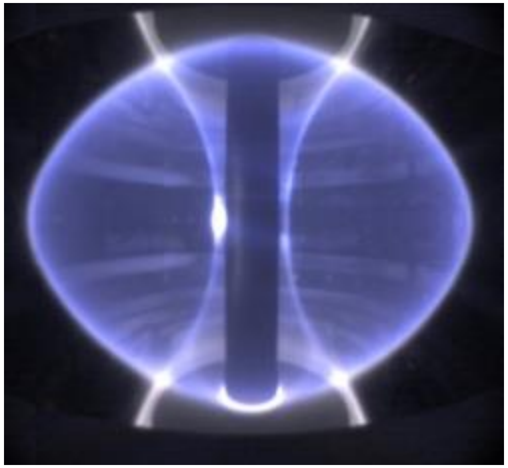
Analysis and prediction of momentum pinch in spherical tokamaks

W. Guttenfelder[1], A. Field[2], I. Lupelli[2], J.-K. Park[1], T. Tala[3],
S.M. Kaye[1], M. Peters[4], Y. Ren[1], W.M. Solomon[1]
[1]PPPL, [2]CCFE, [3]VTT-Finland, [4]Indiana University

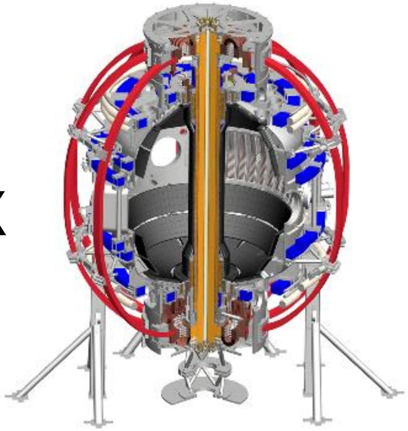
CCFE
CULHAM CENTRE FOR FUSION ENERGY

PPPL



MAST

NSTX



OVERVIEW

- To predict rotation profile (important for macro- and micro-instabilities) need to understand torques sources/sinks and momentum transport

- Momentum transport has well known diffusive and convective (pinch) contribution

- Here, investigating momentum pinch in low aspect ratio, high beta spherical tokamak plasmas (NSTX & MAST) as an additional constraint on theory (stems out of ITPA T&C activity)

Interpretation of toroidal angular momentum transport often assumes diffusive ($-\chi_\varphi \nabla\Omega$) and convective ($V_\varphi\Omega$) components

- Transport equation:

$$\frac{\partial}{\partial t}\left(n_i m_i \left\langle R^2 \right\rangle \Omega\right) + \nabla \cdot \Pi_\varphi = T$$

- *Assumed* transport form:

$$\Pi_\varphi = nmR^2\left(-\underline{\chi_\varphi}\nabla\Omega + \underline{V_\varphi}\Omega + C_{RS}\right)$$

Prandtl number $\boxed{\mathrm{Pr} = \frac{\chi_\varphi}{\chi_i}}$

Pinch parameter $\boxed{\frac{RV_\varphi}{\chi_\varphi}}$

$$\hat{\Pi}_\varphi = \hat{\chi}_\varphi\left(\hat{u}' + \frac{RV_\varphi}{\chi_\varphi}\hat{u} + \frac{C_{RS}}{\chi_\varphi}\right)$$

$$\hat{u}' = \frac{-R^2\nabla\Omega}{c_s} \qquad \hat{u} = \frac{R\Omega}{c_s}$$

- Pinch expected due to Coriolis effect (Peeters, 2007), or equivalently turbulent equipartition (Hahm, 2007) + thermoelectric force (Peeters, 2009)

Interpretation of toroidal angular momentum transport often assumes diffusive ($-\chi_\varphi \nabla\Omega$) and convective ($V_\varphi \Omega$) components

- Transport equation:

$$\frac{\partial}{\partial t}\left(n_i m_i \left\langle R^2 \right\rangle \Omega\right) + \nabla \cdot \Pi_\varphi = T$$

- *Assumed* transport form:

$$\Pi_\varphi = nmR^2\left(-\underline{\chi_\varphi}\nabla\Omega + \underline{V_\varphi}\Omega + \cancel{C_{RS}}\right)$$

Prandtl number

$$\boxed{Pr = \frac{\chi_\varphi}{\chi_i}}$$

Pinch parameter

$$\boxed{\frac{RV_\varphi}{\chi_\varphi}}$$

$$\hat{\Pi}_\varphi = \hat{\chi}_\varphi\left(\hat{u}' + \frac{RV_\varphi}{\chi_\varphi}\hat{u} + \cancel{\frac{C_{RS}}{\chi_\varphi}}\right)$$

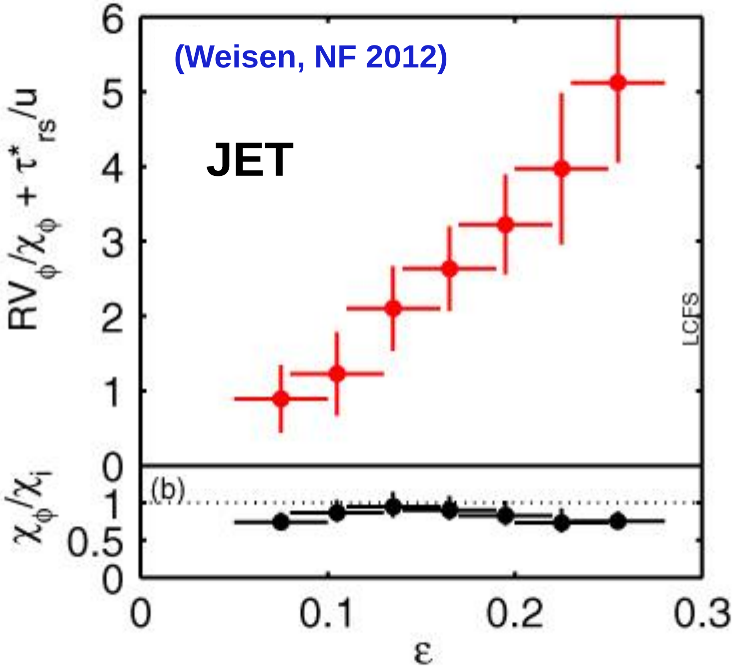
$$\hat{u}' = \frac{-R^2\nabla\Omega}{c_s} \qquad \hat{u} = \frac{R\Omega}{c_s}$$

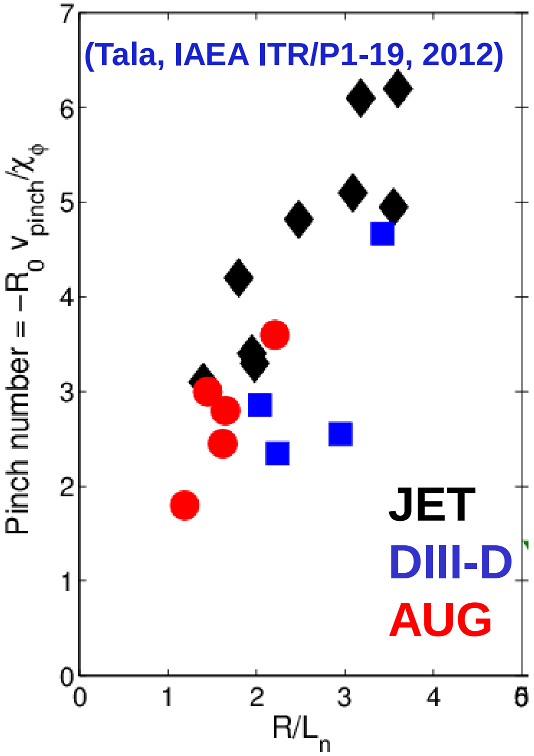
- Pinch expected due to Coriolis effect (Peeters, 2007), or equivalently turbulent equipartition (Hahm, 2007) + thermoelectric force (Peeters, 2009)

Ignoring any possible residual stress (intrinsic torque) contributions

Momentum pinch measured and predicted in conventional tokamaks

- Measurements in many machines from both perturbative experiments (NBI, 3D coils) and statistical regression analysis

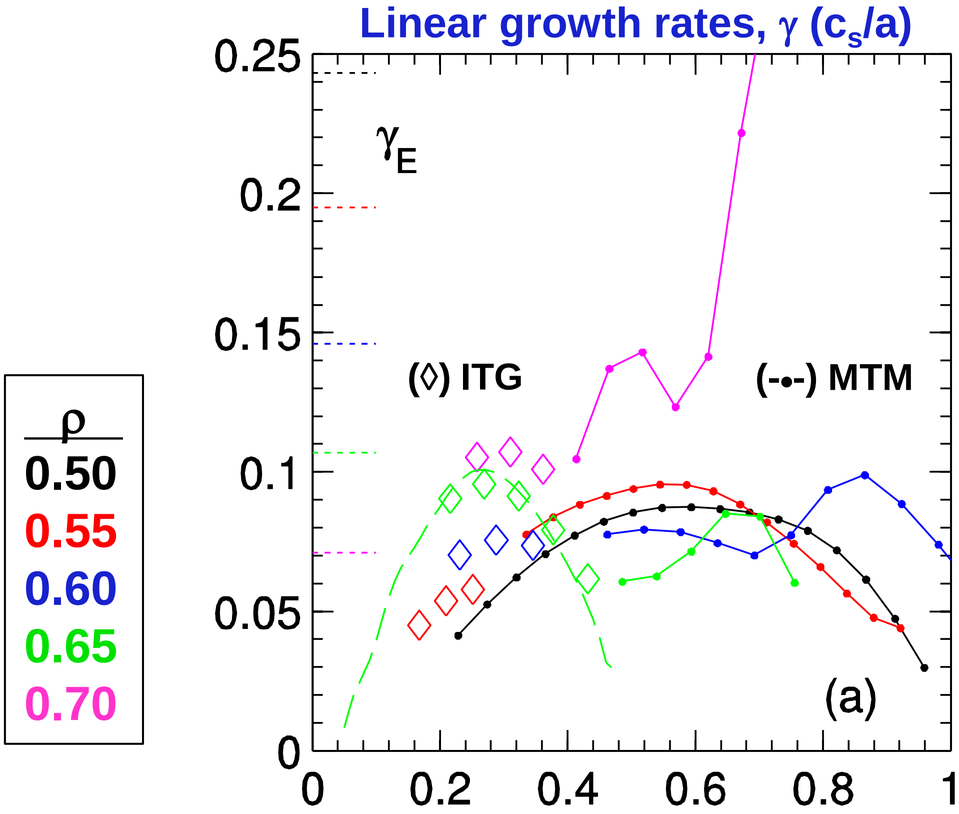

(Weisen, NF 2012)


(Tala, IAEA ITR/P1-19, 2012)

- Increase in (inward) pinch observed with ε=r/R and R/L_n, also predicted by ITG theory (Peeters, PRL 2007; PoP 2009)

Higher beta NSTX H-modes often dominated by microtearing modes (MTM) with sub-dominant ballooning modes

- Most cases have $\gamma_{MTM} > \gamma_{ballooning}$ (Π_φ=0 for MTM)
- Sub-dominant modes can be ITG, KBM or compressional ballooning modes – calculate pinch *assuming* they contribute to transport



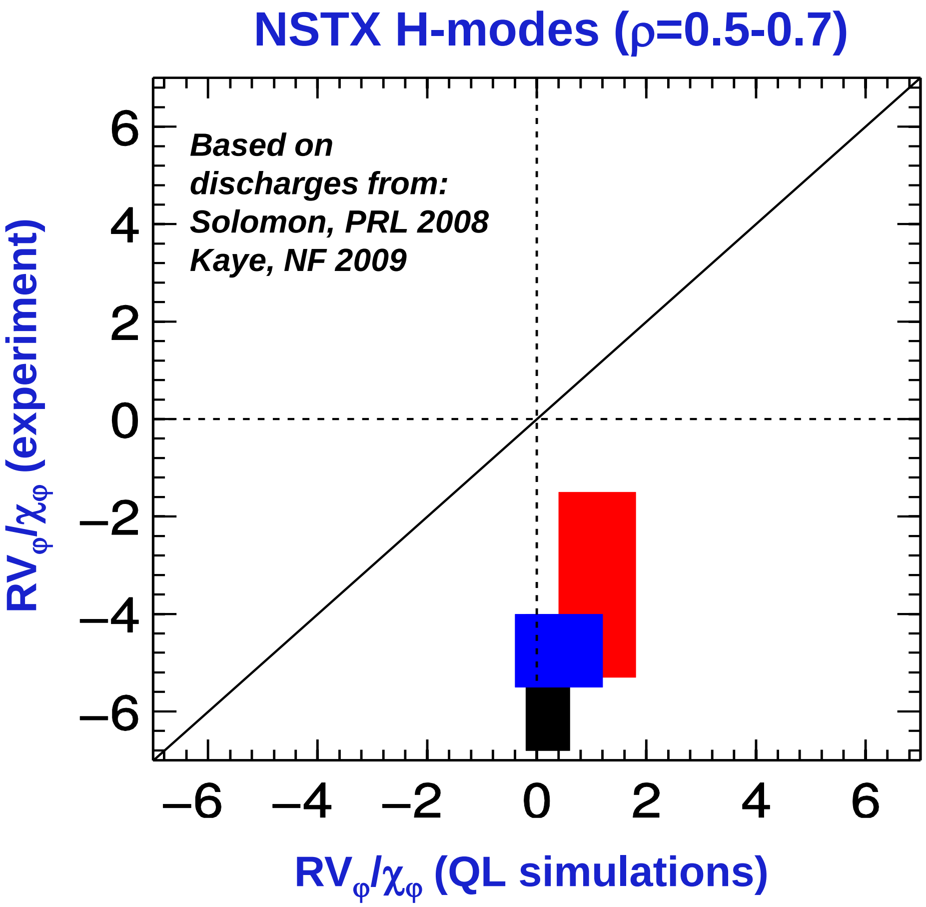
Linear GYRO simulations
(Candy, Waltz, 2003)
3 species: D,C,e
EM: φ, $A_{||}$, $B_{||}$
Equilibrium reconstruction

β_T=12%, β_N=3.5

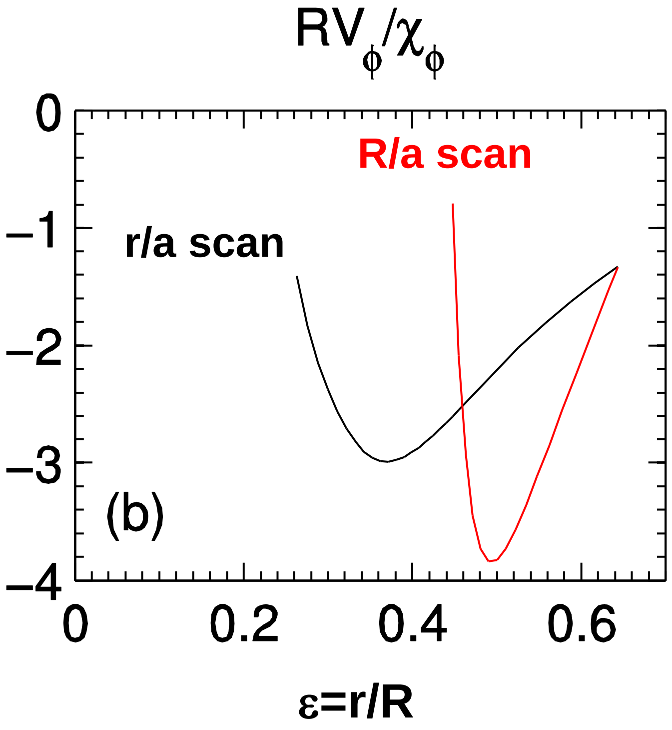
Guttenfelder, 2016 (Phys. Plasmas, *in review*)

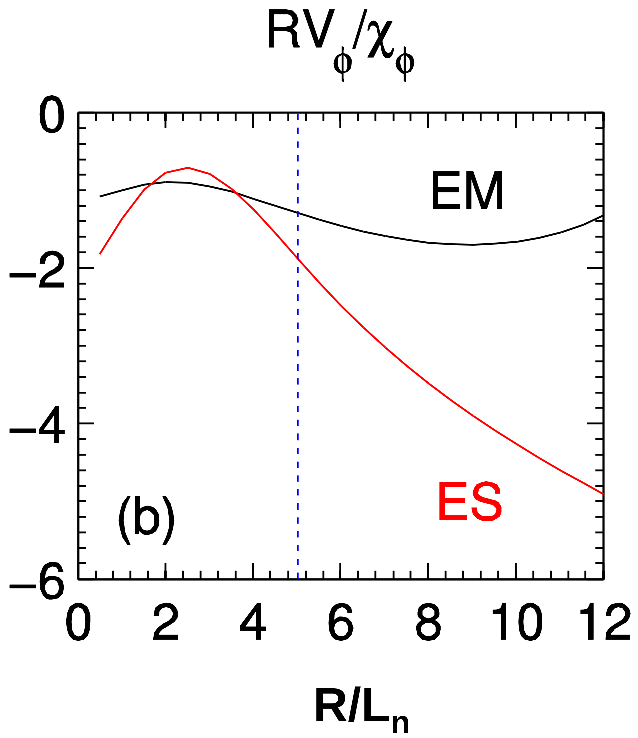
Negligible or outward momentum convection predicted from ES and EM ballooning modes in NSTX

- Weak/outward pinch consequence of parallel mode structure response at high beta, low aspect ratio, see:
  - Peeters, PoP (2009)
  - Kluy, PoP (2009)
  - Hein, PoP (2011)
  - Guttenfelder, PoP (2016, *in review*)



A larger (inward) pinch can be found: (i) at increased aspect ratio, (ii) in purely ES limit at high ∇n





- Non-monotonic dependence on $\varepsilon=r/R$
- Can’t do aspect ratio scan…can try to do similar analyses at lower beta

MAST L-mode experiment conducted in 2013

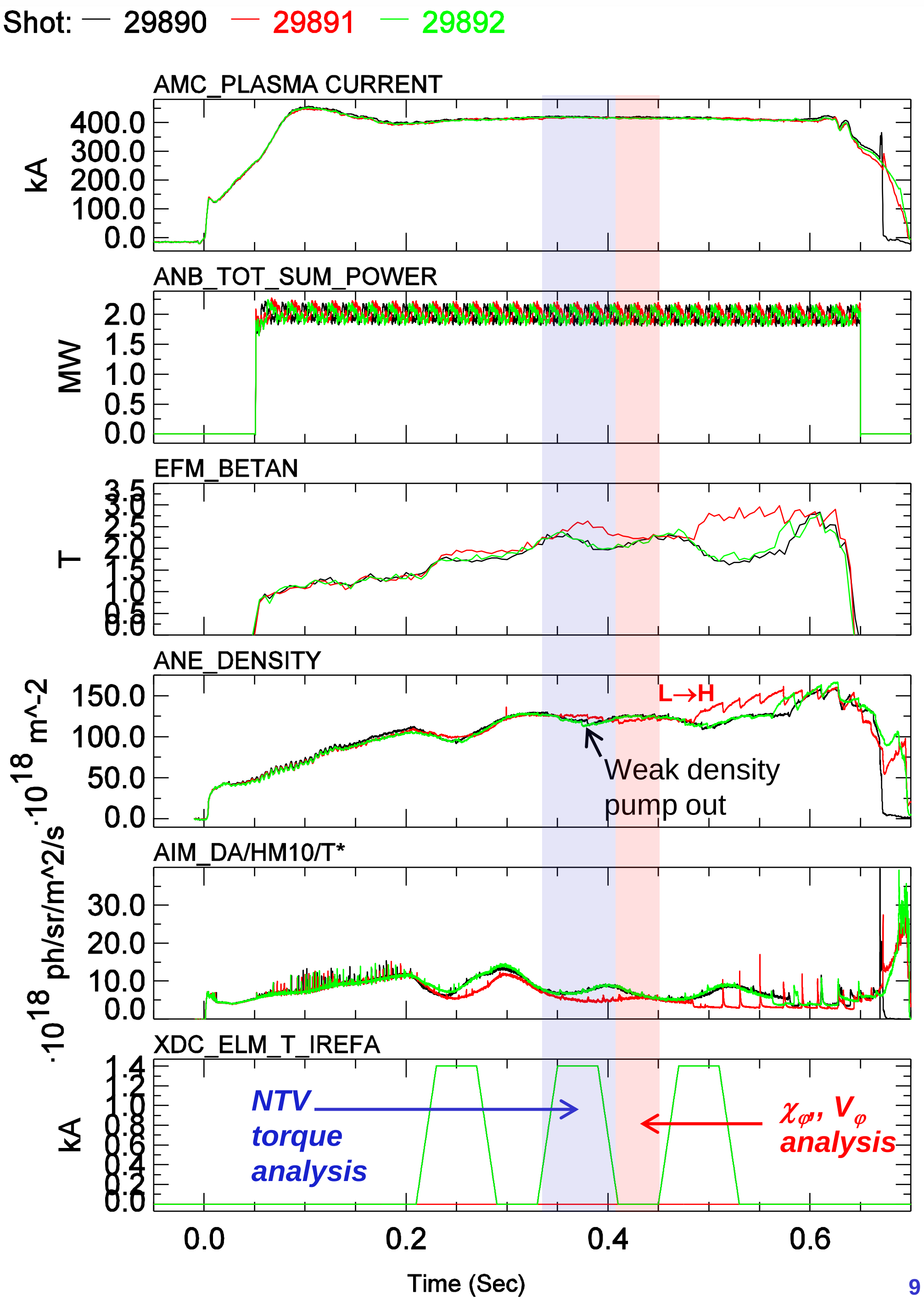
- 2 MW LSN L-mode
  - $\langle n_e \rangle = 2.3\times10^{19}$ m^{-3}
  - B_T=0.5 T, I_p=0.4 MA ($q_{95}\approx5$)
  - **β_N~2, β_T~4%**

- **29890/ 29892 – three n=3 field pulses applied to brake rotation**

  29891 – no nRMP pulses

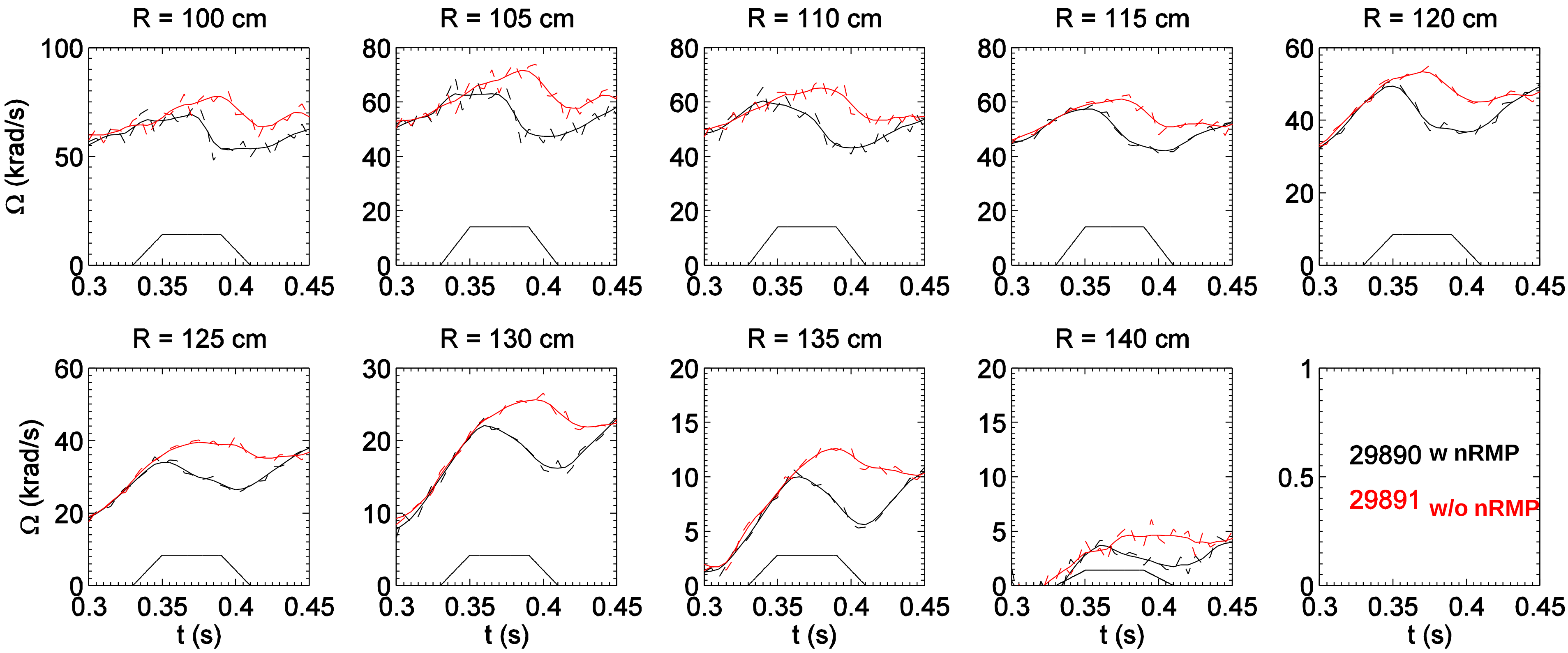
- Weak density pump out w/ nRMP, drop in β_N

- Without RMP, eventual transition into H-mode (t~0.47 s)



Changes in toroidal rotation due to n=3 nRMP clearly observed

- Non-stationary conditions -- control shot (29891) provides a baseline for analysis
- Filtering to remove faster sawteeth oscillations(Δt_{ST}~6-12 ms)
    - $\Delta\Omega_{ST}$~2-6 krad/s < $\Delta\Omega_{3D}$~10-20 krad/s

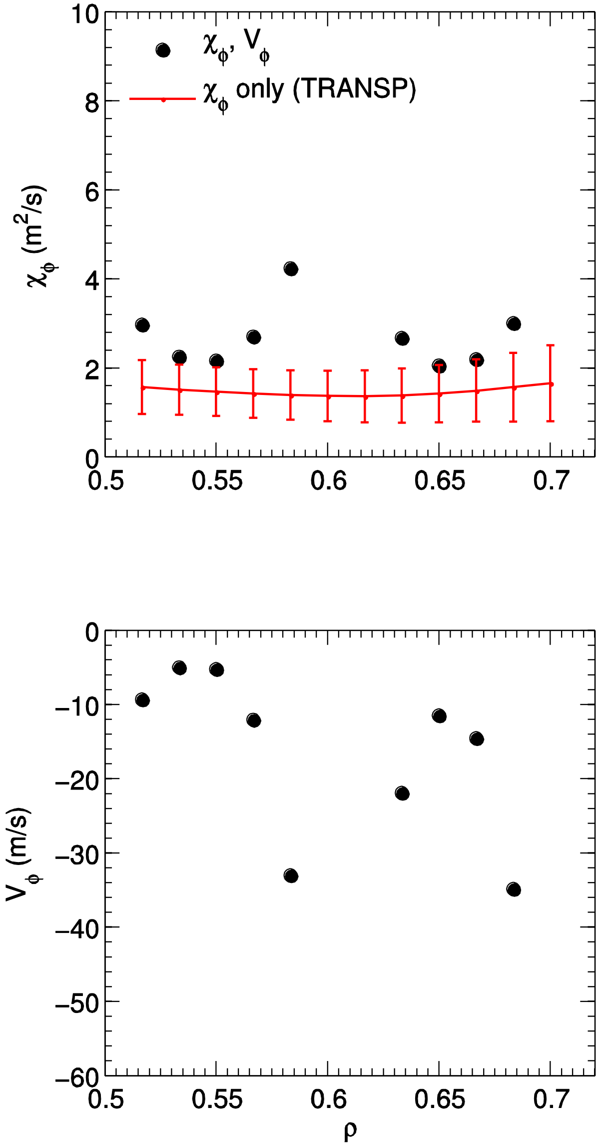


Inward momentum pinch inferred from transient recovery

- **χ_φ, V_φ assumed constant in time**

- Using both χ_φ and V_φ improves the quality of fit (χ_ν^2 smaller than χ_φ-only fit)

- At locations where there is a strong Ω-$\nabla\Omega$ linear correlation, method is ill-posed $\Rightarrow$ χ_φ & V_φ tend to large values



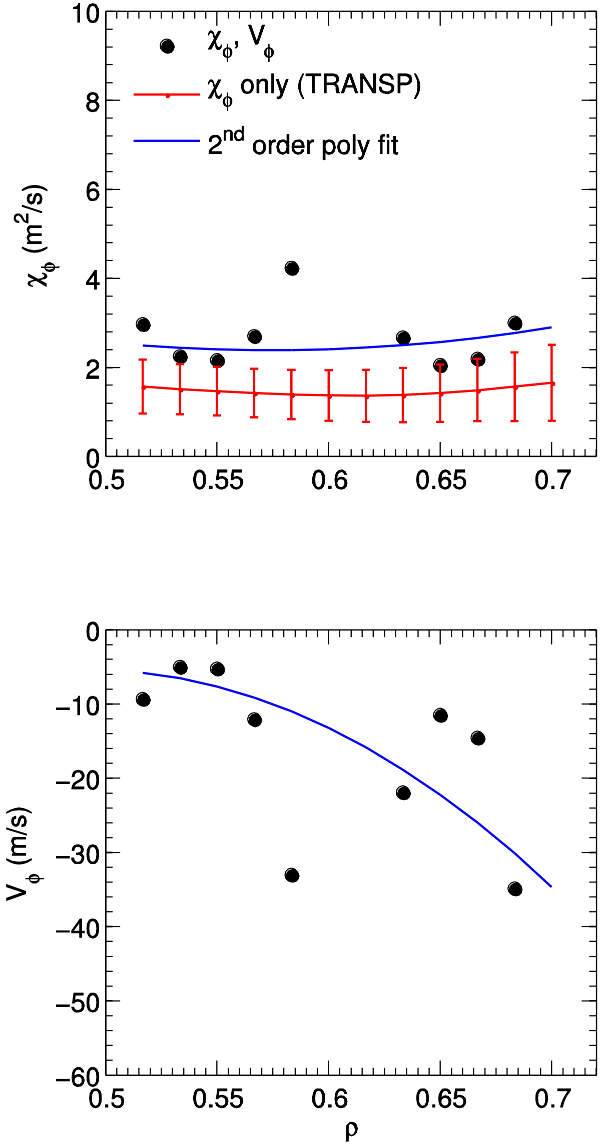
Inward momentum pinch inferred from transient recovery

- **χ_φ, V_φ assumed constant in time**

- Using both χ_φ and V_φ improves the quality of fit (χ_ν^2 smaller than χ_φ-only fit)

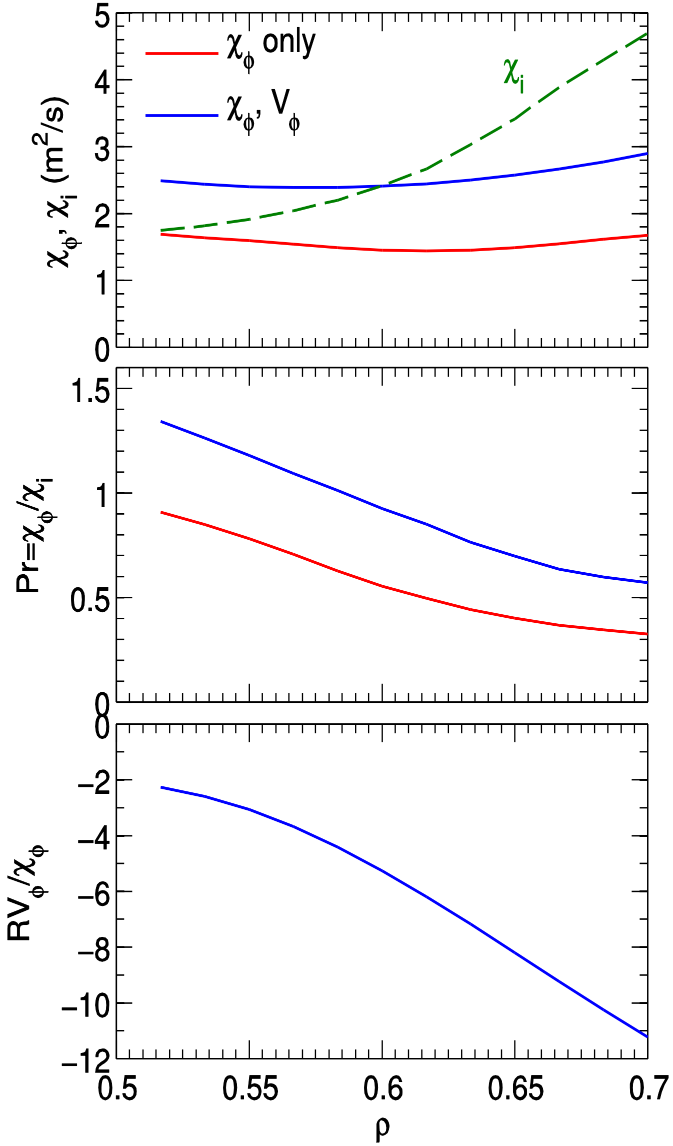
- At locations where there is a strong Ω-$\nabla\Omega$ linear correlation, method is ill-posed $\Rightarrow$ χ_φ & V_φ tend to large values

- Can fit entire analysis region simultaneously using polynomial profiles
  - Best fit (lowest χ_ν^2) using quadratic



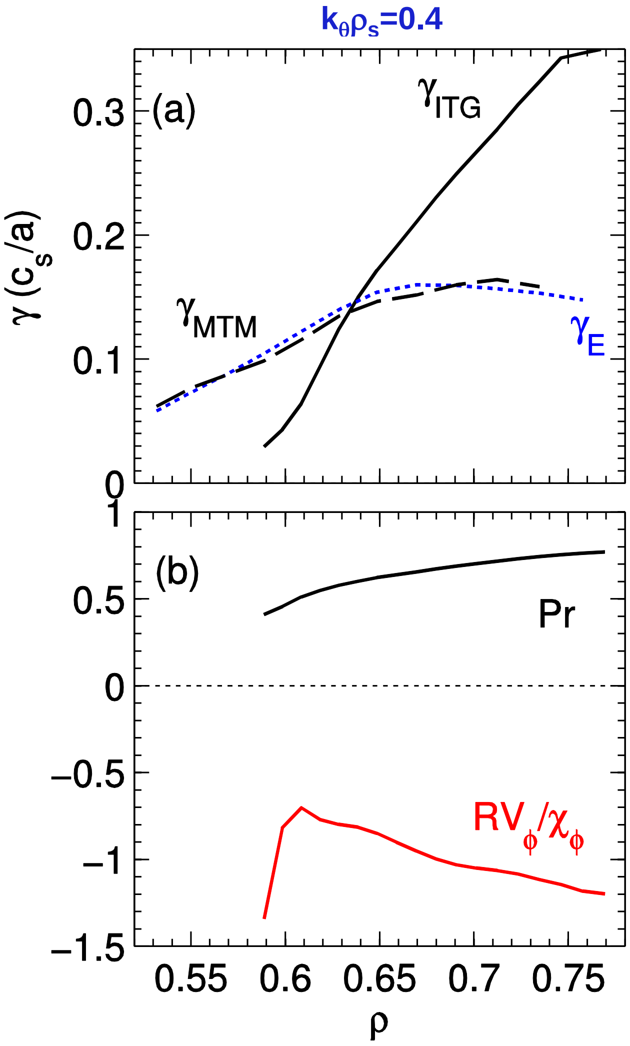
Pinch parameter comparable to conventional tokamaks and those found in NSTX H-modes

- Prandtl number Pr~0.5-1.5
- Pinch parameter RV_φ/χ_φ ~ (-2) to (-11)



Where ITG dominant, predicted Pinch is small ($RV_\varphi/\chi_\varphi \approx -1$)

- Similar discrepancy as in NSTX H-modes
- Expecting parametric dependencies to be similar to NSTX L-mode predictions
- Investigate nonlocal effects at finite $\rho_* \sim 1/100$ due to:
  - Profile shear ~ $\omega_r' \cdot \rho_*$ (Camenen, NF 2011)
  - Intensity shear ~ $d(\gamma_{ITG}-\gamma_E)/dr \cdot \rho_*$ (Gurcan, PoP 2010)
  - Neoclassical flows (Barnes, PRL 2013)



Summary & future work

- Inward momentum pinch inferred from perturbative experiments in both NSTX H-mode and MAST L-modes
  - RV_φ/χ_φ = (-1)-(-10) comparable to conventional tokamaks

- Not reproduced by local, quasilinear GK theory (Coriolis pinch), unlike in in conventional tokamaks

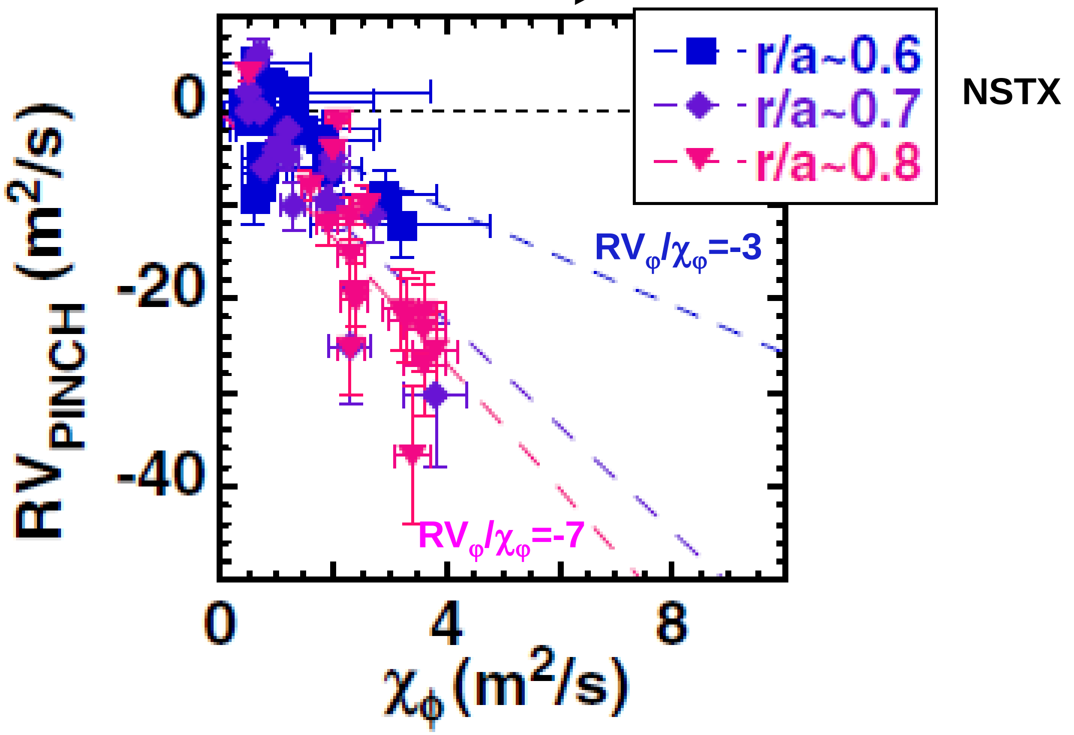
- Investigating other possibilities:
  - Revisit experimental analysis: (i) $\chi_\varphi \sim \chi_i(t)$, (ii) if $V_\varphi \sim 0$, solve instead for Π_{RS}
  - Working on global GYRO and GTS simulations to predict residual stress at finite $\rho_* \sim 1/100$

- NSTX-U L-mode experiments planned for this run campaign (2016)

BACKUP

Perturbative NSTX H-mode experiments indicate existence of an inward momentum pinch, $RV_\varphi/\chi_\varphi \approx -(1\text{-}7)$

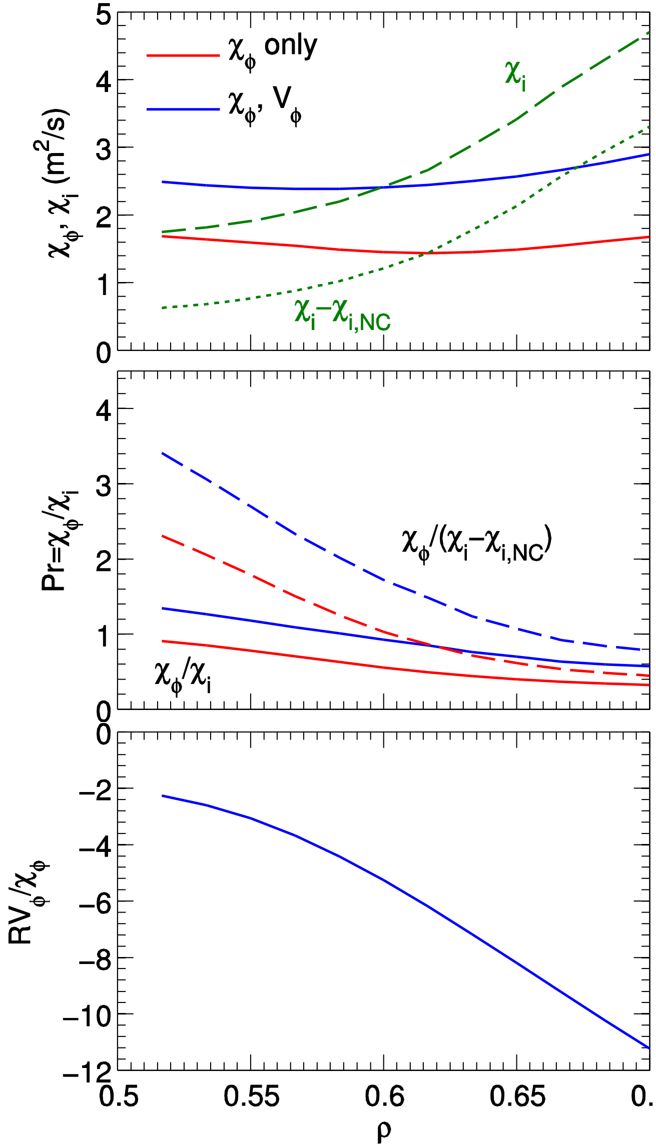
(Solomon, PRL 2008, PoP 2010;
Kaye, NF 2009; Yoshida, NF 2012)



- Local, linear gyrokinetic simulations of ITG turbulence describe pinch and scaling in conventional tokamaks ⇒ **does this hold for STs?**

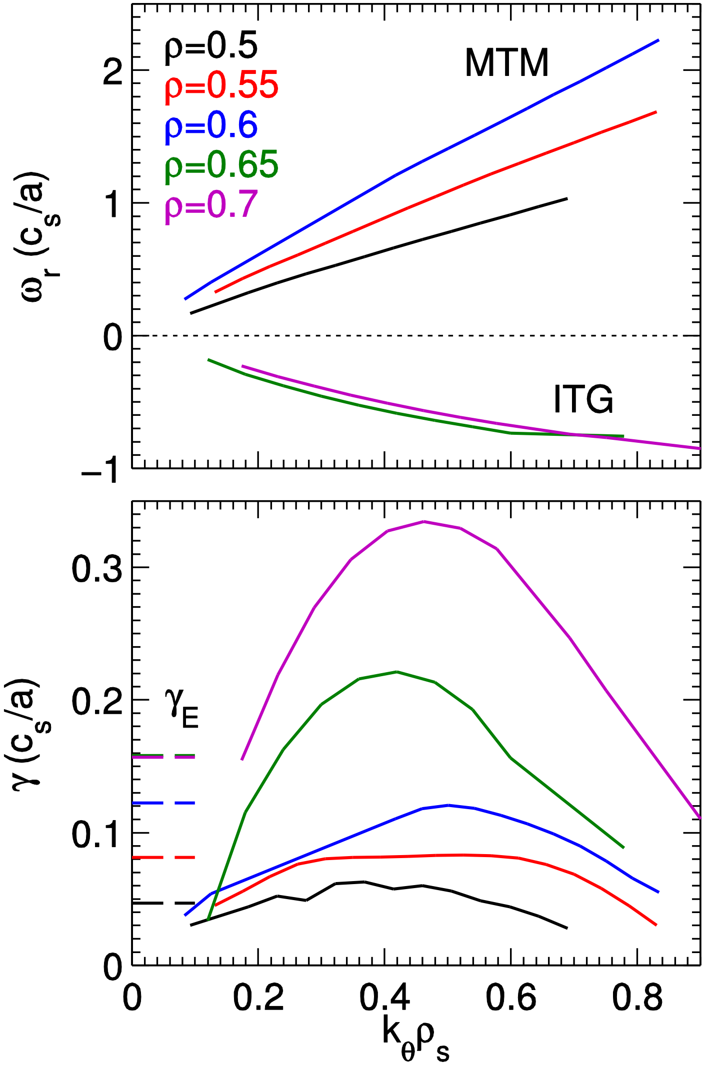
Subtracting neoclassical ion thermal transport leads to larger Pr~0.8-4.0

- In L-mode, $\chi_{i,NC}$ smaller than χ_i but still substantial contribution
- $\chi_e \sim 3 \cdot \chi_i$, additional uncertainty from T_e~T_i collisional energy exchange



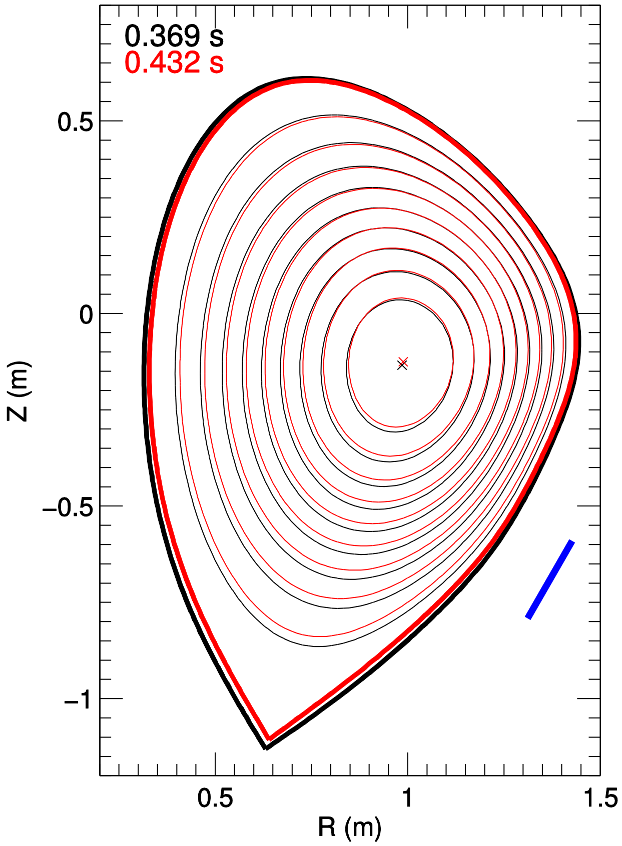
Linear GYRO simulations predict unstable microtearing (ρ=0.5-0.6) and ITG (ρ>0.6)

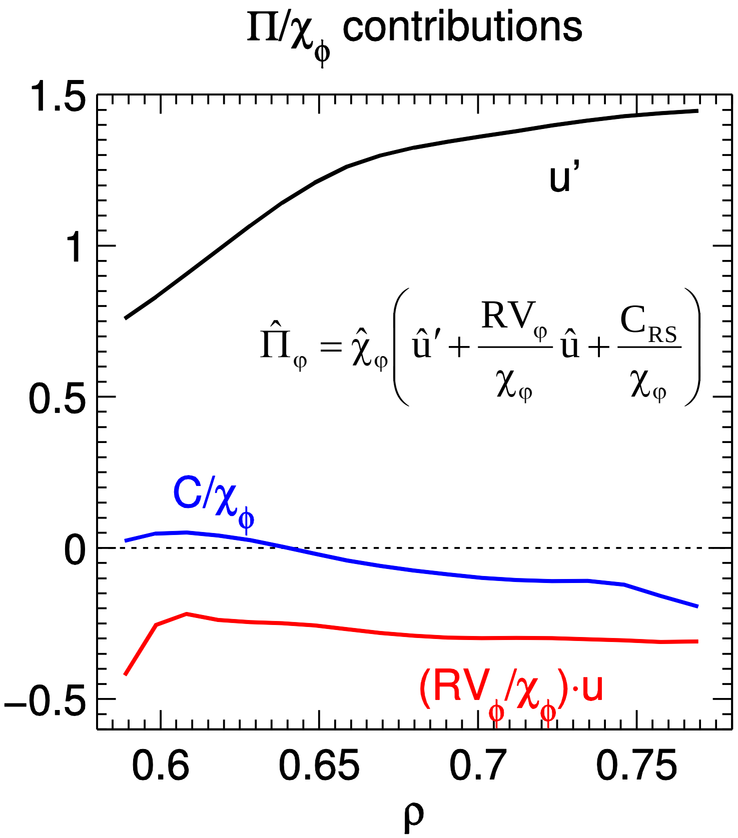
- Was surprised to see this in L-mode!



Increasing level of residual stress contribution towards outer radii due to up-down asymmetry

- Eliminated when surfaces forced to be up-down symmetric
- Small compared to diffusive flux

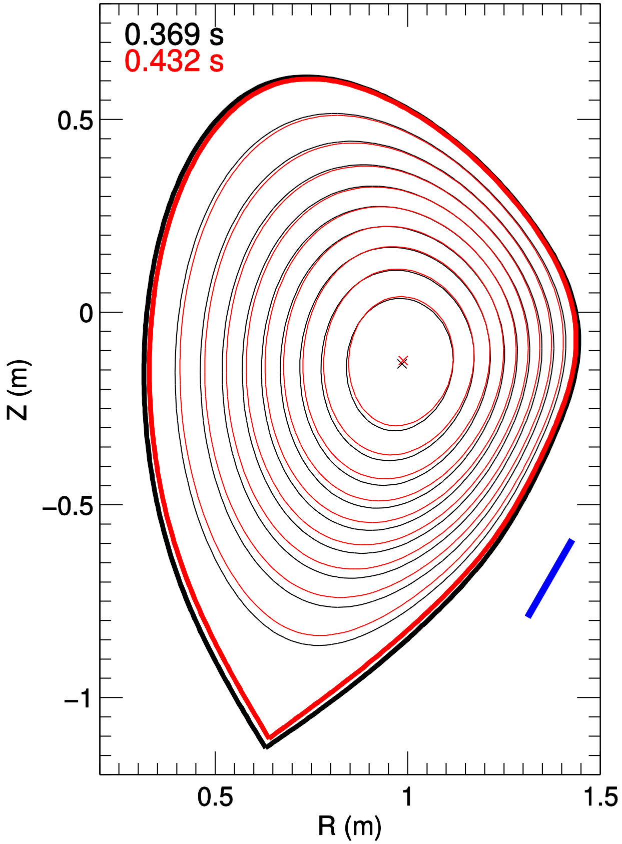


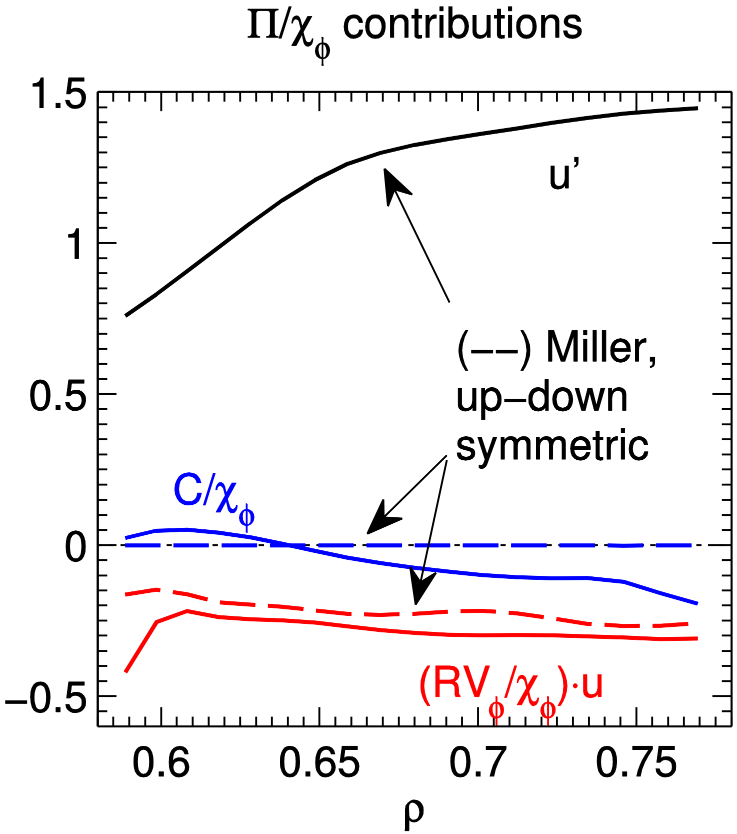


$$\hat{\Pi}_{\varphi} = \hat{\chi}_{\varphi}\left(\hat{u}' + \frac{RV_{\varphi}}{\chi_{\varphi}}\hat{u} + \frac{C_{RS}}{\chi_{\varphi}}\right)$$

Increasing level of residual stress contribution towards outer radii due to up-down asymmetry

- Small compared to diffusive flux
- Eliminated when forced to be up-down symmetric (using Miller, 1998)

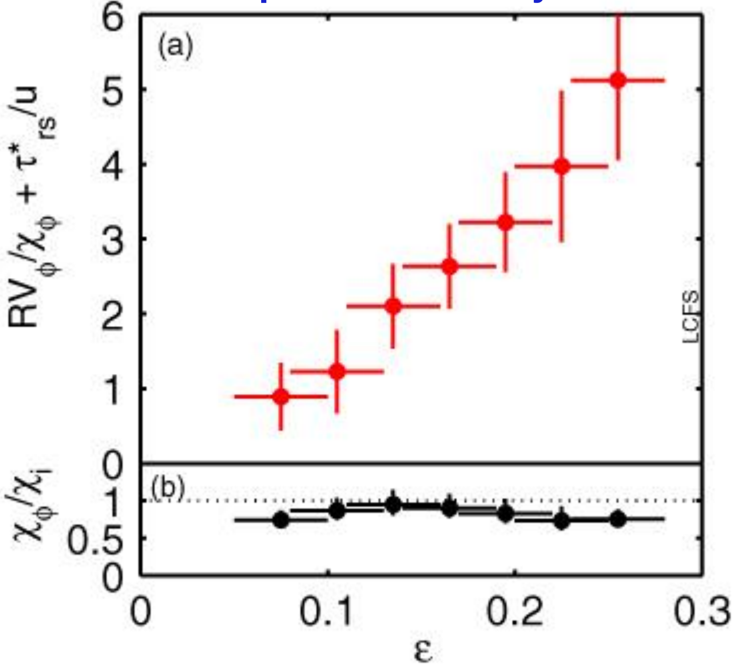




Momentum pinch measured and predicted in conventional tokamaks

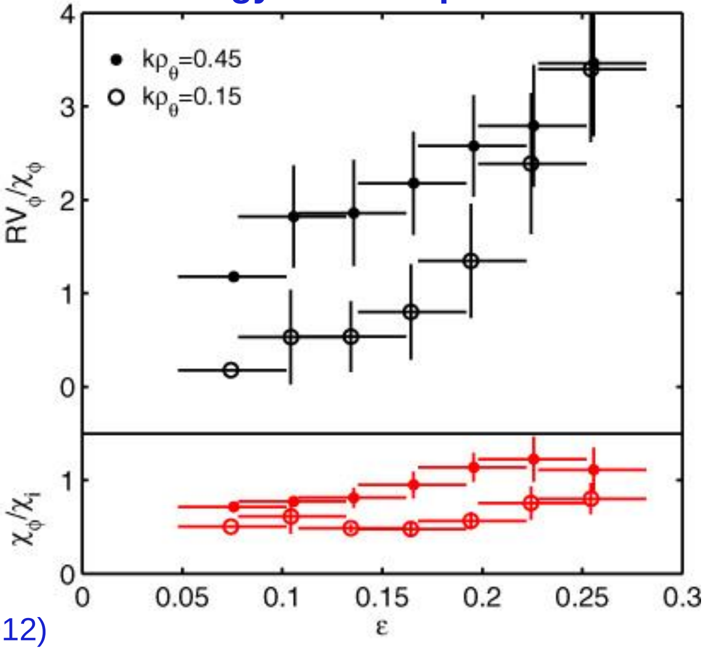
- Measurements in many machines from both perturbative experiments (NBI, 3D coils) and statistical regression analysis

JET
experimental analysis



(Weisen, NF 2012)

JET
regression analysis of
linear gyrokinetic predictions



$$\boxed{(\text{exp.})\quad RV_\varphi/\chi_\varphi = -1.7 - 0.41\cdot R/L_n - 12\cdot\varepsilon^{1/2} - 0.41\cdot q - 1.9\cdot T_i/T_e}$$

$$\boxed{(\text{sim.})\quad RV_\varphi/\chi_\varphi = -2.8 - 0.44\cdot R/L_n - 6.8\cdot\varepsilon^{1/2} - 0.31\cdot q + 0.21\cdot s - 0.077\cdot R/L_{Te}}$$

Momentum pinch coupled to symmetry breaking in parallel mode structure

- Can think of as correction to curvature drift in lab frame

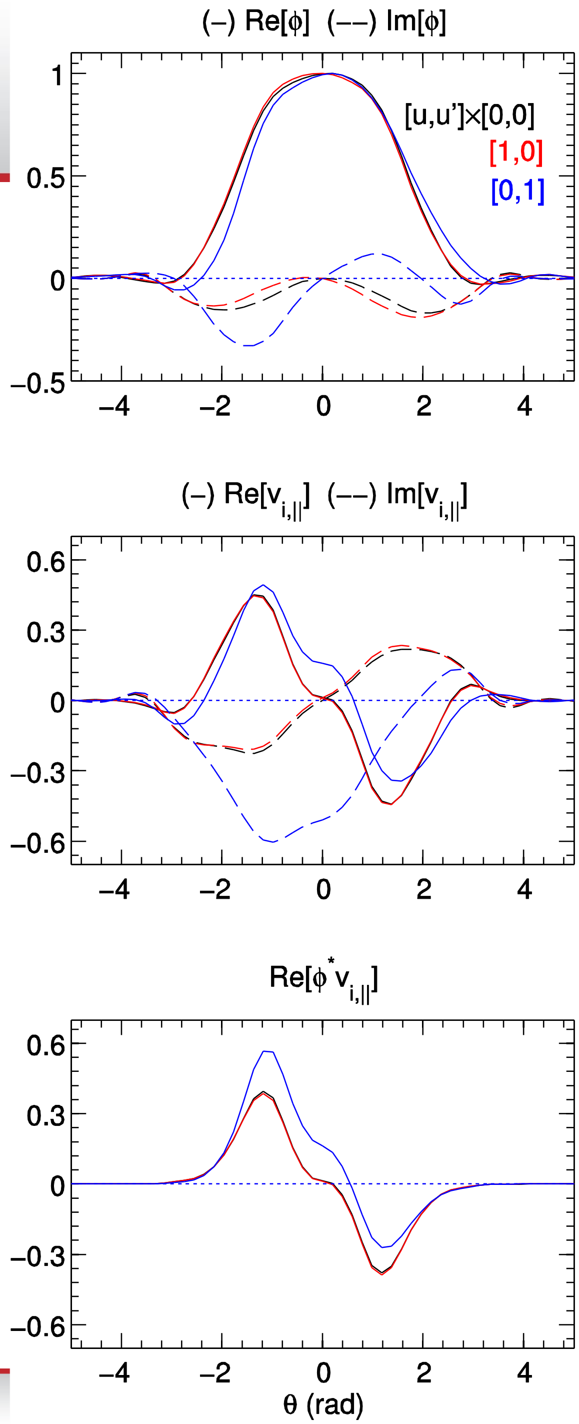
$$v_\kappa \approx \frac{mv_\parallel^2}{eBR} \rightarrow \frac{2m\left(v_\parallel + u_0\right)^2}{eBR} = \frac{mv_\parallel^2}{eBR} + \frac{2mv_\parallel u_0}{eBR} + \frac{mu_0^2}{eBR}$$

Curvature ***Coriolis (M)*** ***Centrifugal (M^2)***

- M<1 smaller than curv. drift, does not influence stability

- But, toroidal flow couples δn, δT with δu → causes momentum transport

- Asymmetry very small due to u>0 in NSTX – little convective transport



Momentum pinch coupled to symmetry breaking in parallel mode structure

- Can think of as correction to curvature drift in lab frame

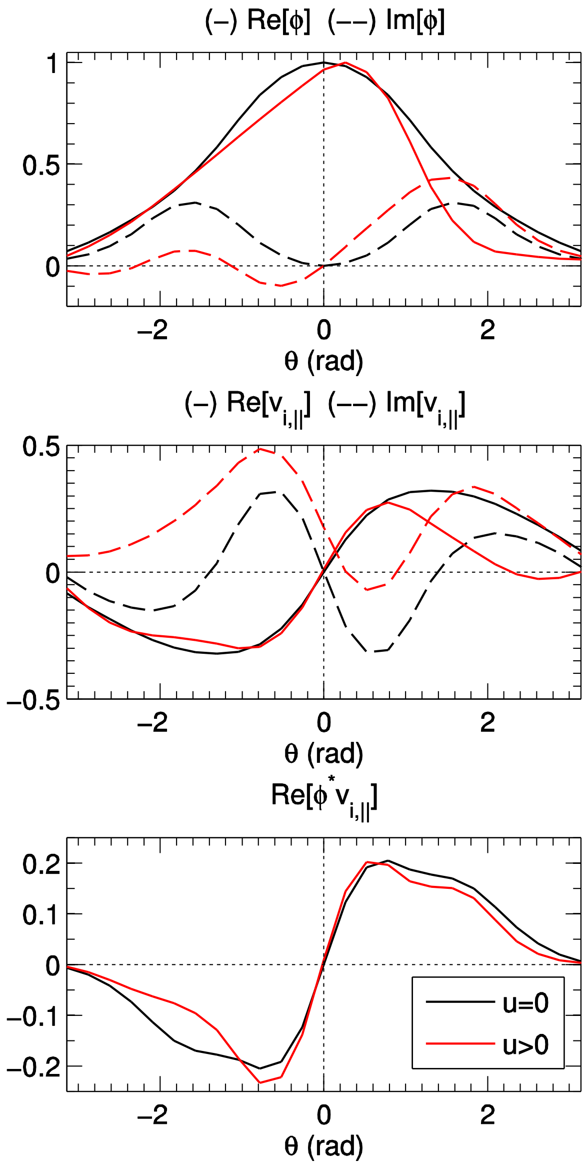
$$\frac{d\mathbf{X}}{dt} = v_{\|}\mathbf{b} + \mathbf{v}_E + \frac{m(v_{\|}^2 + 2v_{\|}u_{\|} + u_{\|}^2) + \mu B}{ZeB_{\|}^*}\frac{\mathbf{B}\times\nabla B}{B^2}. \quad (45)$$

- M<1 smaller than curv. drift, does not influence stability

$$\mathbf{v}_{\rm co} = \frac{2mv_{\|}u_{\|}}{ZeB_{\|}^*}\frac{\mathbf{B}\times\nabla B}{B^2},$$

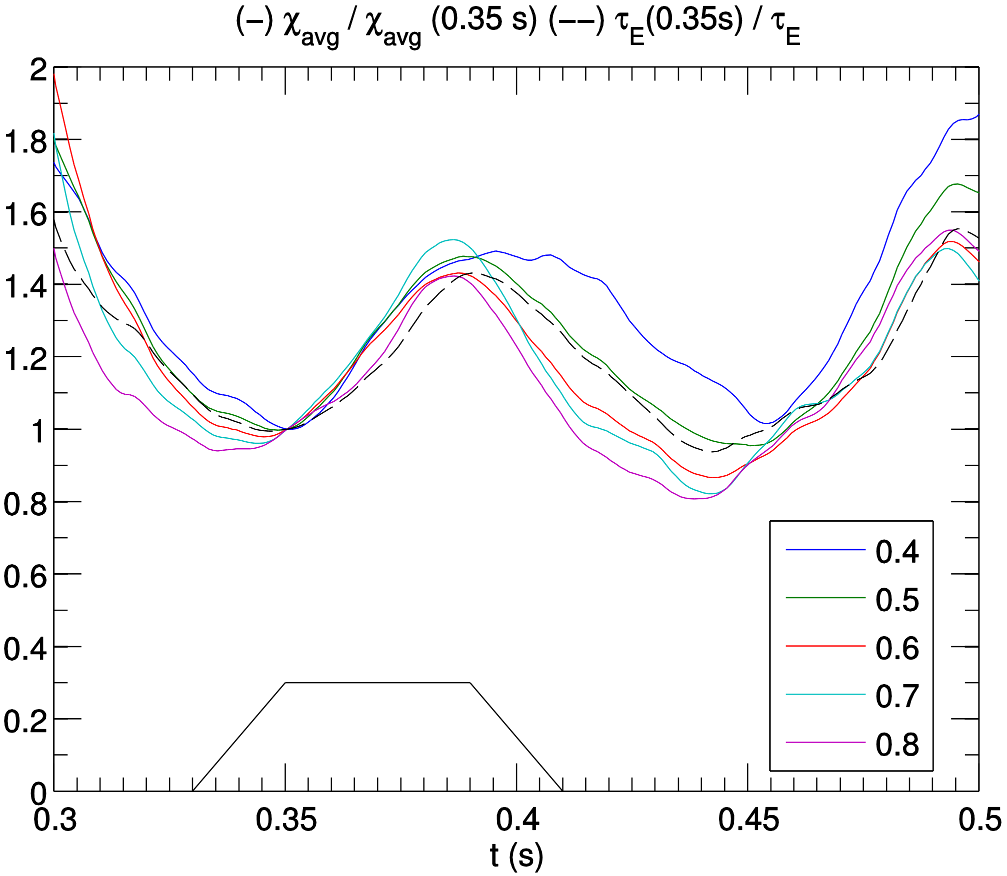
$$\mathbf{v}_{\rm cf} = \frac{mu_{\|}^2}{ZeB_{\|}^*}\frac{\mathbf{B}\times\nabla B}{B^2},$$

- But, toroidal flow couples δn, δT with δu → causes momentum transport



MAST L-mode experiments

- adf



Momentum pinch analysis

Method to infer χ_φ and V_φ from transient rotation response *after* RMP turn-off

- TRANSP solves for momentum flux, Π, using the flux-surface-averaged toroidal angular momentum transport equation (Goldston, Varenna 1985), plus NUBEAM calculations for torque sources & sinks:

$$\frac{\partial}{\partial t}\left(\sum_i n_i m_i \langle R^2 \rangle \Omega\right) + \frac{1}{V'}\frac{\partial}{\partial \rho}\left[V' \cdot \Pi\right] = \sum T_{\mathrm{source}} - \sum T_{\mathrm{sink}}$$

- *Assuming* momentum flux composed of only diffusive and convective contributions:

$$\Pi = \sum_i n_i m_i \left[-\langle R^2 (\nabla\rho)^2 \rangle \chi_\varphi \frac{\partial \Omega}{\partial \rho} + \langle R^2 \rangle \langle \nabla\rho \rangle V_\varphi \Omega \right]$$

we can use $\Pi(\rho,t)$, $d\Omega/d\rho(\rho,t)$, and $\Omega(\rho,t)$ in a nonlinear least squares fit algorithm to determine best fit $\chi_\varphi(\rho)$, $V_\varphi(\rho)$ (assumed constant in time)

- Note: method only valid if $d\Omega/d\rho(t)$ and $\Omega(t)$ are sufficiently decorrelated

Quasi-linear gyrokinetic analysis

Linear GYRO simulations
(Candy, Waltz, 2003)
3 species: D,C,e
EM: φ, $A_{||}$, $B_{||}$
Equilibrium reconstruction

Analysis method of
Peeters, PRL (2007)
Use multiple runs with:
[u,u’]=[0,0],[0,1],[1,0] to infer
$\chi_\varphi = [\Pi(u')-\Pi(u'=0)] / u'$
$V_\varphi = [\Pi(u)-\Pi(u=0)] / u$

Momentum transport is anomalous in NSTX, Prandtl numbers $\chi_\varphi/\chi_i < 1$ for L- and H-modes

- $\mathrm{Pr}=\chi_\varphi/\chi_i \approx 0.3\text{-}1.0$ over many radii and discharges (assumes $V_\varphi=0$)

- $\chi_\varphi > \chi_{\varphi,\mathrm{NC}}$ for both L and H

In L-mode $\chi_i > \chi_{i,\mathrm{NC}}$

$$\mathrm{Pr}=\frac{\chi_\varphi}{\chi_i}\approx\frac{\chi_{\varphi,\mathrm{turb}}}{\chi_{i,\mathrm{turb}}}$$

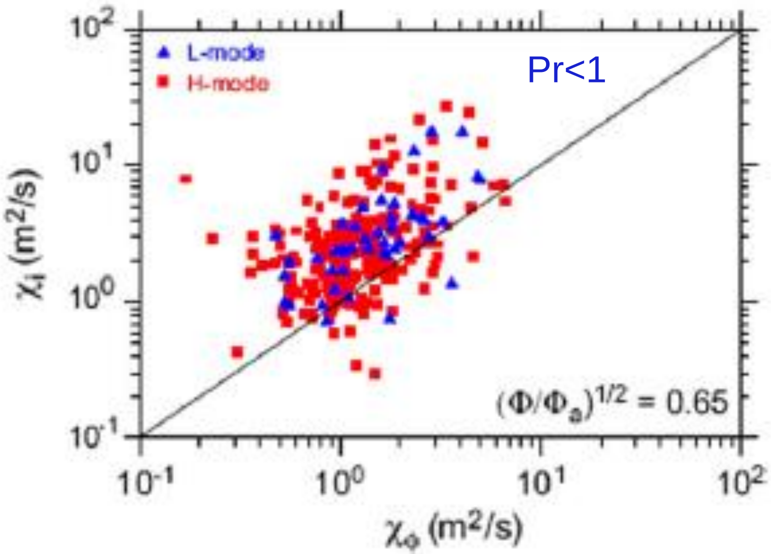
In H-mode $\chi_i \approx \chi_{i,\mathrm{NC}}$

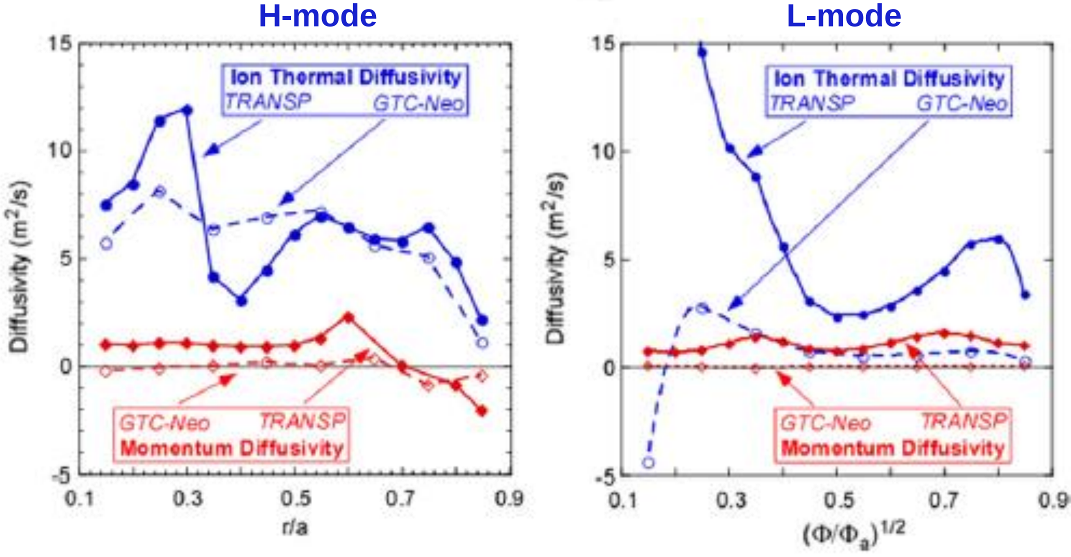
$$\mathrm{Pr}=\frac{\chi_\varphi}{\chi_i}=\frac{\chi_{\varphi,\mathrm{turb}}}{(\chi_{i,\mathrm{NC}}+\cancel{\chi_{i,\mathrm{turb}}})}\ \sim 0$$

$\Rightarrow$ Pr less useful in H-mode?

- RV_φ/χ_φ less ambiguous

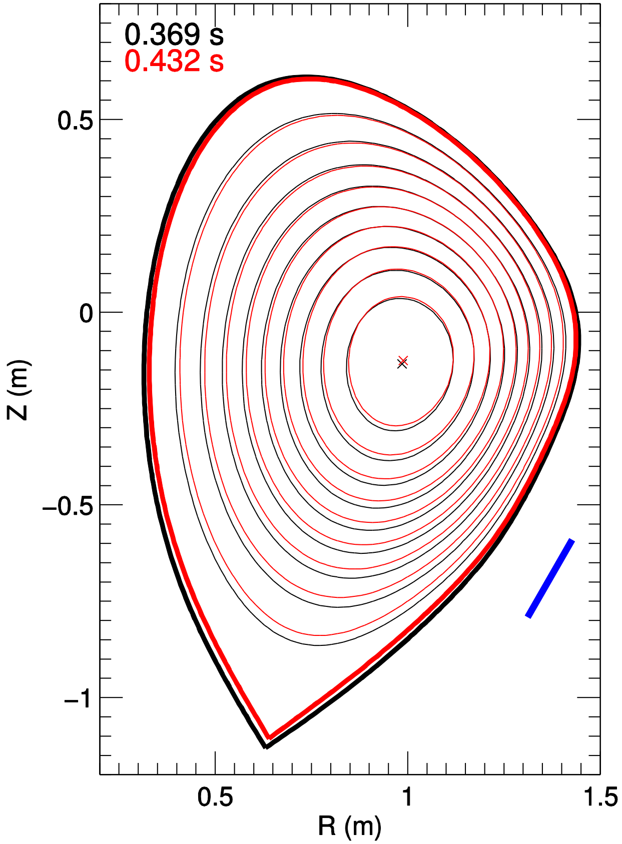
Steady state transport analysis (Kaye et al., 2009)



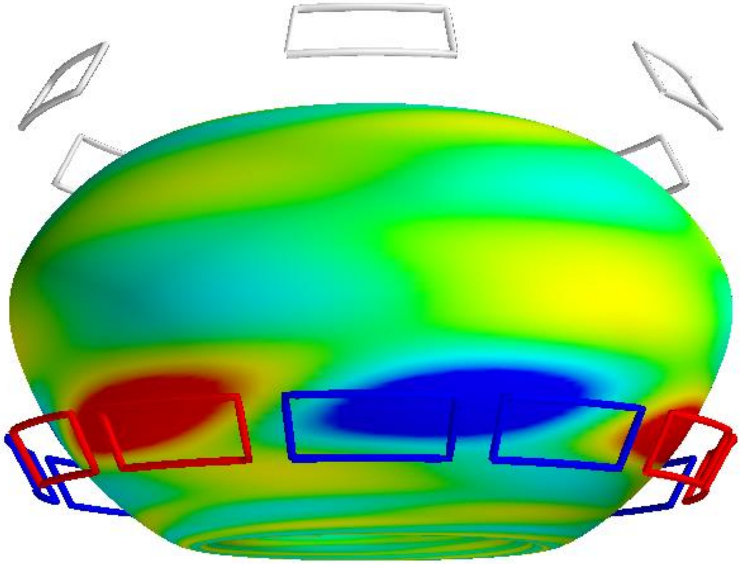


To obtain sufficient rotation braking required strong bias to lower single null

Axisymmetric equilibrium (MSE-constrained EFIT++)

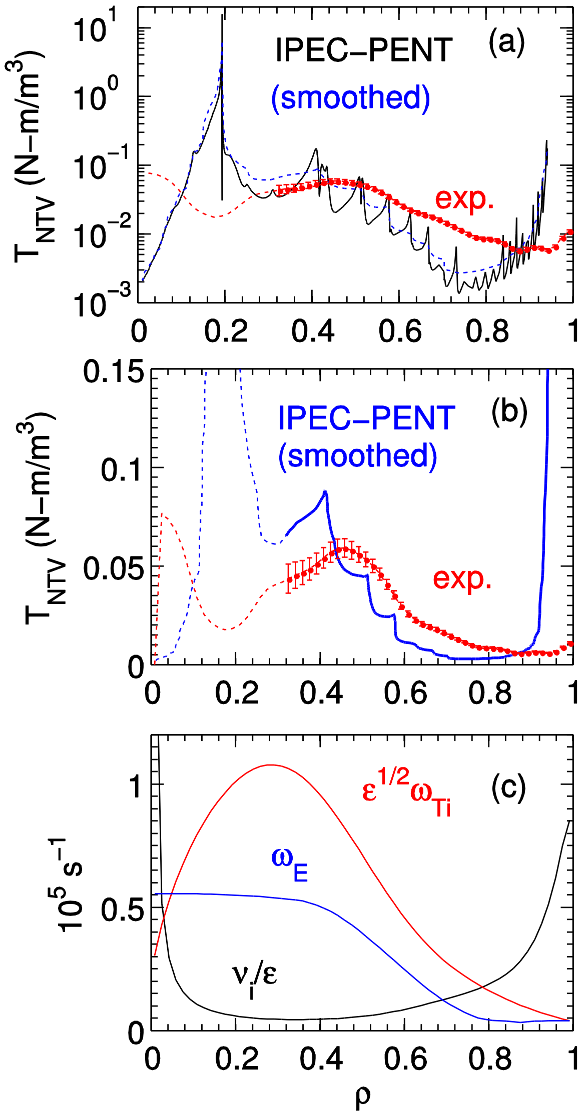


3D perturbation from IPEC



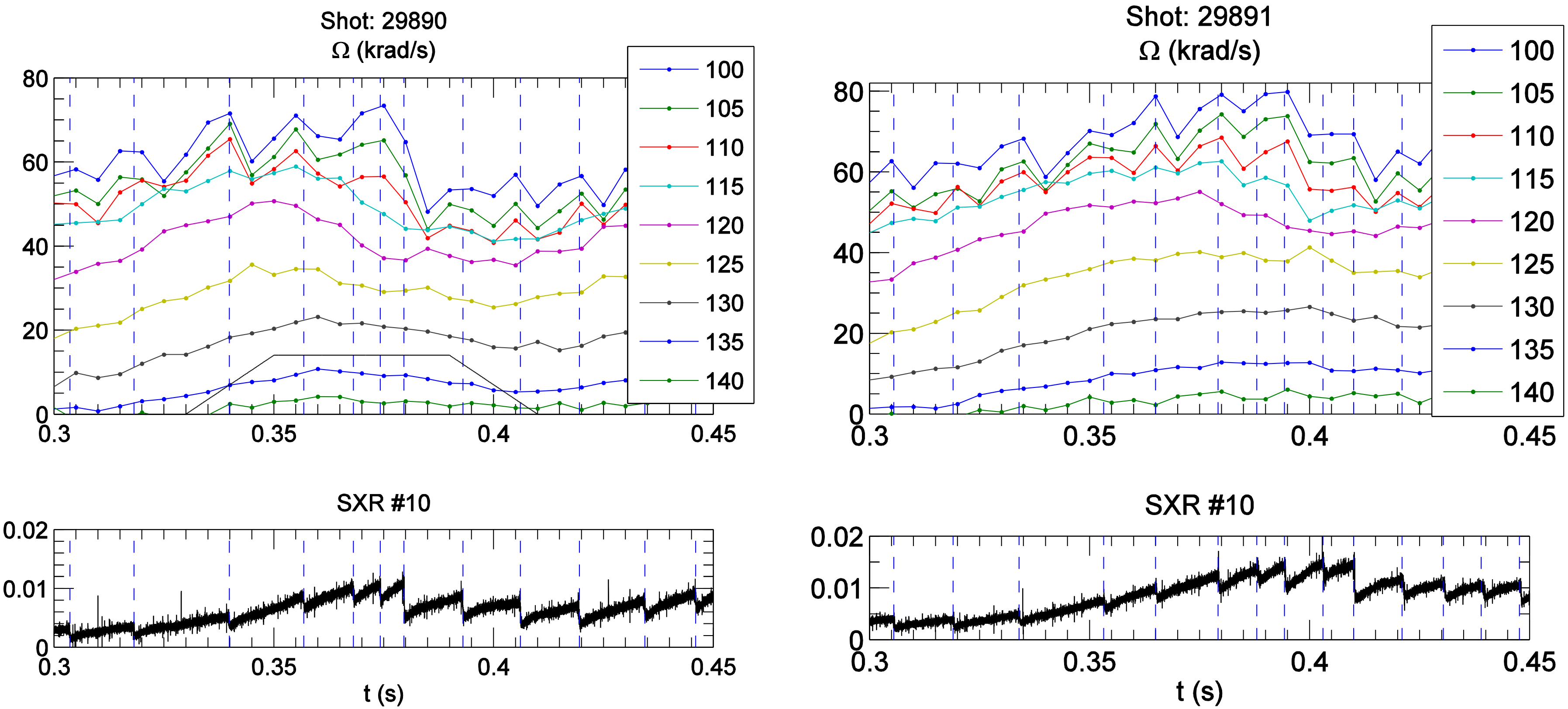
IPEC-PENT modeling predicts similar range of core-dominant NTV torque, but profiles are different due to q=1 subtlety

- Lower coil n=3 configuration generates both resonant and non-resonant components of the field
- Total NTV by trapped and passing ions = 0.13~0.36N-m for q_{min}=0.95~1.05
  - NTV torque is strong at the core by l=1,2 bounce resonances, and also by low enough collisionality due to peaked temperature profile
- But wrong in details: Non-linear saturation of the field inside q<1, potato orbits at the center, and finite-orbit averaging near the peaks can be all important



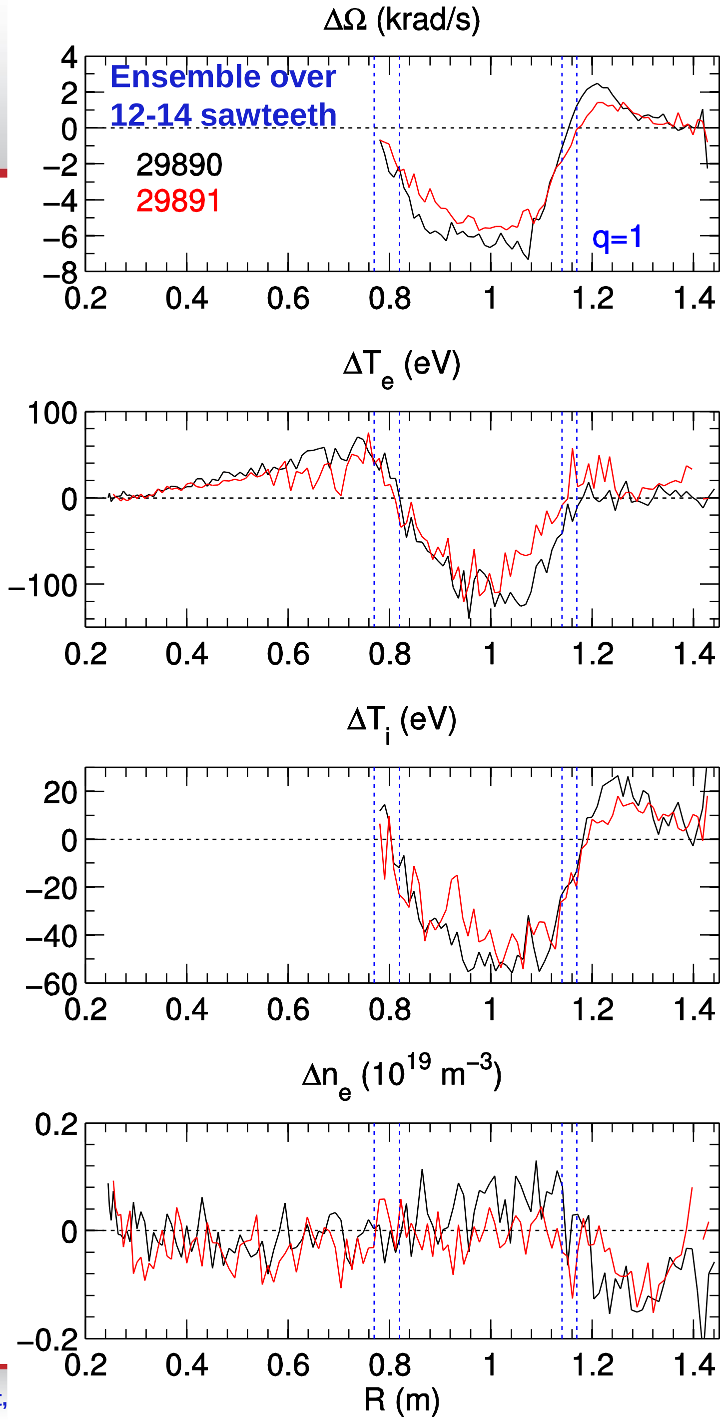
Effect of sawteeth visible in central rotation

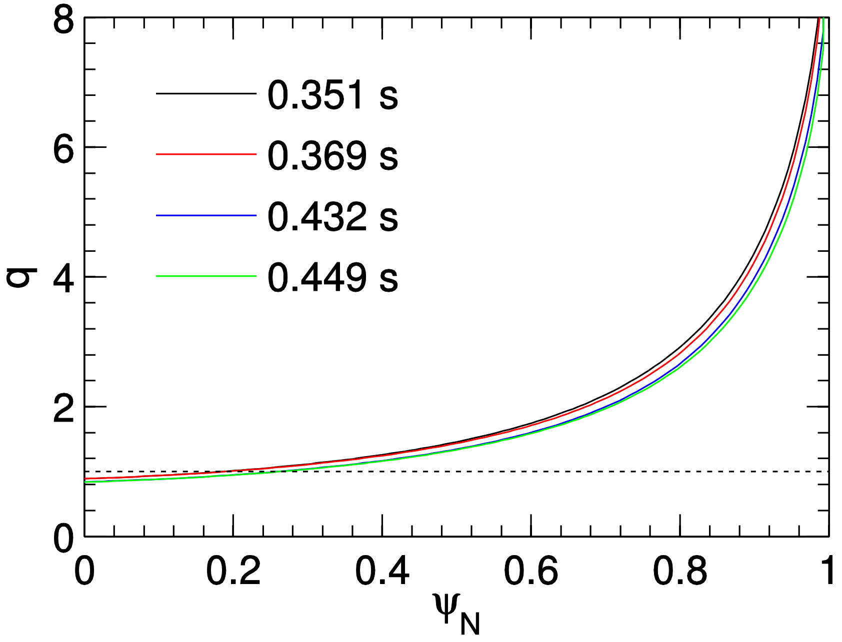
- Sawteeth occur with period $(\Delta t)_{ST}\approx$6-22 ms (average ~12 ms)
- CXRS measurement sampling of Δt=5 ms
- Can ensemble difference just before/after to estimate average $\Delta\Omega_{ST}$, $\Delta T_{e,ST}$, …



Sawteeth cause ~6 krad/s (~8%) deceleration inside inversion radius

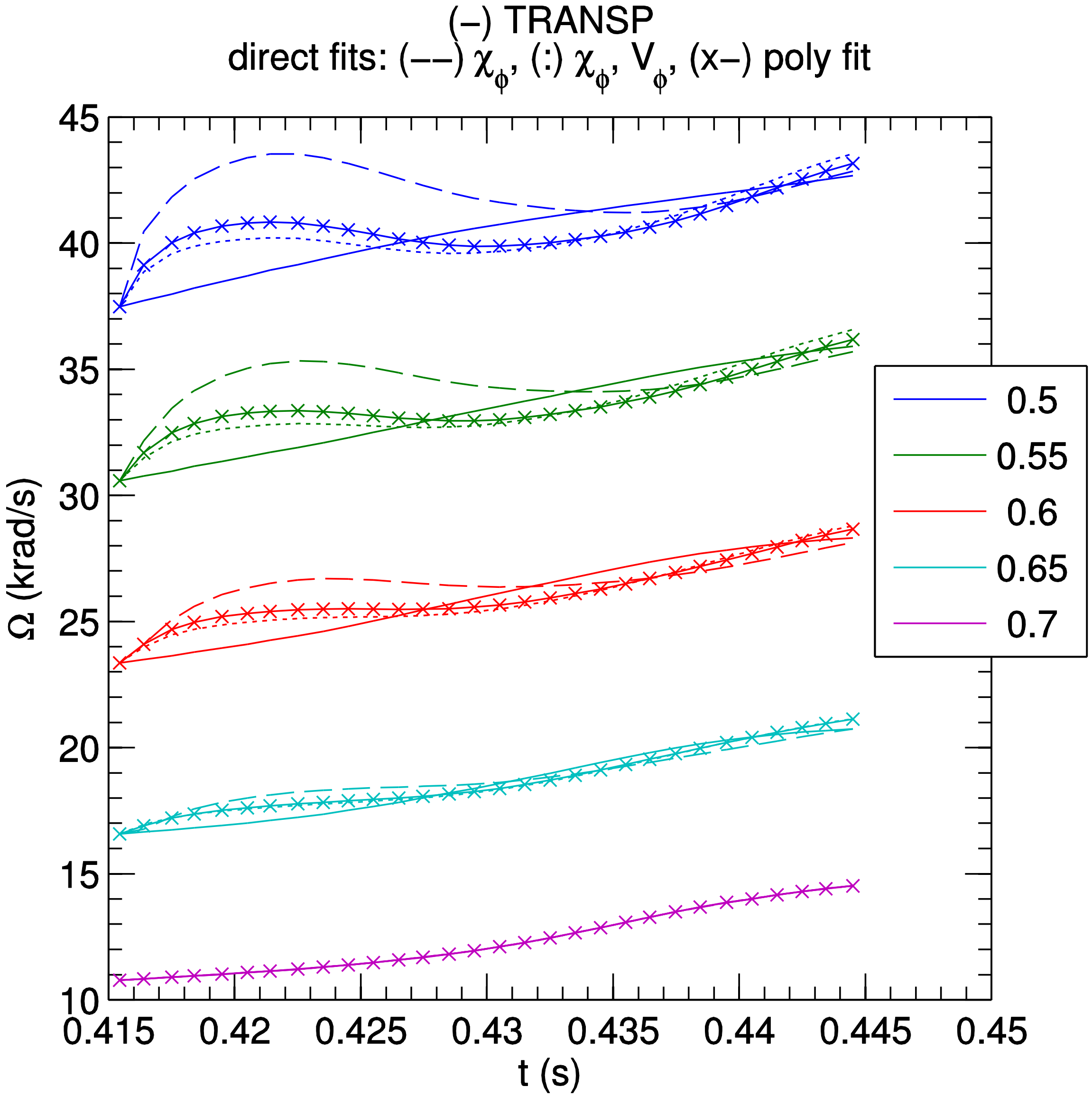
- q=1 surface ψ_N~0.19-0.26 (R_{out}~114-118 cm) consistent with ΔT_e inversion
- ΔT_e ~ 120 eV (~16% of $T_{e,0}$~750)
- ΔT_i ~ 50 eV (~6% of $T_{i,0}$~800)





Predicted Ω(t) response improved when including convection (χ_φ & V_φ) as opposed to diffusion only (χ_φ)

- Details of time response not accurately reproduced



Many theoretical mechanisms to consider for momentum transport

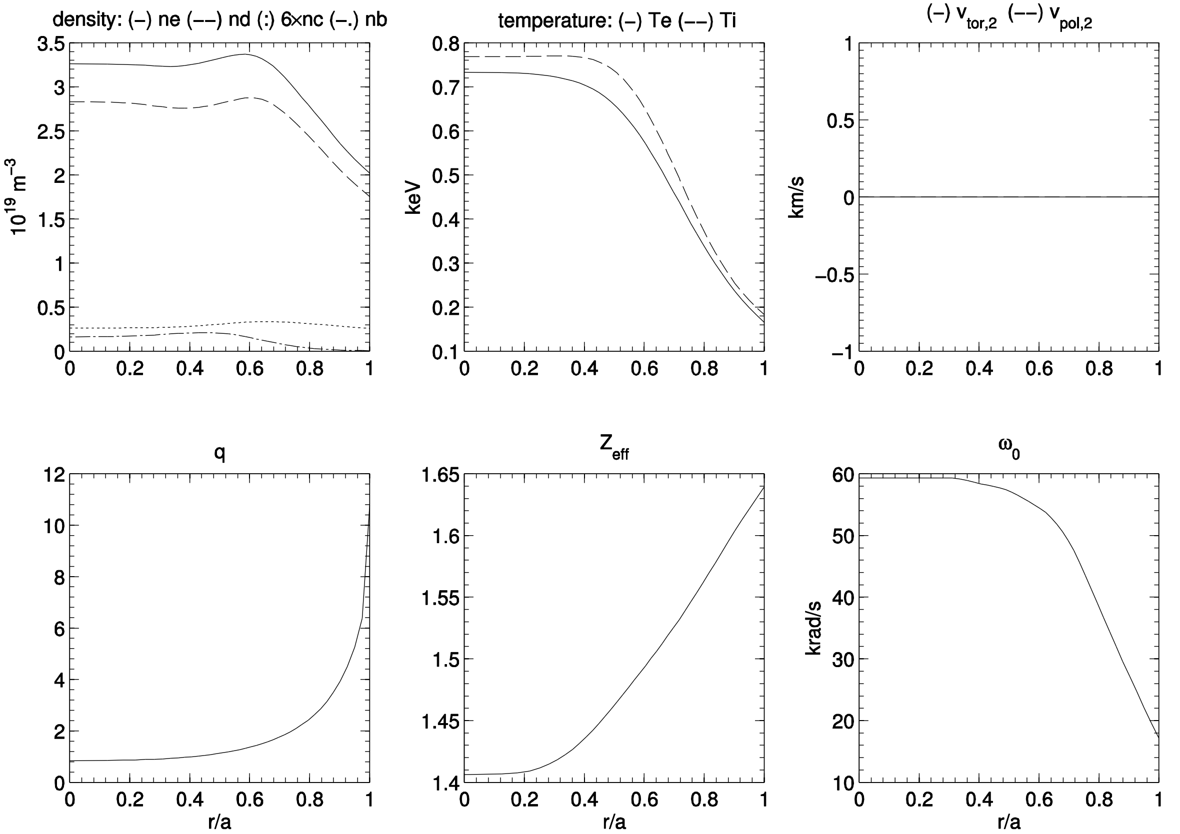
$$\Pi_\varphi = \mathrm{nmR}(\chi_\varphi u' + \chi_{\varphi\perp}\gamma_E) + (\mathrm{nmRV}_\varphi + \mathrm{mR}\Gamma_p)u + C_{UD} + C_{\rho*} + \ldots$$

- More general expression for momentum transport (e.g., Peeters, NF 2011) includes contributions due to:
  - Perpendicular (E×B) flow shear [Casson, 2010; Dominguez, 1993]
  - Particle convection (usually expected to be small)
  - Up-down asymmetry [Camenen, 2009]
  - Finite ρ_*/nonlocal effects (profile shearing, …) [Camenen, 2011]
- Also, important to consider all mechanisms in fully developed nonlinear turbulence (i.e. not just quasi-linear)
- In the core of NSTX NBI plasmas, toroidal flow dominates radial force balance so that $u'=(qR/r)\cdot\gamma_E$ (i.e. negligible v_{pol}, ∇p_i contributions)
  - In theory and codes we can vary u', γ_E, u, ρ_* independently to identify various physical mechanisms

- Have begun to investigate nonlinear, E×B shear and finite ρ_* effects

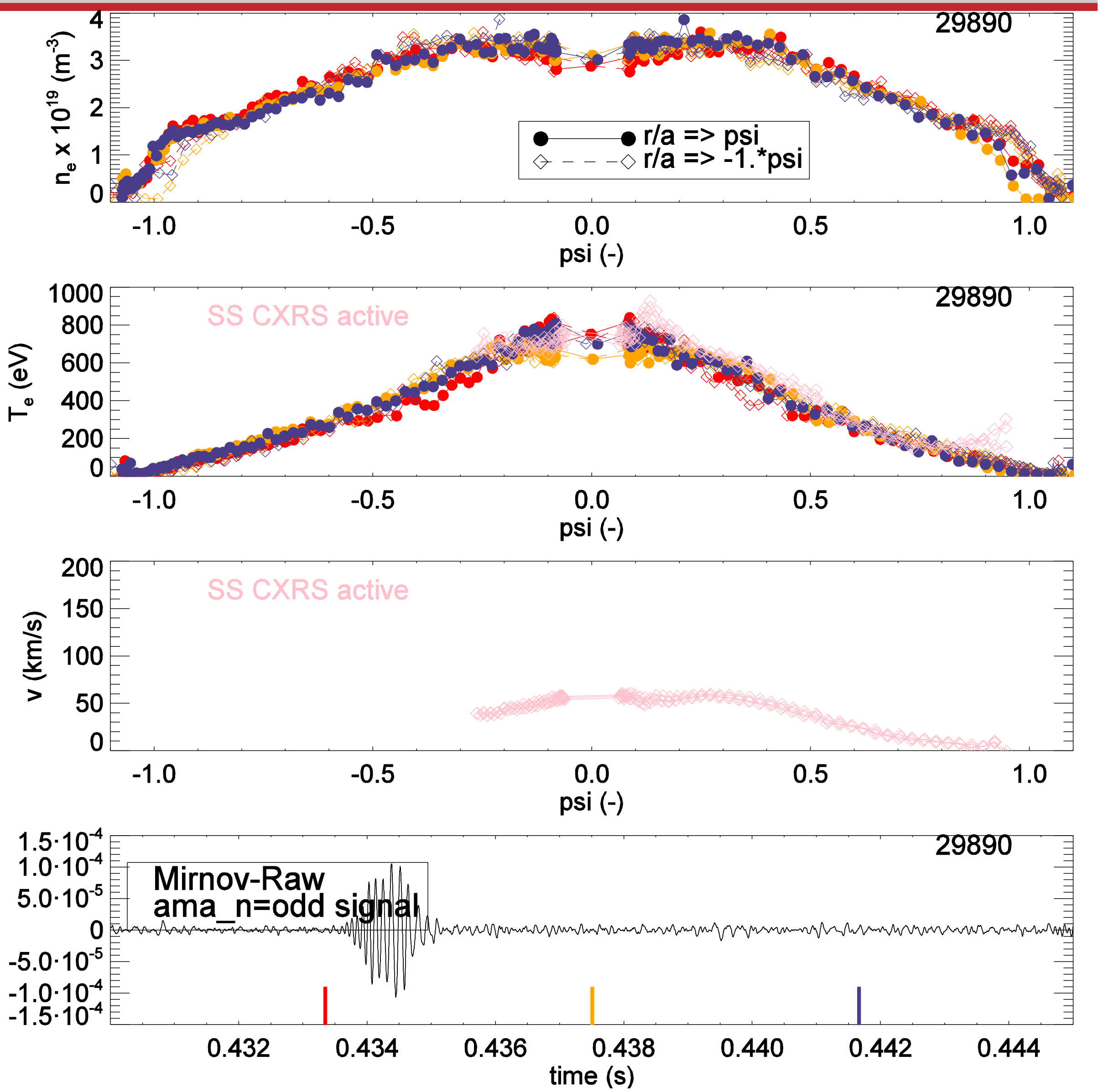
EFIT and profiles – 29890F01, t~0.43 s

- Using MSE-constrained EFIT++ g-file from I. Lupelli (t=0.432 s)
- Using TRANSP profiles from 29890F01, time-averaged t=0.43-0.44 s
- Wrote D, C, D_{beam} information ($T_C=T_D$, $T_{beam}=2/3E_{beam}$)
- Calculated Z_{eff} from species densities – **what about Z_{eff} from Bremsstrahlung (larger than carbon only)?**



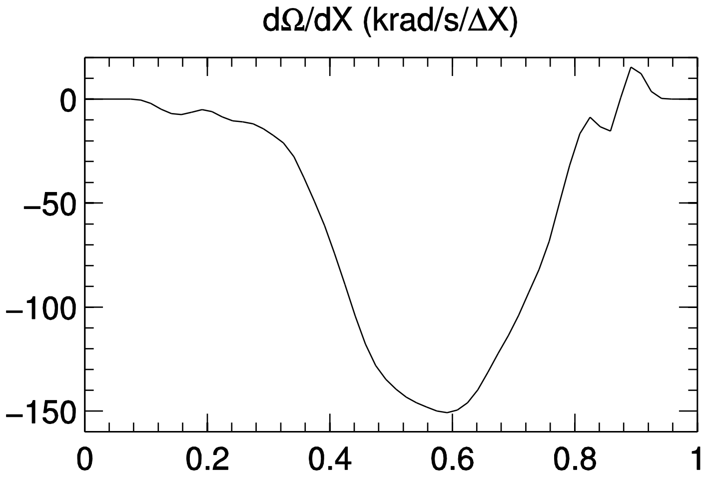
Raw data example vs. ψ_N

- Can see the CXRS T_i get a bit crazy for $\psi_N>0.8$, and rotation has gotten small (probably low carbon density) – flattish rotation profiles used in TRANSP out here



Rotation shear strongest around X=0.6; Peeters rough non-locality condition not particularly big

- asd



(exp.) $RV_\varphi/\chi_\varphi \approx -1.7 + 0.41\cdot R/L_n + 0.41\cdot q + 12\cdot\varepsilon^{1/2} - 1.9\cdot T_i/T_e$

(sim.) $RV_\varphi/\chi_\varphi \approx -2.8 + 0.44\cdot R/L_n + 0.31\cdot q + 6.8\cdot\varepsilon^{1/2} + 0.077\cdot R/L_{Te} - 0.21\cdot s$