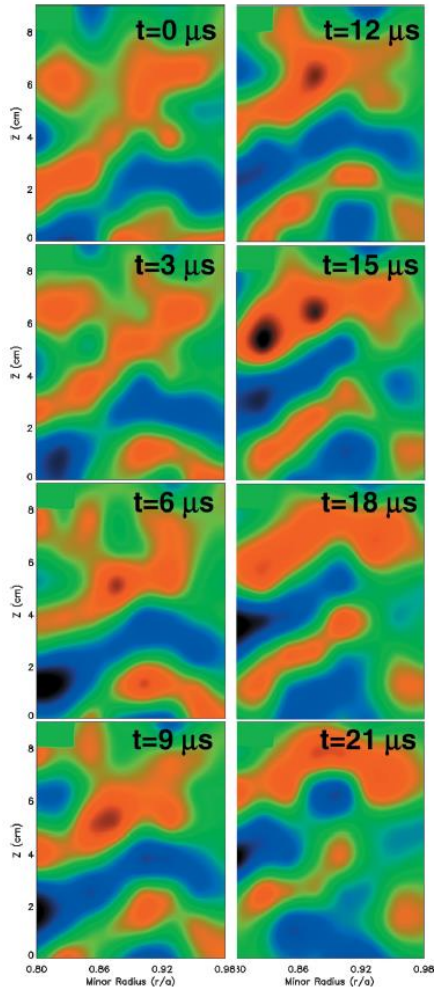
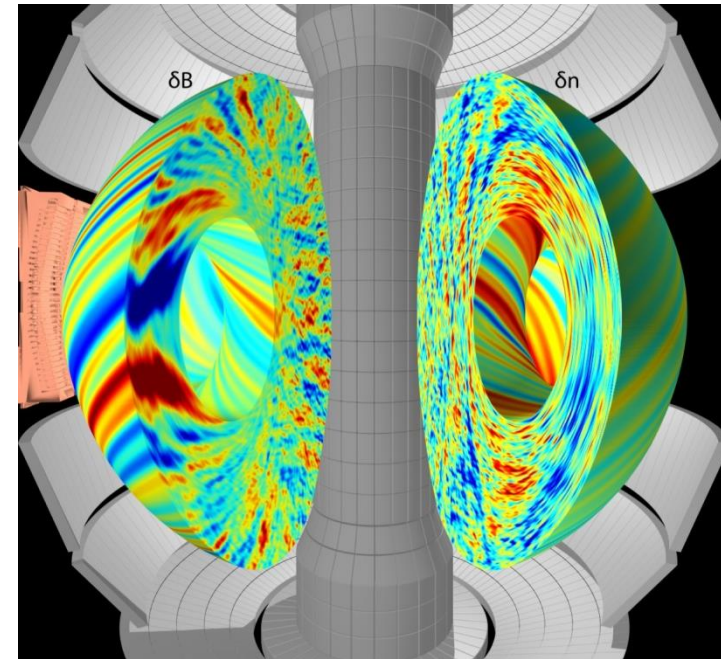


# Intro to magnetized plasma turbulence

Beam emission spectroscopy (BES) measurements in DIII-D



Gyrokinetic turbulence simulation in NSTX



**Walter Guttenfelder**  
*Princeton Plasma Physics Laboratory*

UT-Knoxville lectures  
Oct. 16 & 18, 2018

## 2 slide summary of some turbulent transport concepts in magnetized fusion plasmas (1)

- For fusion gain  $Q \sim nT\tau_E$  (& 100% non-inductive tokamak operation) we need excellent energy confinement,  $\tau_E$
- Energy confinement depends on turbulence ( $\tau_E \sim a^2/\chi_{\text{turb}}$ )
  - As does particle, impurity & momentum transport
- Core turbulence generally accepted to be drift wave in nature
  - Quasi-2D ( $L_{\perp} \sim \rho_i$ ,  $\rho_e \ll L_{\parallel} \sim qR$ )
  - Driven by  $\nabla T$  &  $\nabla n$
  - Frequencies  $\sim$  diamagnetic drift frequency ( $\omega \sim \omega_* \sim k_{\theta} \rho_i \cdot c_s/L_{n,T}$ )
  - Drift wave transport generally follows gyroBohm scaling  $\chi_{\text{turb}} \sim \chi_{\text{GB}} \sim \rho_i^2 v_{Ti}/a$ , *however...*
  - Thresholds and stiffness are critical, i.e.  $\chi_{\text{turb}} \sim \chi_{\text{GB}} \cdot F(\dots) \cdot (\nabla T - \nabla T_{\text{crit}})$
- Toroidal ion temperature gradient (ITG) drift wave is a key instability for controlling confinement in current tokamaks
  - Unstable due to interchange-like toroidal drifts, analogous to Rayleigh-Taylor instability
  - Threshold influenced by magnetic equilibrium ( $q$ ,  $s$ ) and other parameters
  - Nonlinear saturated transport depends on zonal flows & perpendicular ExB sheared flow

## 2 slide summary of some turbulent transport concepts in magnetized fusion plasmas (2)

- Reduced models are constructed by quasi-linear calculations + “mixing-length” estimates for nonlinear saturation
  - We rely heavily on direct numerical simulation using gyrokinetic codes to guide model development
  - Reasonably predict confinement scaling and core profiles
- Many other flavors of turbulence exist (TEM, ETG, PVG, MTM, KBM)
  - $\rho_i$  or  $\rho_e$  scale
  - Electrostatic or electromagnetic (at increasing beta)
  - Different physical drives, parametric dependencies, & influence on transport channels ( $\Gamma$  vs.  $Q$  vs.  $\Pi$ )
- Things get more complicated for edge / boundary turbulence
  - Changing topology (closed flux surfaces  $\rightarrow$  X-point (poloidal field null)  $\rightarrow$  open field lines & sheaths at physical boundary)
  - Larger gyroradius / banana widths,  $\rho_{\text{banana}}/\Delta_{\text{ped}} \sim 1 \rightarrow$  orbit losses & non-local effects
  - Large amplitude fluctuations,  $\delta n/n_0 \sim 1$  ( $\delta f \rightarrow$  full-F simulations)
  - Neutral particles, radiation, other atomic physics...

# Some additional sources & references

- Greg Hammett has a lot of great introductory material to fusion, tokamaks, drift waves, ITG turbulence, gyrokinetics, etc... ([w3.pppl.gov/~hammett](http://w3.pppl.gov/~hammett))
- Greg & I recently gave five 90 minute lectures on turbulence at the 2018 Graduate Summer School ([gss.pppl.gov](http://gss.pppl.gov))
- See the following for broader reviews and thousands of useful references
- Transport & Turbulence reviews:
  - Liewer, Nuclear Fusion (1985)
  - Wootton, Phys. Fluids B (1990)
  - Carreras, IEEE Trans. Plasma Science (1997)
  - Wolf, PPCF (2003)
  - Tynan, PPCF (2009)
  - ITER Physics Basis (IPB), Nuclear Fusion (1999)
  - Progress in ITER Physics Basis (PIPb), Nuclear Fusion (2007)
- Drift wave reviews:
  - Horton, Rev. Modern Physics (1999)
  - Tang, Nuclear Fusion (1978)
- Gyrokinetic simulation review:
  - Garbet, Nuclear Fusion (2010)
- Zonal flow/GAM reviews:
  - Diamond et al., PPCF (2005)
  - Fujisawa, Nuclear Fusion (2009)
- Measurement techniques:
  - Bretz, RSI (1997)

# OUTLINE

## Lecture #1 (Tuesday, 10/16)

- Fusion, confinement, tokamaks, transport
- General turbulence examples
- Turbulence in magnetized plasma
- Drift waves
- ITG instability

## Lecture #2 (Thursday, 10/18)

- Other flavors of microinstability (TEM, ETG, MTM, KBM, PVG/KH)
- Turbulent transport, nonlinear saturation
- Zonal flows & geodesic acoustic modes (GAMs)
- ExB shear suppression
- Modeling turbulent transport

## Extra

- Edge turbulence considerations (L-H transition, H-mode pedestal turbulence, scrape off layer/divertor turbulence)
- Stellarator turbulence considerations

# **FUSION, CONFINEMENT, TOKAMAKS, TRANSPORT**

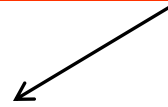
We desire fusion gain  $> 1$ , more fusion power out than power to heat the plasma

**Fusion gain**

$$Q = \frac{\text{fusion power}}{\text{heating power}}$$

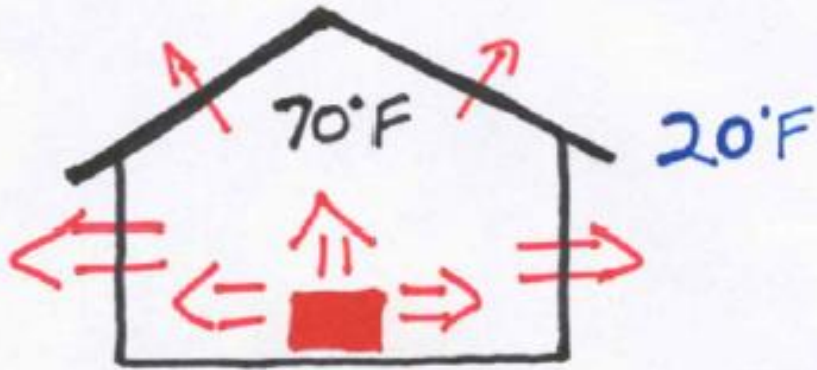
Fusion power  $\sim (\text{pressure})^2 \times \text{volume}$

$$Q \sim (\text{pressure}) \times (\text{confinement time})$$



$$\text{confinement time} \sim \frac{\text{pressure} \times \text{volume}}{\text{heating power}}$$

# Confinement time is a measure of how well insulated the plasma is from the surrounding boundary



$$\text{confinement time} \sim \frac{\text{energy in plasma (Joules)}}{\text{heating power (Watts)}}$$

## For ignition (a self-sustaining, “burning plasma”)

$Q \sim \text{pressure} \times \text{confinement time} > \underline{8 \text{ atm}\cdot\text{s}}$  (at  $\sim 150$  million C)

**pressure**  $\sim 2\text{-}4 \times$  atmospheric pressure (limited by MHD stability,  $\beta$  limits)

**energy confinement time,  $\tau_E$**   $\sim 2\text{-}4$  seconds

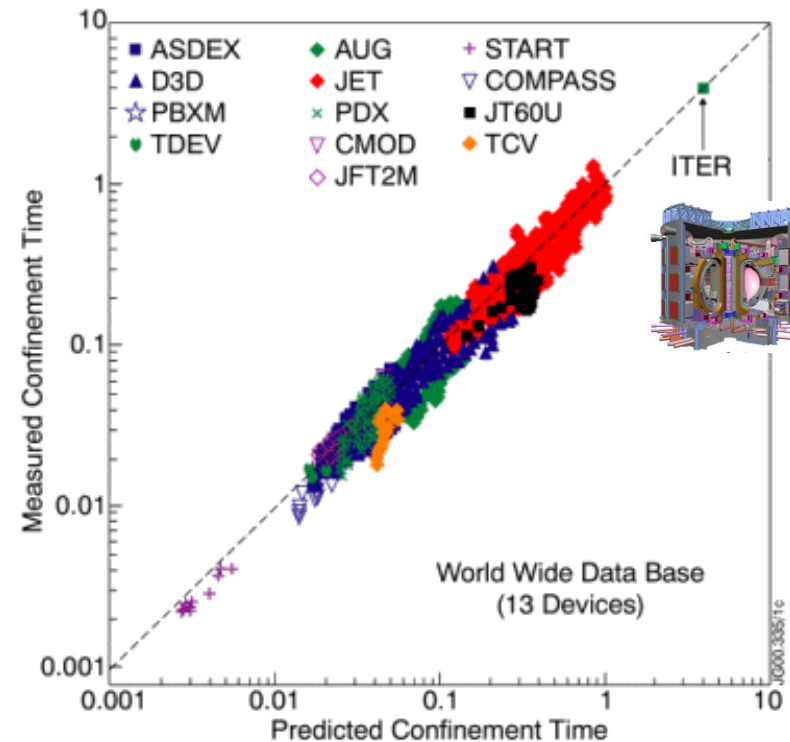


# Machine design / extrapolation often relies on empirical scaling of energy confinement

- E.g “ITER H-mode scaling”,  $\tau_{IPB,H98(y,2)}$

$$\tau_H = 0.145 \frac{I_M^{0.93} R_0^{1.39} a^{0.58} \kappa^{0.78} \bar{n}_{20}^{-0.41} B_0^{0.15} A^{0.19}}{P_M^{0.69}} \text{ s,}$$

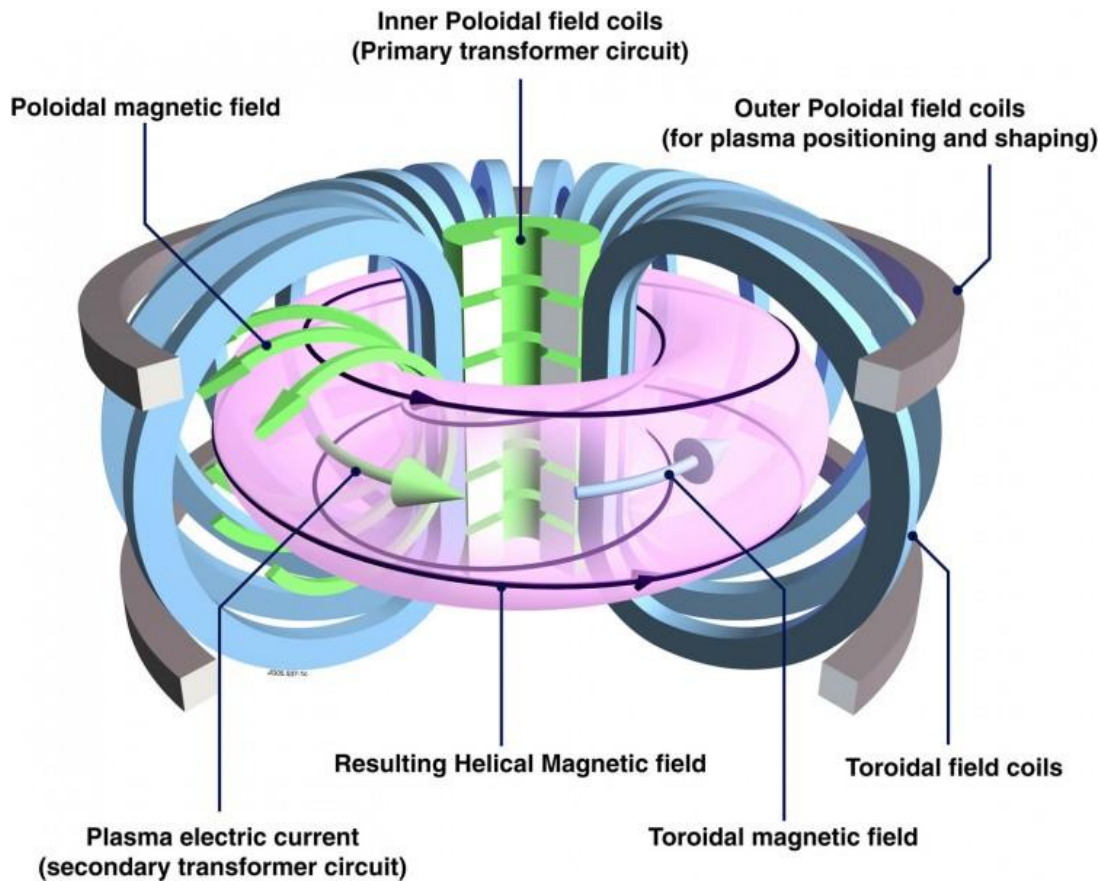
- Tokamak reactor design studies that enforce 100% non-inductive (stationary) **require excellent confinement, i.e.  $H_{98} > 1$  for  $\tau_E = H_{98} \cdot \tau_{IPB98(y,2)}$**



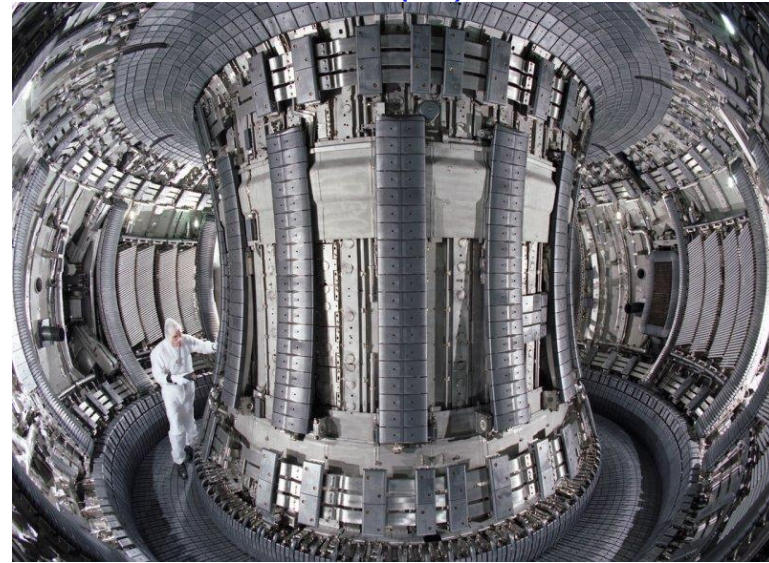
- Empirical confinement scalings are very useful, but have known pitfalls (power laws may not be appropriate, strong collinearity in some variables, ...)
- Can we understand (turbulent) transport losses to optimize, or at least improve confidence in, next step MFE device performance?**

# Tokamaks

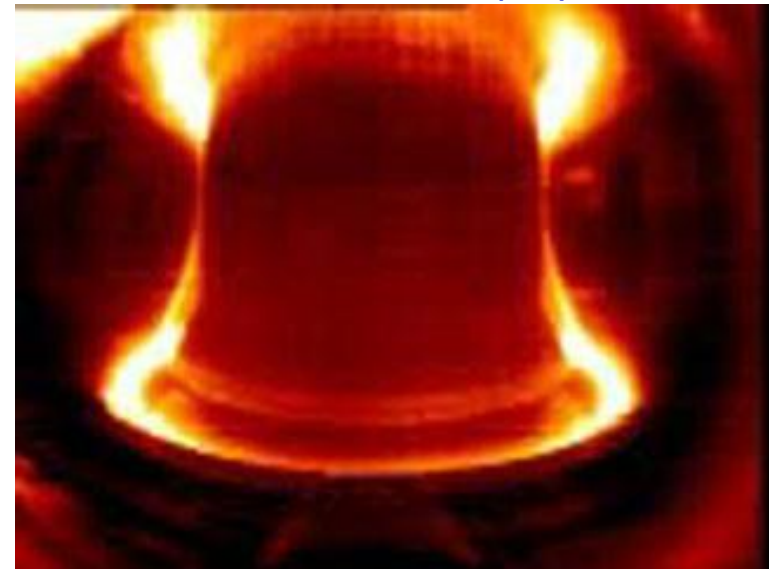
- Axisymmetric
- Helical field lines confine plasma



*JET (UK)*

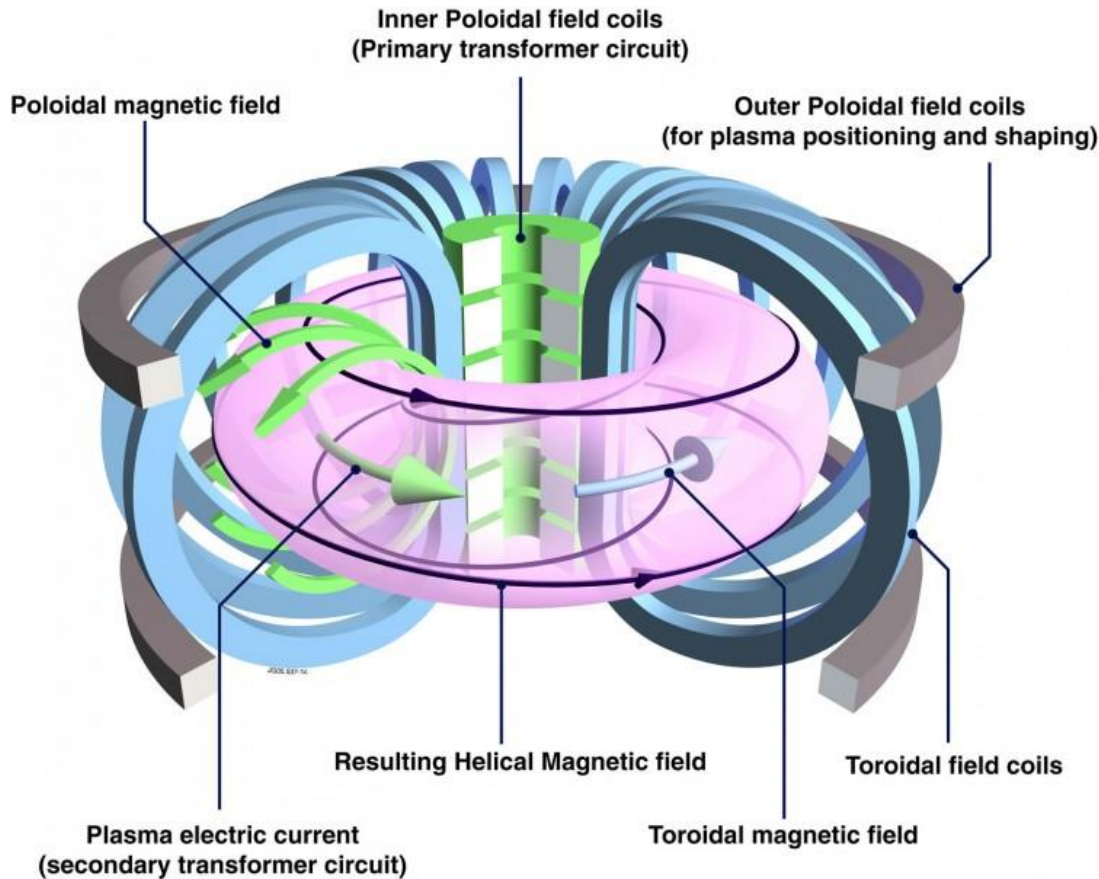


*Alcator C-Mod (MIT)*

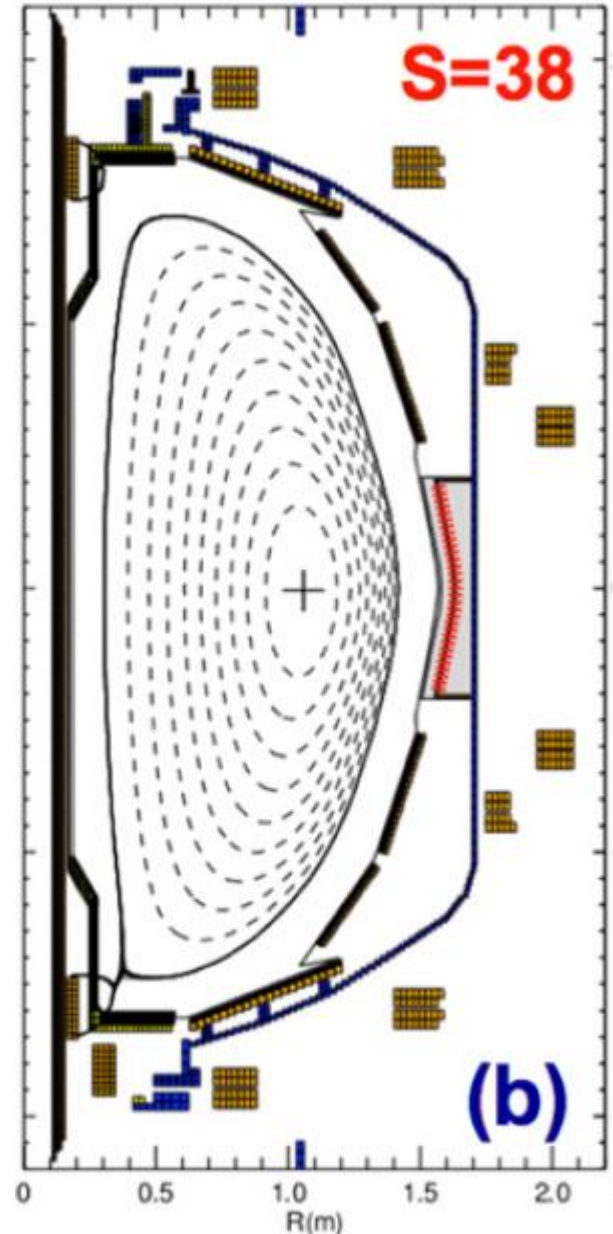


# Tokamaks

- Axisymmetric
- Helical field lines confine plasma
- Closed, nested flux surfaces

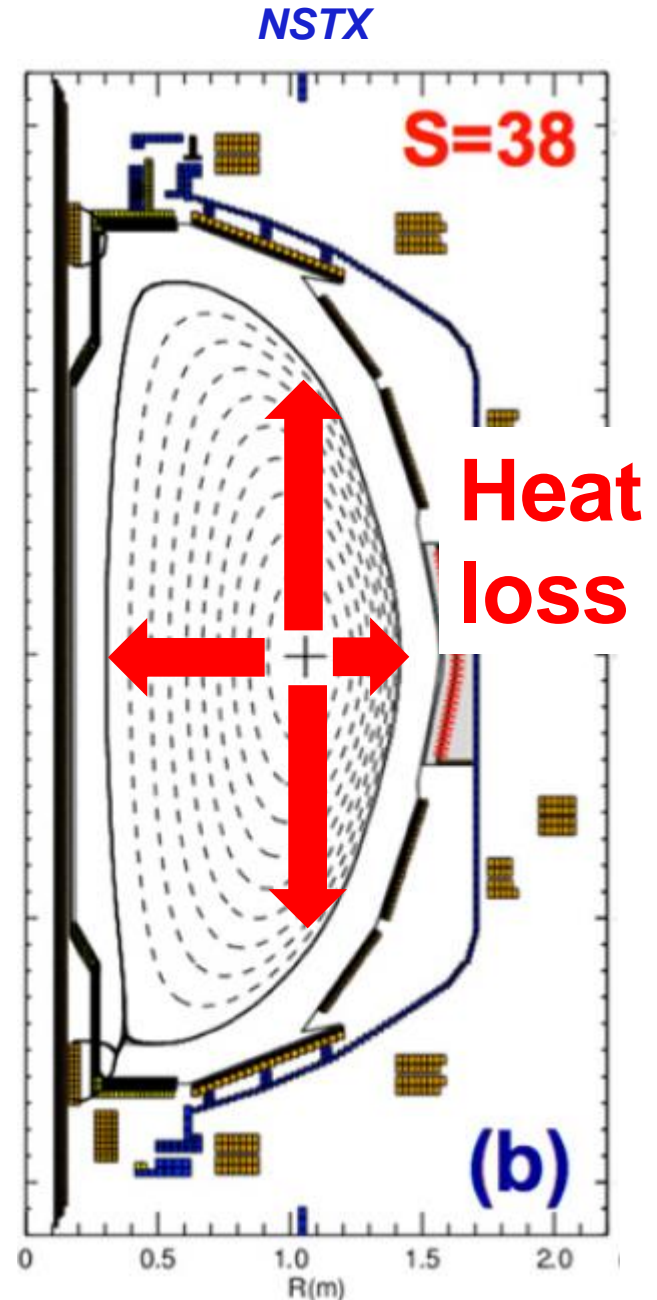
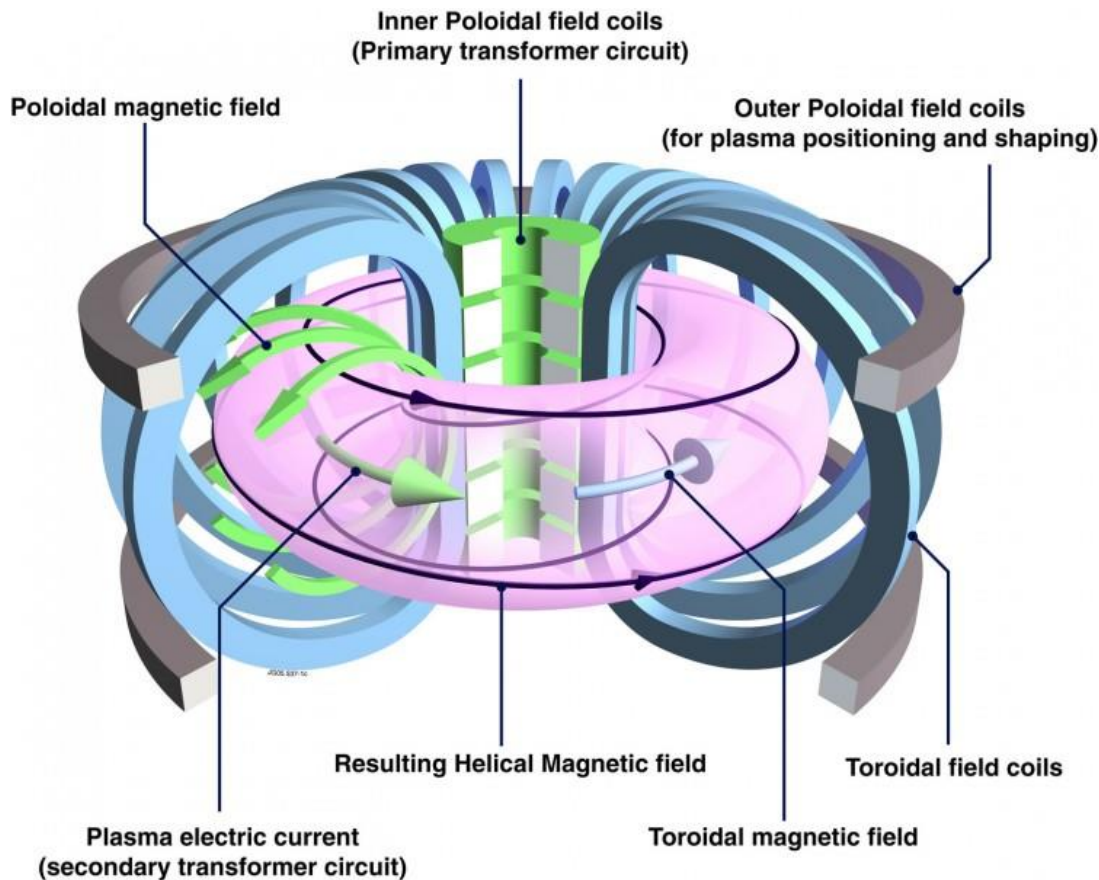


NSTX



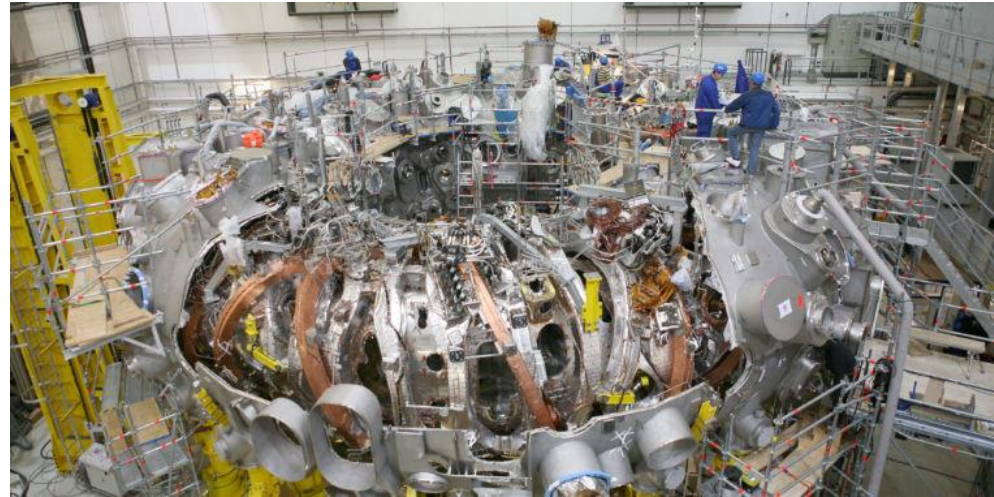
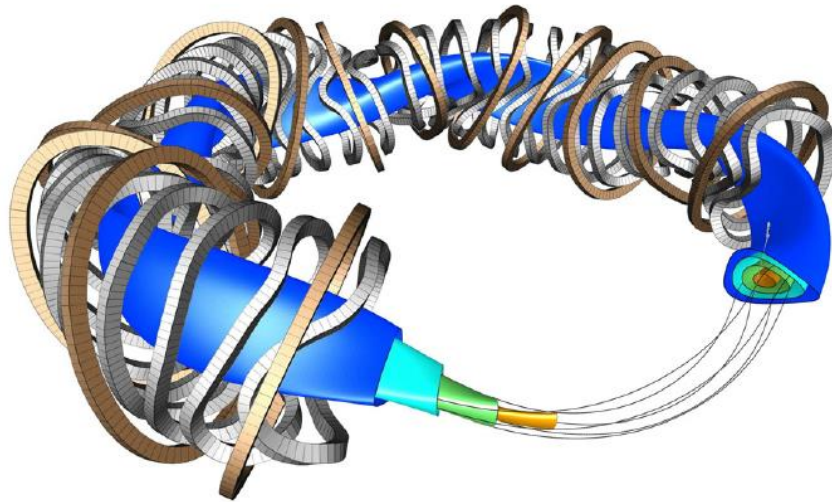
# Tokamaks

- Axisymmetric
- Helical field lines confine plasma
- Closed, nested flux surfaces

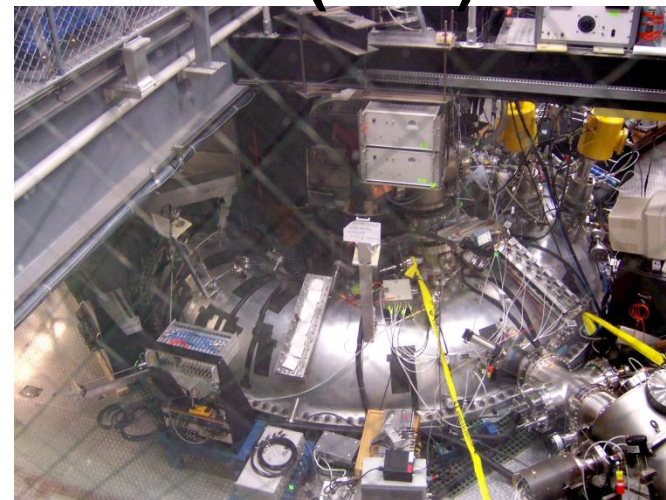
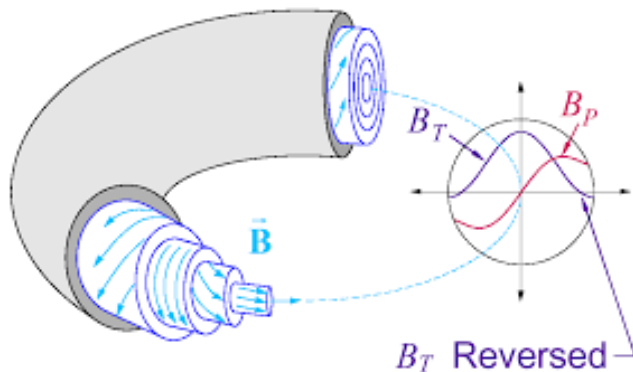


For what we're going to discuss, general phenomenology also important for stellarators or any toroidal B field

## W7-X stellarator



## MST Reversed Field Pinch (RFP)



## We use 1D transport equations to interpret experiments

- Take moments of kinetic equation
- Flux surface average, i.e. everything depends only on flux surface label ( $\rho$ )

$$\frac{3}{2} n(\rho, t) \frac{\partial T(\rho, t)}{\partial t} + \nabla \cdot Q(\rho, t) = \dot{P}_{\text{source}}(\rho, t) - \dot{P}_{\text{sink}}(\rho, t)$$

# We use 1D transport equations to interpret experiments

- Take moments of kinetic equation
- Flux surface average, i.e. everything depends only on flux surface label ( $\rho$ )
- Average over short space and time scales of turbulence (assume sufficient scale separation, e.g.  $\tau_{\text{turb}} \ll \tau_{\text{transport}}, L_{\text{turb}} \ll L_{\text{machine}}$ )  $\rightarrow$  macroscopic transport equation for evolution of equilibrium (non-turbulent) plasma state

$$\frac{3}{2} n(\rho, t) \frac{\partial T(\rho, t)}{\partial t} + \nabla \cdot Q(\rho, t) = \dot{P}_{\text{source}}(\rho, t) - \dot{P}_{\text{sink}}(\rho, t)$$

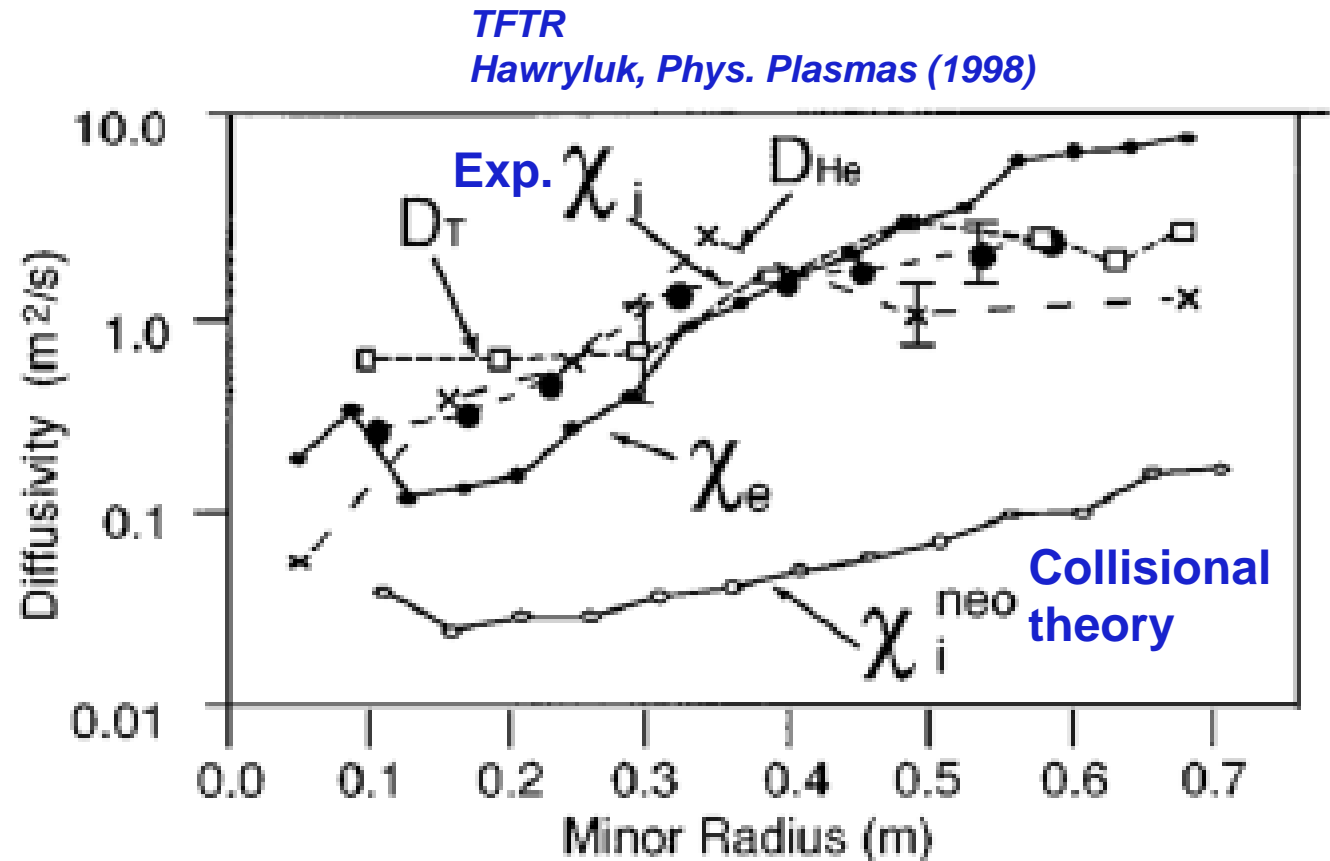
- To infer experimental transport,  $Q_{\text{exp}}$ :
  - Measure profiles (Thomson Scattering, CHERS)
  - Measure / calculate sources (NBI, RF)
  - Measure / calculate losses ( $P_{\text{rad}}$ )

# Inferred experimental transport larger than collisional (neoclassical) theory – extra “anomalous” contribution

$$D = -\frac{\Gamma}{\nabla n}$$

$$\chi = -\frac{Q}{n\nabla T}$$

- Reporting transport as diffusivities – does not mean the transport processes are collisionally diffusive!



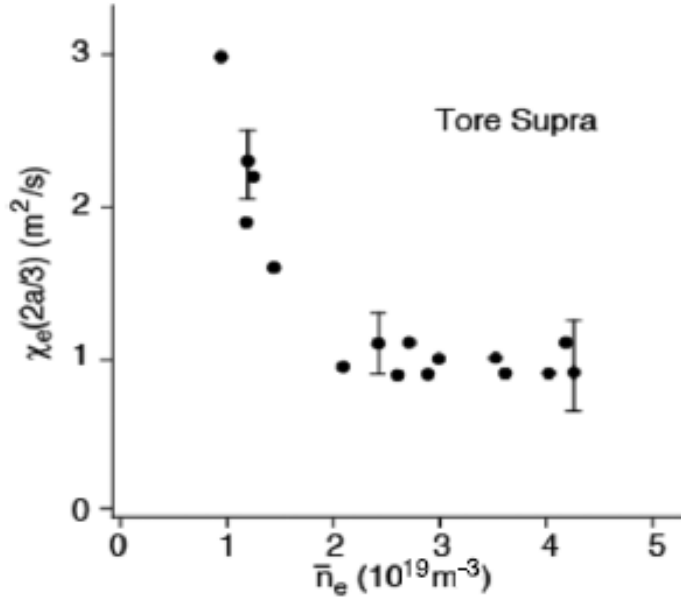
**Figure 1.** Results from TFTR showing ion thermal, momentum, and electron diffusivities in an L-mode discharge; reprinted with permission from the American Institute of Physics.



# Correlation between local transport and density fluctuations hints at turbulence as source of anomalous transport

Garbet, *Nuclear Fusion* (1992)  
 Tynan, *PPCF* (2009)

$$\chi = -\frac{Q}{n\nabla T}$$

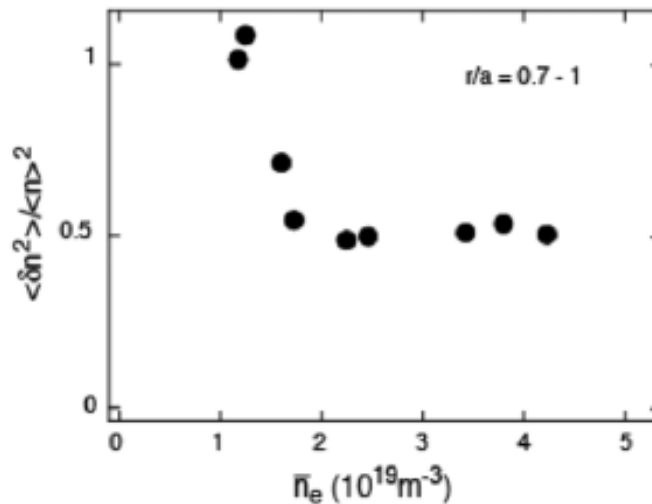


$$Q_{\text{exp}} = Q_{\text{collisions}} + Q_{\text{turbulence}}$$



*Our goal is to understand this*

$$\frac{\langle \delta n^2 \rangle}{\langle n \rangle^2}$$



# What is turbulence?

# What is turbulence? *I know it when I see it (maybe)...*

- M. Lesieur (2004) gives the following tentative definition:

(Hammett class notes)

- “Firstly, a turbulent flow must be *unpredictable*, in the sense that a small uncertainty as to its knowledge at a given initial time will amplify so as to render impossible a precise deterministic prediction of its evolution”. [I.e, turbulence is “chaotic”, it may occur in a formally deterministic system, but exhibits apparently random behavior because of extreme sensitivity to initial/boundary conditions.]
  - “Secondly, it has to satisfy the increased mixing property”, i.e., turbulent flows “should be able to mix transported quantities much more rapidly than if only molecular [collisional] diffusion processes were involved.” This property is of most interest for practical applications to calculate turbulent heat diffusion or turbulent drag.
  - “Thirdly, it must involve a wide range of spatial wave lengths”
- Also, turbulence is not a property of the *fluid*, it’s a feature of the *flow*

## Turbulence is an advective process

- Transport a result of finite average correlation between perturbed drift velocity ( $\delta v$ ) and perturbed fluid moments ( $\delta n$ ,  $\delta T$ ,  $\delta v$ )
  - Particle flux,  $\Gamma = \langle \delta v \delta n \rangle$
  - Heat flux,  $Q = 3/2 n_0 \langle \delta v \delta T \rangle + 3/2 T_0 \langle \delta v \delta n \rangle$
  - Momentum flux,  $\Pi \sim \langle \delta v \delta v \rangle$  (Reynolds stress, just like Navier Stokes)
- Electrostatic turbulence often most relevant in tokamaks  $\rightarrow$   $E \times B$  drift from potential perturbations:  $\delta v_E = B \times \nabla(\delta \phi) / B^2 \sim k_\theta(\delta \phi) / B$
- Can also have magnetic contributions at high beta,  $\delta v_B \sim v_{||}(\delta B_r / B)$  (magnetic “flutter” transport)

# Concepts of turbulence to remember

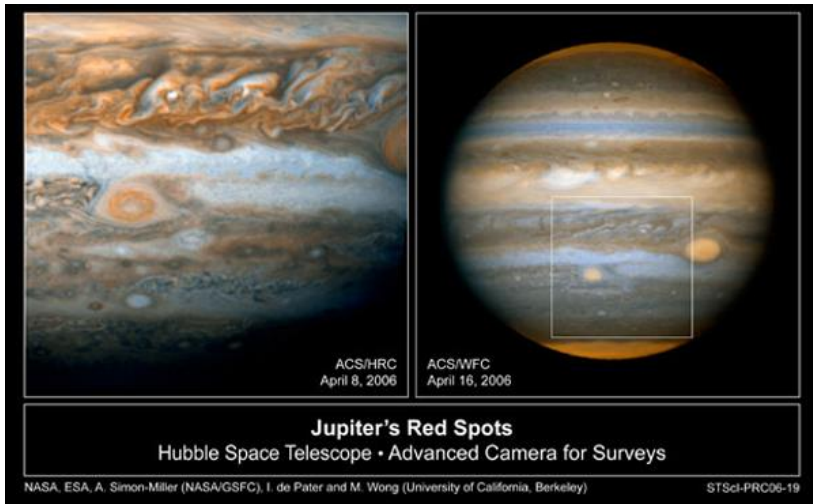
- Turbulence is deterministic yet unpredictable (chaotic), appears random
  - We often treat & diagnose statistically, but also rely on first-principles direct numerical simulation (DNS)
- Turbulence causes transport larger than collisional transport
  - **Transport** is the key application of why we care about turbulence
- Turbulence spans a wide range of spatial and temporal scales
  - Or in the case of hot, low-collisionality plasma, a wide range of scales in 6D phase-space ( $\mathbf{x}, \mathbf{v}$ )

# Concepts of turbulence to remember

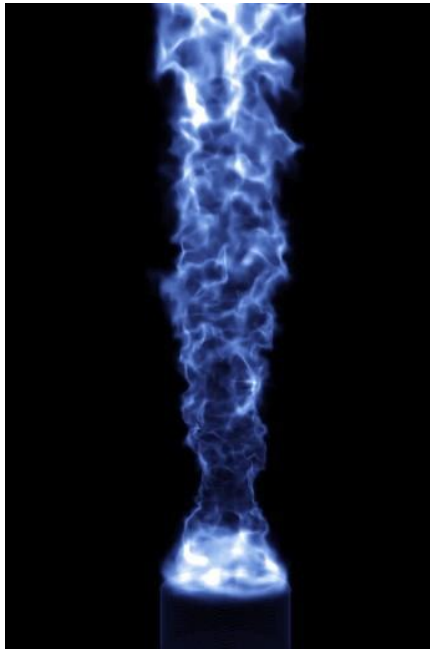
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- Turbulence spans a wide range of spatial and temporal scales
  - Or in the case of hot, low-collisionality plasma, a wide range of scales in 6D phase-space ( $\mathbf{x}, \mathbf{v}$ )
- It's cool! “Turbulence is the most important unsolved problem in classical physics” (~Feynman)

# **Turbulence examples (that you can see with your eyes)**

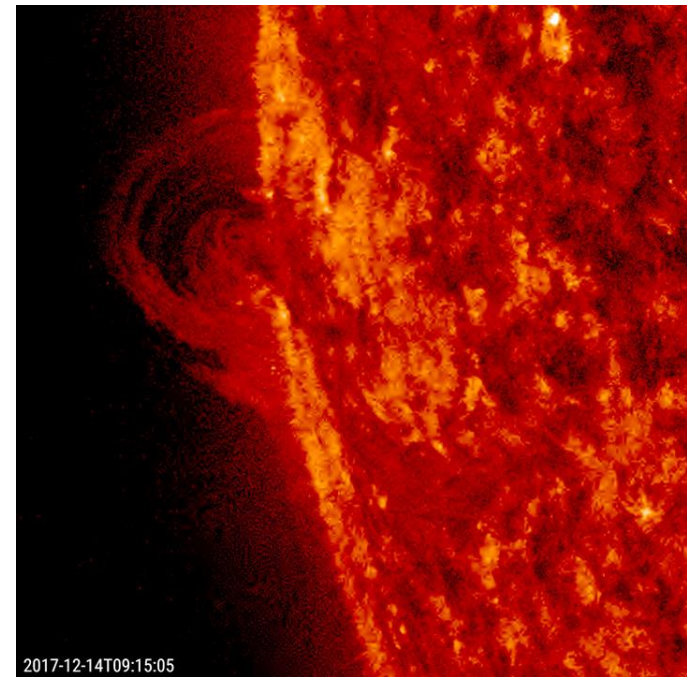
# Turbulence found throughout the universe



Steve Morri



Universität Duisburg-Essen



2017-12-14T09:15:05

<https://sdo.gsfc.nasa.gov/gallery>



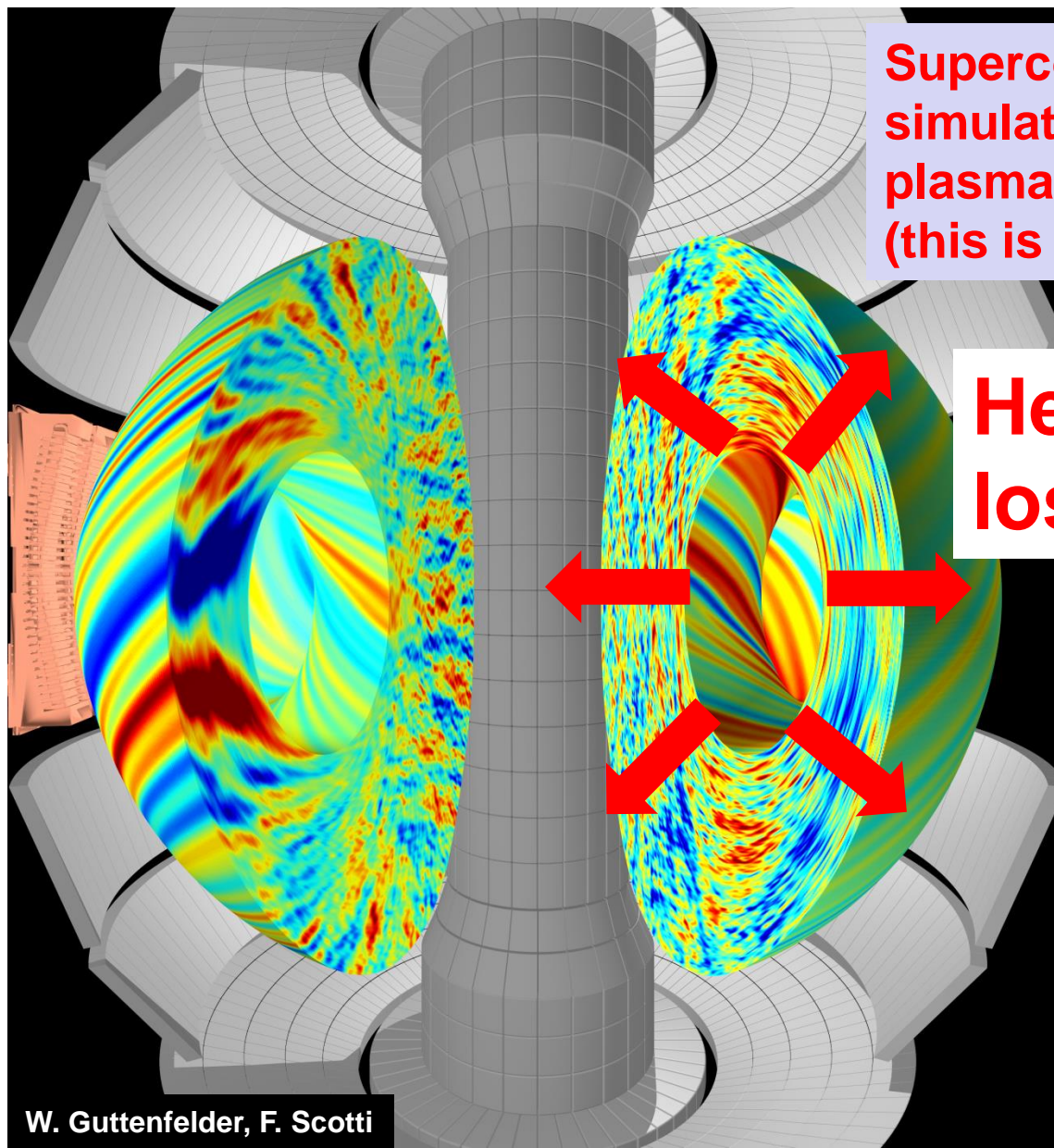
# Turbulence is ubiquitous throughout planetary atmospheres



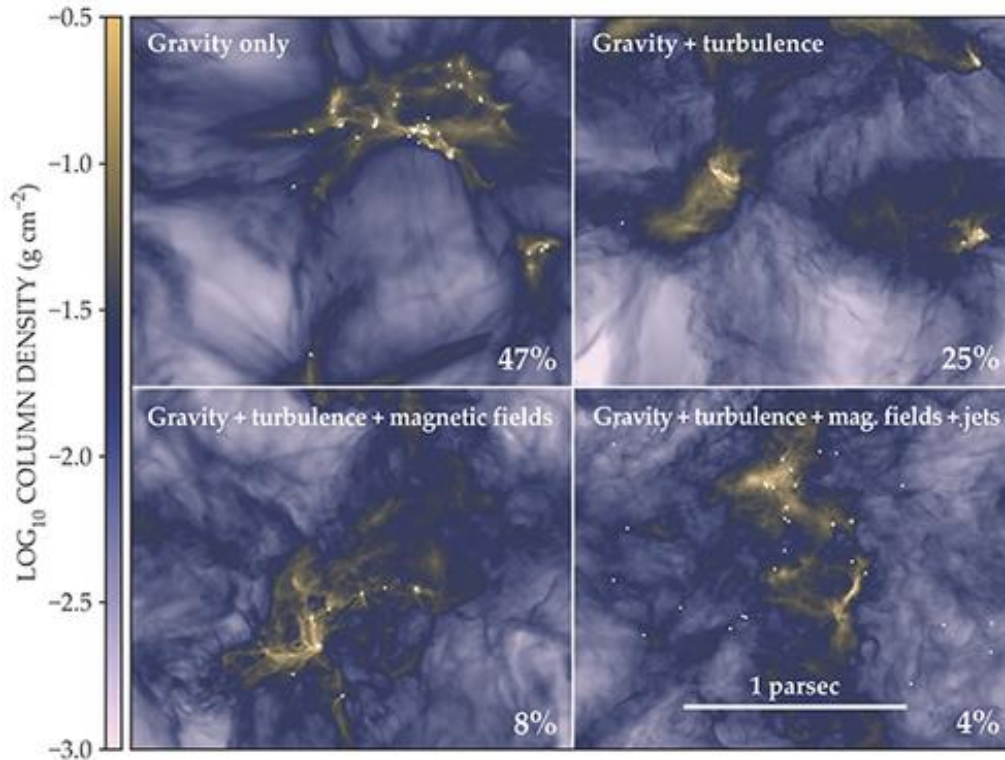
# Plasma turbulence determines energy confinement / insulation in magnetic fusion energy devices

Supercomputer simulation of plasma turbulence (this is what I do 😊)

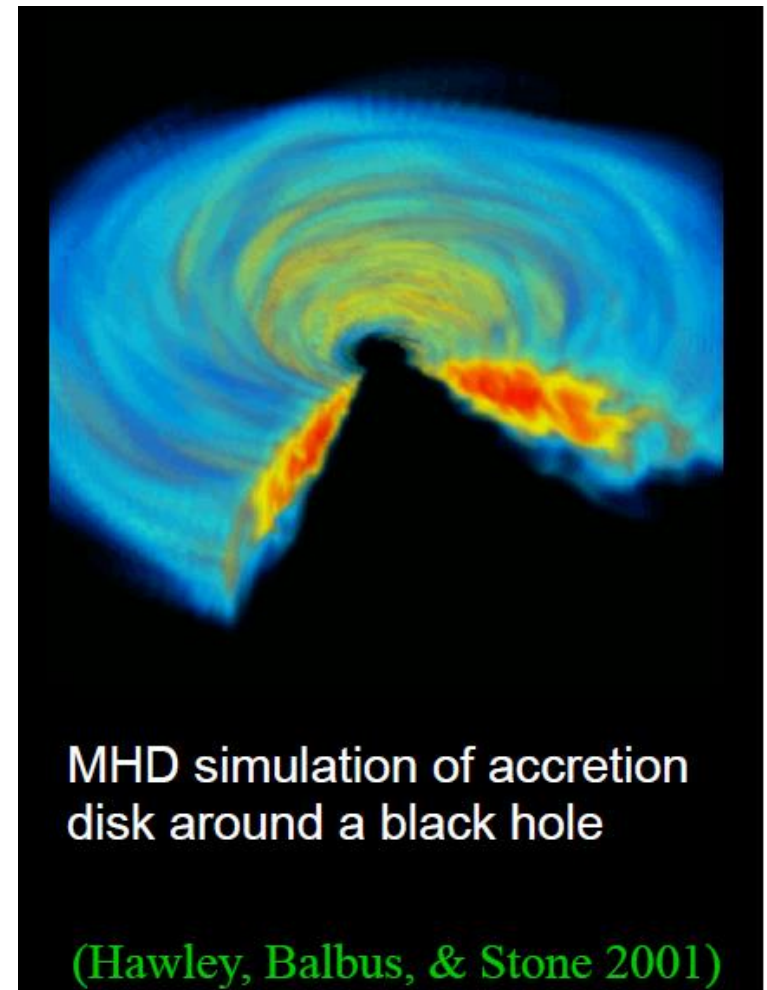
Heat loss



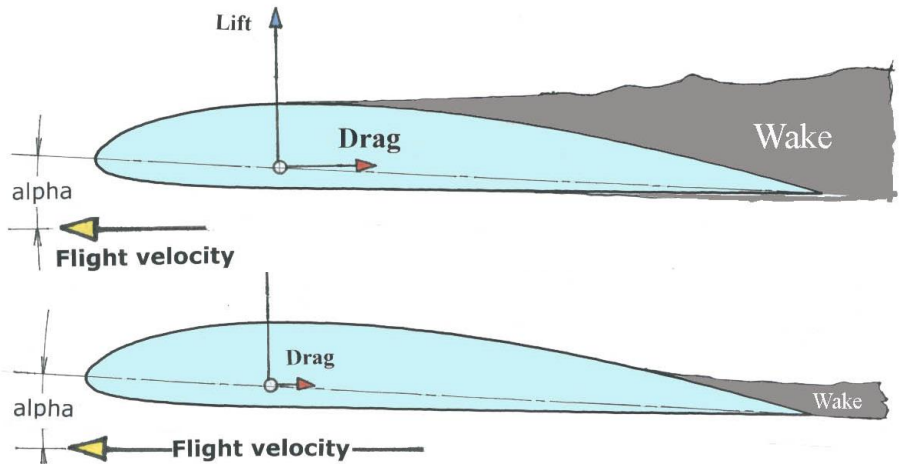
# Turbulence is important throughout astrophysics



- Plays a role in star formation (C. Federrath, Physics Today, June 2018)



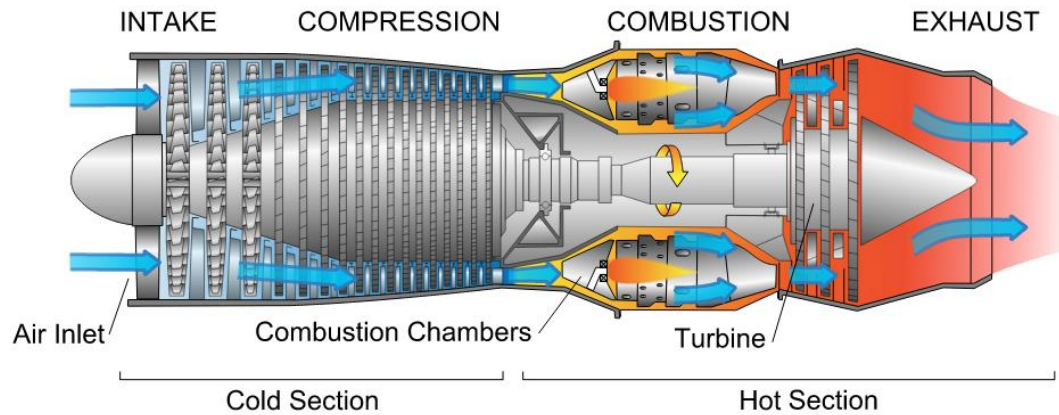
# Turbulence is crucial to lift, drag & stall characteristics of airfoils



Turbulence generators

Increased turbulence on airfoil helps minimize boundary-layer separation and drag from adverse pressure gradient

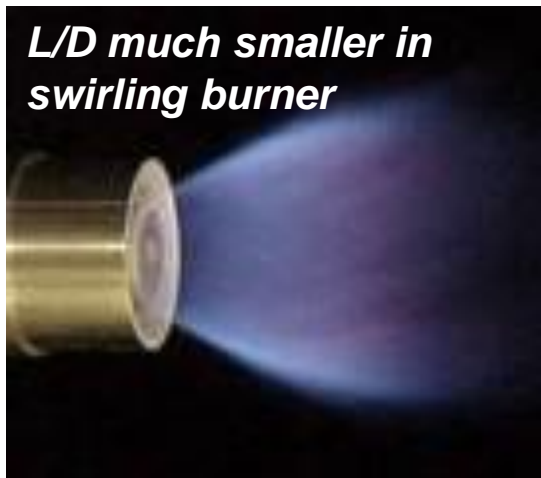




*L/D ~ 100-200 in non-premixed jet flames*



*L/D much smaller in swirling burner*



Turbulent mixing of fuel and air is critical for efficient & economical jet engines

**Turbulence in oceans crucial to the climate,  
important for transporting heat, salinity and carbon**

## **Perpetual Ocean (NASA, MIT)**

nasa.gov

mitgcm.org

# Fun with turbulence in art

## *Starry Night*, Van Gogh (1889)





# Leonardo da Vinci (1508), *turbolenza*



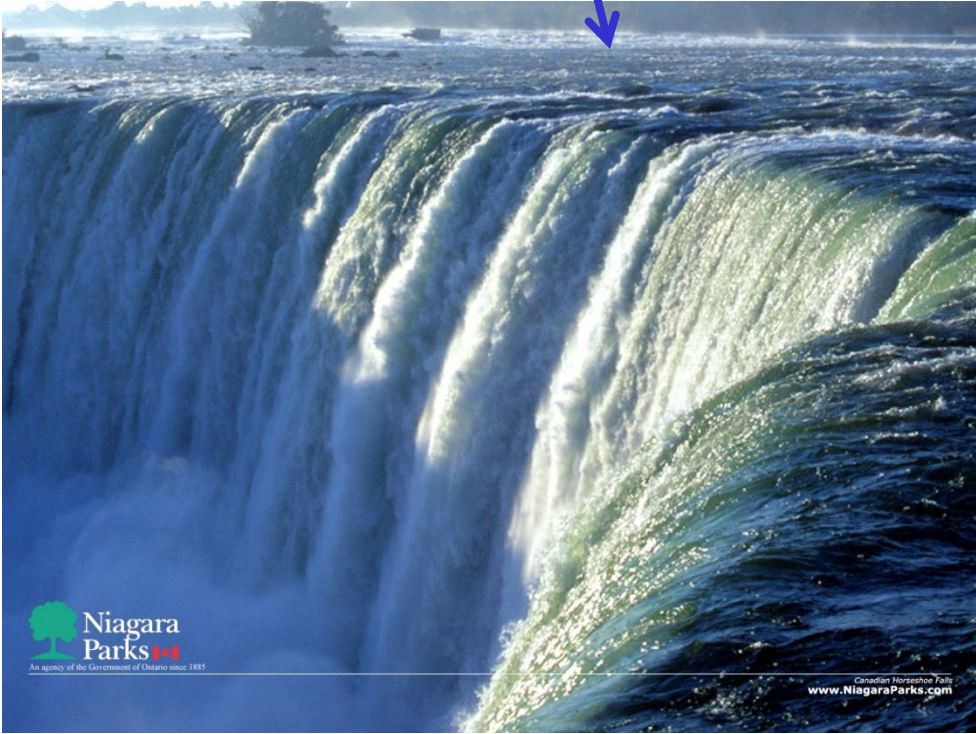
# The Great Wave off Kanagawa, Hokusai (1831)



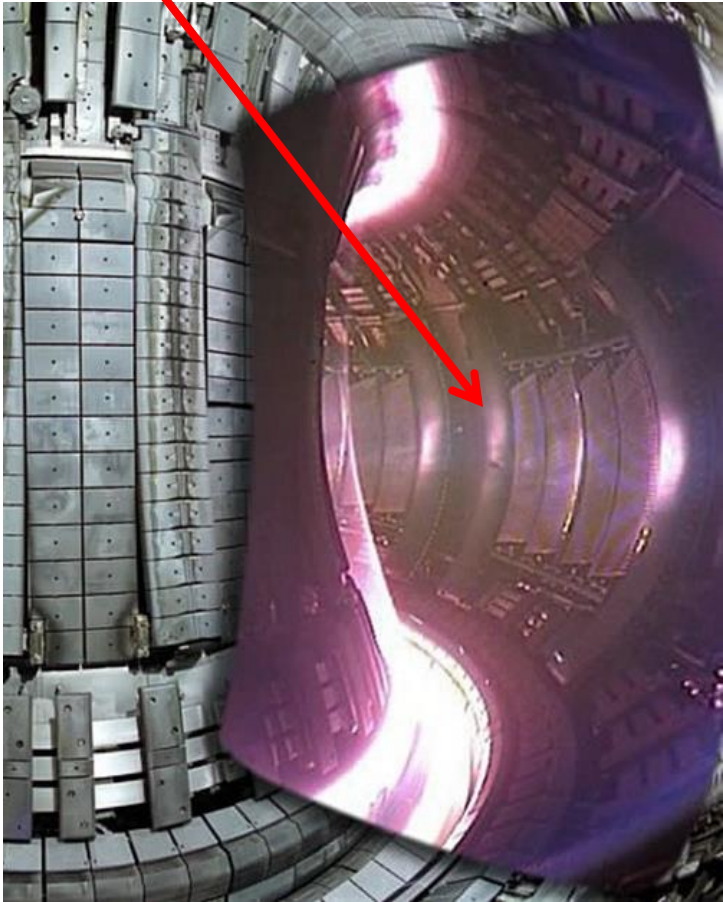
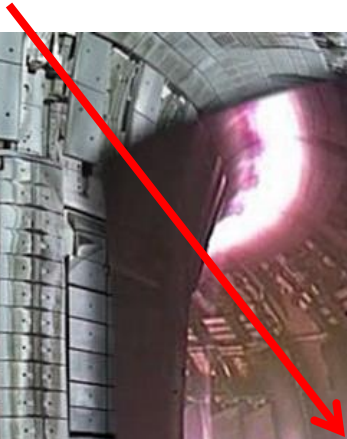
# Observing turbulence in tokamaks

# Very challenging to diagnose turbulence at 100 million degrees...

300 C



100,000,000 C

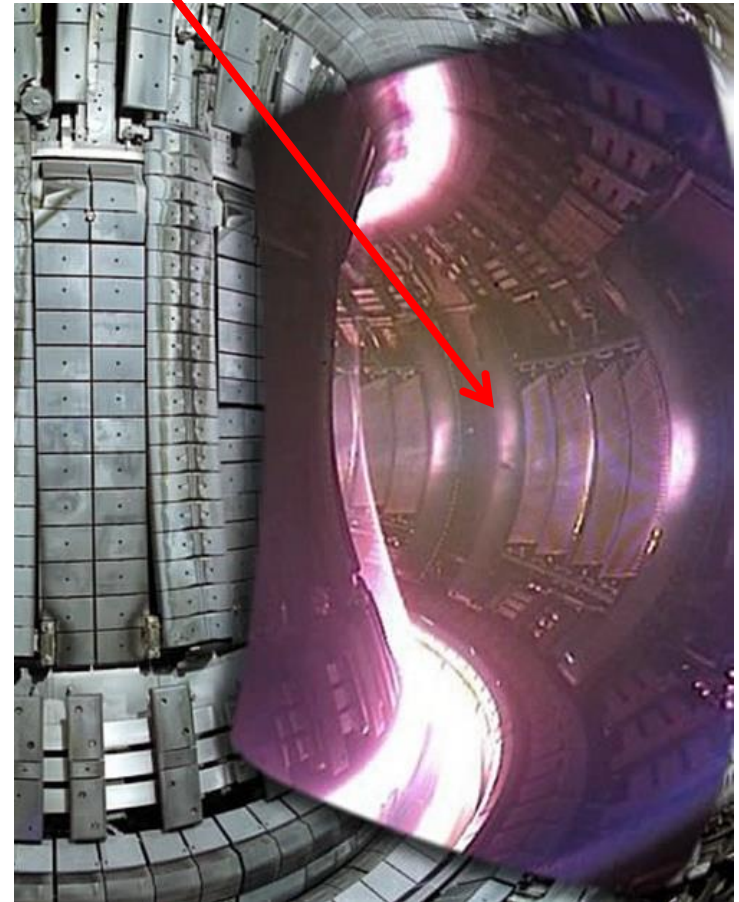
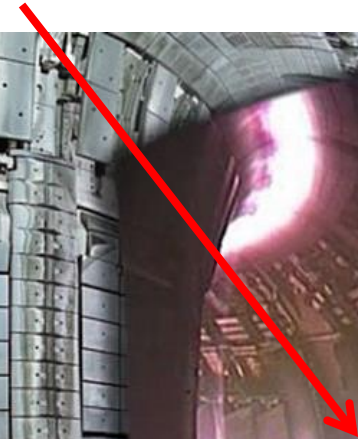


# Very challenging to diagnose turbulence at 100 million degrees...

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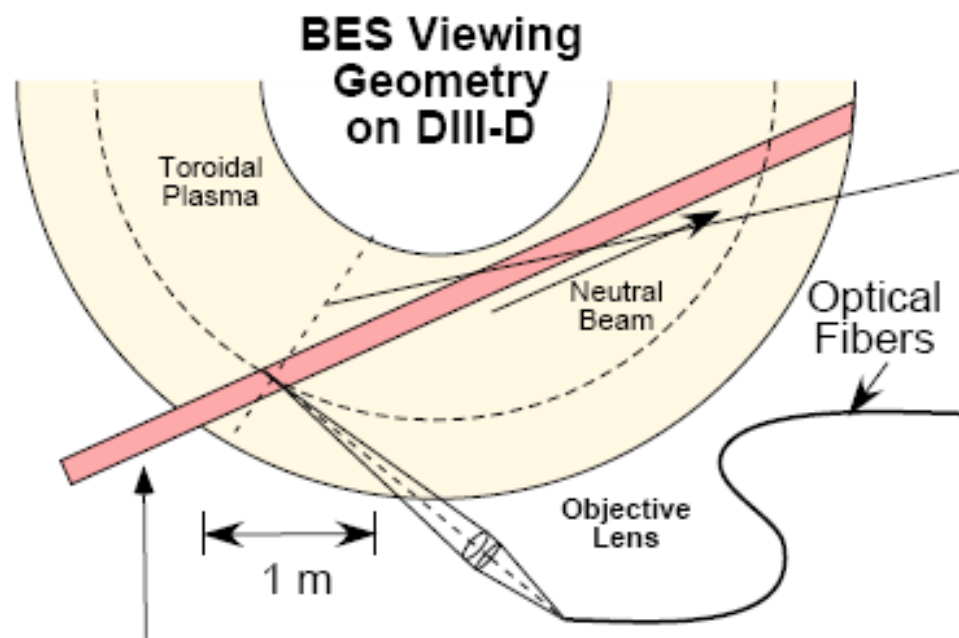
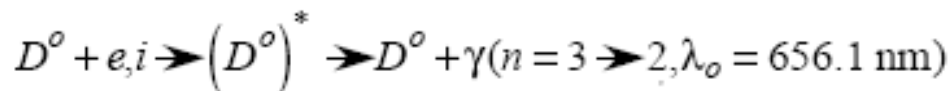
100,000,000 C



Physical probes don't work for hot core plasmas, instead → spectroscopy, reflectometry,  $\mu$ wave scattering, ...

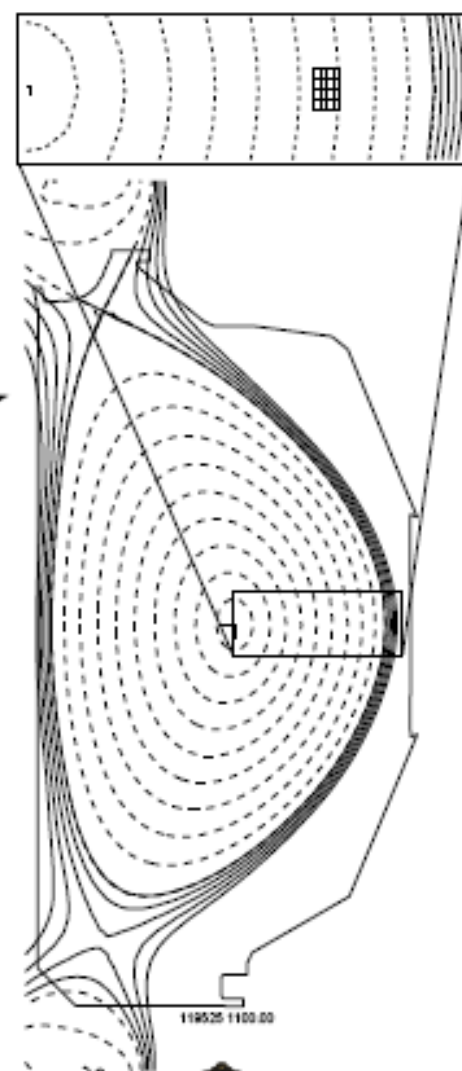
# BEAM EMISSION SPECTROSCOPY MEASUREMENT OF LOCALIZED, LONG-WAVELENGTH ( $k_{\perp}\rho_i < 1$ ) DENSITY FLUCTUATIONS

Collisionally-excited, Doppler-shifted neutral beam fluorescence



75 KeV  $D^0$  Neutral Beam  
(150 L (R))

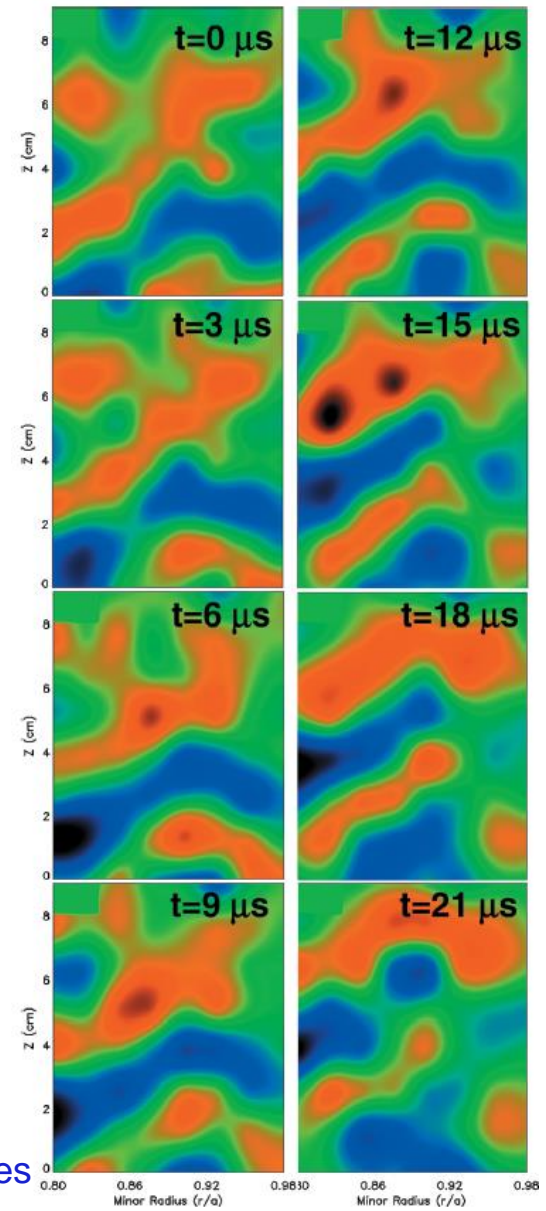
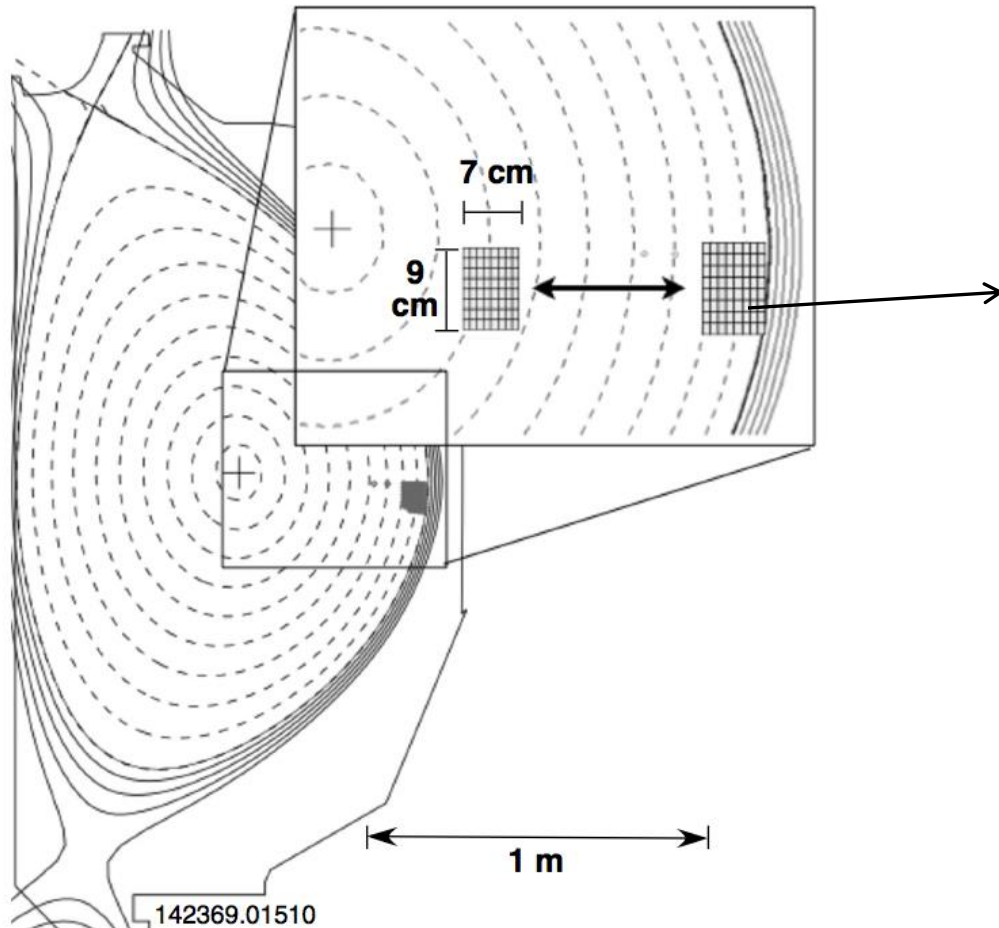
$$\frac{\tilde{I}}{I} \mu \frac{\tilde{n}}{n}$$



# Spectroscopic imaging provides a 2D picture of turbulence in tokamaks: cm spatial scales, $\mu\text{s}$ time scales, $<1\%$ amplitude

- Utilize interaction of neutral atoms with charged particles to measure density

DIII-D tokamak (General Atomics)



Movies at: <https://fusion.gat.com/global/BESMovies>

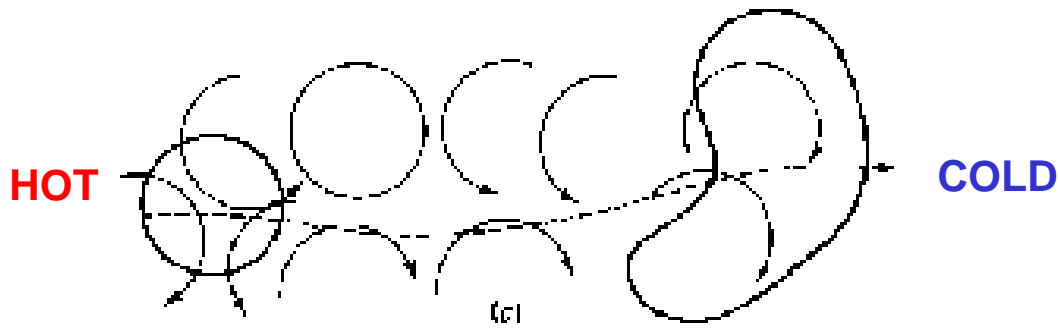
## BES videos

<https://fusion.gat.com/global/BESMovies>

(University of Wisconsin; General Atomics)



# Rough estimate of turbulent diffusivity indicates it's a plausible explanation for confinement

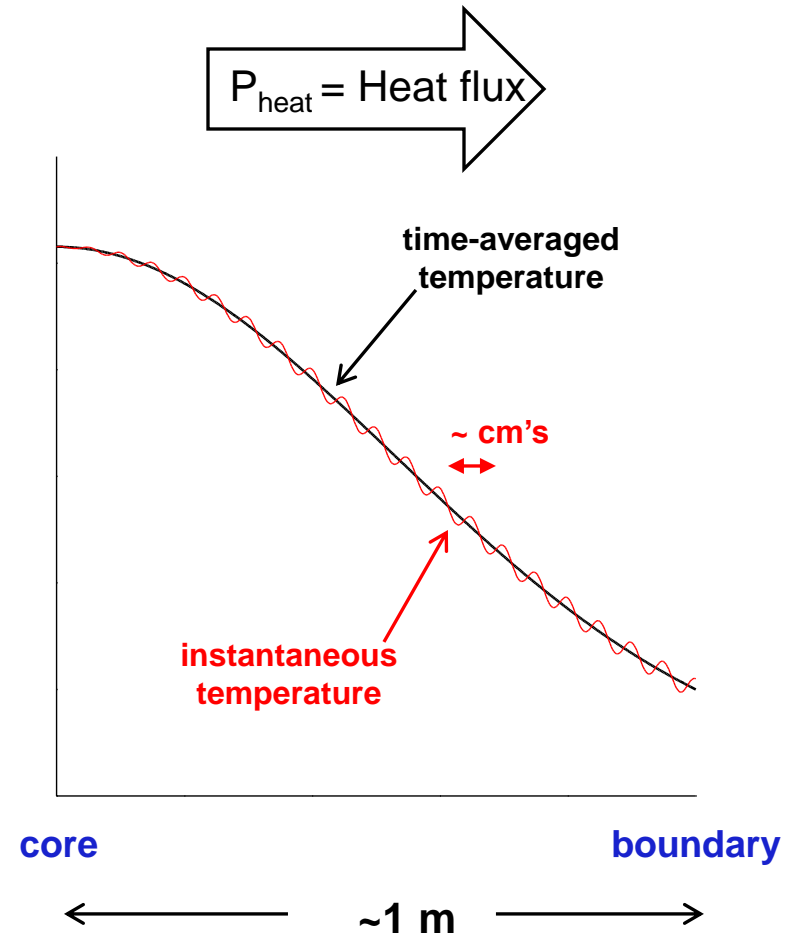


- $D_{\text{turbulence}} \sim (\text{step size})^2 \times \text{decorrelation rate}$

step size  $\sim 5\text{-}10$  gyroradii  $\sim$  few cm's

decorrelation rate  $\sim 100$  kHz

$$\text{confinement time} \sim \frac{1}{D_{\text{turbulence}}}$$



Turbulence confinement time estimate  $\sim 0.1$  s

Experimental confinement time  $\sim 0.1$  s

# Turbulence advects/mixes/transport energy, particles and momentum

- Turbulence provides a highly nonlinear flux-gradient relationship due to sources of free energy

$$\begin{bmatrix} \Gamma \\ \Pi_\phi \\ Q_i \\ Q_e \end{bmatrix} = - \begin{bmatrix} \text{flux - gradient} \\ \text{relationship} \\ \text{matrix} \end{bmatrix} \cdot \begin{bmatrix} \nabla n \\ R \nabla \Omega \\ \nabla T_i \\ \nabla T_e \end{bmatrix}$$

- I realize I'm largely referring to energy transport, but just as important for a self-consistent reactor solution is:
  - Particle transport  $\rightarrow$  need to fuel D & T in reactors
  - Impurity transport  $\rightarrow$  expelling He ash; avoiding impurity accumulation from e.g. sputtering high-Z (e.g. tungsten) walls
  - Momentum transport  $\rightarrow$  rotation is critical to macrostability (RWM/NTM) and part of self-consistent turbulence solution via  $E \times B$  sheared flows (*more later*)

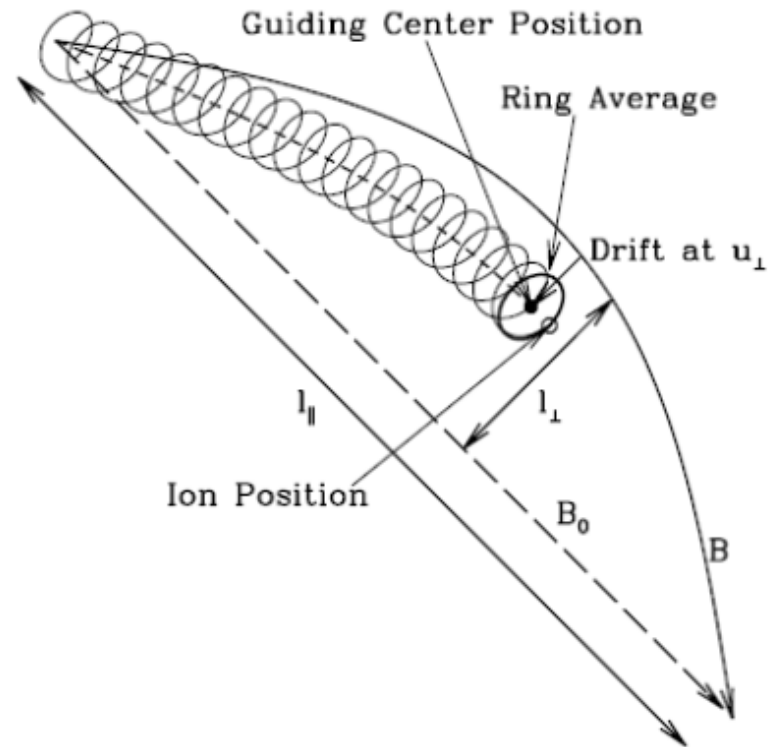
**Measurements are challenging  
and limited – also use theory and  
simulation to help improve  
understanding**

# Gyrokinetics in brief – evolving 5D gyro-averaged distribution function

$$\frac{\omega}{\Omega} \ll 1$$

$$f(\bar{\mathbf{x}}, \bar{\mathbf{v}}, t) \xrightarrow{\text{gyroaverage}} f(\bar{\mathbf{R}}, v_{\parallel}, v_{\perp}, t)$$

- Average over fast gyro-motion  $\rightarrow$  evolve a distribution of gyro-rings



Howes et al., *Astro. J.* (2006)

# Gyrokinetics in brief – evolving 5D gyro-averaged distribution function

$$\frac{\omega}{\Omega}, \frac{\rho}{L}, \frac{\delta f}{f_0}, \frac{k_{\parallel}}{k_{\perp}} \ll 1$$

$$f(\bar{\mathbf{x}}, \bar{\mathbf{v}}, t) \xrightarrow{\text{gyroaverage}} f(\bar{\mathbf{R}}, v_{\parallel}, v_{\perp}, t) \quad f = F_M + \delta f$$

$$\frac{\partial(\delta f)}{\partial t} + \underbrace{v_{\parallel} \hat{\mathbf{b}} \cdot \nabla \delta f}_{\text{Fast parallel motion}} + \underbrace{\bar{\mathbf{v}}_d \cdot \nabla \delta f}_{\text{Slow perpendicular toroidal drifts}} + \underbrace{\delta \bar{\mathbf{v}} \cdot \nabla F_M}_{\text{Advection across equilibrium gradients}} + \underbrace{\bar{\mathbf{v}}_{E0}(\mathbf{r}) \cdot \nabla \delta f}_{\text{Dopper shift due to sheared equilibrium } E_r(r)} + \underbrace{\delta \bar{\mathbf{v}} \cdot \nabla \delta f}_{\text{Perpendicular non-linearity}} = C(\delta f)$$

Fast parallel motion

Slow perpendicular toroidal drifts

Advection across equilibrium gradients  
( $\nabla T_0, \nabla n_0, \nabla V_0$ )

Dopper shift due to sheared equilibrium  $E_r(r)$

Perpendicular non-linearity

Collisions

$$\bar{\mathbf{v}}_{\kappa} = m v_{\parallel}^2 \frac{\hat{\mathbf{b}} \times \bar{\boldsymbol{\kappa}}}{qB}$$

$$\bar{\mathbf{v}}_{\nabla B} = \frac{m v_{\perp}^2}{2} \frac{\hat{\mathbf{b}} \times \nabla B / B}{qB}$$

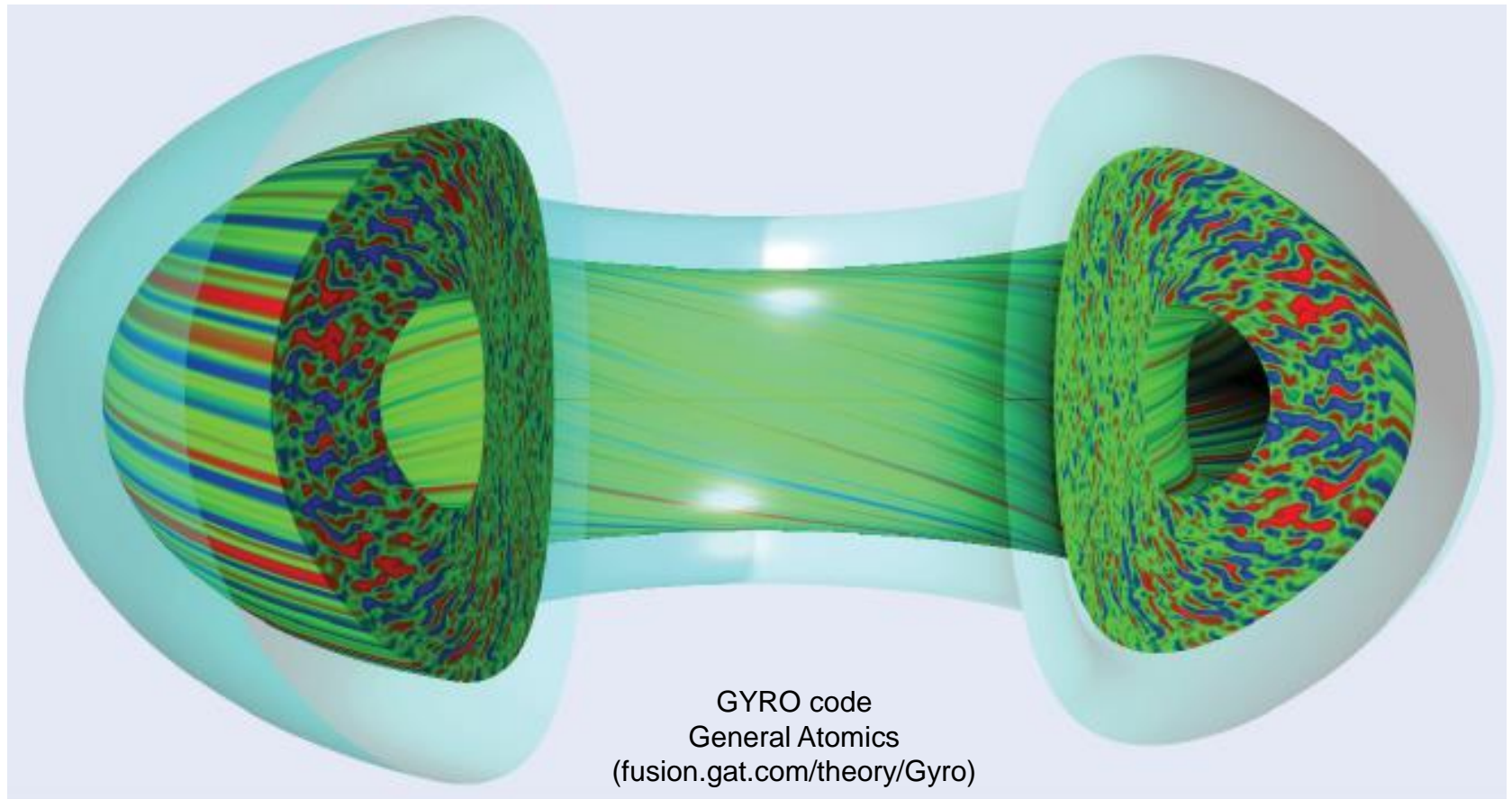
$$\delta \mathbf{v}_a \doteq \frac{c}{B} \mathbf{b} \times \nabla \Psi_a$$

$$\Psi_a(\mathbf{R}) \doteq \left\langle \delta \phi(\mathbf{R} + \boldsymbol{\rho}) - \frac{1}{c} (\mathbf{V}_0 + \mathbf{v}) \cdot \delta \mathbf{A}(\mathbf{R} + \boldsymbol{\rho}) \right\rangle_{\mathbf{R}}$$

- Must also solve gyrokinetic Maxwell equations self-consistently to obtain  $\delta \phi, \delta B$

# Direct numerical simulations of 5D gyrokinetic turbulence enabled by supercomputing

- 3D space + 2D particle motion, self-consistent electric and magnetic fields
  - 100's millions of grid points, or 10's billions of particle markers
  - Millions of cpu-hours, exploiting up to 200,000 cpu's (nersc.gov, nccs.gov)

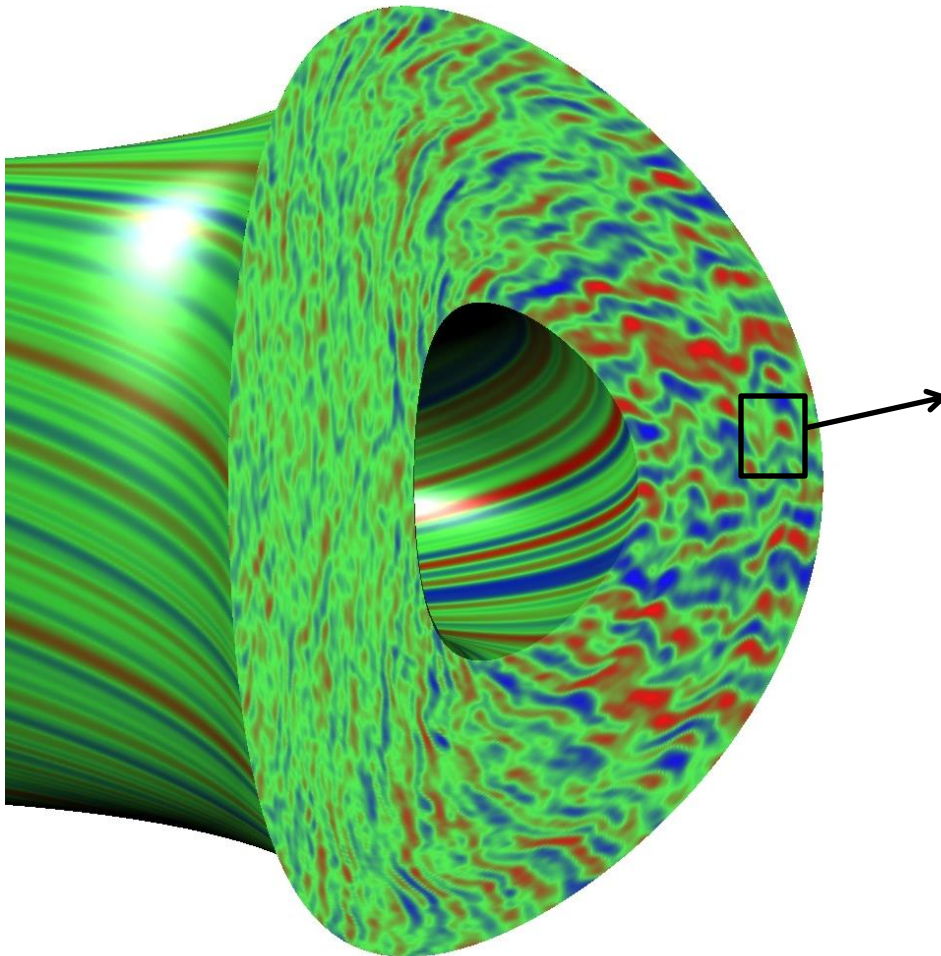


**Code: GYRO**

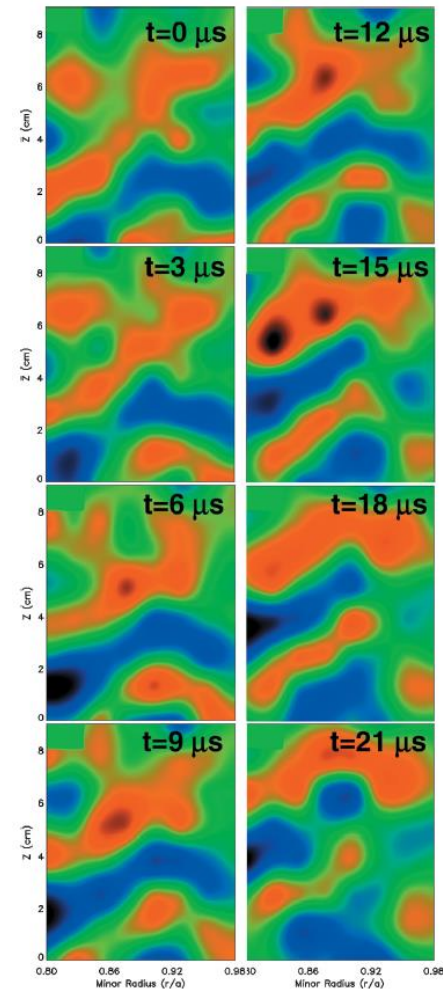
**Authors: Jeff Candy and Ron Waltz**

# Physically realistic turbulence simulations now capable of reproducing measured behavior

Simulation



Experiment

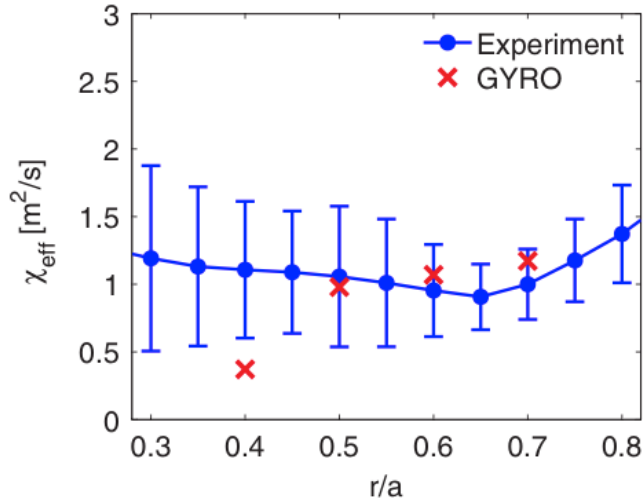


Movies at: <https://fusion.gat.com/theory/Gyromovies>

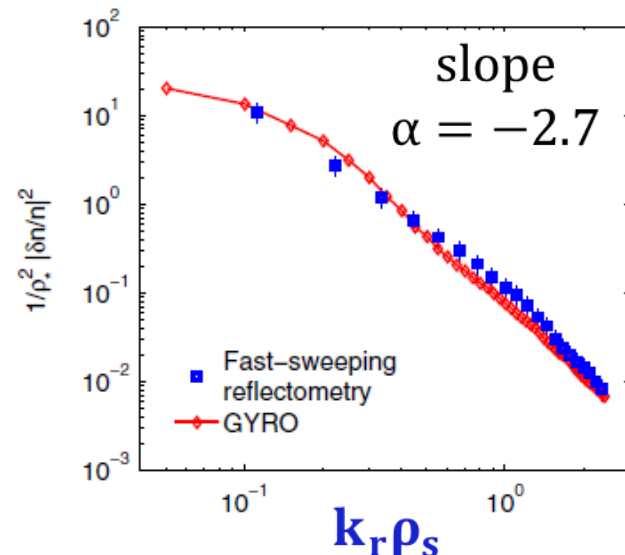
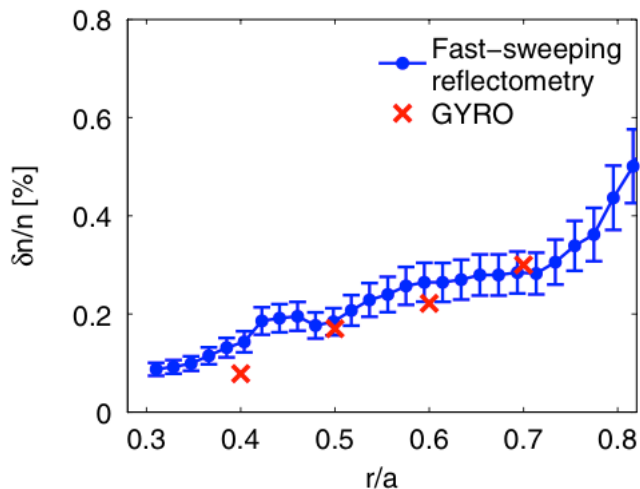
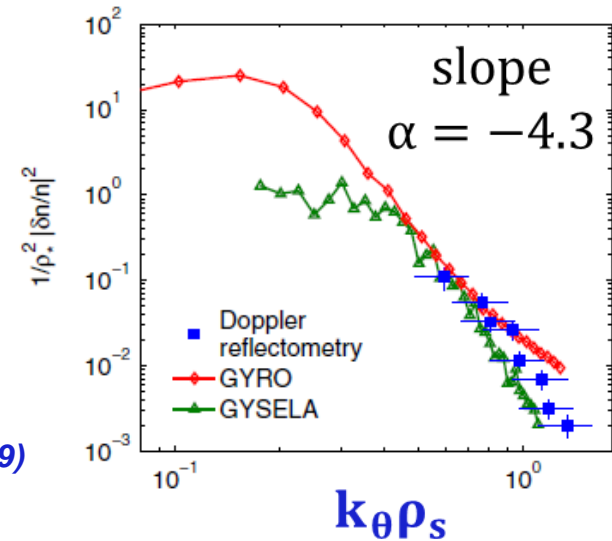


## Example of a validation study in Tore Supra

- Transport, density fluctuation amplitude (from reflectometry) and spectral characteristics all consistent with nonlinear gyrokinetic simulations (GYRO, GYSELA)
- Provides confidence in theoretical understanding of key turbulence mechanism (ITG in this case, more on ITG later)



Casati, PRL (2009)

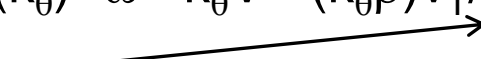


**What is the nature of  
turbulence dynamics in  
tokamaks?**

**Drift waves**

# 40+ years of theory predicts turbulence in magnetized plasma should often be drift wave in nature

## General predicted drift wave characteristics:

- Finite-frequency drifting waves,  $\omega(k_\theta) \sim \omega_* \sim k_\theta V_* \sim (k_\theta \rho) v_T / L_n$
- Driven by  $\nabla n, \nabla T$  ( $1/L_n = -1/n \cdot \nabla n$ ) 
- Can propagate in ion or electron diamagnetic direction, depending on conditions/dominant gradients
- Quasi-2D, elongated along the field lines ( $L_{\parallel} \gg L_{\perp}, k_{\parallel} \ll k_{\perp}$ )
  - Particles can rapidly move along field lines to smooth out perturbations
- Perpendicular sizes linked to local gyroradius,  $L_{\perp} \sim \rho_{i,e}$  or  $k_{\perp} \rho_{i,e} \sim 1$
- Correlation times linked to acoustic velocity,  $\tau_{\text{cor}} \sim c_s / R$
- In a tokamak expected to be “ballooning”, i.e. stronger on outboard side
  - Due to “bad curvature”/“effective gravity” pointing outwards from symmetry axis
  - Often only measured at one location (e.g. outboard midplane)
- Fluctuation strength loosely follows mixing length scaling ( $\delta n / n_0 \sim \rho_s / L_n$ )
- Transport has gyrobohm scaling,  $\chi_{\text{GB}} = \rho_i^2 v_{Ti} / R$ 
  - But other factors important! I.e.  $\chi_{\text{turb}} \sim \chi_{\text{GB}} \cdot F(\dots) \cdot [R/L_T - R/L_{T,\text{crit}}]$

# The Simplest Drift Wave

(too simple)

4

Classic drift wave ordering:

$$v_{\perp i} \ll \frac{\omega}{k_{\parallel}} \ll v_{\perp e}$$

Fluid ions

ions don't move  
much along  $\underline{B}$

In fact, we're assuming  
 $T_i = \nabla T_i = 0$

wave parallel  
phase speed

$$\underline{k} = k_{\parallel} \hat{b} + \underline{k}_{\perp}$$

will find  
 $k_{\parallel} \ll k_{\perp}$

Kinetic electrons

"adiabatic" or  
Boltzmann  
response

$$n_e = G e^{|\phi|/T_{e0}}$$

$$\tilde{n}_e = n_{e0} \frac{e\phi}{T_{e0}}$$

# Insert perturbed $\mathbf{E} \times \mathbf{B}$ drift into ion continuity

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_E) = 0$$

$$\begin{aligned} \mathbf{v}_E &= \frac{c}{B^2} \mathbf{E} \times \mathbf{B} \\ &= \frac{c}{B} \hat{z} \times \nabla \phi \end{aligned}$$

$$n_i(\underline{x}, t) = n_0(r) + \tilde{n}_i(\underline{x}, t)$$

$$\frac{\partial \tilde{n}_i}{\partial t} + \mathbf{v}_E \cdot \nabla (n_0 + \tilde{n}_i)$$

linear                  nonlinear

$$+ n_i \underbrace{\nabla \cdot \mathbf{v}_E}_{= 0 \text{ in slab limit}} = 0$$

Perturbed advection of background gradient is key element of electrostatic drift waves

$$\frac{\partial n_0}{\partial x} \equiv - \frac{n_0(r)}{L_n(r)}$$

Linear term becomes:

$$\underline{v}_E \cdot \nabla n_0 = - \underline{v}_E \cdot \hat{x} \frac{n_0}{L_n} = - \frac{c}{B} (\hat{b} \times \nabla \phi) \cdot \hat{x} \frac{n_0}{L_n}$$

$$= \frac{c}{B} i k_y \phi \frac{n_0}{L_n}$$

$$\underline{v}_E \cdot \nabla n_0 = i \omega_{*e} \frac{e \phi}{T_{e0}} n_0$$

the "diamagnetic frequency" =

$$\omega_{*e} = \frac{c T_{e0}}{e B} \frac{k_y}{L_n} = \frac{c_s}{L_n} k_y \rho_s = k_y v_{xe}$$

$$\frac{c T_{e0}}{e B} = \rho_s c_s = 16 \times (\text{Bohm Diffusion coefficient})$$

diamagnetic  
fluid velocity  
of electrons  
( $\neq$  a particle  
drift)

$$c_s = \sqrt{\frac{T_e}{m_i}} = \text{"sound speed"} \\ (\text{in cold ion limit, } T_i = 0)$$

$$\rho_s = \frac{c_s}{\Omega_{ci}} = \text{"sound gyroradius"} \quad (\text{ion gyroradius at electron temperature}).$$

## Quasineutrality:

$$\tilde{n}_i = \tilde{n}_e = n_{e0} \frac{e\phi}{T_{e0}}$$

$$\underbrace{\frac{\partial}{\partial t} \left( n_{e0} \frac{e\phi}{T_{e0}} \right) + i\omega_{*e} \frac{e\phi}{T_{e0}} n_{e0}}_{=0!} + \underbrace{\frac{c}{B} \hat{z} \times \nabla \phi \cdot \nabla \left( n_{e0} \frac{e\phi}{T_{e0}} \right)}_{=0!} = 0$$

$$\omega = \omega_{*e}$$

classic linear drift wave  
dispersion relation

$$\omega = (k_y \rho_s) \cdot c_s / L_n$$

Nonlinearity  
vanishes!

Need to include  
next order corrections  
to electron response  
and/or next order  
corrections to ion response



- No instability in this simple model because of Boltzmann (adiabatic) electrons & no ion temperature gradient
- We will illustrate the “toroidal ion temperature gradient (ITG)” instability in the next section

# Finite gyroradius effects limit characteristic size to ion-gyroradius ( $k_{\perp}\rho_i \sim 1$ )

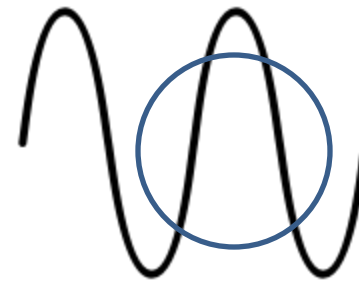
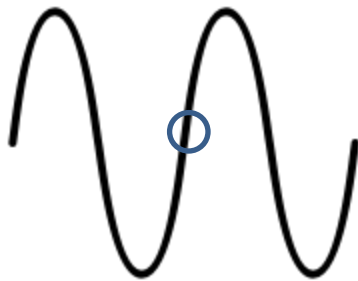
- Drift velocity increases with smaller wavelength (larger  $k_{\perp}\rho_i$ )

$$\vec{v}_E = \frac{\hat{b} \times \nabla\phi}{B} = -ik_{\perp} \frac{\phi}{B} = -ik_{\perp} \left(\frac{\phi}{T_i}\right) \left(\frac{T_i}{B}\right) = -i(k_{\perp}\rho_i) \left(\frac{\phi}{T_i}\right) v_{Ti}$$

- If wavelength approaches ion gyroradius ( $k_{\perp}\rho_i \geq 1$ ), average electric field experienced over fast ion-gyromotion is reduced:

$$\langle \nabla\phi \rangle_{\text{gyro-average}} \sim \nabla\phi$$

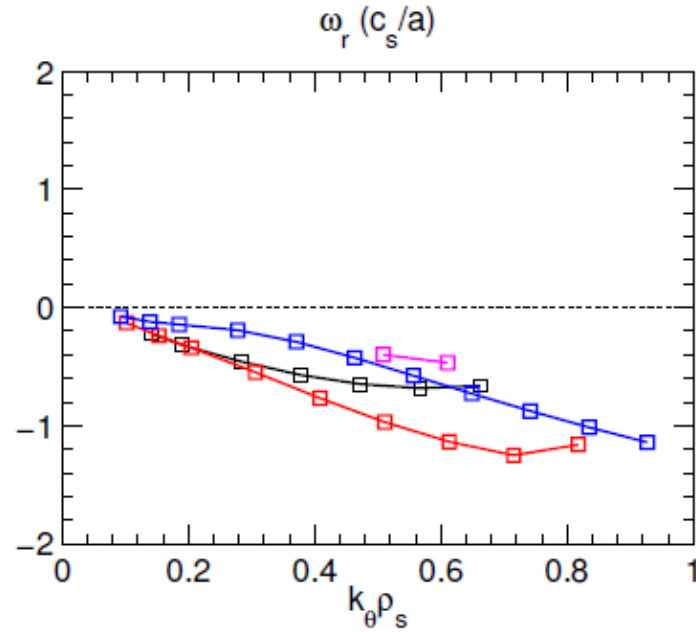
$$\langle \nabla\phi \rangle_{\text{gyro-average}} \sim \nabla\phi [1 - (k_{\perp}\rho_i)^2]$$



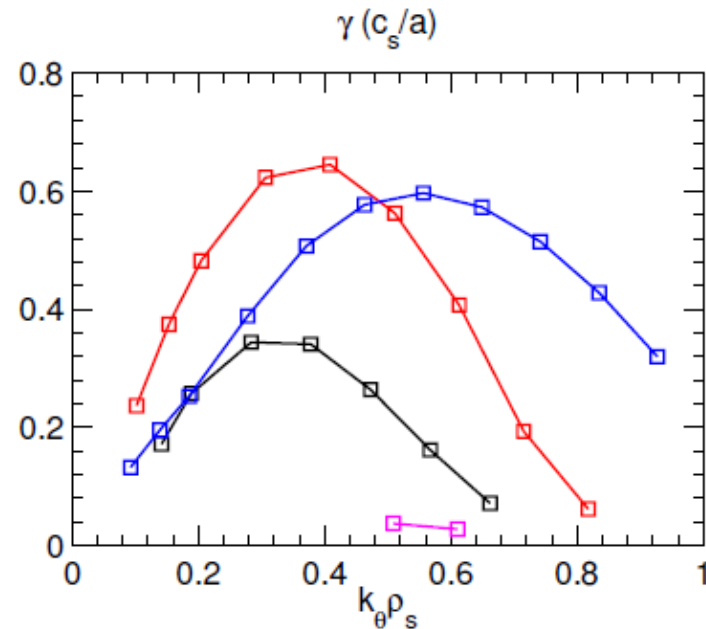
**$\Rightarrow$  Maximum growth rates (and typical turbulence scale sizes) occur for  $(k_{\perp}\rho_i) \leq 1$**

# Example linear gyrokinetic simulation results (MAST tokamak)

*Different colors represent different radii in the plasma*



**Real frequencies**



**Linear growth rates**

# Why do micro-instabilities & turbulence develop in tokamaks?

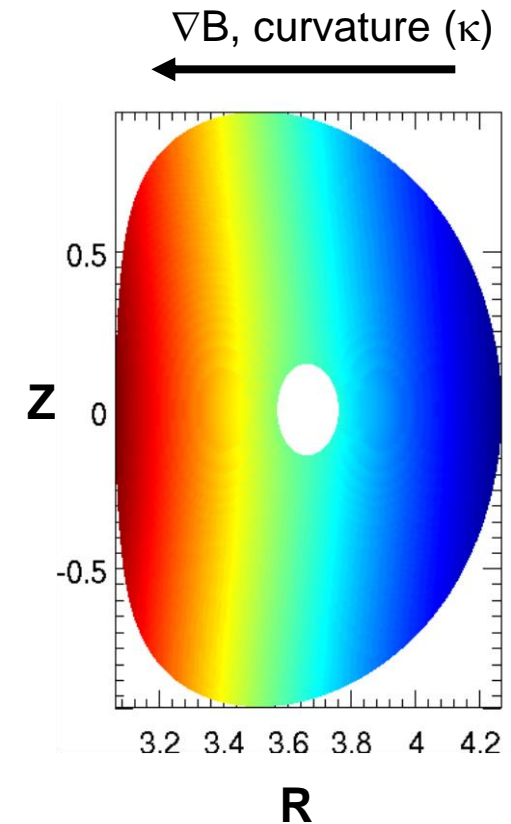
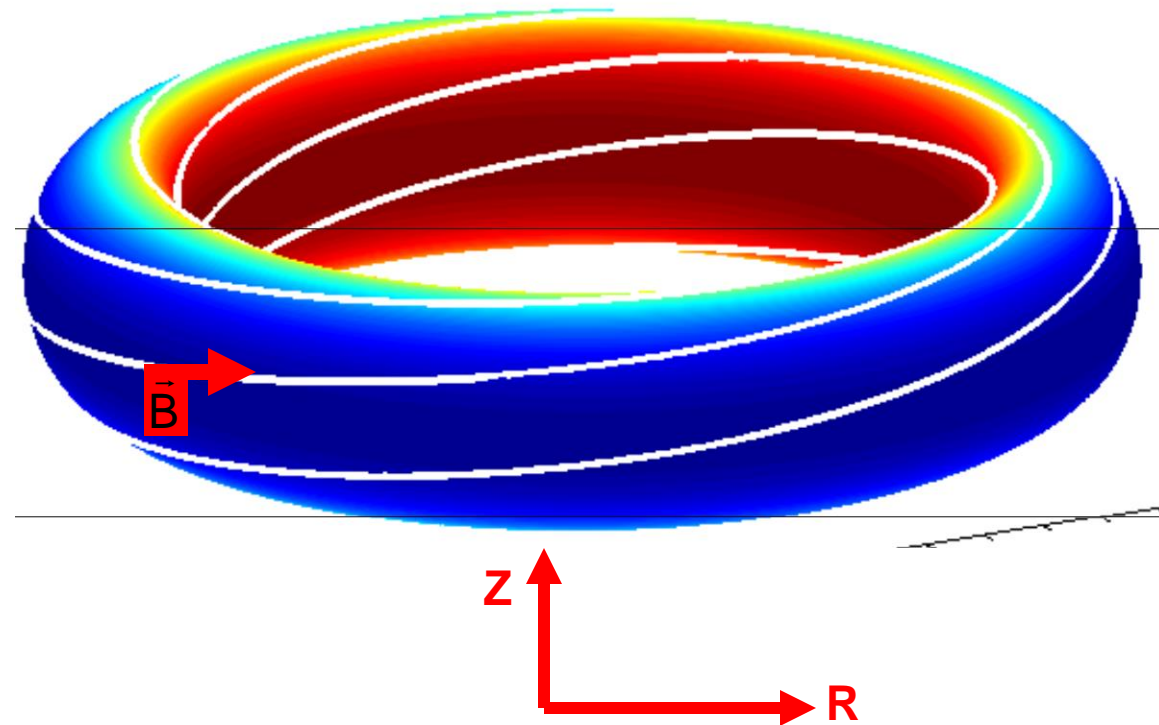
**Example:** Linear stability analysis of Ion Temperature Gradient (ITG) “ballooning” micro-instability (expected to dominate in ITER)

# Toroidicity Leads To Inhomogeneity in $|B|$ , gives $\nabla B$ and curvature ( $\kappa$ ) drifts

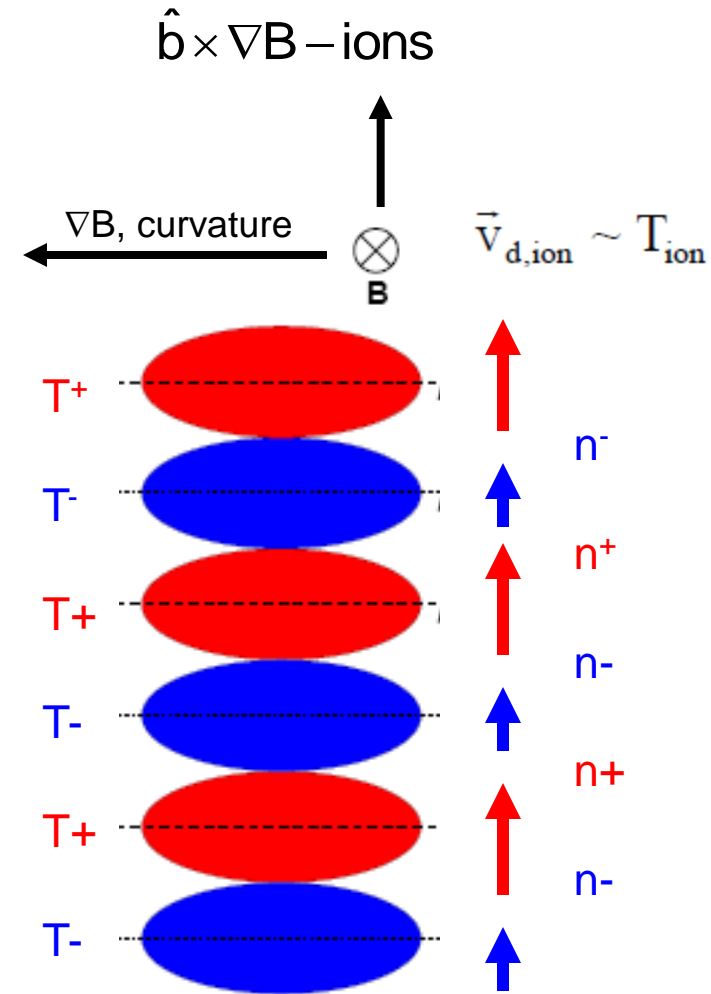
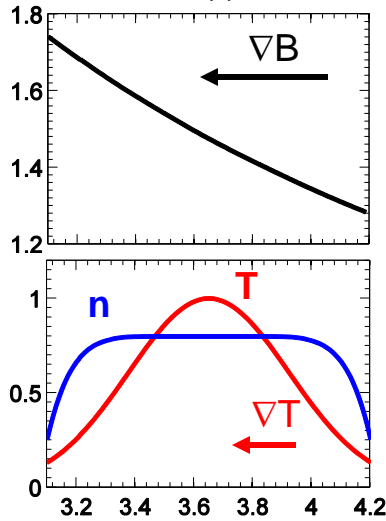
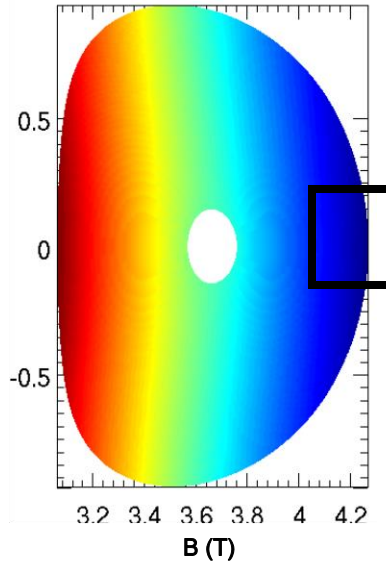
$$\bar{v}_\kappa = mv_\parallel^2 \frac{\hat{b} \times \bar{\kappa}}{qB} \sim T_\parallel$$

$$\bar{v}_{\nabla B} = \frac{mv_\perp^2}{2} \frac{\hat{b} \times \nabla B / B}{qB} \sim T_\perp$$

- What happens when there are small perturbations in  $T_\parallel$ ,  $T_\perp$ ?  $\Rightarrow$  Linear stability analysis...



# Temperature perturbation ( $\delta T$ ) leads to compression ( $\nabla \cdot \mathbf{v}_{di}$ ), density perturbation – $90^\circ$ out-of-phase with $\delta T$



- Fourier decompose perturbations in space ( $k_\theta \rho_i \leq 1$ )
- Assume small  $\delta T$  perturbation

# Dynamics Must Satisfy Quasi-neutrality

- Quasi-neutrality (Poisson equation,  $k_{\perp}^2 \lambda_D^2 \ll 1$ ) requires

$$-\nabla^2 \tilde{\phi} = \frac{1}{\epsilon_0} \sum_s e Z_s \int d^3 v f_s$$

$$\tilde{n}_i = \tilde{n}_e$$

$$(k_{\perp}^2 \lambda_D^2) \frac{\tilde{\phi}}{T} = \frac{\tilde{n}_i - \tilde{n}_e}{n_0}$$

- For this ion drift wave instability, parallel electron motion is very rapid

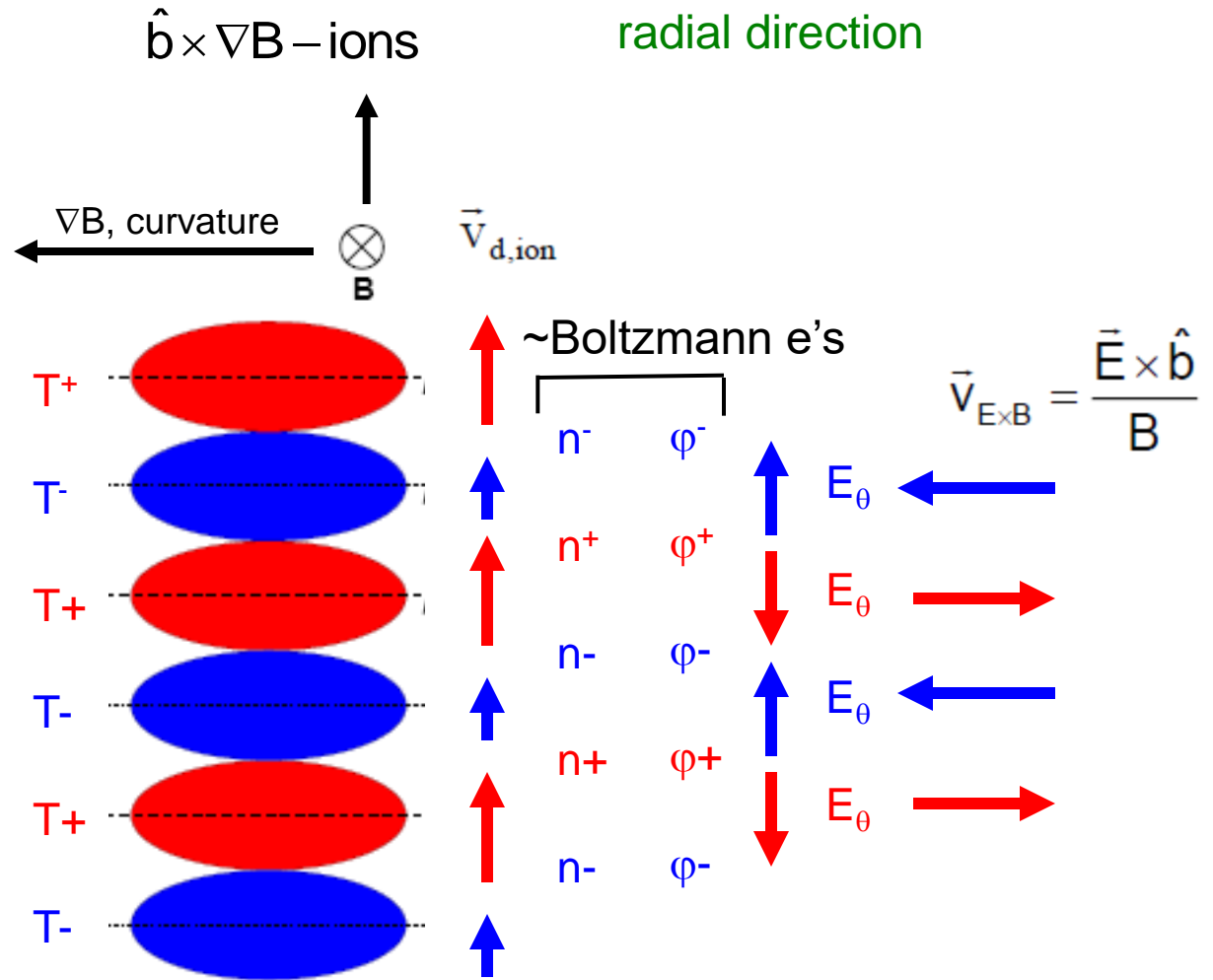
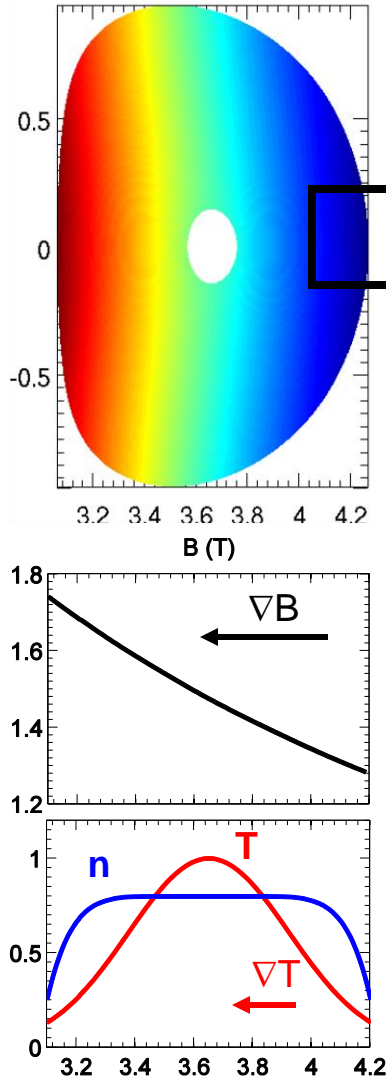
$$\omega < k_{\parallel} v_{Te} \rightarrow 0 = -T_e \nabla \tilde{n}_e + n_e e \nabla \tilde{\phi}$$

⇒ Electrons (approximately) maintain a Boltzmann distribution

$$(n_0 + \tilde{n}_e) = n_0 \exp(e\tilde{\phi}/T_e)$$

$$\tilde{n}_e \approx n_0 e\tilde{\phi}/T_e \Rightarrow \tilde{n}_e \approx \tilde{\phi}$$

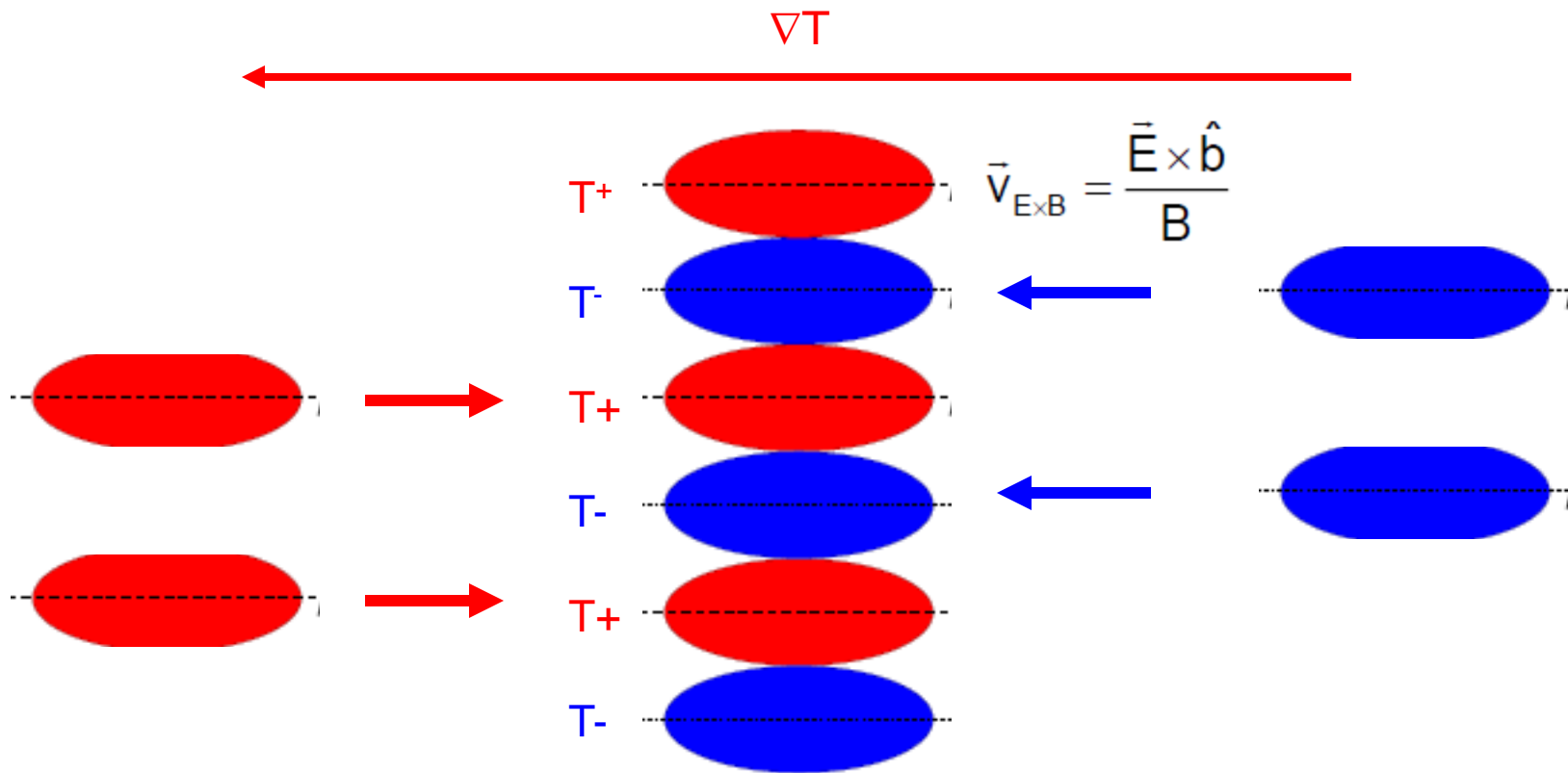
# Perturbed Potential Creates $E \times B$ Advection



- Advection occurs in the radial direction



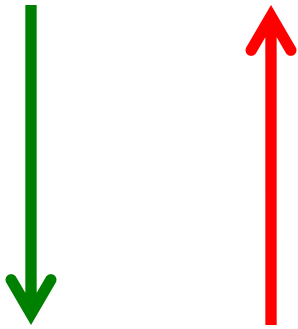
# Background Temperature Gradient Reinforces Perturbation $\Rightarrow$ Instability



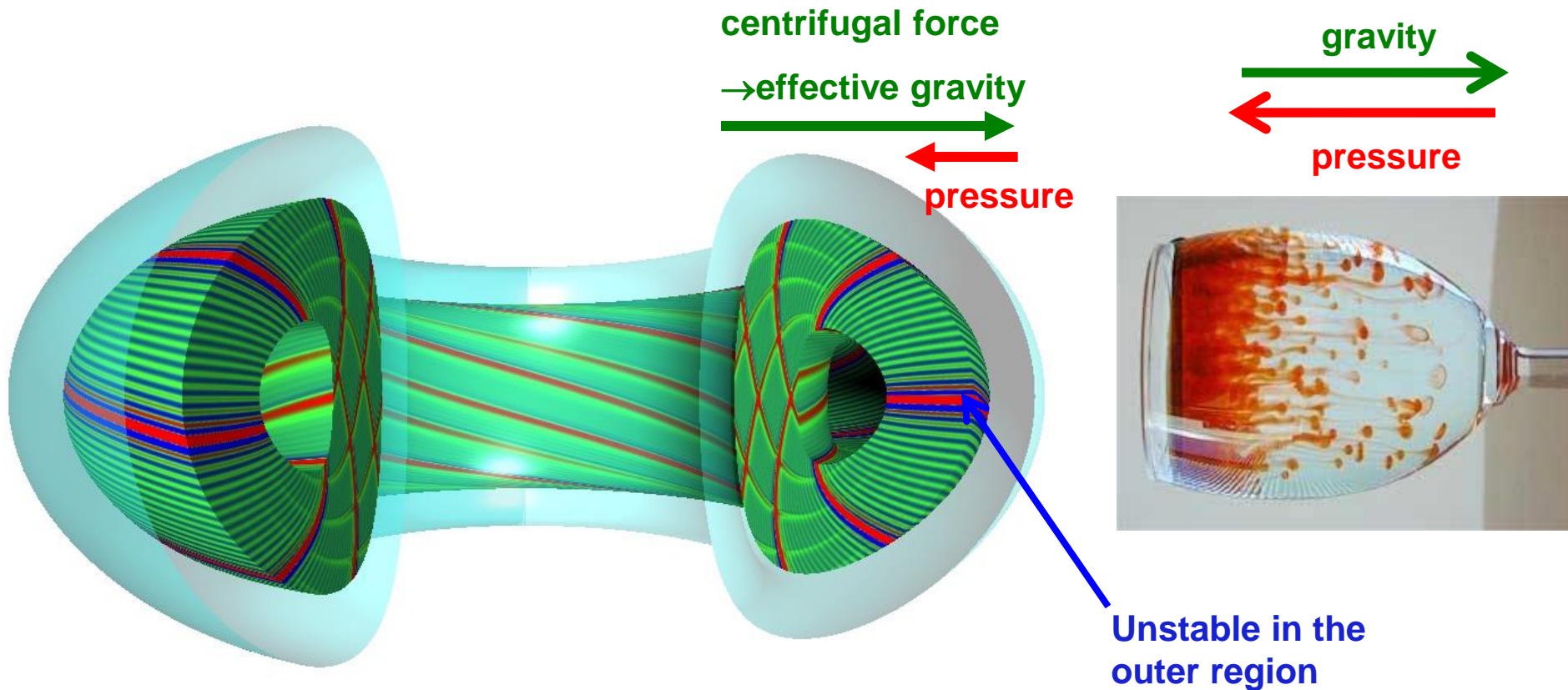
# Analogy for turbulence in tokamaks – Rayleigh-Taylor instability

- Higher density on top of lower density, with gravity acting downwards

gravity density/pressure



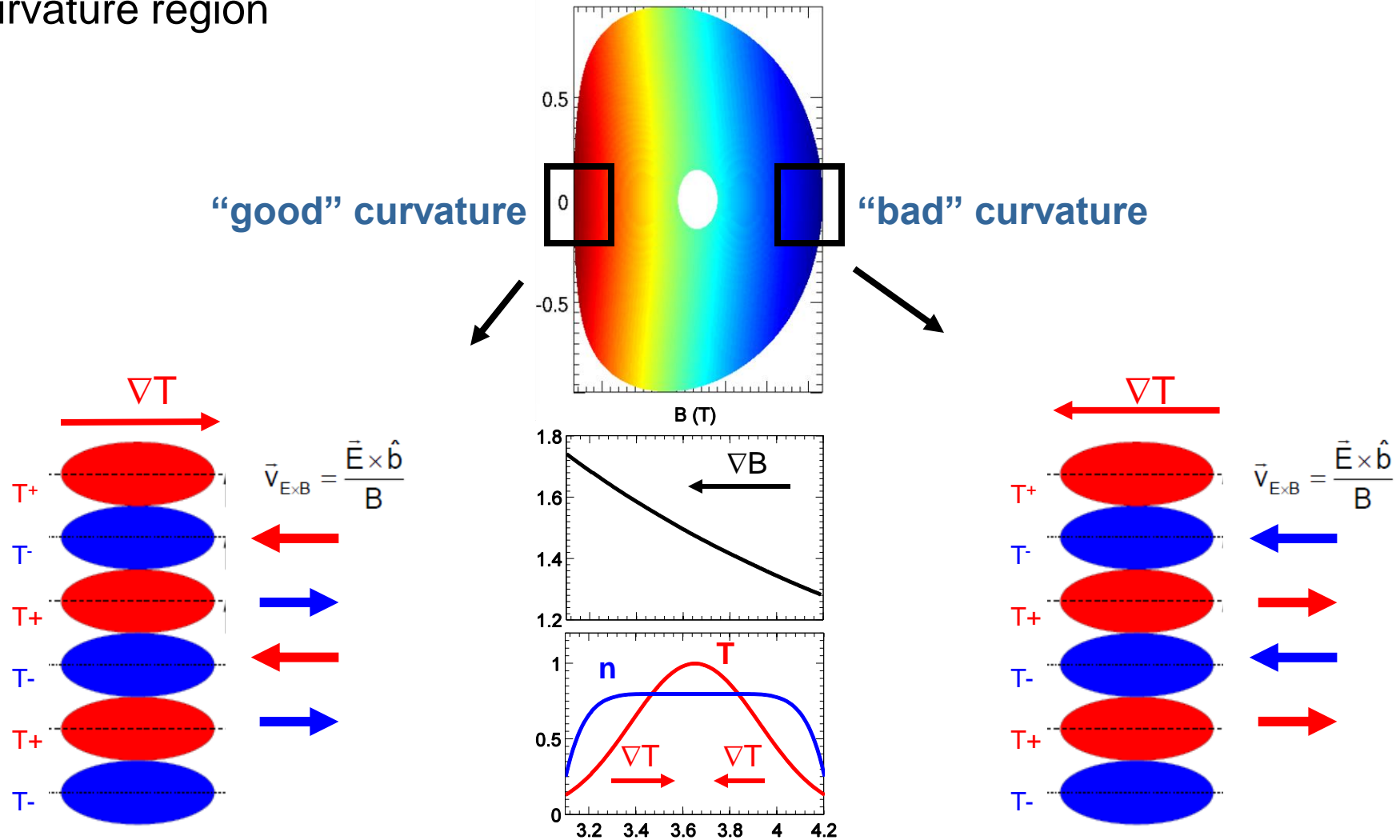
# Inertial force in toroidal field acts like an effective gravity



GYRO code  
<https://fusion.gat.com/theory/Gyro>

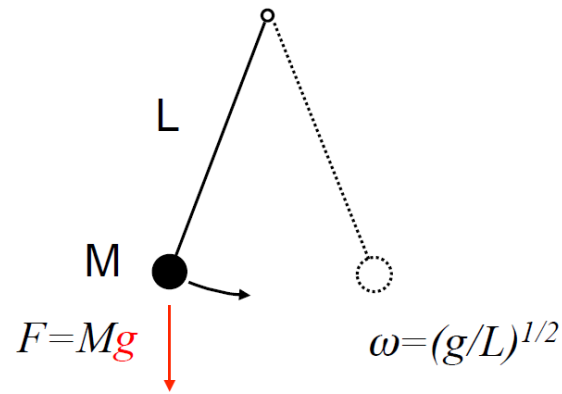
# Same Dynamics Occur On Inboard Side But Now Temperature Gradient Is Stabilizing

- Advection with  $\nabla T$  counteracts perturbations on inboard side – “good” curvature region

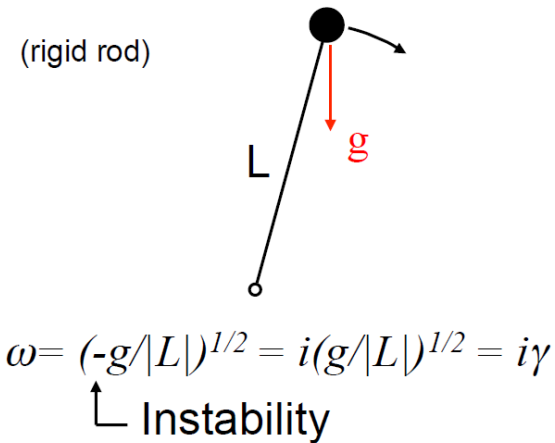


# Similar to comparing stable / unstable (inverted) pendulum

## Stable Pendulum

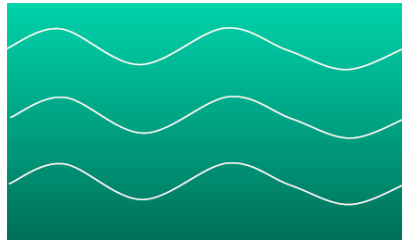


## Unstable Inverted Pendulum



## Density-stratified Fluid

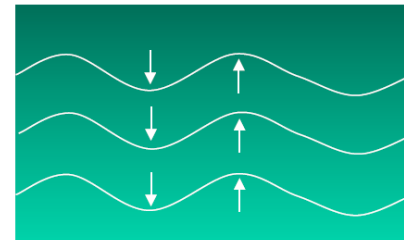
$$\rho = \exp(-y/L)$$



stable  $\omega=(g/L)^{1/2}$

## Inverted-density fluid ⇒ Rayleigh-Taylor Instability

$$\rho = \exp(y/L)$$



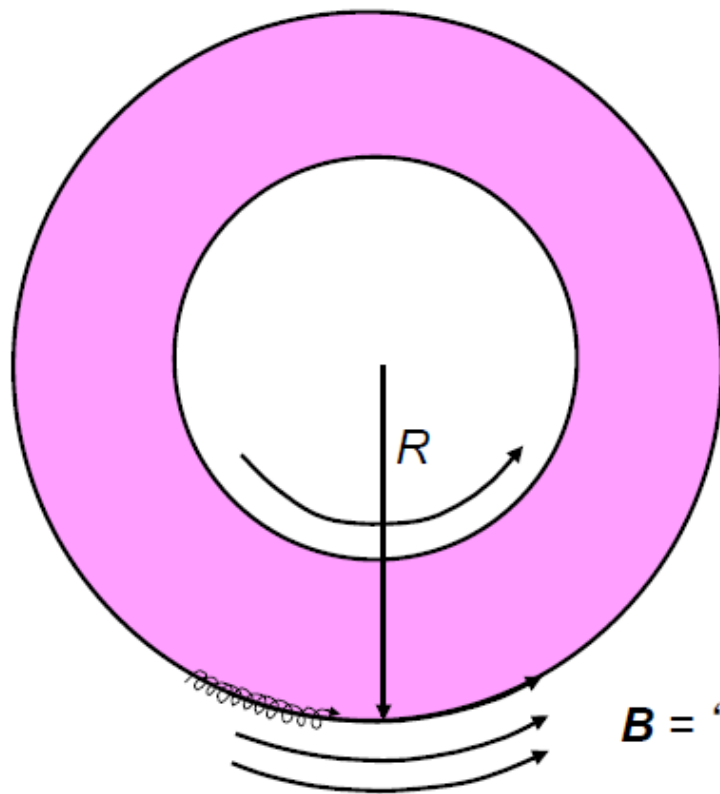
Max growth rate  $\gamma=(g/L)^{1/2}$

21

(Hammett notes)

# “Bad Curvature” instability in plasmas $\approx$ Inverted Pendulum / Rayleigh-Taylor Instability

Top view of toroidal plasma:



plasma = heavy fluid

$B$  = “light fluid”

$g_{\text{eff}} = \frac{v^2}{R}$  centrifugal force

Growth rate:

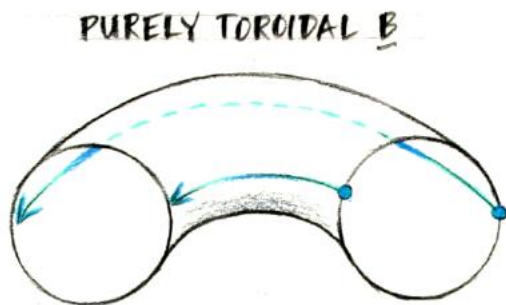
$$\gamma = \sqrt{\frac{g_{\text{eff}}}{L}} = \sqrt{\frac{V_t^2}{RL}} = \frac{V_t}{\sqrt{RL}}$$

Similar instability mechanism  
in MHD & drift/microinstabilities

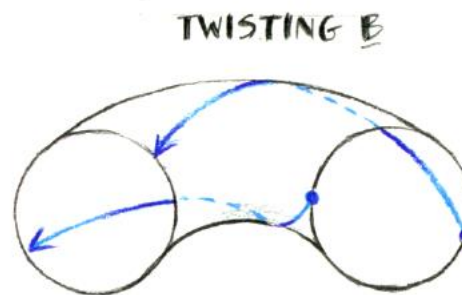
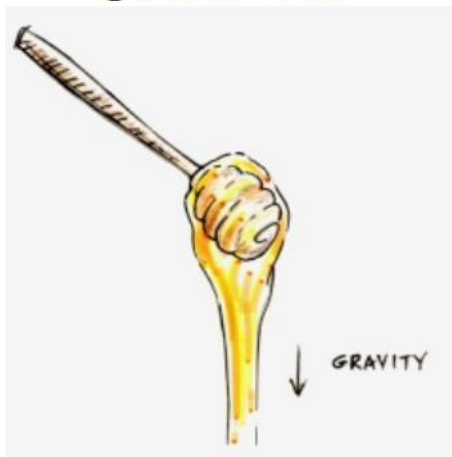
$1/L = |\nabla p|/p$  in MHD,  
 $\propto$  combination of  $\nabla n$  &  $\nabla T$   
in microinstabilities.

# The Secret for Stabilizing Bad-Curvature Instabilities

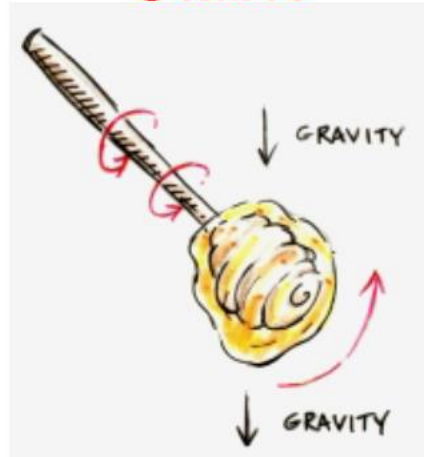
Twist in  $\mathbf{B}$  carries plasma from bad curvature region to good curvature region:



Unstable



Stable



(Hammett notes)

Similar to how twirling a honey dipper can prevent honey from dripping.

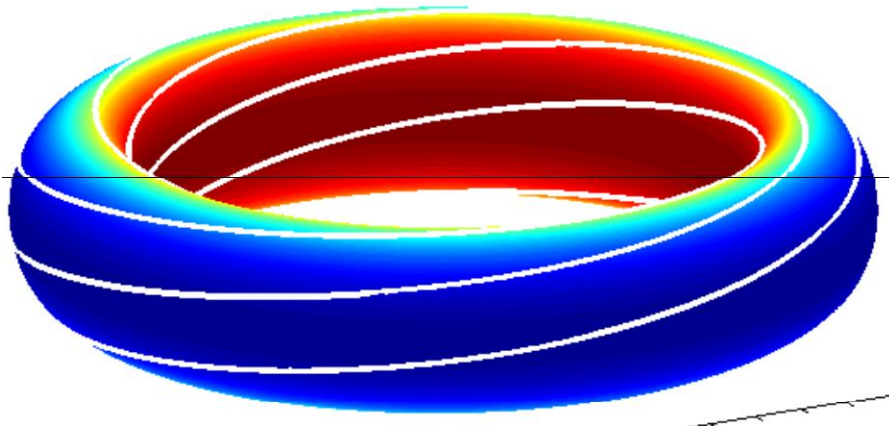
# Fast Parallel Motion Along Helical Field Line Connects Good & Bad Curvature Regions

- Approximate growth rate on outboard side

$$\gamma_{\text{instability}} \sim \frac{V_{\text{th}}}{\sqrt{RL_T}} \quad 1/L_T = -1/T \cdot \nabla T$$

- Parallel transit time

$$\gamma_{\text{parallel}} \sim \frac{V_{\text{th}}}{qR}$$

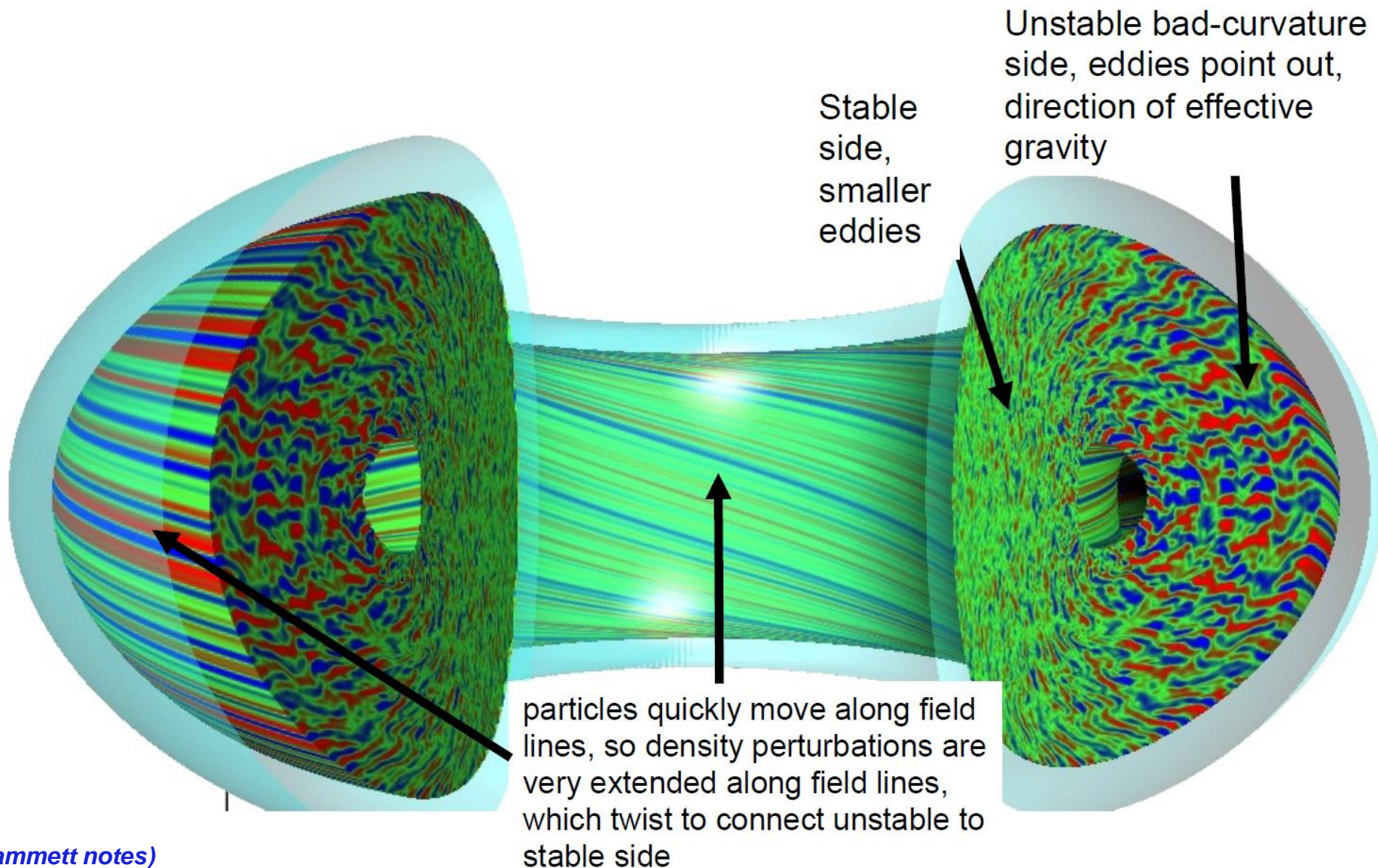


- Expect instability if  $\gamma_{\text{instability}} > \gamma_{\text{parallel}}$ , or

$$\left( \frac{R}{L_T} \right)_{\text{threshold}} \approx \frac{1}{q^2}$$

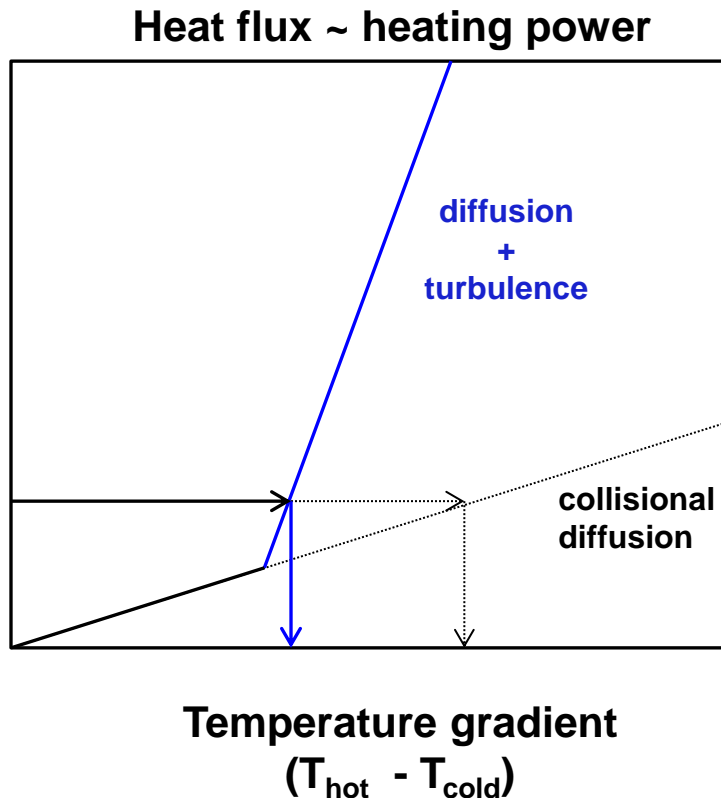


# Ballooning nature observed in simulations



# Threshold-like behavior analogous to Rayleigh-Benard instability

Analogous to convective transport when heating a fluid from below ... boiling water (before the boiling)



Rayleigh, Benard, early 1900's

Threshold gradient for temperature gradient driven instabilities have been characterized over parameter space with gyrokinetic simulations

# Critical gradient for ITG determined from many linear gyrokinetic simulations (guided by theory)

$$\frac{R_o}{L_{Tcrit}} = \text{Max} \left[ \left(1 + \frac{T_i}{T_e}\right) \left(1.33 + 1.91 \frac{\hat{S}}{q}\right) \left(1 - 1.5 \frac{r}{R_o}\right) \left(1 + 0.3 \frac{rdk}{dr}\right), \right. \\ \left. 0.8 \frac{R_o}{L_n} \right]$$

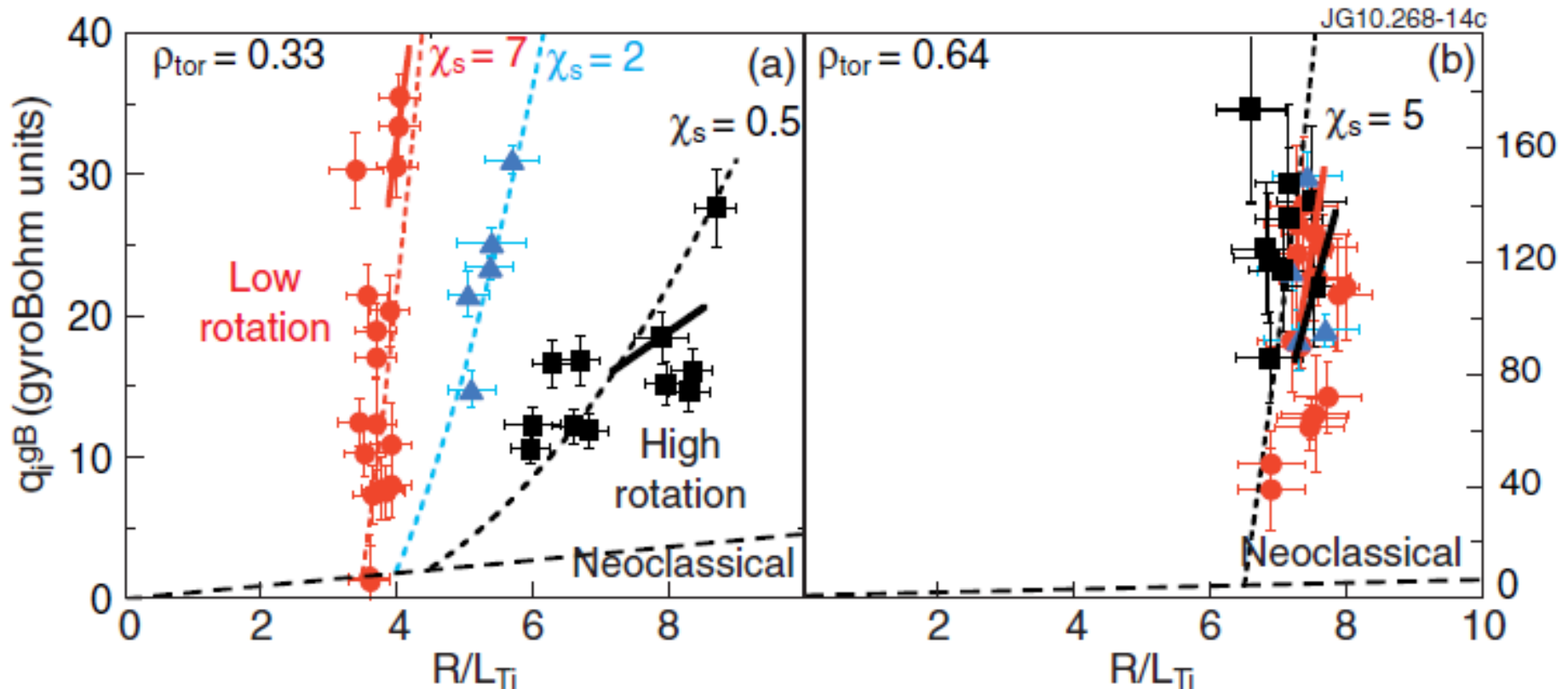
- $R/L_T = -R/T \cdot \nabla T$  is the normalized temperature gradient
- Natural way to normalize gradients for toroidal drift waves, i.e. ratio of diamagnetic-to-toroidal drift frequencies:

$$\omega_{*T} = k_y (\mathbf{B} \times \nabla p) / nqB^2 \quad \rightarrow (k_\theta \rho_i) v_T / L_T \quad \rightarrow \omega_{*T} / \omega_D = R/L_T$$

$$\omega_D = k_y (\mathbf{B} \times m v_\perp^2 \nabla B / 2B) / qB^2 \quad \rightarrow (k_\theta \rho_i) v_T / R$$

# Threshold-like behavior observed experimentally

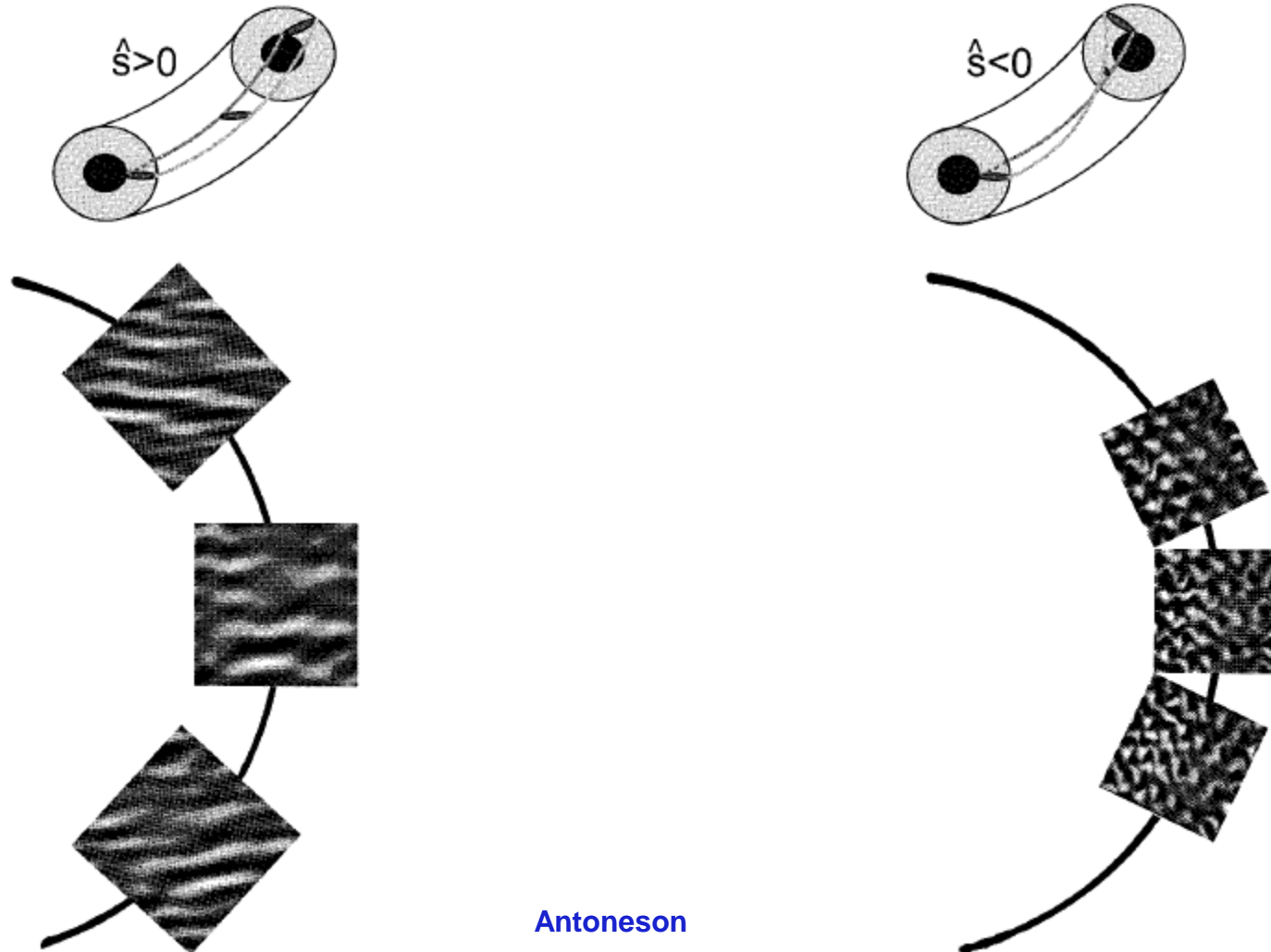
- Experimentally inferred threshold varies with equilibrium, plasma rotation, ...
- Stiffness ( $\sim dQ/d\nabla T$  above threshold) also varies
- $\chi = -Q/n\nabla T$  highly nonlinear (also use perturbative experiments to probe stiffness)



JET  
Mantica, PRL (2011)

# With physical understanding, can try to manipulate/optimize microstability

- E.g., magnetic shear influences stability by twisting radially-elongated instability to better align (or misalign) with bad curvature drive

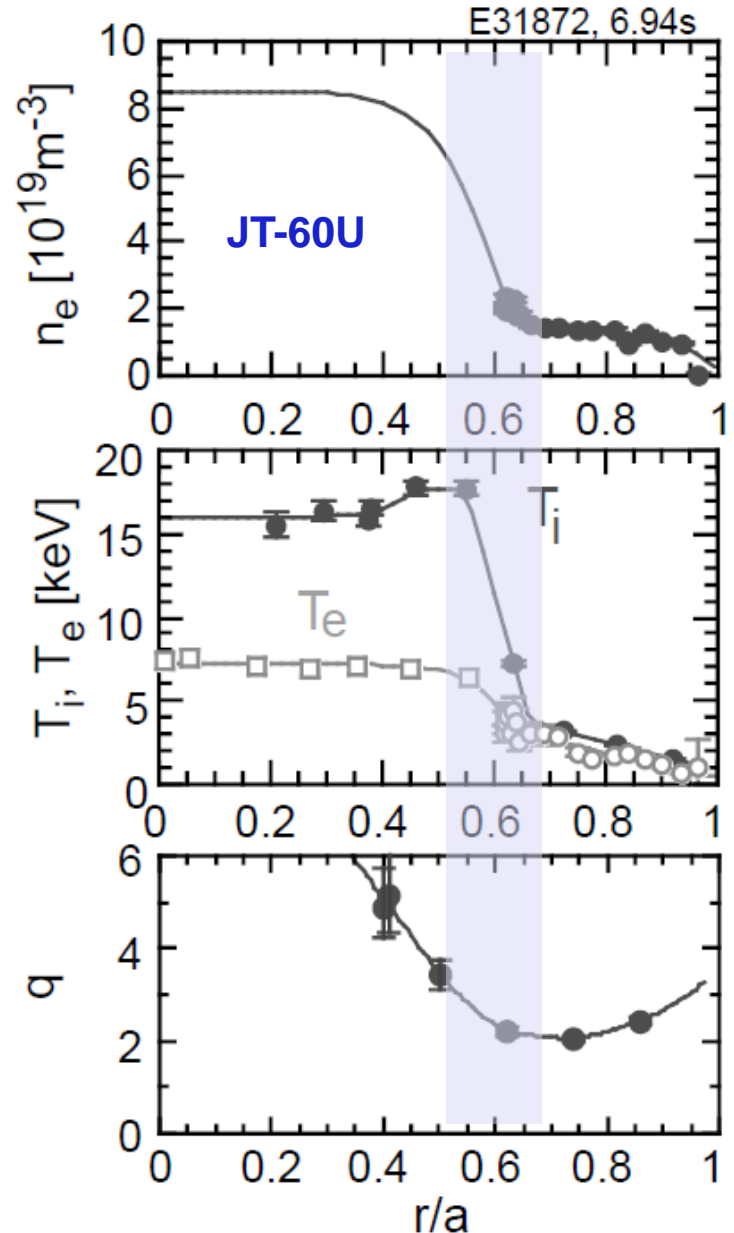


# Reverse magnetic shear can lead to internal transport barriers (ITBs)

- ITBs established on numerous devices
- Used to achieve “equivalent”  $Q_{DT,eq} \sim 1.25$  in JT-60U (in D-D plasma)
- $\chi_i \sim \chi_{i,NC}$  in ITB region (complete suppression of ion scale turbulence)



Ishida, NF (1999)



**Very simple growth rate derivation of  
previous toroidal ITG cartoon picture**

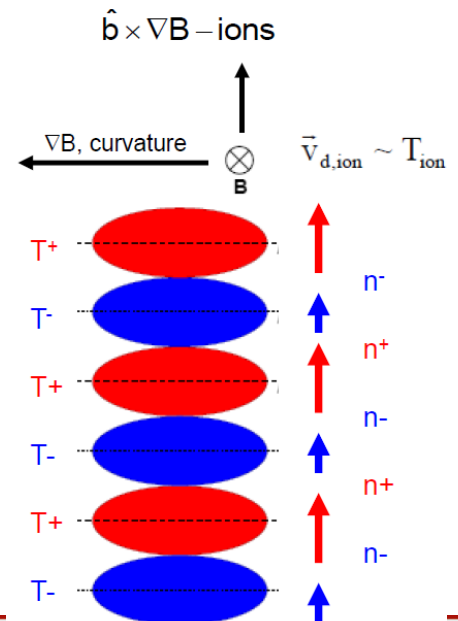
Temperature perturbation ( $\delta T$ ) leads to compression ( $\nabla \cdot \mathbf{v}_{di}$ ), density perturbation  $-90^\circ$  out-of-phase with  $\delta T$

$$dn/dt + \nabla \cdot (n\mathbf{v}) = 0$$

$$-i\omega\delta n \text{ from } -n_0\nabla \cdot \delta\mathbf{v}_d \sim -n_0\nabla \cdot (\delta T_\perp \mathbf{b} \times \nabla B/B)/B \sim -n_0 ik_y \delta T / BR$$

$$-i\omega(\delta n/n_0) \sim -ik_y(\delta T/T_0) T/BR \sim -i(k_y V_D) (\delta T/T_0) \sim -i\omega_D (\delta T/T_0)$$

$$-i(\omega_r + i\gamma)(\delta n/n_0) = -i\omega_D (\delta T/T_0)$$



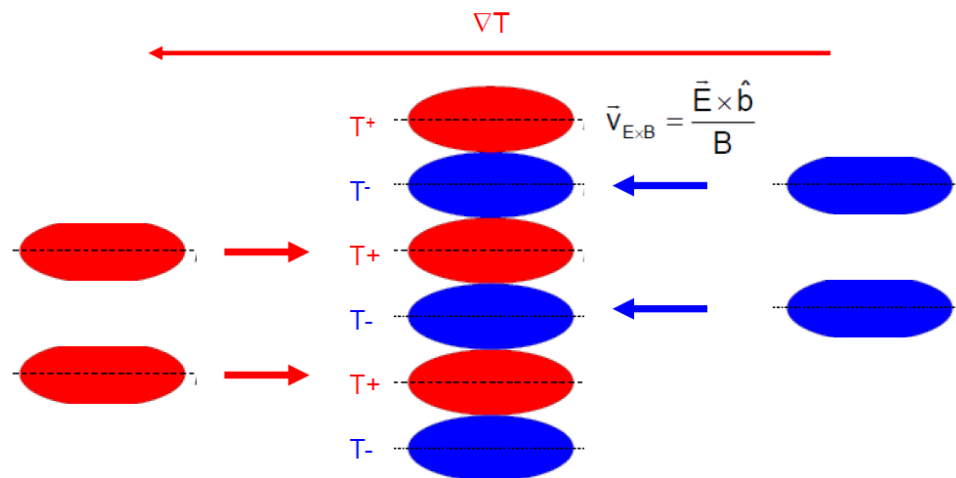


# Background Temperature Gradient Reinforces Perturbation $\Rightarrow$ Instability

$$-i\omega\delta T \text{ from } -\delta\mathbf{v}_E \cdot \nabla T_0 \sim -(\mathbf{b} \times \nabla \delta\phi / B) \cdot \nabla T_0 \sim ik_y \delta\phi / B \cdot \nabla T_0 \sim ik_y \delta\phi (T/B) / L_T$$

$$-i\omega(\delta T/T) \sim ik_y(\delta\phi/T) T / B L_T \sim i(k_y V_{*T})(\delta\phi/T) \sim i\omega_{*T}(\delta\phi/T)$$

$$-i(\omega_r + i\gamma)(\delta T/T) = i\omega_{*T}(\delta\phi/T)$$



# Simplest dispersion from these 3 terms

(1) Compression from toroidal drifts

$$\omega(\delta n_i/n_0) = \omega_{Di} (\delta T_i/T_{i0})$$

(2) Quasi-neutrality + Boltzmann electron response

$$(\delta n_i/n_0) = (\delta n_e/n_0) = (\delta\phi/T_{e0}) = (\delta\phi/T_{i0})(T_i/T_e)$$

(3)  $E \times B$  advection of background gradient

$$-\omega(\delta T_i/T_{i0}) = \omega_{*T}(\delta\phi/T_i)$$

$$(1)+(2): \quad \omega(T_i/T_e)(\delta\phi/T_{i0}) = \omega_{Di} (\delta T_i/T_{i0})$$

$$(+3): \quad \omega(T_i/T_e) = -\omega_{Di} \omega_{*T} / \omega$$

$$\omega^2 = -(k_y \rho_i)^2 v_{Ti}^2 / RL_T \quad (\text{assume } T_e = T_i)$$

$$\omega = \pm i (k_y \rho_i) v_{Ti} / (RL_T)^{1/2}$$

**How do we go from linear  
stability to turbulent  
transport?**

# Transport depends on a spectrum of amplitude fluctuations and cross-phases

- E.g. particle flux from electrostatic perturbations

$$\Gamma(x, t) = \langle \overline{\delta n \delta v_r} \rangle$$

- As a function of wavenumber:

$$\Gamma_{k_\theta} = \frac{n T_e}{B} k_\theta \left| \frac{N^*(k_\theta)}{n} \right| \left| \frac{\Phi_r(k_\theta)}{T_e} \right| \sin \{ \alpha_{n\phi}(k_\theta) \}$$

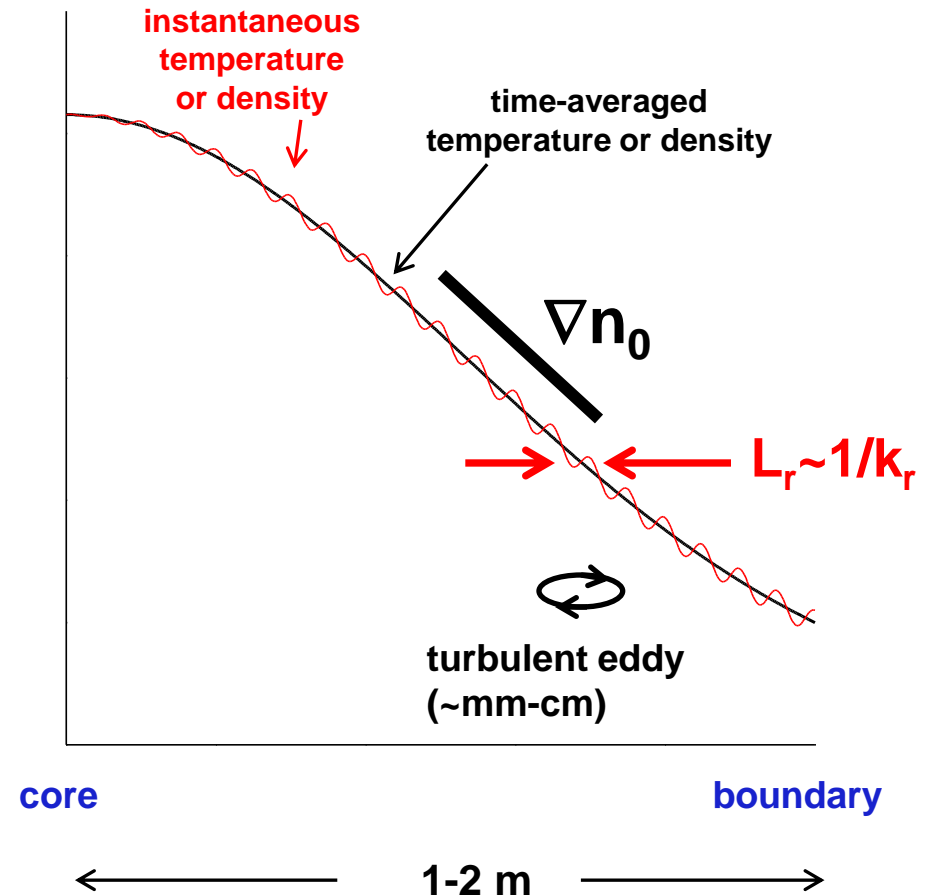
*Amplitude spectra*

*Cross phase*

- Except for Langmuir probes in cool edge plasma, we never (?) have been able to measure all the quantities needed to directly infer *turbulent transport* (especially cross phase)

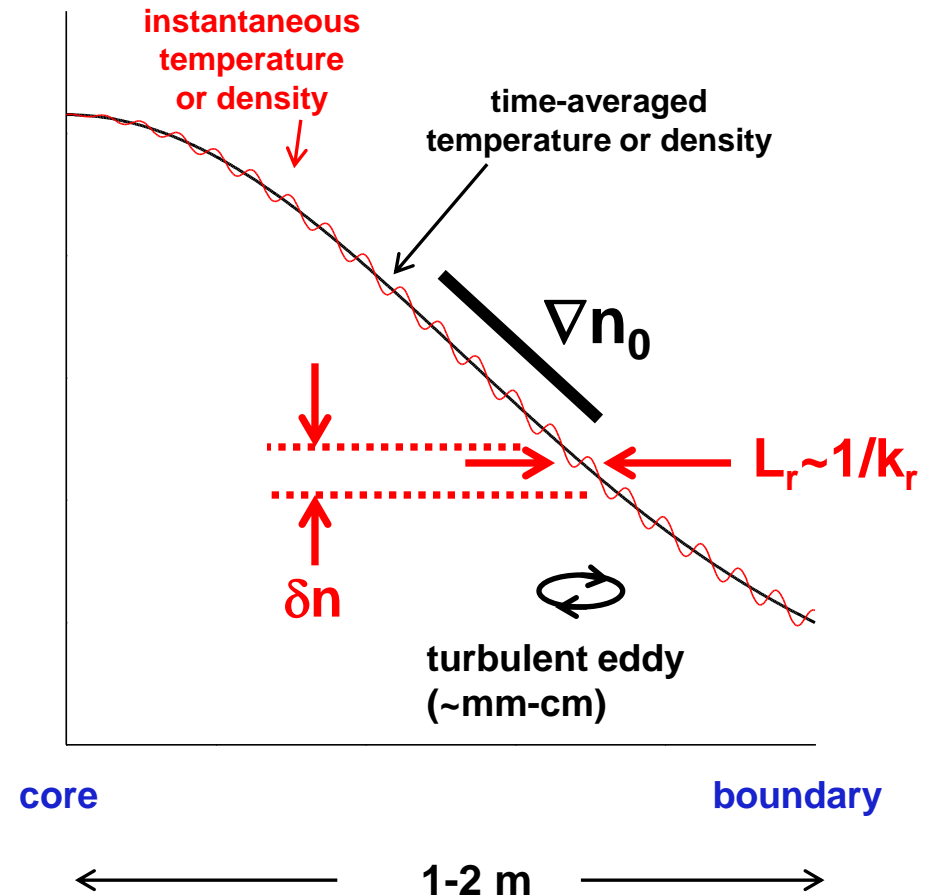
# Mixing length estimate for fluctuation amplitude

- In the presence of an equilibrium gradient,  $\nabla n_0$ , turbulence with radial correlation  $L_r$  will mix regions of high and low density



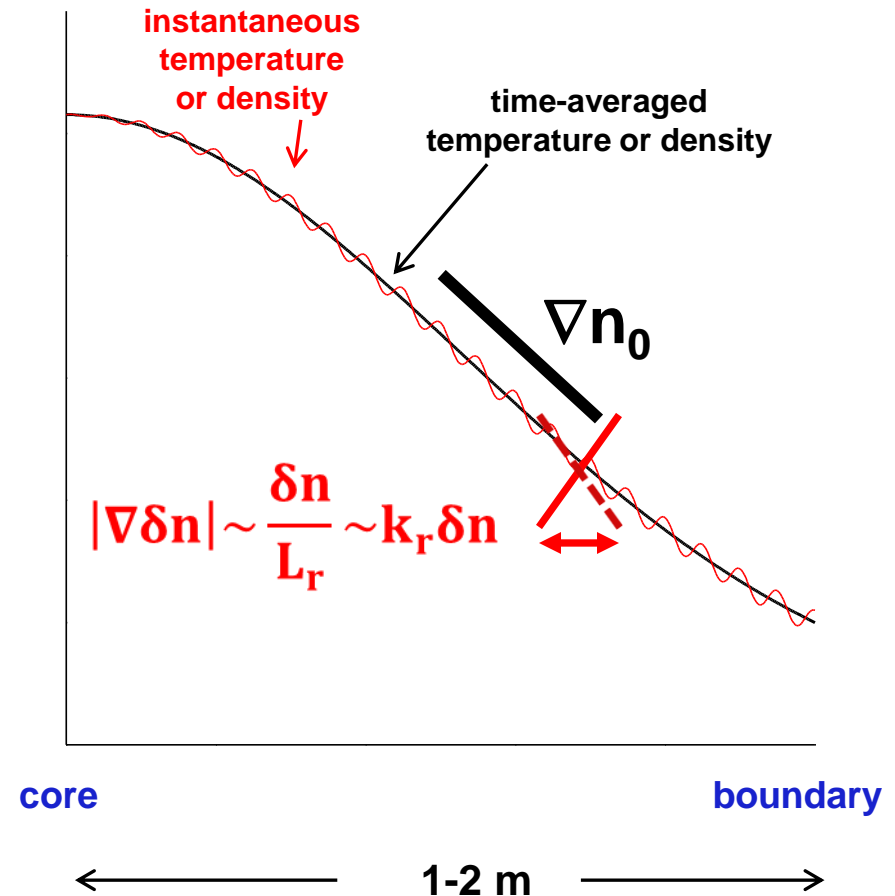
# Mixing length estimate for fluctuation amplitude

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- Leads to fluctuation  $\delta n$



# Mixing length estimate for fluctuation amplitude

- In the presence of an equilibrium gradient,  $\nabla n_0$ , turbulence with radial correlation  $L_r$  will mix regions of high and low density
- Leads to fluctuation  $\delta n$
- Another interpretation: local, instantaneous gradient limited to equilibrium gradient



# Mixing length estimate for fluctuation amplitude

$$\delta n \approx \nabla n_0 \cdot \mathbf{L}_r$$

$$\frac{\delta n}{n_0} \approx \frac{\nabla n_0}{n_0} \cdot \mathbf{L}_r \approx \frac{L_r}{L_n} \quad (1/L_n = \nabla n_0 / n_0)$$

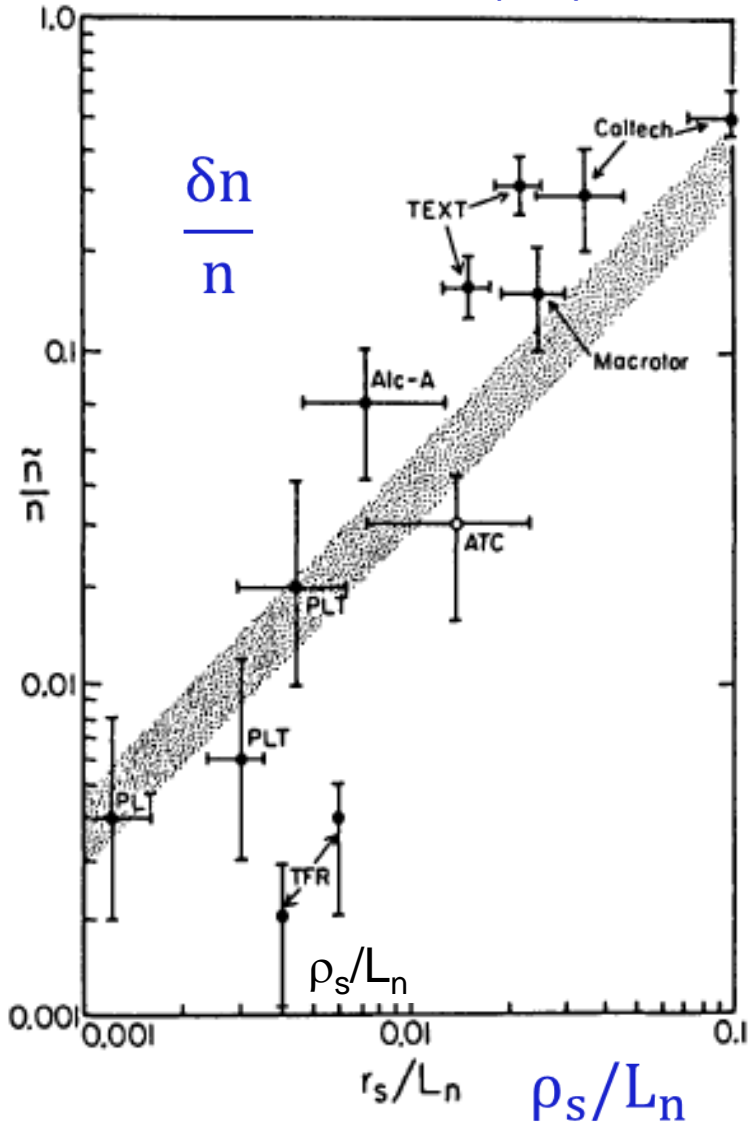
$$\frac{\delta n}{n_0} \sim \frac{1}{k_{\perp} L_n} \sim \frac{\rho_s}{L_n} \quad (k_{\perp}^{-1} \sim L_r; k_{\perp} \rho_s \sim \text{const } t)$$

**Expect  $\delta n/n_0 \sim \rho_s/L$**

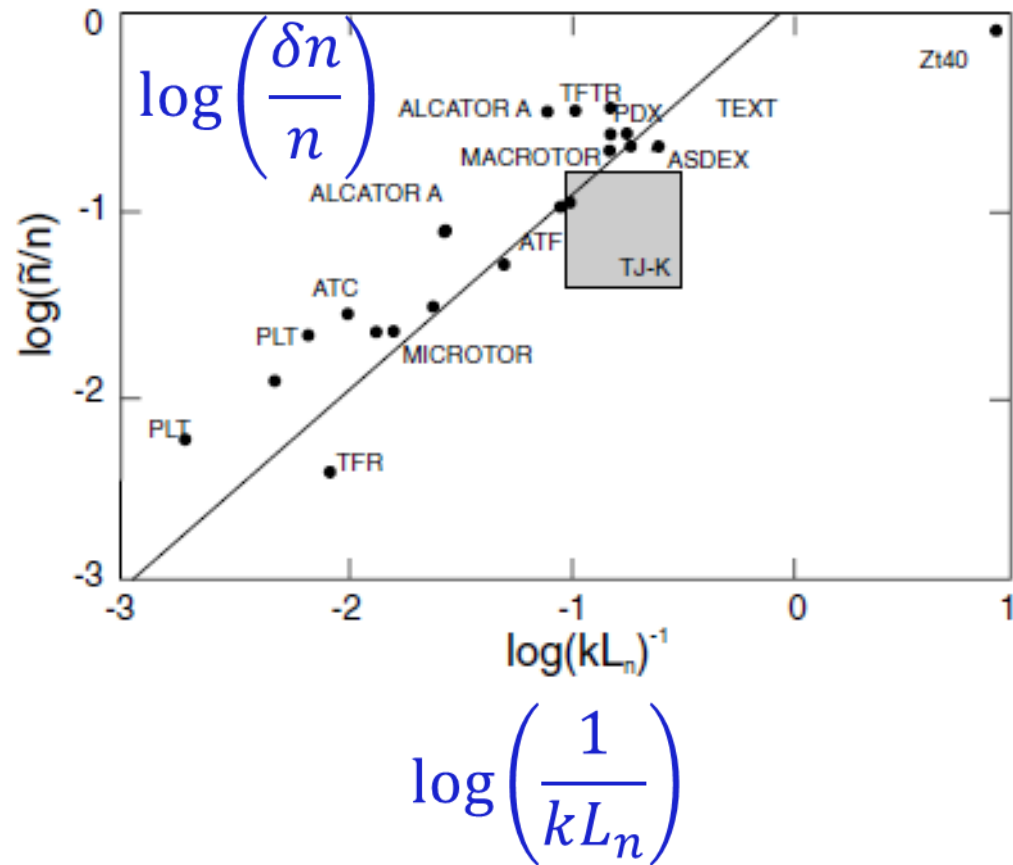


# Fluctuation intensity across machines loosely scales with mixing length estimate, reinforces local $\rho_s$ drift nature

Liewer, Nuclear Fusion (1985)



Lechte, New J. of Physics (2002)



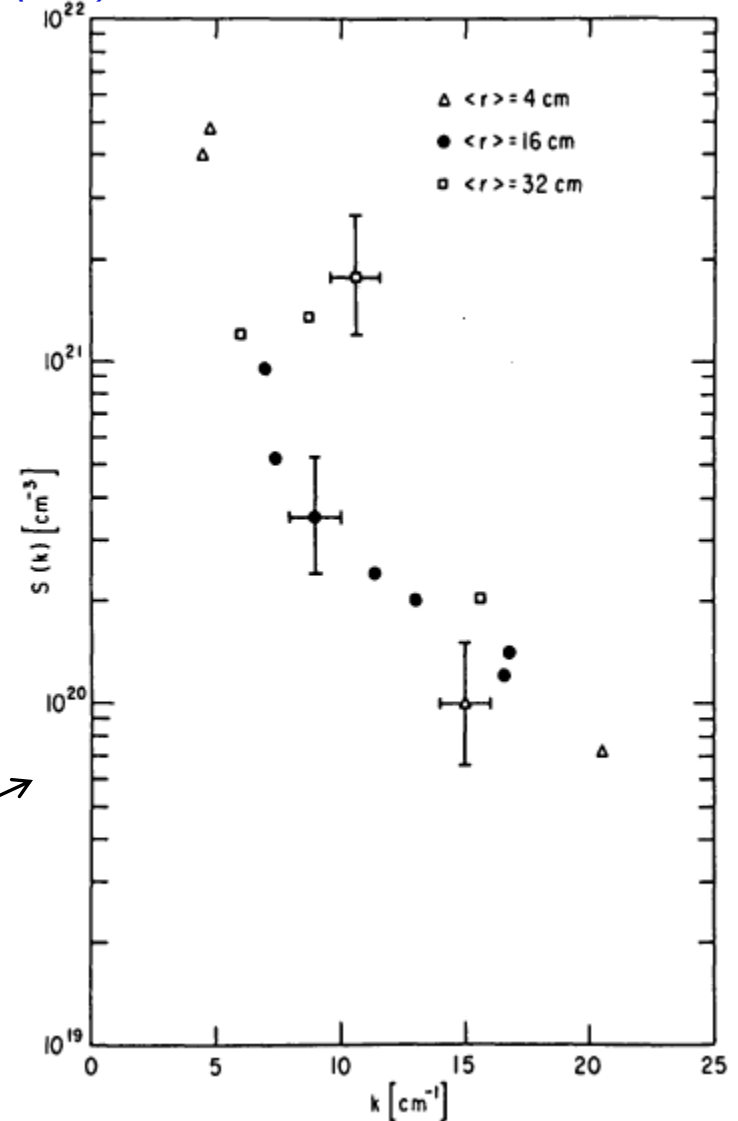
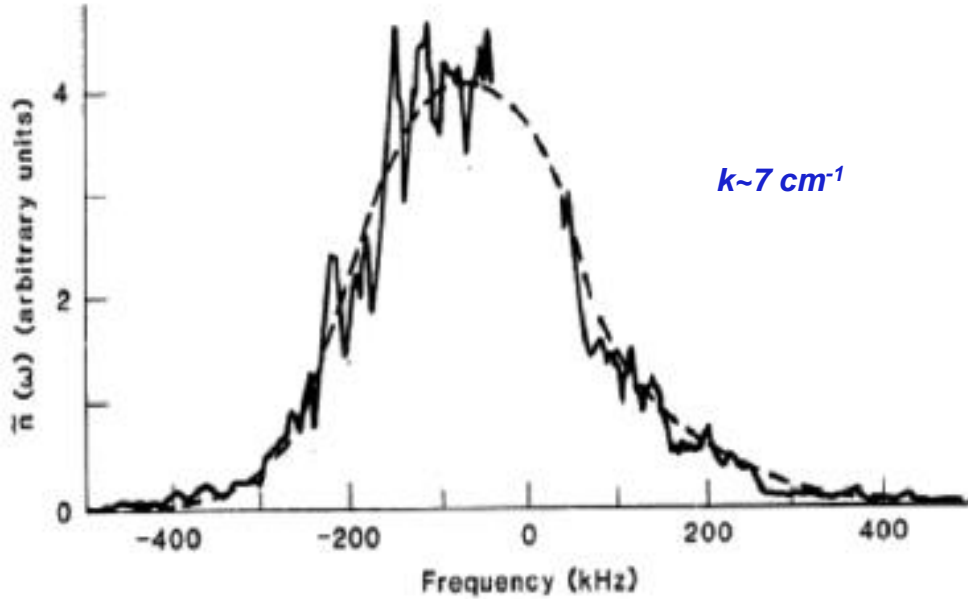
# Broad frequency and wavenumber spectra measured, e.g. from microwave scattering

Mazzucato, PRL (1982)

Surko & Slusher, Science (1983)

Princeton Large Torus (PLT)

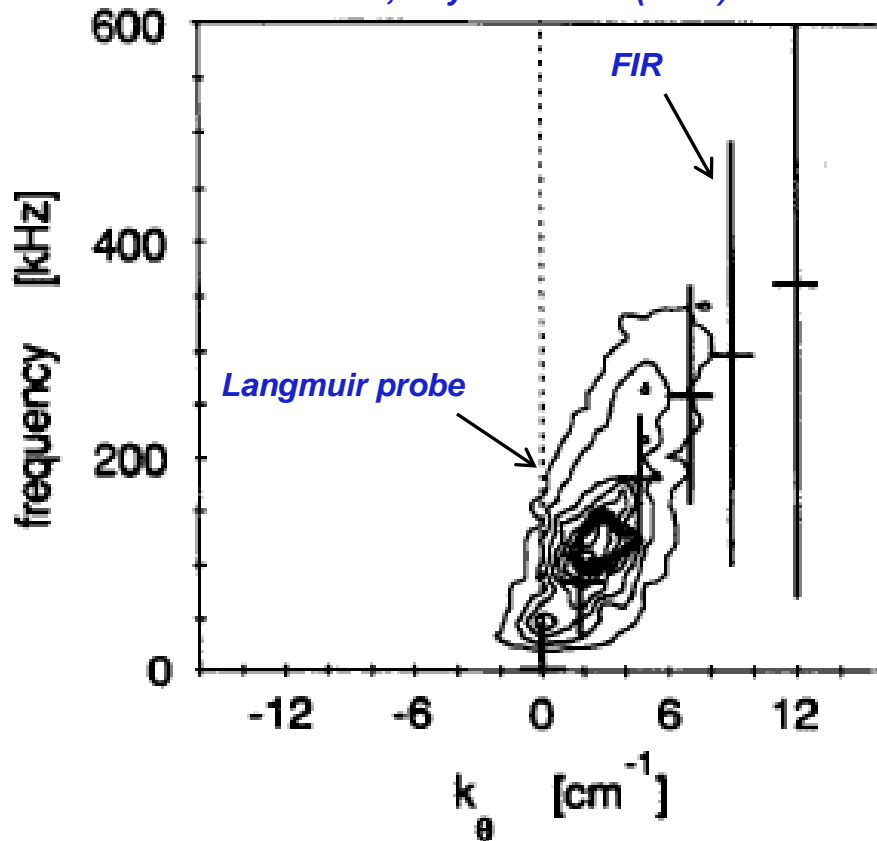
$k \sim 7 \text{ cm}^{-1}$



- Different scattering angles measure different  $k$ , observe spectral decay in wavenumber

# Broad drift wave turbulent spectrum verified simultaneously with Langmuir probes and FIR scattering

TEXT, Ritz, Nuclear Fusion (1987)  
Wooton, Phys. Fluids B (1990)



- Illustrates drift wave dispersion
- However, real frequency almost always dominated by Doppler shift

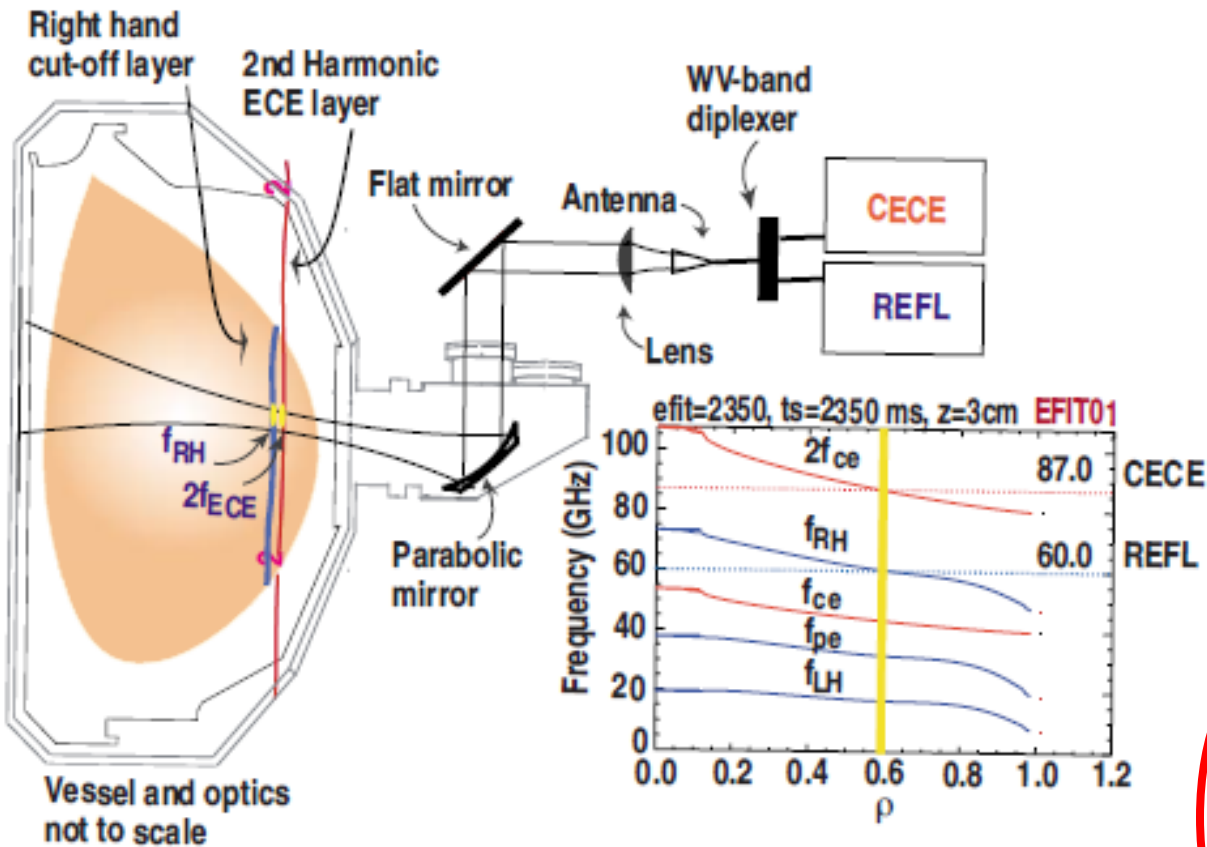
$$\omega_{\text{lab}} = \omega_{\text{mode}}(k_{\theta}) + k_{\theta} v_{\text{doppler}}$$

- Often challenging to determine mode frequency (in plasma frame) within uncertainties

FIG. 1. The  $S(k_{\theta}, \omega)$  spectrum at  $r = 0.255$  m in TEXT, from Langmuir probes (contours) and FIR scattering (bars indicate FWHM).

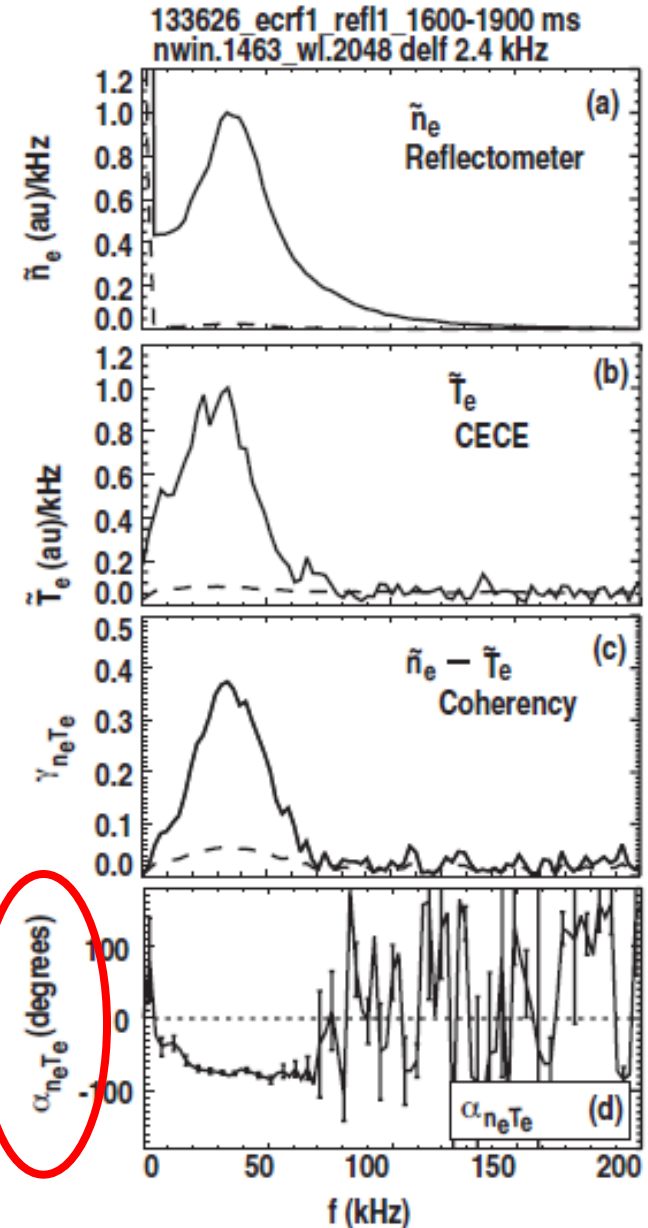
# Simultaneous measurement of $n_e$ and $T_e$ using same beam path allows for cross-phase measurement

- Not directly the cross-phase relate to transport (e.g.  $\phi$ -T).

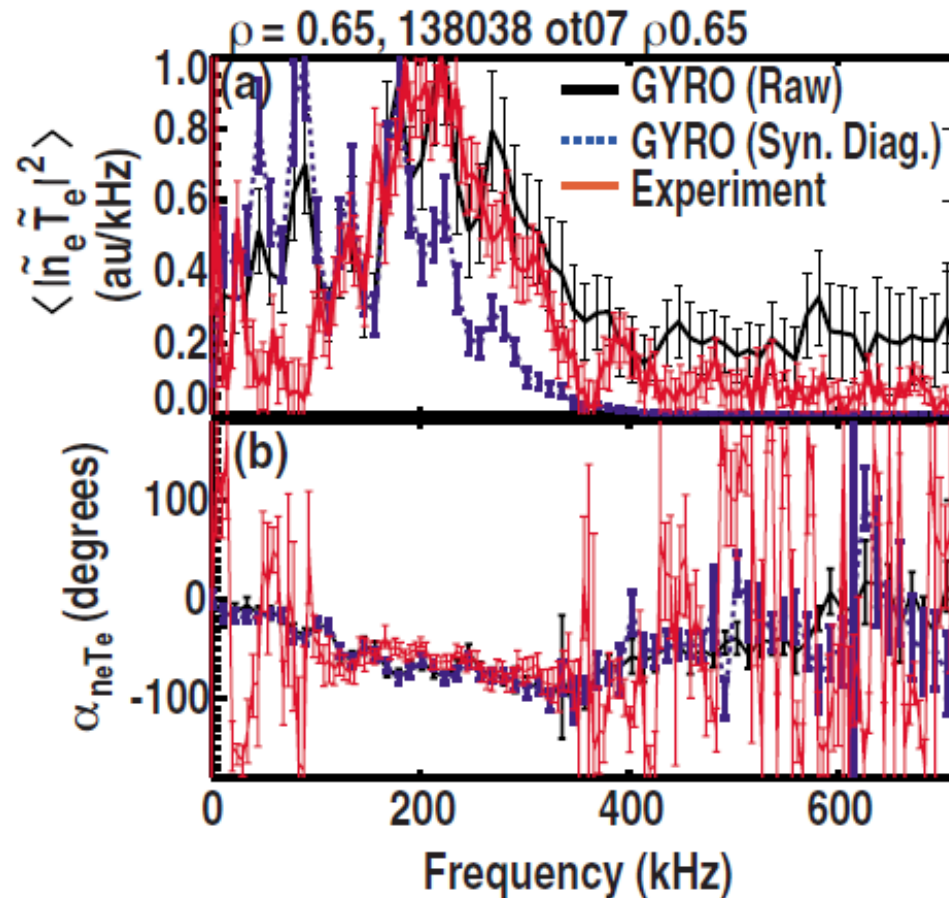


$$\gamma_{n_e T_e}(f) = \frac{|\langle S_{\tilde{n}_e}^* S_{\tilde{T}_e} \rangle|}{|\langle S_{\tilde{n}_e} \rangle|^2 |\langle S_{\tilde{T}_e} \rangle|^2}$$

DIII-D  
White, PoP (2010)



# $n_e$ - $T_e$ cross phases agree amazingly well with simulations!



- Concept of “validation hierarchy” – validate theory with high level quantities (transport) + components [ $\delta n(\omega, k)$ ,  $\delta T(\omega, k)$ , cross-phases]
- Provides (stronger) constraint to validate theory & physical understanding

# Spectrum shape / distribution governed by nonlinear three-wave interactions

- Linearly unstable modes grow,  $\delta n(\mathbf{k}) \sim \exp[i\mathbf{k} \cdot \mathbf{x} + \gamma(\mathbf{k})t]$
- At large amplitude, interact via nonlinear advection,  $\delta \mathbf{v}_E \cdot \nabla \delta n$   
I.e. “three-wave” coupling in wavenumber space

$$d/dt(\delta n) \sim \delta \mathbf{v}_E \cdot \nabla \delta n$$

$$d/dt[\delta n(\mathbf{k}_3)] \sim \sum_{\mathbf{k}_1, \mathbf{k}_2} [(\mathbf{b} \times \mathbf{k}_1 \delta \varphi) \cdot \mathbf{k}_2 \delta n]$$

summed over all  $(\mathbf{k}_1, \mathbf{k}_2)$  for  $\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_3$

- Energy gets distributed across  $\mathbf{k}$  space (& velocity space) until damped by stable modes (& collisions)  $\rightarrow$  saturation

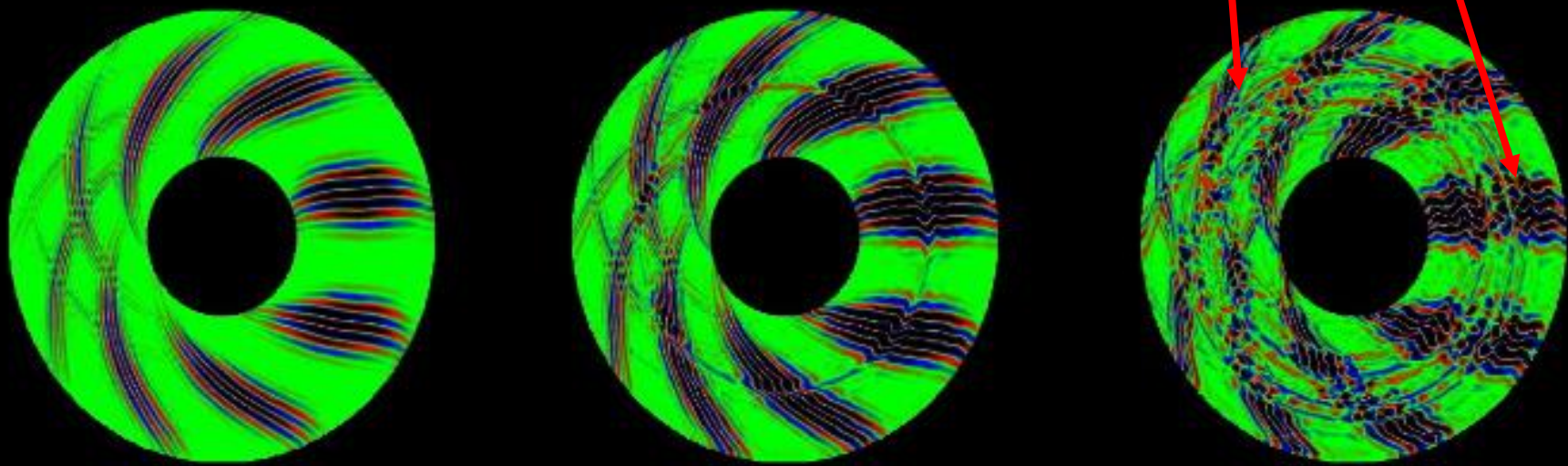
# Self-generated “zonal flows” also impacts saturation of turbulence and overall transport (esp. ITG)

- Potential perturbations uniform on flux surfaces, near zero frequency ( $f \sim 0$ )
- Predator-prey like behavior: turbulence drives ZF (linearly stable), which regulates/clamps turbulence; if turbulence drops enough, ZF drive drops, allows turbulence to grow again...

Linear instability stage demonstrates structure of fastest growing modes

Large flow shear from instability cause perpendicular “zonal flows”

Zonal flows help moderate the turbulence!!!



*Rayleigh-Taylor like instability ultimately driving Kelvin-Helmholtz-like instability → non-linear saturation*

**Code: GYRO**

**Authors: Jeff Candy and Ron Waltz**



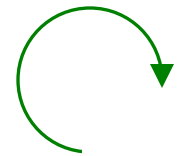
# Generation of zonal flows in tokamaks similar to “Kelvin-Helmoltz” instability found throughout nature



Variation of flows in one direction...



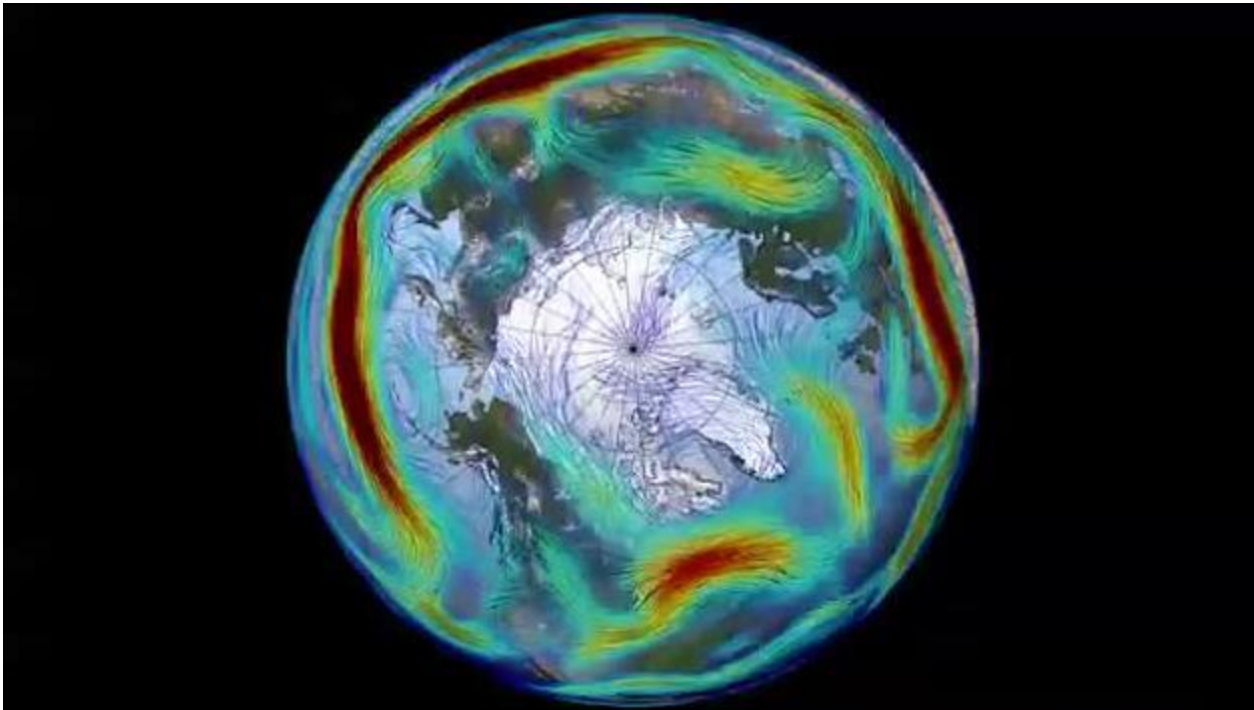
lead to flows in another direction



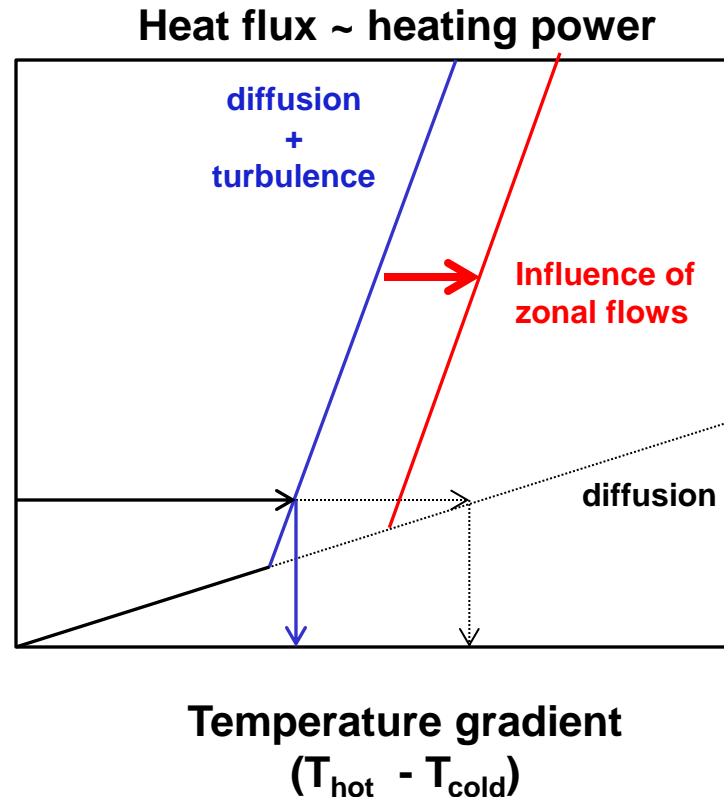
(potential contours → stream functions)

# The Jet Stream is a zonal flow (or really, vice-versa)

- NASA/Goddard Space Flight Center Scientific Visualization Studio



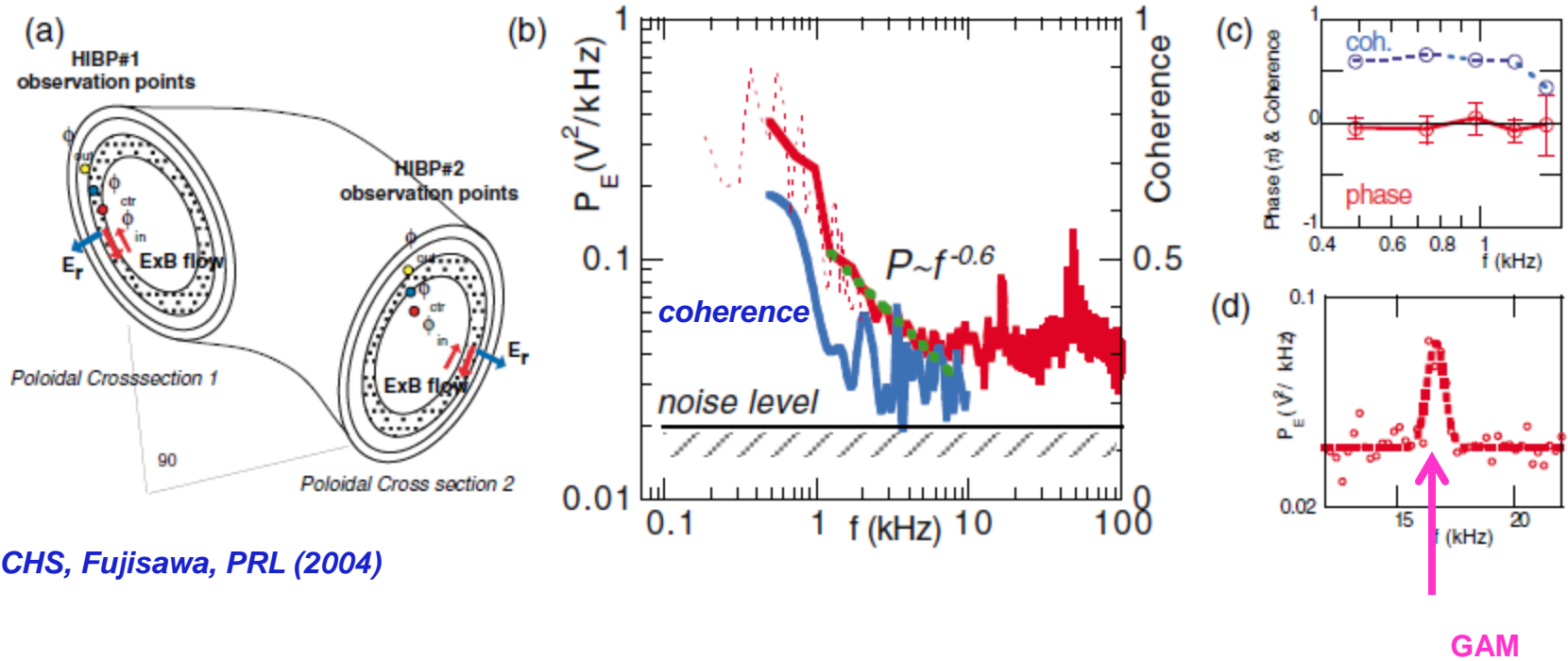
# Zonal flows reduce the heating power required to maintain a given temperature (than had they not been there)



- So-called “Dimitis shift” [A. Dimitis et al, Phys. Plasmas 7, 969 (2000)]

# ZF also leads to Geodesic Acoustic Mode (GAM) oscillation, also contributes to nonlinear saturation

- Zonal flow potential  $\phi$  is uniform on a flux surface
- $v_{E,ZF} = \nabla_r \phi / B$  varies like  $\cos(\theta)$  from  $1/B$
- Compressibility ( $\nabla \cdot v_{E,ZF}$ ) gives rise to  $\sim$ coherent geodesic acoustic mode ( $\omega_{GAM} \approx c_s/R$ ) from associated ( $n=0, m=1$ ) pressure perturbation

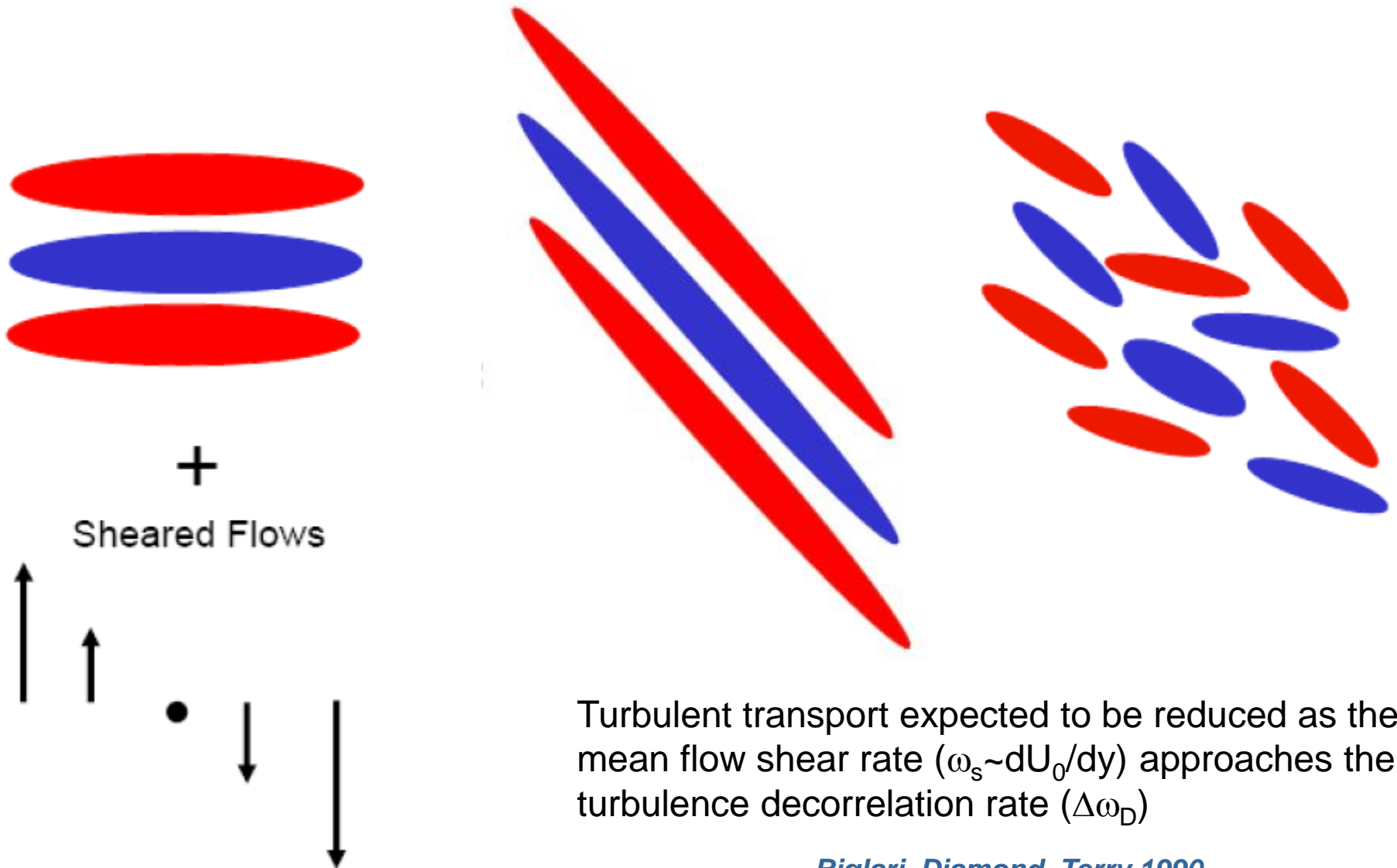


CHS, Fujisawa, PRL (2004)

**GAMs are easier to measure, have been identified in numerous tokamaks and stellarators, consistent with theory predictions**

# Suppression of turbulence via sheared perpendicular ( $E \times B$ ) flow

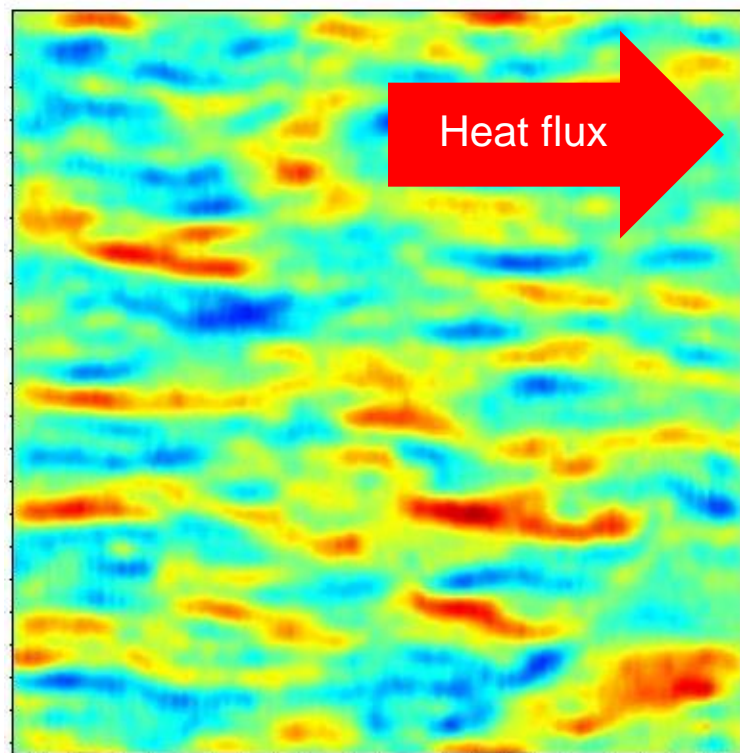
# Large scale sheared flows can tear apart turbulent eddies, reduce turbulence, mixing and transport



# Large scale sheared flows can tear apart turbulent eddies, reduce turbulence → improve confinement

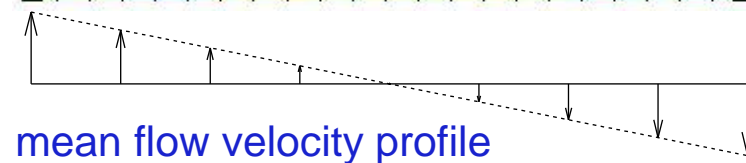
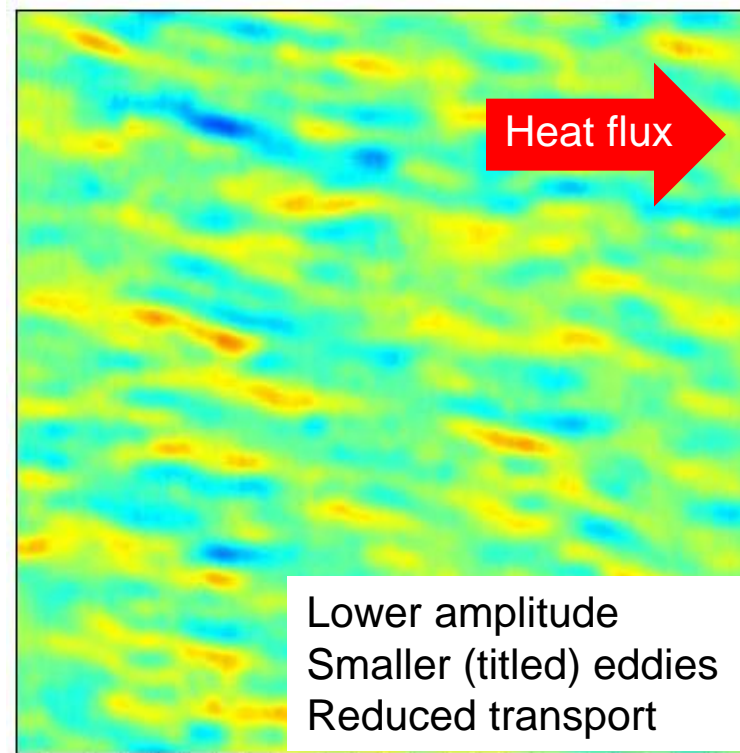
## NSTX simulations

Snapshot of density without flow shear

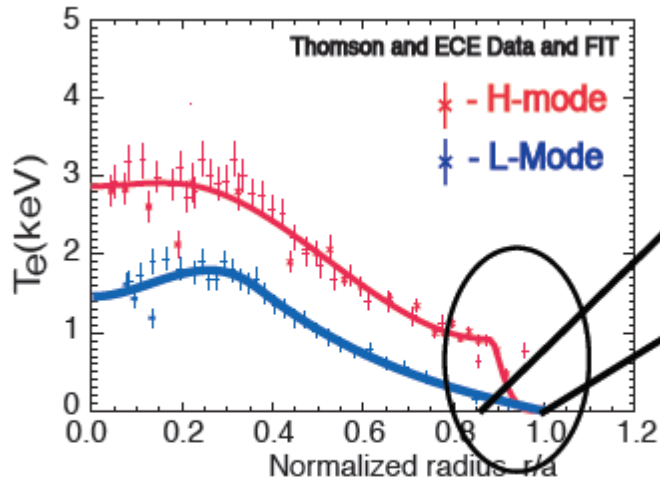
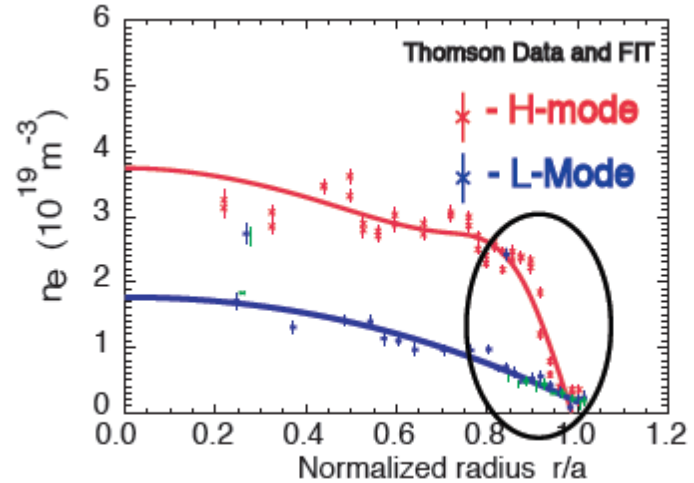


← 100 ion radii  
6,000 electron radii →  
~50 cm

Snapshot of density with flow shear

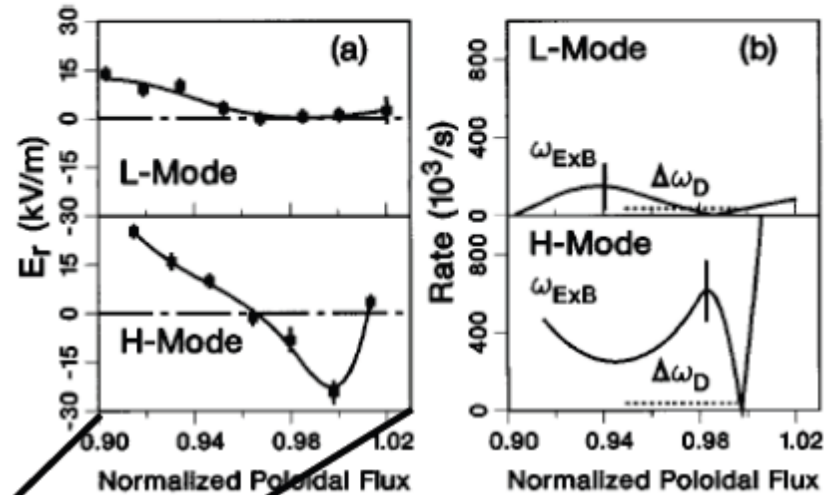


# Spontaneous “H-mode” edge transport barrier can form with sufficient heating power → improved confinement



Data from DIII-D

(from Carter, 2013)



Burrell 1997

- Correlated with strong shear in equilibrium radial electric field ( $E_r$ )
- Suppression of turbulence predicted when equilibrium shearing rate ( $\omega_{E \times B}$ ) > turbulence decorrelation rate ( $\Delta\omega_D$ ) [Biglari, 1990; Hahm, 1994]

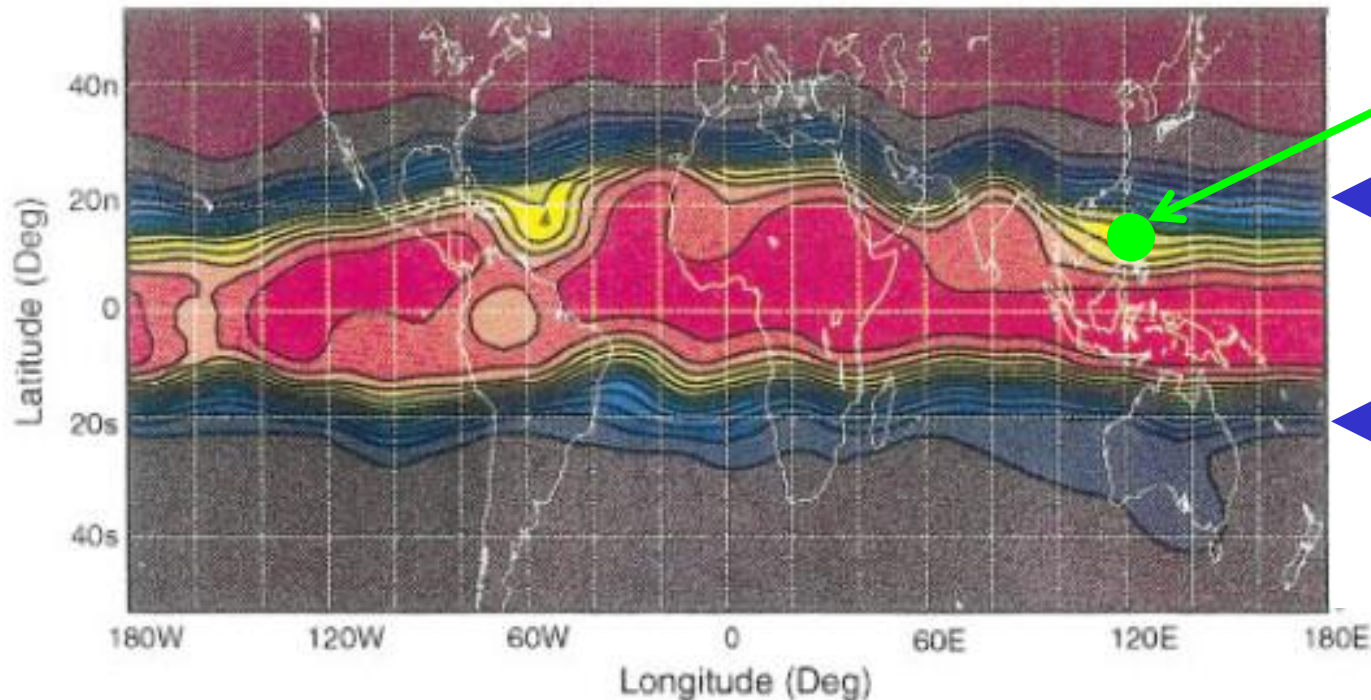


# In neutral fluids, sheared flows are usually the source of free energy to drive turbulence

- Thin (quasi-2D) atmosphere in axisymmetric geometry of rotating planets similar to tokamak plasma turbulence
- Stratospheric ash from Mt. Pinatubo eruption (1991) spread rapidly around equator, **but confined in latitude by flow shear**



Aerosol concentration



Large shear in stratospheric equatorial jet

(Trepte, 1993)

# REDUCED MODELS

# Have learned a lot from validating first-principles gyrokinetic simulations with experiment

- But the simulations are expensive (1 local multi-scale simulation ~ 20M cpu-hrs)
- Desire a model capable of reproducing flux-gradient relationship that is far quicker, so we can do integrated predictive modeling (“flight simulator”)
- All physics based models are local & gradient-driven, i.e. given gradients from a single flux surface they predict fluxes:

$$\begin{bmatrix} \Gamma \\ \Pi_\phi \\ Q_i \\ Q_e \end{bmatrix} = - \begin{bmatrix} \text{flux - gradient} \\ \text{relationship} \\ \text{matrix} \end{bmatrix} \cdot \begin{bmatrix} \nabla n \\ R \nabla \Omega \\ \nabla T_i \\ \nabla T_e \end{bmatrix}$$

that can be used in solving the 1D transport equation predictively

$$\frac{3}{2} n(\rho, t) \frac{\partial T(\rho, t)}{\partial t} + \nabla \cdot Q(\rho, t) = \dot{P}_{\text{source}}(\rho, t) - \dot{P}_{\text{sink}}(\rho, t)$$

# Is local assumption appropriate?

- If  $\rho_* = \rho_i / L$  is small enough ( $< \sim 1/300$ ), local is good  $\rightarrow$  OK for ITER and most reactor designs (at least in the core, *not the edge*)
- Challenges: In the edge, additional effects may change how we model transport / gradient relationship
  - Large, intermittent edge fluctuations with strong non-local effects may demand full-F gyrokinetic simulations (XGC-1, Gkeyll)
  - Local transport time scale, i.e. evolution of  $T(\rho, t)$ , is increasingly fast relative to turbulence
  - Related -- edge turbulence should perhaps more realistically be thought of as source driven vs. gradient driven (think external forcing vs. linear instability)
    - We're heating the plasma and watching the temperature respond, not experimentally prescribing a temperature gradient
  - **Unclear how to incorporate these effects in reduced models**

# Illustration of how to develop a simple plasma turbulence drift wave transport model

- Decompose flux expressions into wavenumber, amplitude spectra, and cross-phases

$$\Gamma_{k_\theta} = \frac{nT_e}{B} k_\theta \left| \frac{N^*(k_\theta)}{n} \right| \left| \frac{\Phi_r(k_\theta)}{T_e} \right| \sin\{\alpha_{n\phi}(k_\theta)\}$$

- Amplitude could be estimated using mixing-length hypothesis:

$$\frac{\tilde{n}}{n} = \frac{l}{k_r L_n} \sim \frac{\rho_s}{L_n}$$

# Using dispersion relation, we recover gyroBohm scaling factor

$$\gamma \approx \delta \omega_{*e} = \delta \cdot k_{\theta} T_e / B L_n$$

$$k_{\theta} \rho_s \sim k_r \rho_s \sim 1$$

- $k_{\theta} \rho_s$  for expected peak  $\gamma$
- Assuming isotropic

$$D_{\text{turb}} = \frac{\gamma}{k_r^2} = \delta \frac{\rho_s}{L_n} \frac{T_e}{B}$$

$$D_{\text{turb}} \approx \delta \cdot \chi_{GB}$$

- *In the local (small  $\rho_*$ ) limit, all transport quantities have leading order gyroBohm scaling*
- **But linear stability ( $\delta$ ) still matters (e.g. thresholds & stiffness)**

## Early models (60's-80's) used analytic fluid or gyrokinetic theory to evaluate linear stability

- Fancy non-linear theories also used to refine model for saturated fluctuation amplitudes
- A turning point in model sophistication was the advent of gyrofluid equations & increased computational power
  - Hammett, Perkins, Dorland, Beer, Waltz, ....
- Take fluid moments of gyrokinetic equation
- Pick suitable kinetic closures
- Tweak closure free parameters to best match linear gyrokinetic simulations
  - Linear GK simulations became routine in mid-90's, but expensive and slow relative to gyrofluid

# Breakthrough in understanding (90's...) was recognition of threshold and stiffness

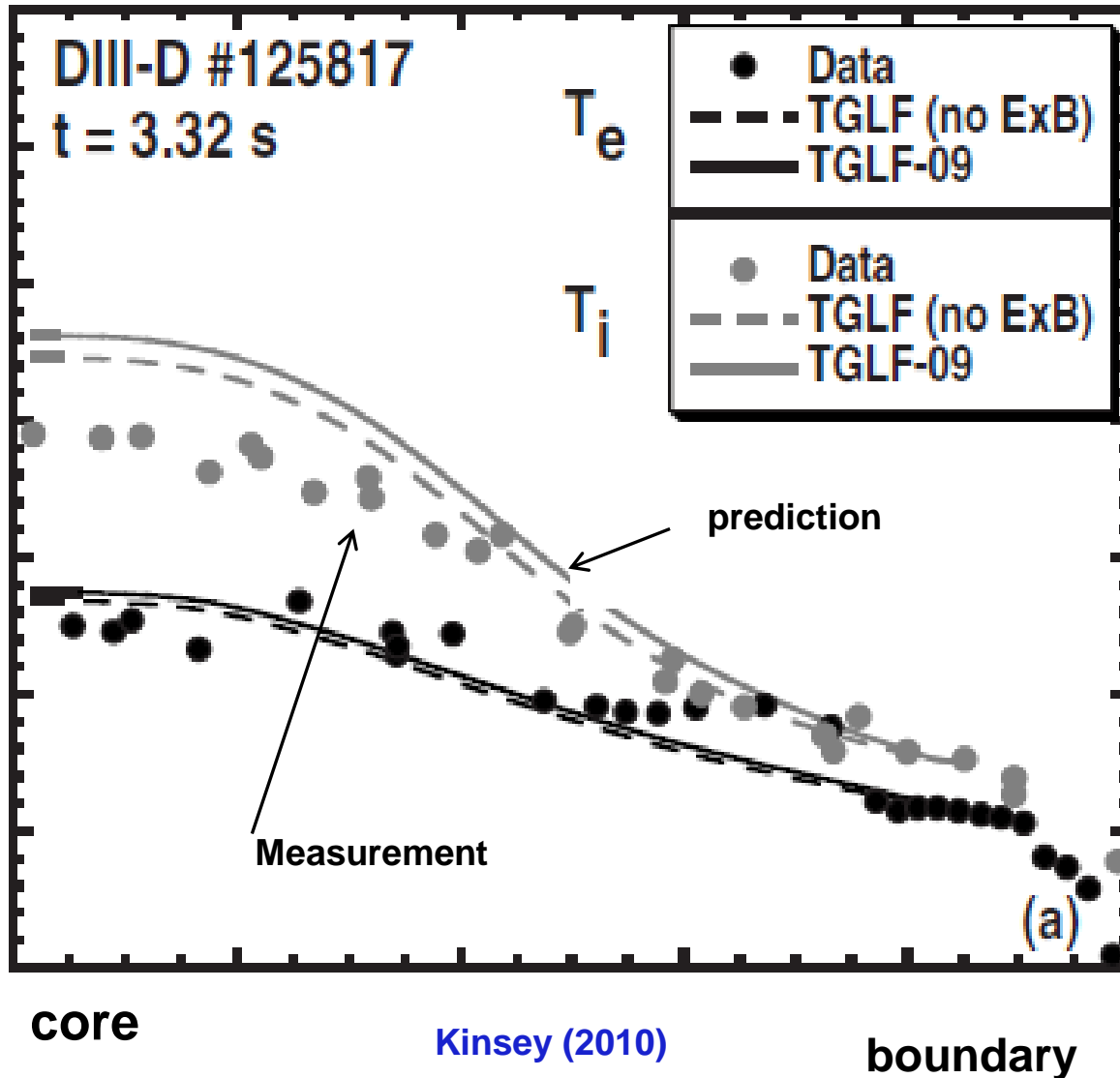
$$Q_{\text{model}} = Q_{\text{GB}} \cdot F(s, q, \dots) \cdot \left( \frac{R}{L_T} - \frac{R}{L_{T,\text{crit}}} \right)^\alpha$$

- All local models have gyroBohm prefactor ( $Q_{\text{GB}}$ )
- First modern model approaches fit coefficients in above equation to large numbers of GF and/or GK simulations
  - $R/L_{T,\text{crit}}$  from linear simulations
  - Additional scaling coefficient  $F(s,q,\dots)$  from nonlinear simulations
- *A bunch of fit coefficients, but entirely from first principles*
- **Modern transport models: IFS-PPPL, GLF23, TGLF, QualiKiz, ...**

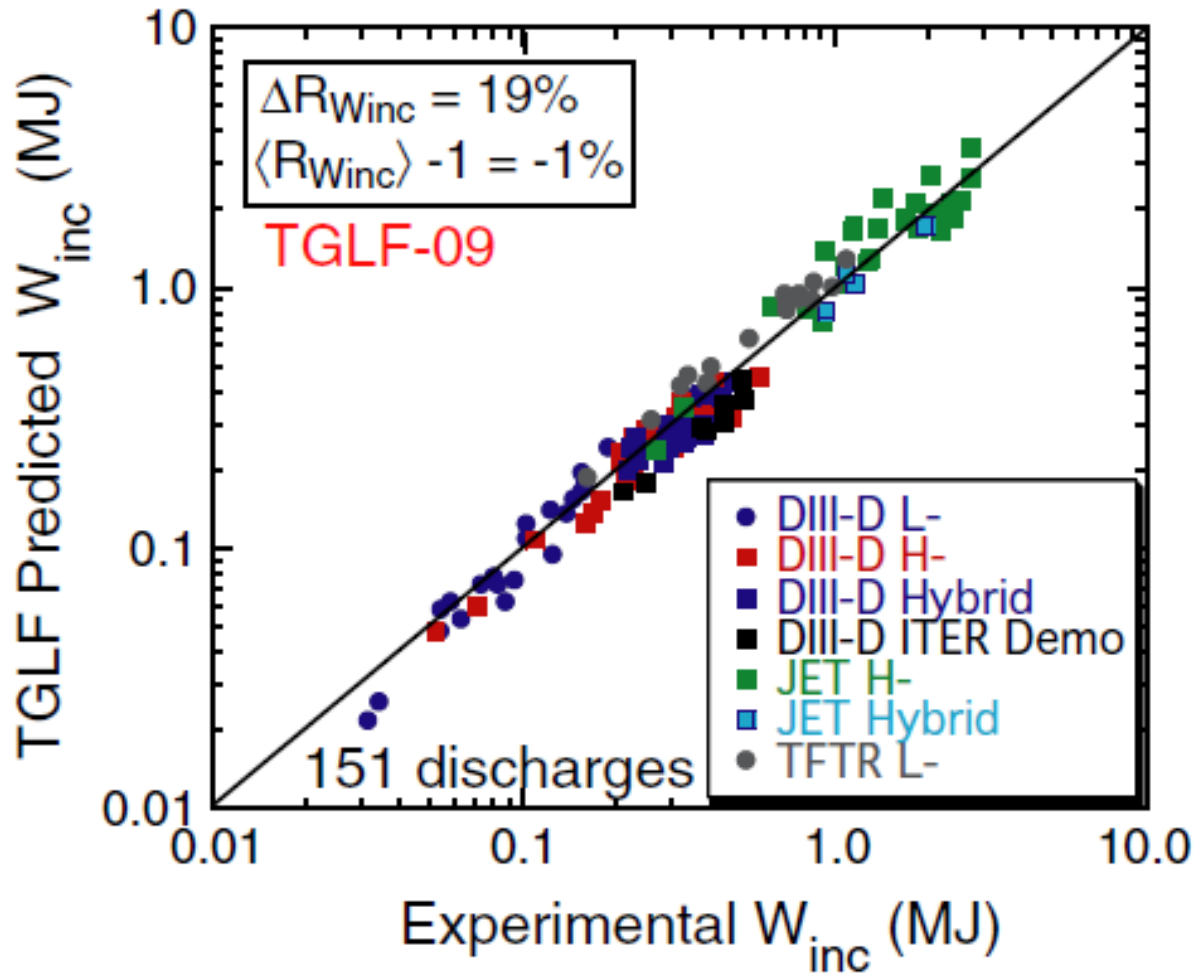


# Some success in profile predictions (TGLF model on DIII-D)

Temperature



# Good agreement in predicted energy confinement over database of discharges



Kinsey (2011)

**There are many flavors of micro-  
instabilities/turbulence**

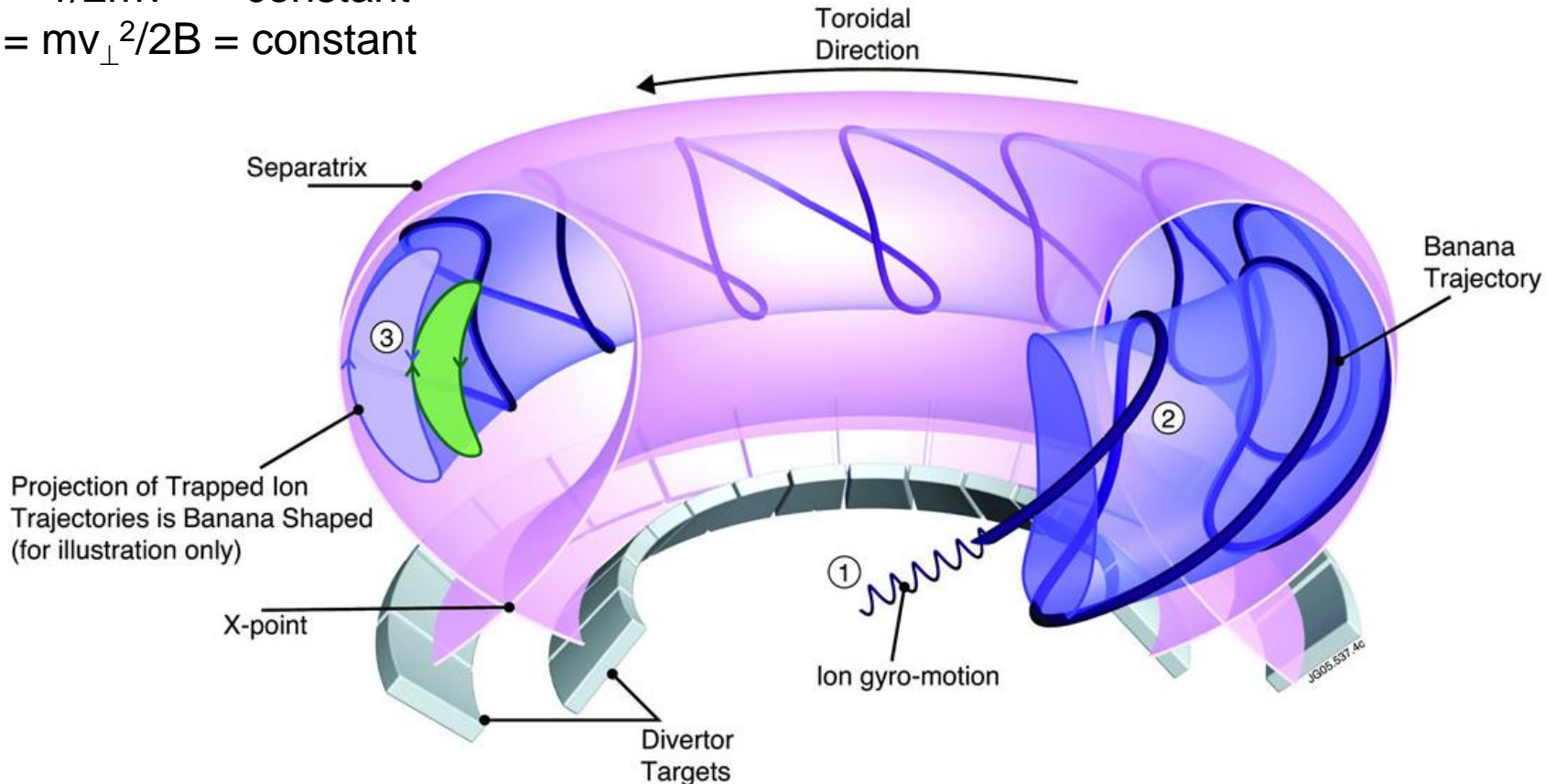
# Beyond general characteristics, there are many theoretical “flavors” of drift waves possible in tokamak core & edge

- Usually think of drift waves as gradient driven ( $\nabla T_i$ ,  $\nabla T_e$ ,  $\nabla n$ )
  - Often exhibit threshold in one or more of these parameters
- Different theoretical “flavors” exhibit different parametric dependencies, predicted in various limits, depending on gradients,  $T_e/T_i$ ,  $v$ ,  $\beta$ , geometry, location in plasma...
  - Electrostatic, ion scale ( $k_\theta \rho_i \leq 1$ )
    - Ion temperature gradient (ITG) – driven by  $\nabla T_i$ , weakened by  $\nabla n$
    - Trapped electron mode (TEM) – driven by  $\nabla T_e$  &  $\nabla n_e$ , weakened by  $v_e$
    - Parallel velocity gradient (PVG) – driven by  $R\nabla\Omega$  (like Kelvin-Helmholtz)
  - Electrostatic, electron scale ( $k_\theta \rho_e \leq 1$ )
    - Electron temperature gradient (ETG) - driven by  $\nabla T_e$ , weakened by  $\nabla n$
  - Electromagnetic, ion scale ( $k_\theta \rho_i \leq 1$ )
    - Kinetic ballooning mode (KBM) - driven by  $\nabla \beta_{\text{pol}} \sim \alpha_{\text{MHD}}$
    - Microtearing mode (MTM) – driven by  $\nabla T_e$ , at sufficient  $\beta_e$

**Trapped electrons enhance  
ITG and lead to new instability:  
trapped electron mode (TEM)**

# Inhomogeneous magnetic field causes trapped particles to precess toroidally

$$E = \frac{1}{2}mv^2 = \text{constant}$$
$$\mu = \frac{mv_{\perp}^2}{2B} = \text{constant}$$



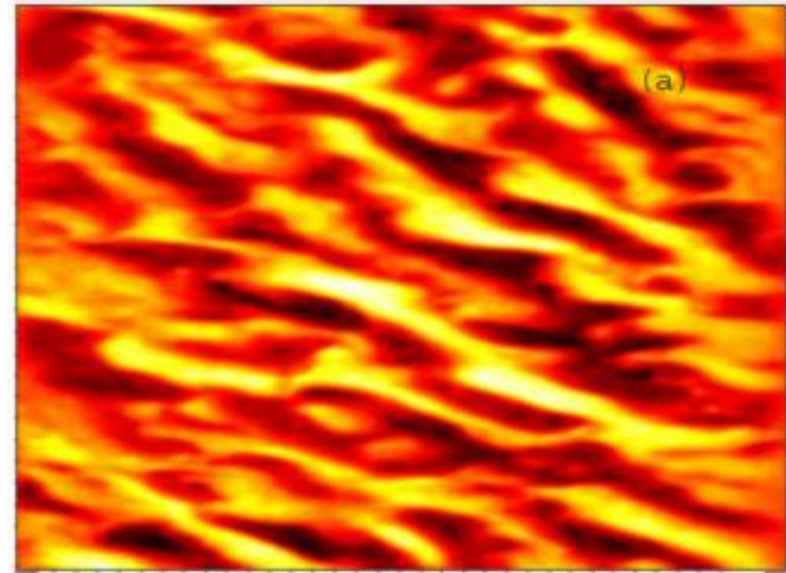
Trapped electron precession frequencies can be comparable to drift wave frequency ( $\omega \sim v_{Ti}/R$ )  $\Rightarrow$  resonance can enhance ITG instability and lead to distinct **trapped electron mode (TEM) instabilities driven by  $\nabla T_e$ ,  $\nabla n_e$**

**Turbulence at electron  
gyroradius can also be important**

# Electron scale ( $\sim$ mm) turbulence can dominate when ITG/TEM suppressed

- Electron temperature gradient (ETG) instability “isomorphic” to ITG, same ballooning instability mechanism but reversed role of ions and electrons
- $L_{\perp} \sim \rho_e$ ,  $\omega \sim v_{Te}/R$  ( $\sim 60$  times smaller,  $\sim 60$  times faster than ITG)
- Characteristic gyroBohm transport expected to be 1/60 of ITG transport  
 $\chi_{ETG} \sim (\Delta x)^2/\Delta t \sim \rho_e^2 v_{Te}/R \sim (1/60) \cdot \rho_i^2 v_{Ti}/R$
- “Streamers” can exist nonlinearly (Jenko, Dorland, 2000, 2001)  
 $\Delta x \sim L_r > L_{\theta}$  ( $k_{\theta} \gg k_r$ )  
 $\Rightarrow$  **Much larger transport than expected**

*density fluctuations from ETG simulation*

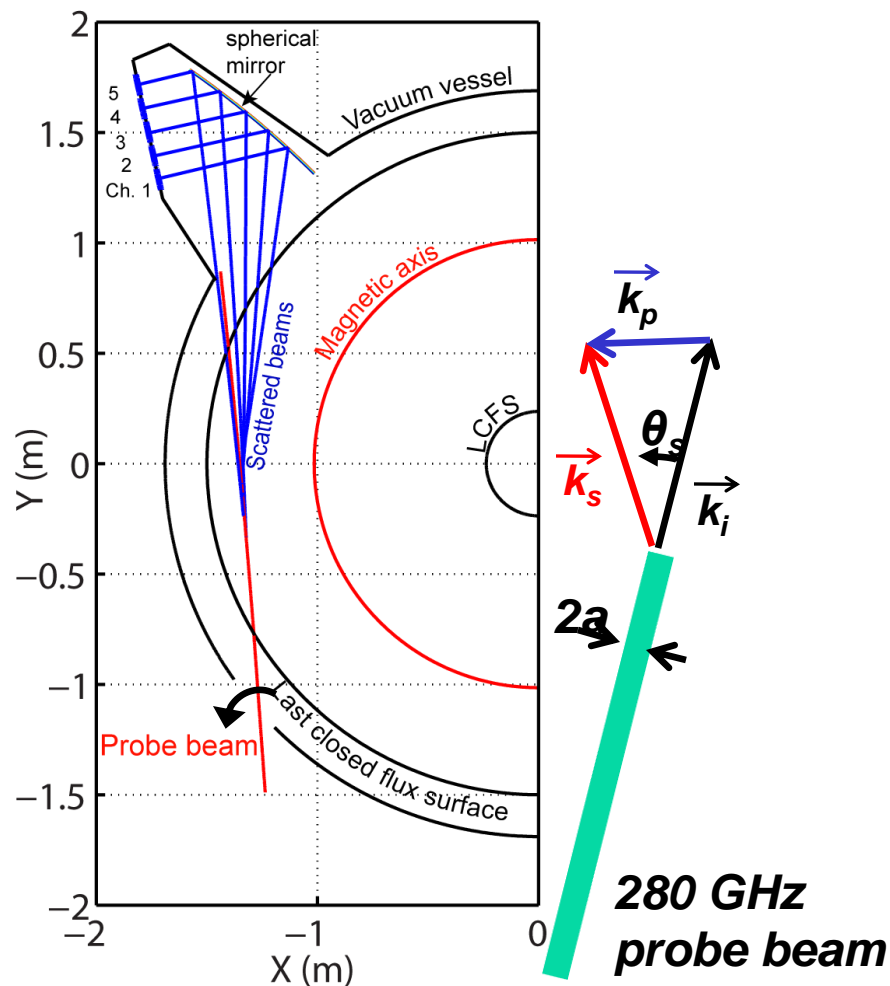


← **6 ion radii**  
**360 electron radii** →  
 **$\sim 2$  cm**

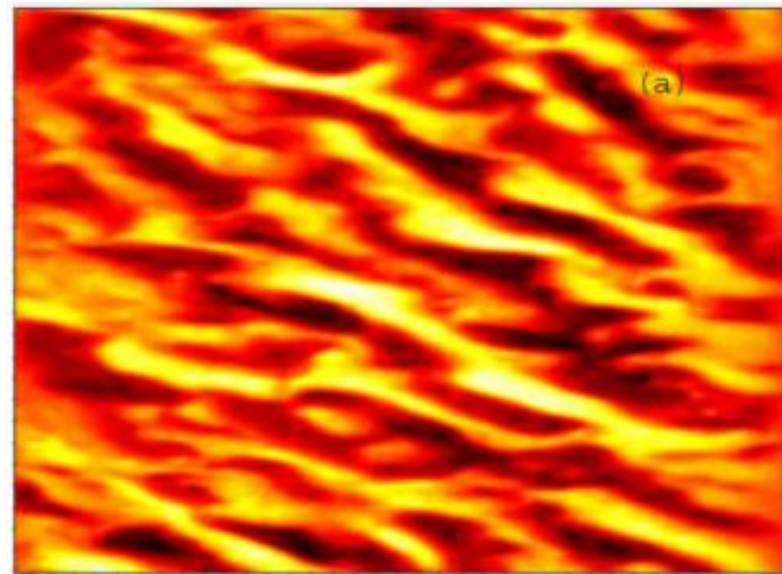
*Guttenfelder, PoP (2011)*



# Not easy to image electron scale (mm) fluctuations → “microwave scattering” used to detect high-k fluctuations



*density fluctuations from ETG simulation*



← **6 ion radii**  
**360 electron radii** →  
**~2 cm**

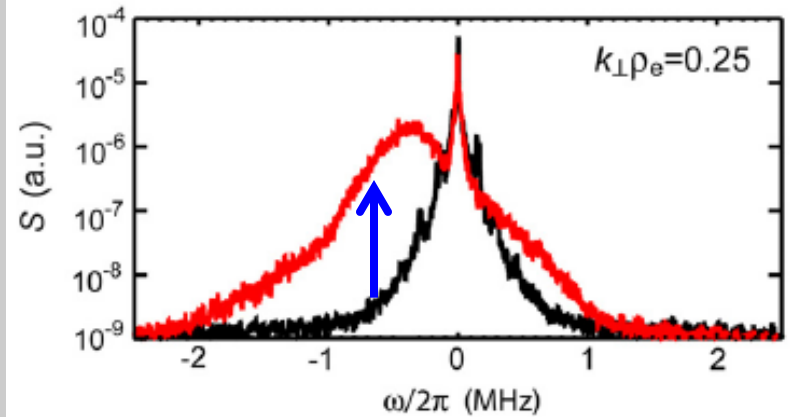
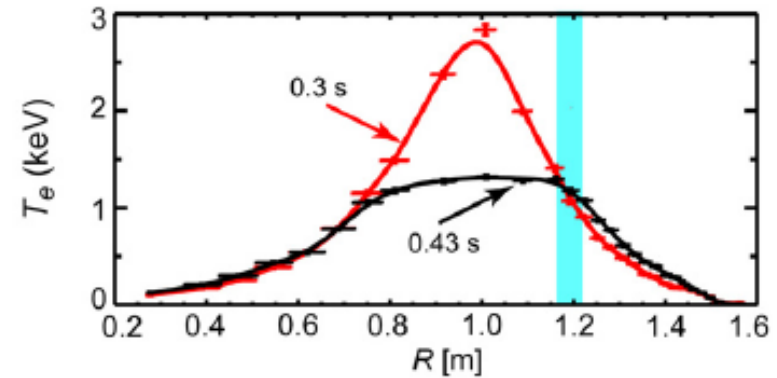
*Smith, RSI (2008)*

# Correlation observed between high-k scattering fluctuations and $\nabla T_e$

- Applying RF heating to increase  $T_e$
- Fluctuations increase as expected for ETG turbulence

• **Other trends measured that are consistent with ETG expectations, e.g. reduction of high-k scattering with:**

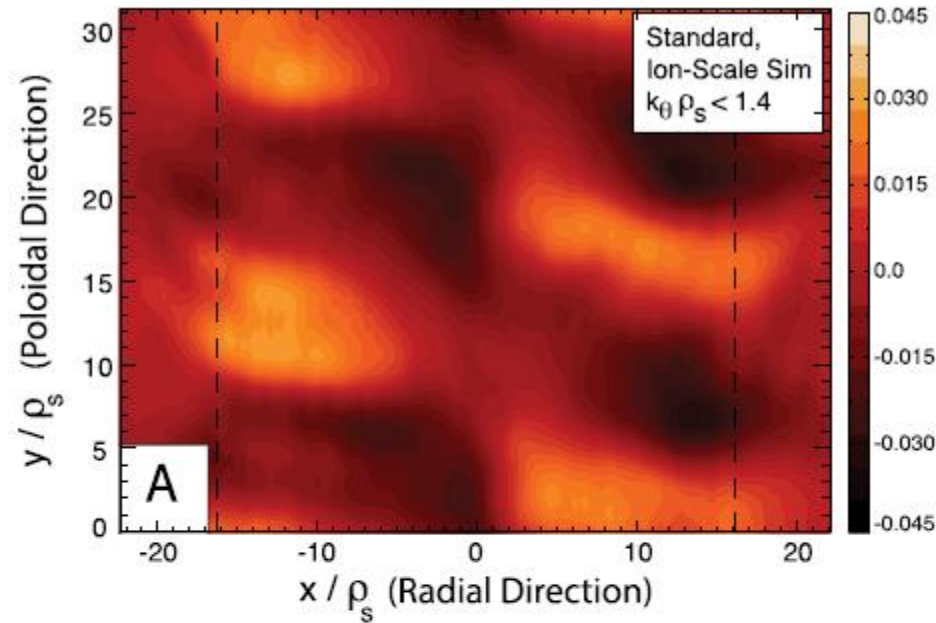
1. **Strongly reversed magnetic shear (Yuh, PRL 2011)**
  - Simulations predict comparable suppression (Peterson, PoP 2012)
2. **Increasing density gradient (Ren, PRL 2011)**
  - Simulations predict comparable trend (Ren, PoP 2012, Guttenfelder NF, 2013, Ruiz PoP 2015)
3. **Sufficiently large  $E \times B$  shear (Smith, PRL 2009)**
  - Observed in ETG simulations (Roach, PPCF 2009; Guttenfelder, PoP 2011)



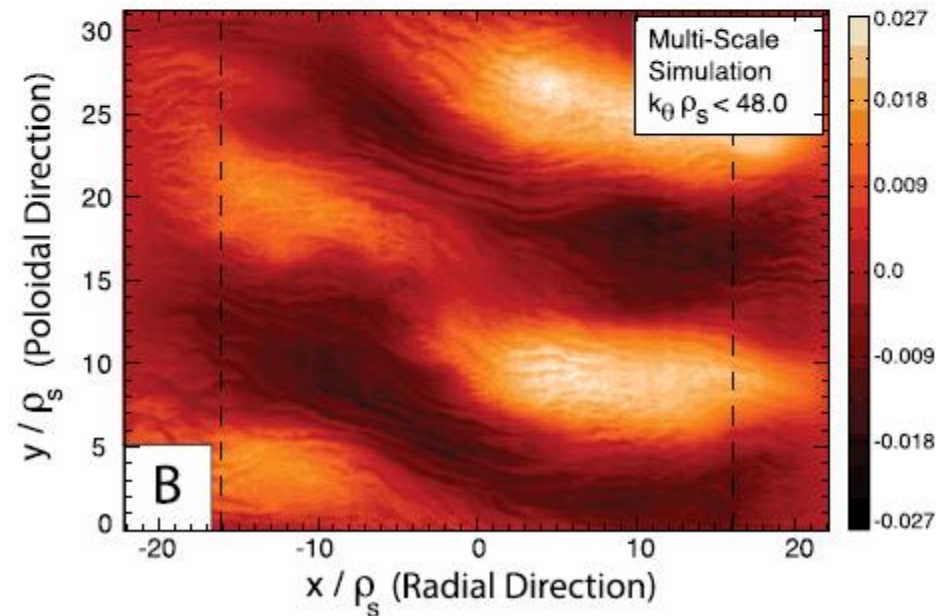
E. Mazzucato et al., NF (2009)

# MULTI-SCALE TURBULENCE (FROM $\rho_i$ TO $\rho_e$ SCALES)

# ETG-like “streamers” predicted to exist on top of ion scale turbulence



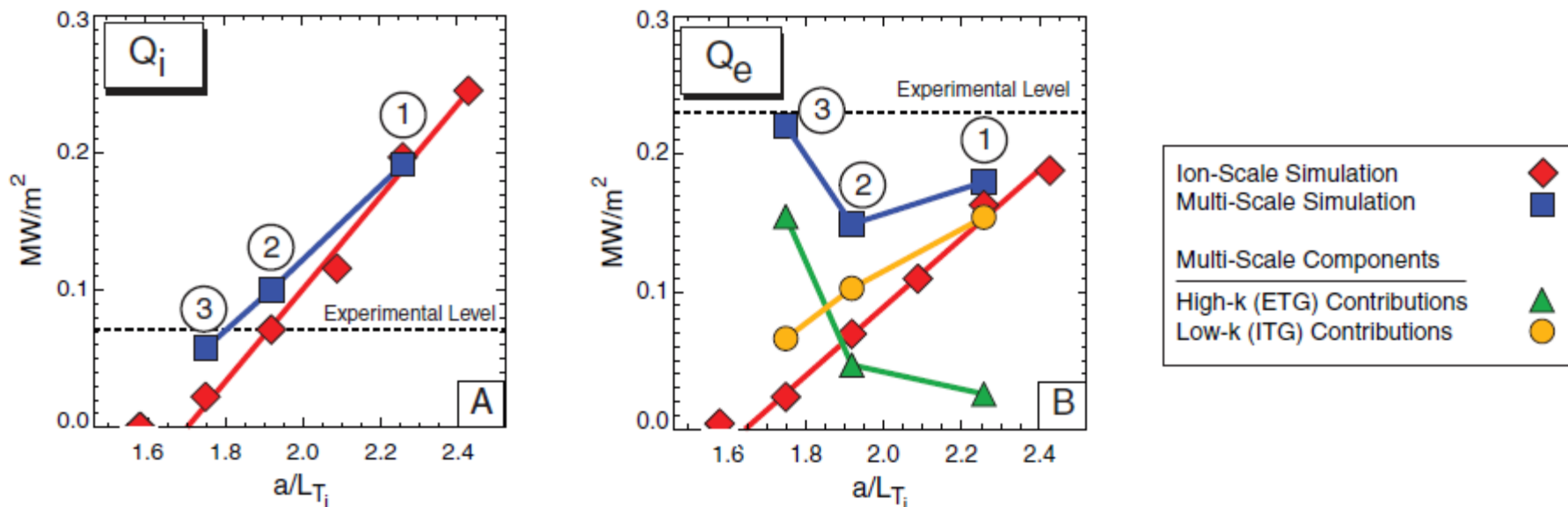
*Howard, PoP (2014)*



# Non-intuitive change in predicted transport due to cross-scale coupling between $\sim\rho_i$ and $\sim\rho_e$

- As  $a/L_{T_i}$  ( $=-R\nabla T_i/T_i$ ) is reduced towards ITG threshold,  $Q_i$  decreases while electron transport increases due to very small scale ( $k_\theta\rho_i > 1$ ,  $k_\theta\rho_e < 1$ ) turbulence
- *can match experiment*

Howard, NF (2016)



## 2 slide summary of some turbulent transport concepts in magnetized fusion plasmas (1)

- For fusion gain  $Q \sim nT\tau_E$  (& 100% non-inductive tokamak operation) we need excellent energy confinement,  $\tau_E$
- Energy confinement depends on turbulence ( $\tau_E \sim a^2/\chi_{\text{turb}}$ )
  - As does particle, impurity & momentum transport
- Core turbulence generally accepted to be drift wave in nature
  - Quasi-2D ( $L_{\perp} \sim \rho_i$ ,  $\rho_e \ll L_{\parallel} \sim qR$ )
  - Driven by  $\nabla T$  &  $\nabla n$
  - Frequencies  $\sim$  diamagnetic drift frequency ( $\omega \sim \omega_* \sim k_{\theta} \rho_i \cdot c_s / L_{n,T}$ )
  - Drift wave transport generally follows gyroBohm scaling  $\chi_{\text{turb}} \sim \chi_{\text{GB}} \sim \rho_i^2 v_{Ti} / a$ , *however...*
  - Thresholds and stiffness are critical, i.e.  $\chi_{\text{turb}} \sim \chi_{\text{GB}} \cdot F(\dots) \cdot (\nabla T - \nabla T_{\text{crit}})$
- Toroidal ion temperature gradient (ITG) drift wave is a key instability for controlling confinement in current tokamaks
  - Unstable due to interchange-like toroidal drifts, analogous to Rayleigh-Taylor instability
  - Threshold influenced by magnetic equilibrium ( $q$ ,  $s$ ) and other parameters
  - Nonlinear saturated transport depends on zonal flows & perpendicular  $E \times B$  sheared flow

## 2 slide summary of some turbulent transport concepts in magnetized fusion plasmas (2)

- Many other flavors of turbulence exist (TEM, ETG, PVG, MTM, KBM)
  - $\rho_i$  or  $\rho_e$  scale
  - Electrostatic or electromagnetic (at increasing beta)
  - Different physical drives, parametric dependencies, & influence on transport channels ( $\Gamma$  vs.  $Q$  vs.  $\Pi$ )
- Reduced models are constructed by quasi-linear calculations + “mixing-length” estimates for nonlinear saturation
  - We rely heavily on direct numerical simulation using gyrokinetic codes to guide model development
  - Reasonably predict confinement scaling and core profiles
- Things get more complicated for edge / boundary turbulence
  - Changing topology (closed flux surfaces  $\rightarrow$  X-point (poloidal field null)  $\rightarrow$  open field lines & sheaths at physical boundary)
  - Larger gyroradius / banana widths,  $\rho_{\text{banana}}/\Delta_{\text{ped}} \sim 1 \rightarrow$  orbit losses & non-local effects
  - Large amplitude fluctuations,  $\delta n/n_0 \sim 1$  ( $\delta f \rightarrow$  full-F simulations)
  - Neutral particles, radiation, other atomic physics...

**THE END**

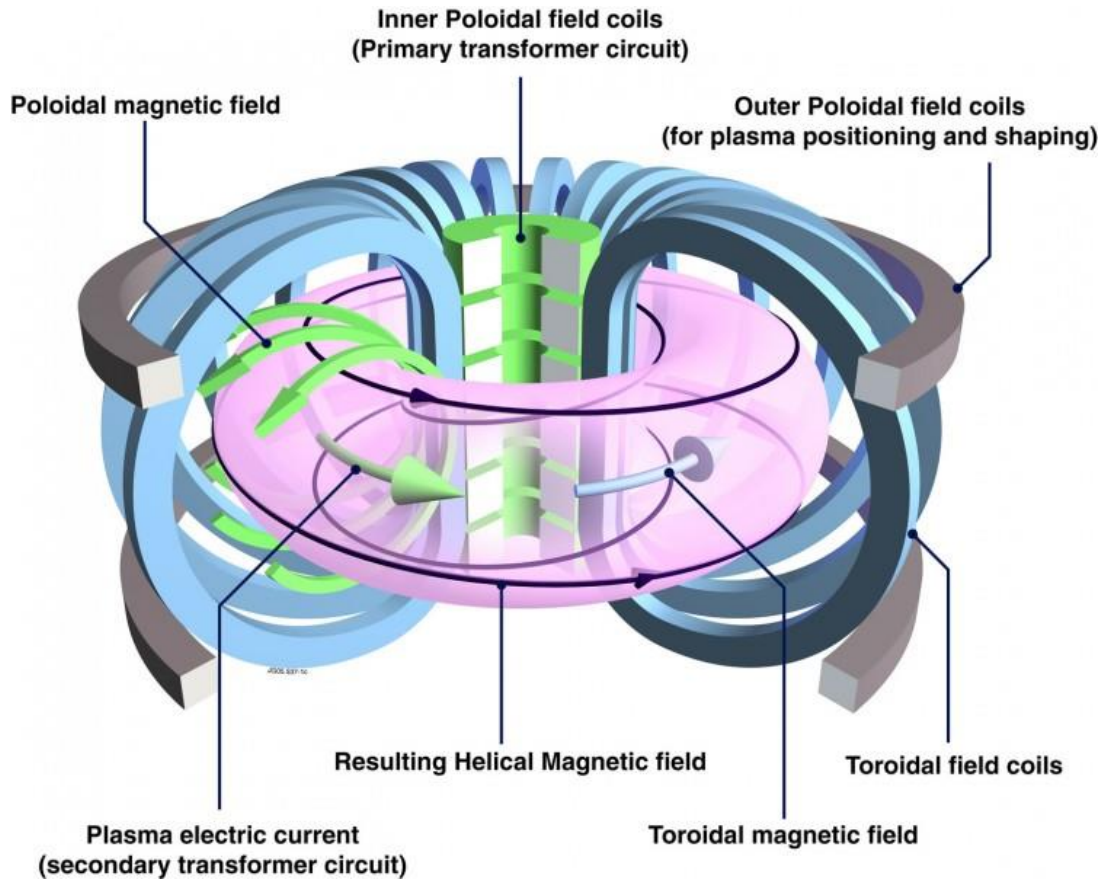




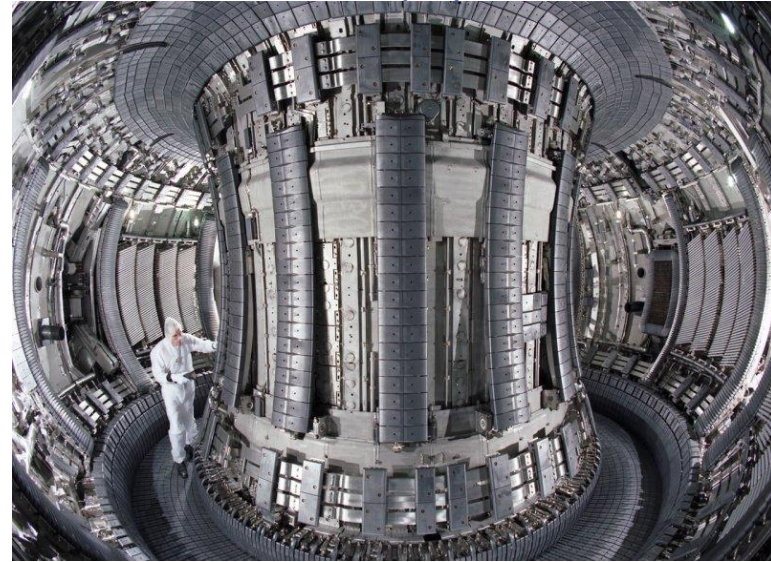


# Tokamaks

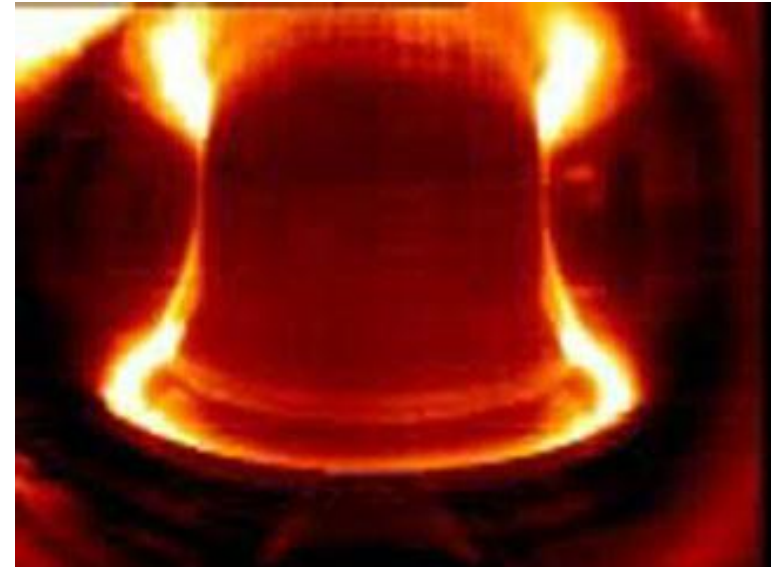
- Axisymmetric
- Helical field lines confine plasma



*JET (UK)*

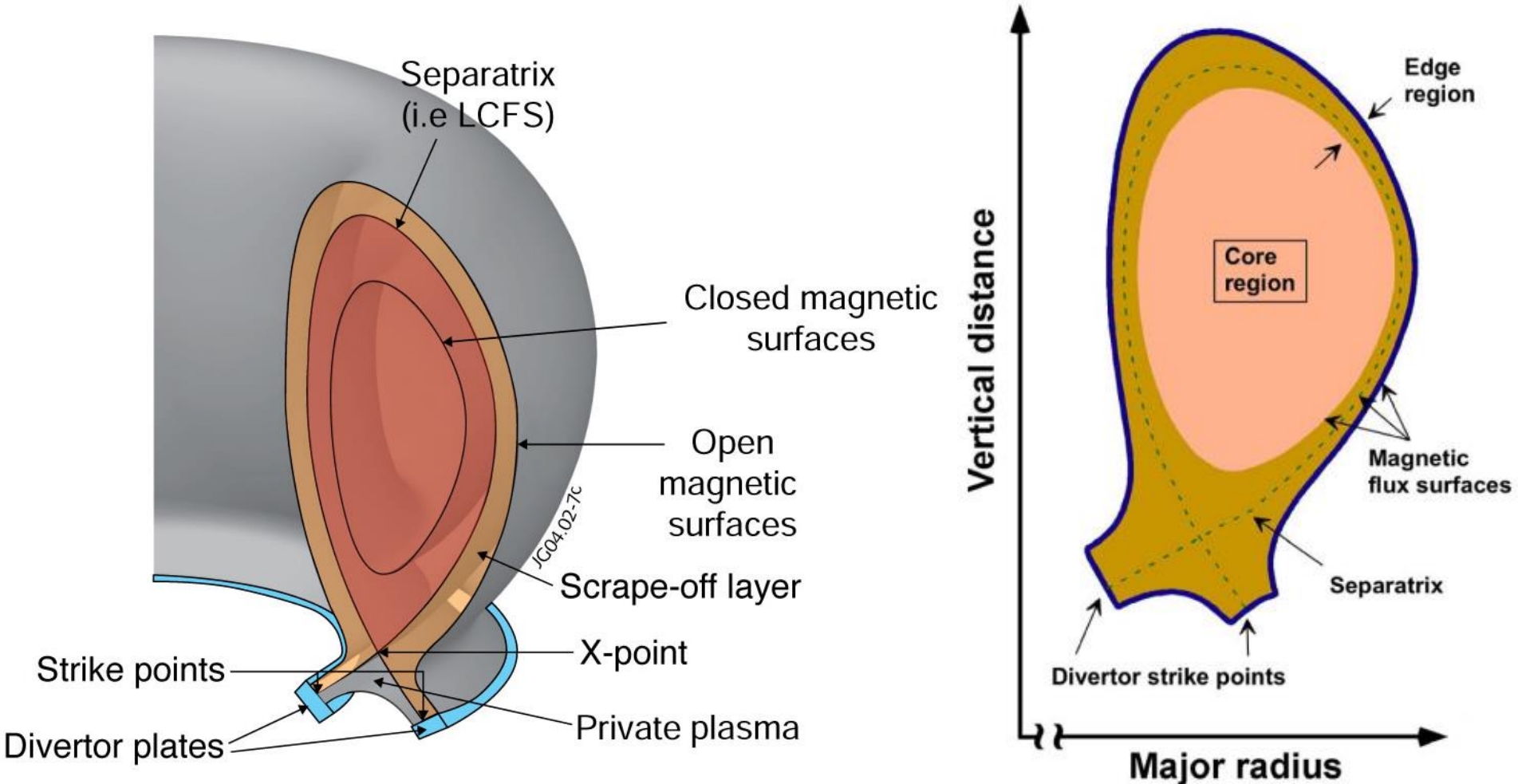


*Alcator C-Mod (MIT)*



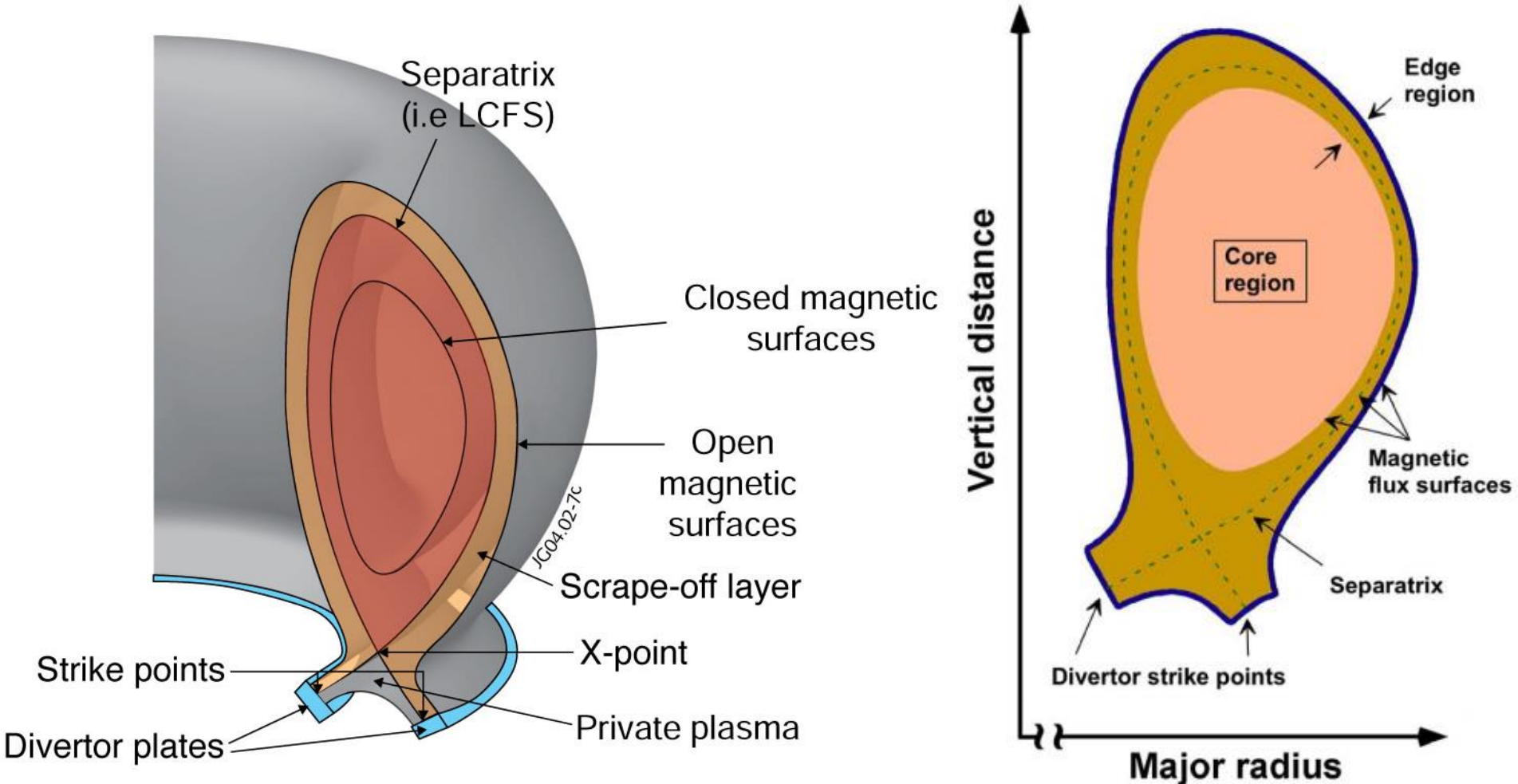
# Going to refer to different spatial regions in the tokamaks

- Especially **core** (~100% ionized), **edge** (just inside separatrix), and **scrape-off layer** (SOL, just outside separatrix)



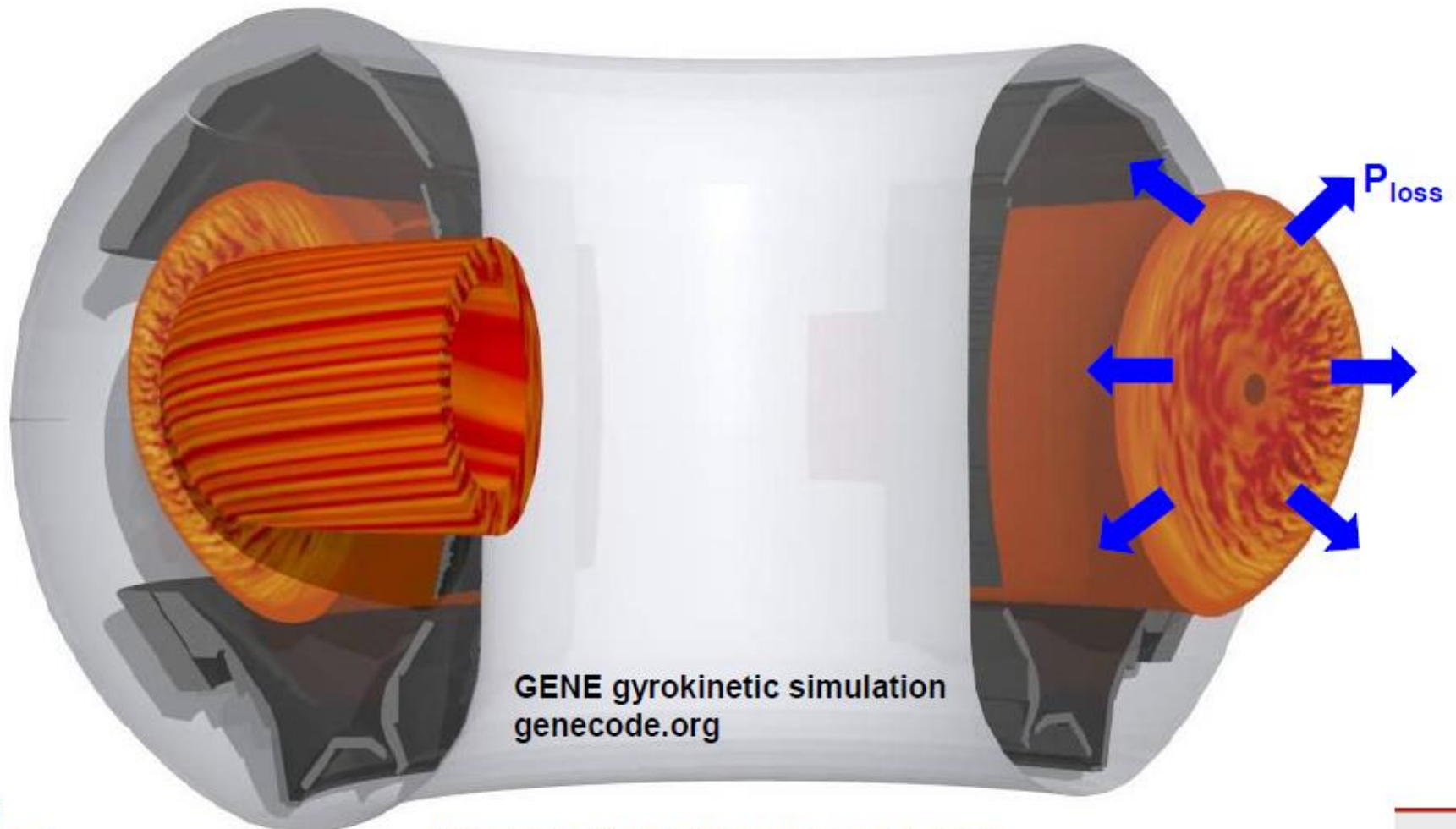
# Going to refer to different spatial regions in the tokamaks

- Especially **core** (~100% ionized), **edge** (just inside separatrix), and **scrape-off layer** (SOL, just outside separatrix)



# Increasing gradients eventually cause small scale micro-instability $\rightarrow$ turbulence

- Quasi-2D dynamics: small perpendicular scales ( $L_{\perp} \sim \rho_i$ ), elongated along field lines
- Small amplitude ( $\delta n/n < 1\%$ ), **still effective at transport, limiting  $\tau_E = 3nT/P_{\text{loss}}$**



# Increasing gradients eventually cause small scale micro-instability → turbulence

- Quasi-2D dynamics: small perpendicular scales ( $L_{\perp} \sim \rho_i$ ), elongated along field lines
- Small amplitude ( $\delta n/n < 1\%$ ), **still effective at transport, limiting  $\tau_E = 3nT/P_{\text{loss}}$**

- Turbulence measurements in ~100 Million C plasma will always be challenging and incomplete
- I'm going to show a lot of results from gyrokinetic turbulence simulations, as they help develop the physics basis to explain and predict
- Such simulations are being used more frequently to predict first and guide experiments

loss



GENE gyrokinetic simulation  
genecode.org

# GENERAL **CORE** TURBULENCE CHARACTERISTICS



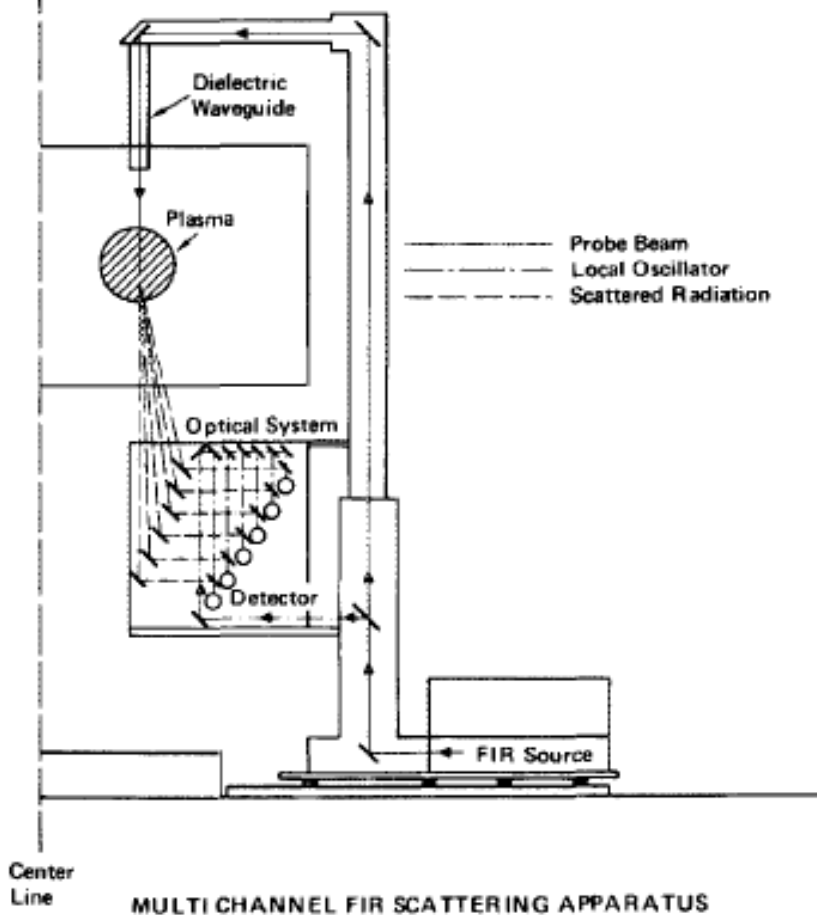
# 40+ years of theory predicts turbulence in magnetized plasma should often be drift wave in nature

## General predicted drift wave characteristics

- Fluctuations in EM fields ( $\phi$ ,  $B$ ) and fluid quantities ( $n, v, T$ ) (although really kinetic at high temperature/low collisionality)
- Finite-frequency drifting waves,  $\omega(k_\theta) \sim \omega_* \sim (k_\theta \rho) v_T / L$ 
  - Can propagate in ion or electron diamagnetic direction, depending on conditions/dominant gradients
- Perpendicular sizes linked to local gyroradius,  $L_\perp \sim \rho_{i,e}$  or  $k_\perp \rho_{i,e} \sim 1$
- Correlation times linked to acoustic velocity,  $\tau_{cor} \sim c_s / R$
- Quasi-2D, elongated along the field lines ( $L_\parallel \gg L_\perp$ ,  $k_\parallel \ll k_\perp$ )
  - Particles can rapidly move along field lines to smooth out perturbations
- in a tokamak expected to be “ballooning”, i.e. stronger on outboard side
  - Due to “bad curvature”/“effective gravity” pointing outwards from symmetry axis
  - Often only measured at one location (e.g. outboard midplane)

# Microwave & far-infrared (FIR) scattering used extensively for density fluctuation measurements

Park, RSI (1985)



- Geometry and frequency determine measurable  $\omega$ ,  $k$

$$\omega_{\text{meas}} = \omega_{\text{scat}} - \omega_{\text{incident}}$$

$$k_{\text{meas}} = k_{\text{scat}} - k_{\text{incident}}$$

- Can be configured for forward scattering, backscattering, reflectometry, ...

FIG. 1. Scannable multichannel FIR scattering apparatus employed on the TEXT tokamak.

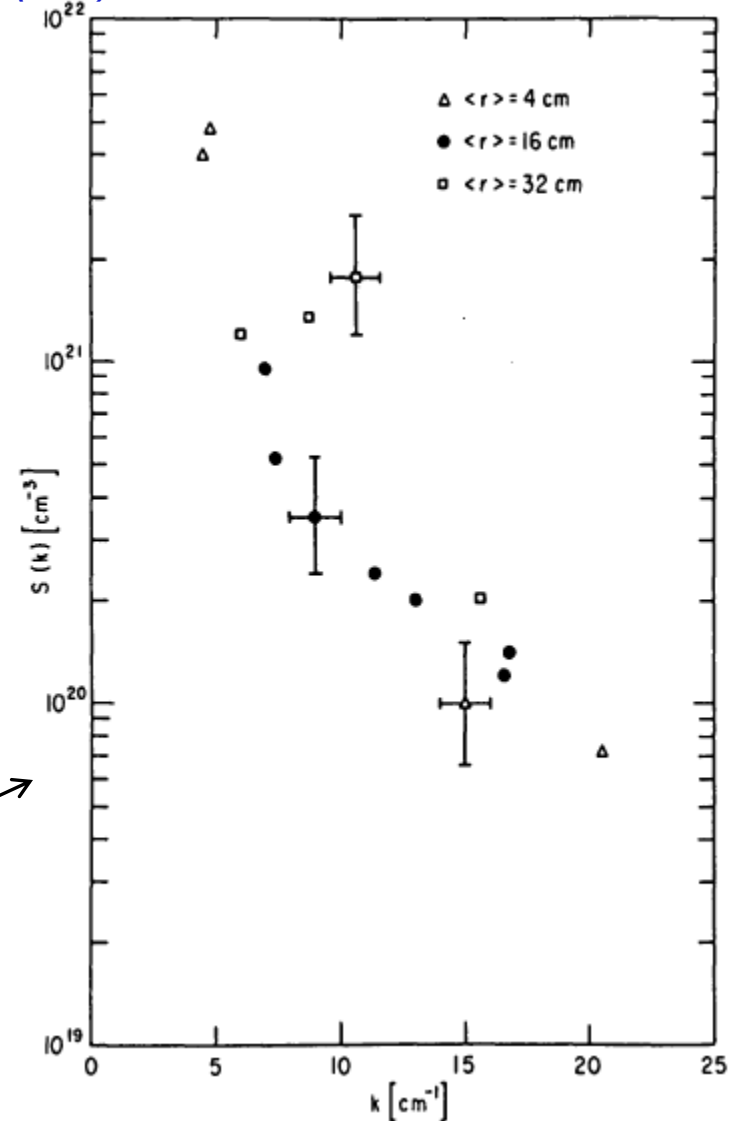
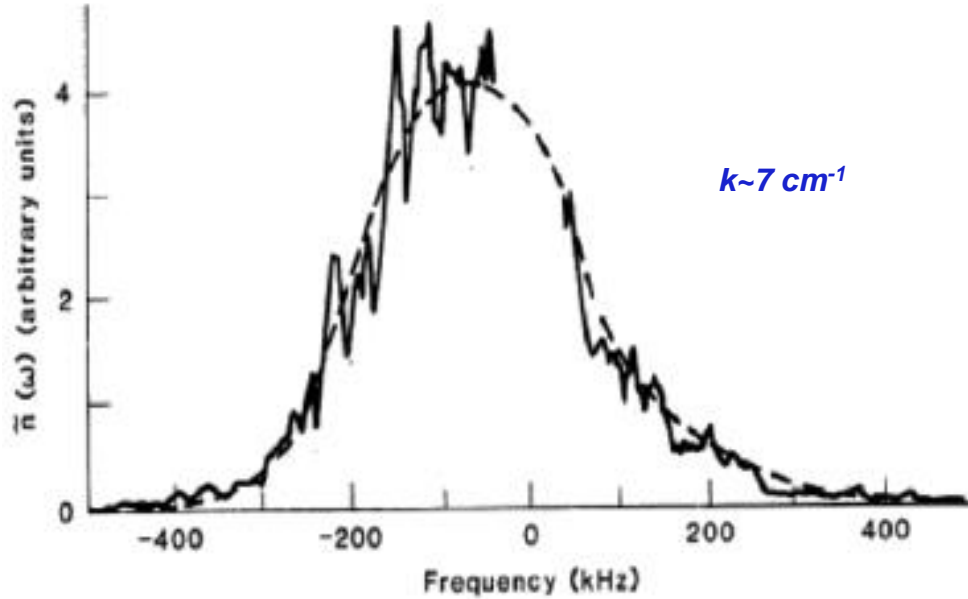
# Broad frequency spectra measured for given scattering wavenumber

Mazzucato, PRL (1982)

Surko & Slusher, Science (1983)

Princeton Large Torus (PLT)

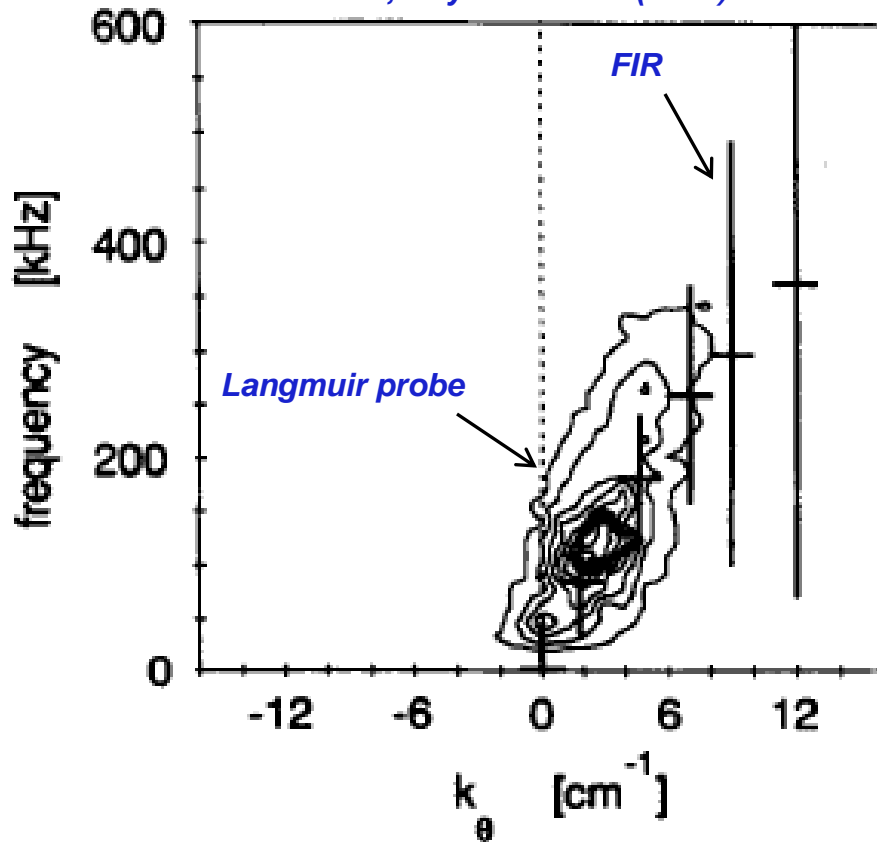
$k \sim 7 \text{ cm}^{-1}$



- Different scattering angles measure different  $k$ , observe spectral decay in wavenumber

# Broad drift wave turbulent spectrum verified simultaneously with Langmuir probes and FIR scattering

TEXT, Ritz, Nuclear Fusion (1987)  
Wooton, Phys. Fluids B (1990)



- Illustrates drift wave dispersion
- However, real frequency almost always dominated by Doppler shift

$$\omega_{\text{lab}} = \omega_{\text{mode}}(k_{\theta}) + k_{\theta} v_{\text{doppler}}$$

- Often challenging to determine mode frequency (in plasma frame) within uncertainties

FIG. 1. The  $S(k_{\theta}, \omega)$  spectrum at  $r = 0.255$  m in TEXT, from Langmuir probes (contours) and FIR scattering (bars indicate FWHM).

# Small normalized fluctuations in core ( $\leq 1\%$ ) increasing to the edge

- Combination of diagnostics used to measure fluctuation amplitudes

*ATF stellarator, Hanson, Nuclear Fusion (1992)*

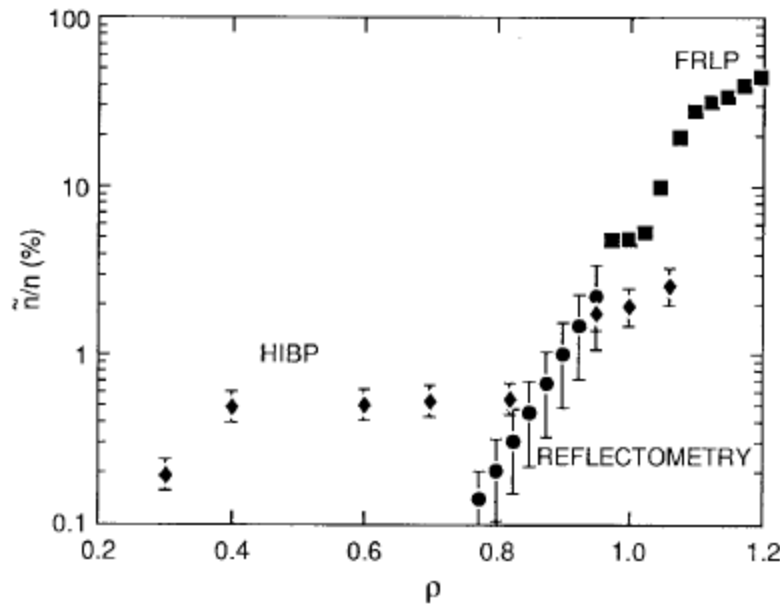


Fig. 4. Radial profile of density fluctuations (in %) in ATF stellarator obtained by combining results from different diagnostics [177].

- Measurements also often show  $\delta n/n_0 \sim \delta\phi/T_0$  (within factor  $\sim 2$ ), expected for

*TEXT tokamak, Wooton, PoFB (1990)*

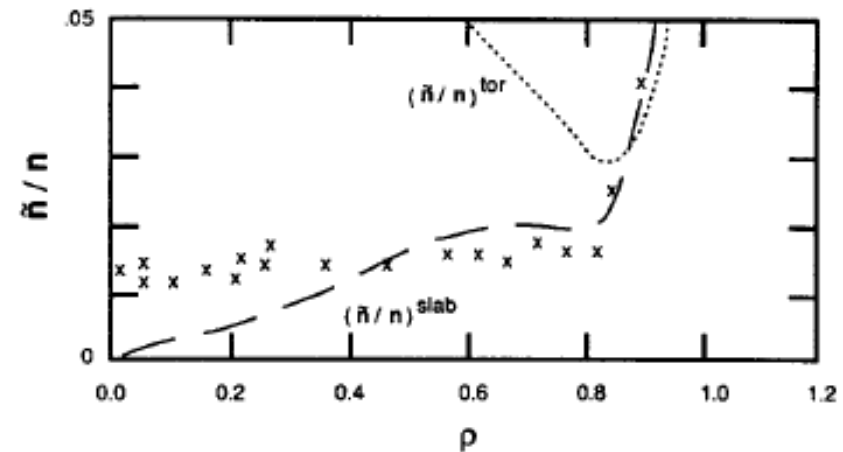


FIG. 6. The spatial variation of  $\tilde{n}/n$  from TEXT ( $B_0 = 2$  T,  $I_p = 200$  kA,  $\bar{n}_e = 2$  to  $3 \times 10^{19} \text{ m}^{-3}$ ,  $\text{H}^+$ ), shown as crosses (HIBP). Also shown are the predictions of two mixing length estimates,  $(\tilde{n}/n)^{\text{tor}}$  and  $(\tilde{n}/n)^{\text{slab}}$ . Both electron feature  $\tilde{n}/n$  and  $k_\theta$  ( $\bar{k}_\theta \rho_e = 0.1$ ) are interpreted assuming no ion feature is present.

# Mixing length estimate for fluctuation amplitude

- In the presence of an equilibrium gradient,  $\nabla n_0$ , turbulence with radial correlation  $L_r$  will mix regions of high and low density

$$\delta n \approx \nabla n_0 \cdot L_r$$

$$\frac{\delta n}{n_0} \approx \frac{\nabla n_0}{n_0} \cdot L_r \approx \frac{L_r}{L_n} \quad (1/L_n = \nabla n_0 / n_0)$$

$$\frac{\delta n}{n_0} \sim \frac{1}{k_{\perp} L_n} \sim \frac{\rho_s}{L_n} \quad (k_{\perp}^{-1} \sim L_r; k_{\perp} \rho_s \sim \text{const } t)$$

- Another interpretation: local, instantaneous gradient limited to equilibrium gradient

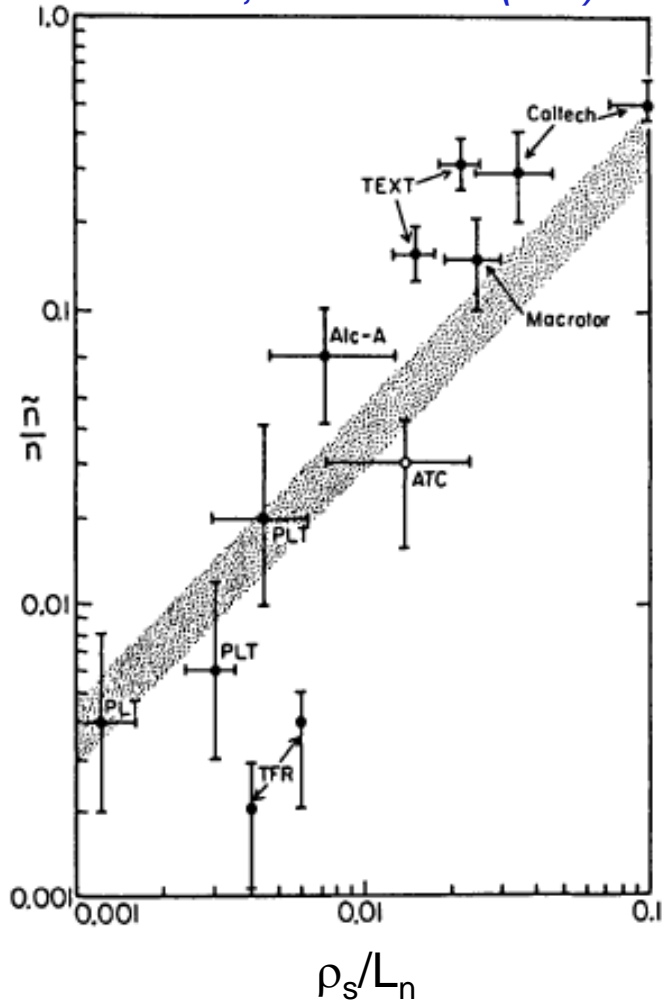
$$\nabla \tilde{n} \approx \nabla n_0$$

$$k_r \tilde{n} \approx \nabla n_0$$

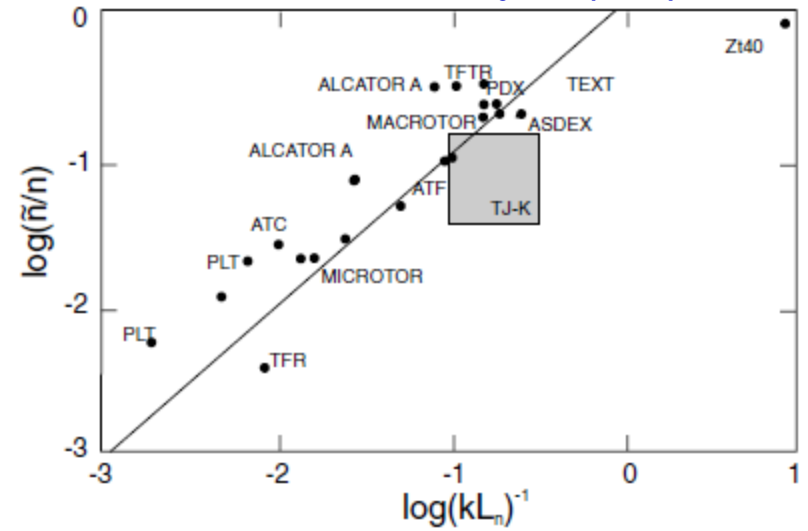
**IF turbulence scale length linked to  $\rho_s$ , would loosely expect  $\delta n/n_0 \sim \rho_s/L_n$**

# Fluctuation intensity across machines loosely scales with mixing length estimate, reinforces local $\rho_s$ drift nature

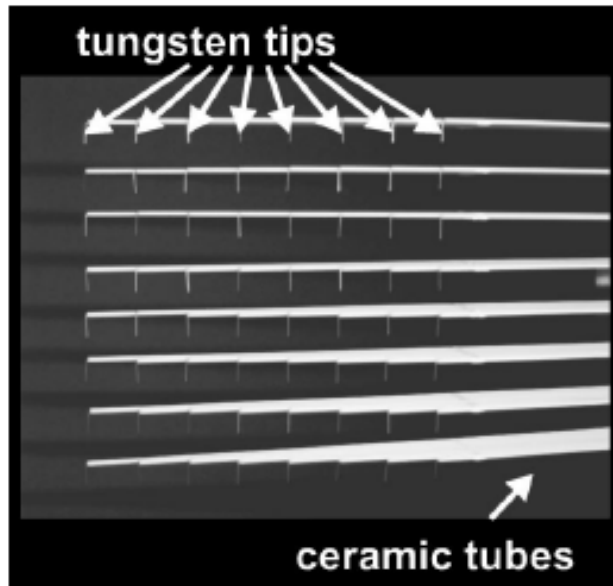
Liewer, Nuclear Fusion (1985)



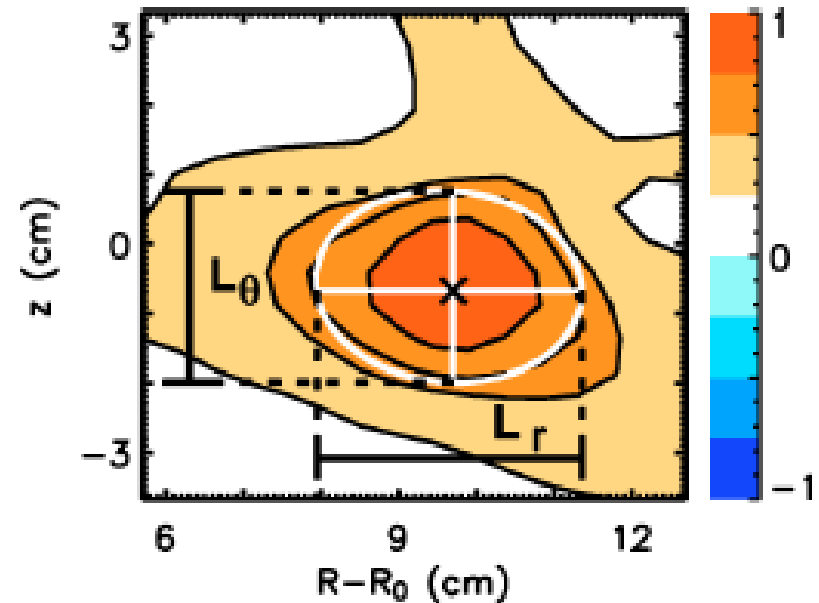
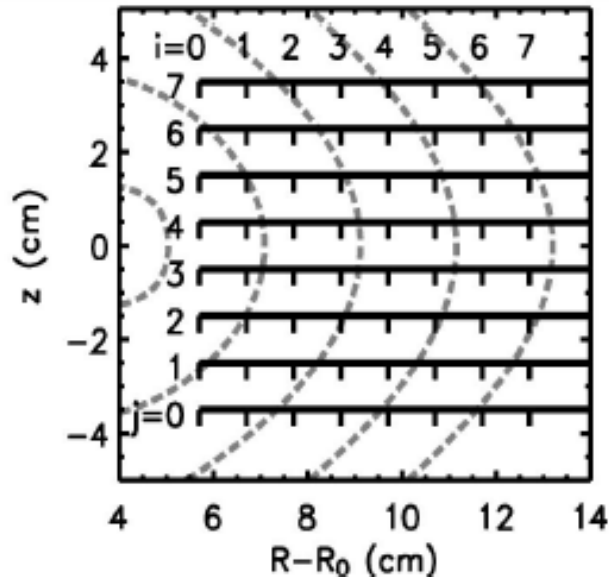
Lechte, New J. of Physics (2002)



# 2D Langmuir probe array in TJ-K stellarator used to directly measure spatial and temporal structures



- Simultaneously acquiring 64 time signals – can directly calculate 2D correlation, with time
- Caveat – relatively cool ( $T \sim 10$  eV) compared to fusion performance plasmas ( $T \sim 10$  keV)



TJ-K [Ramisch, PoP (2005)]

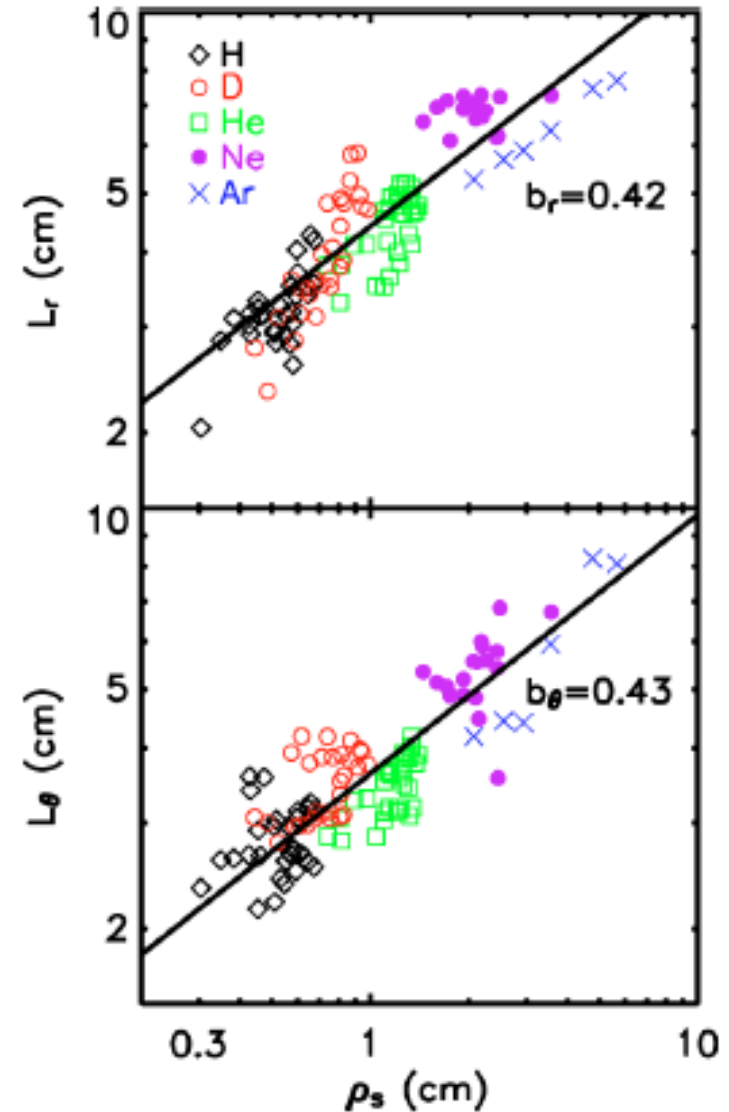


# Radial and poloidal correlation lengths scale with $\rho_s$ reinforcing drift wave nature

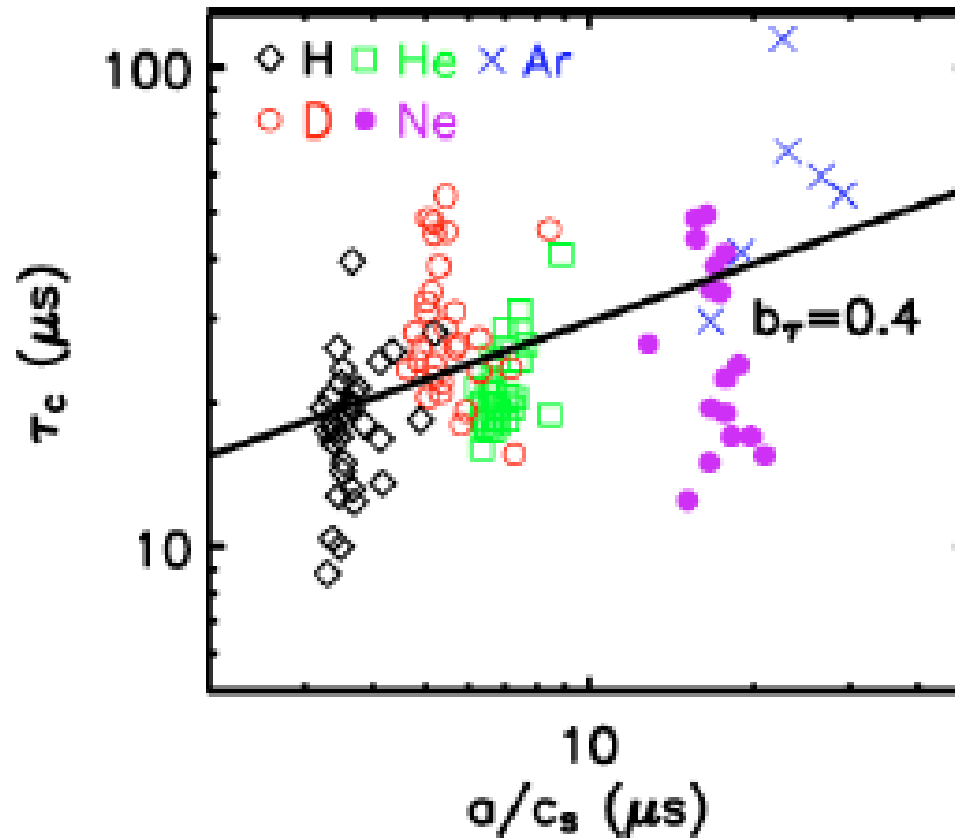
*TJ-K [Ramisch, PoP (2005)]*

- Turbulence close to isotropic

$$L_r \sim L_\theta$$

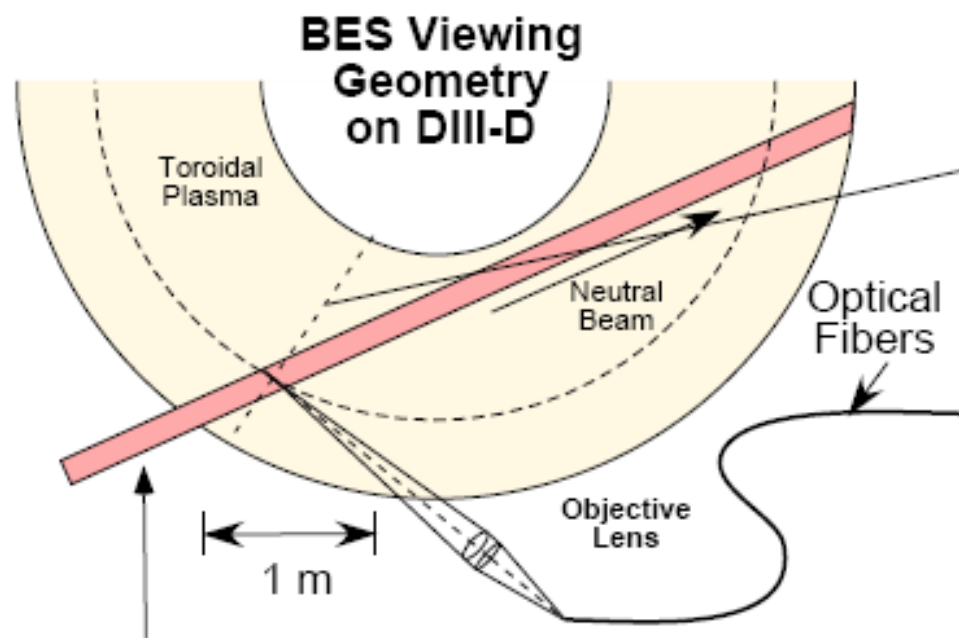
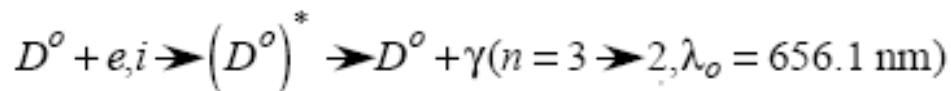


# Temporal scales loosely correlated with acoustic times $c_s/a$



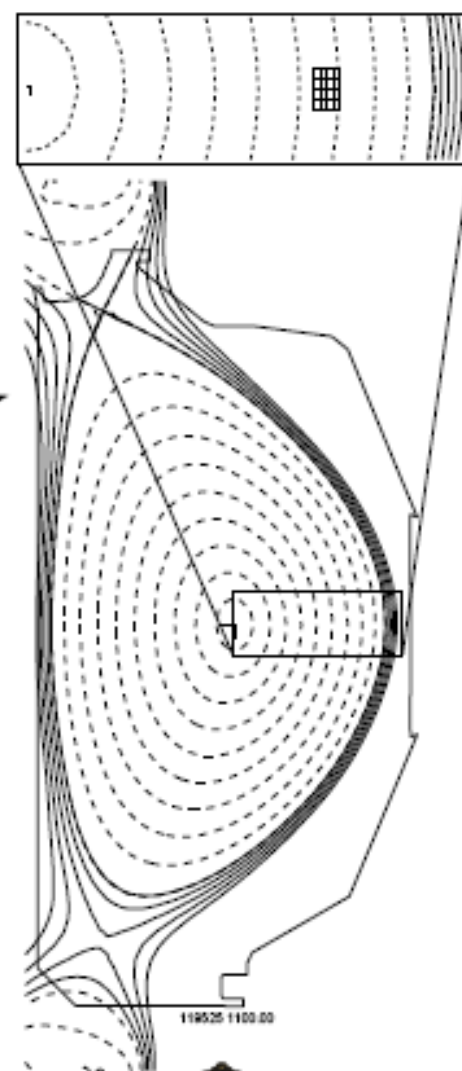
# BEAM EMISSION SPECTROSCOPY MEASUREMENT OF LOCALIZED, LONG-WAVELENGTH ( $k_{\perp}\rho_i < 1$ ) DENSITY FLUCTUATIONS

Collisionally-excited, Doppler-shifted neutral beam fluorescence



75 KeV  $D^0$  Neutral Beam  
(150 L (R))

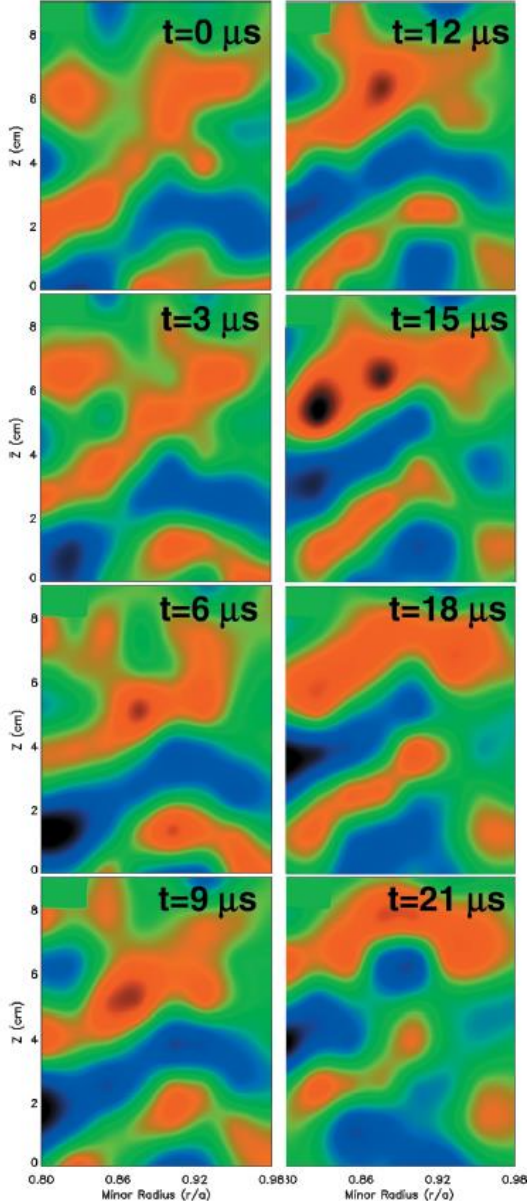
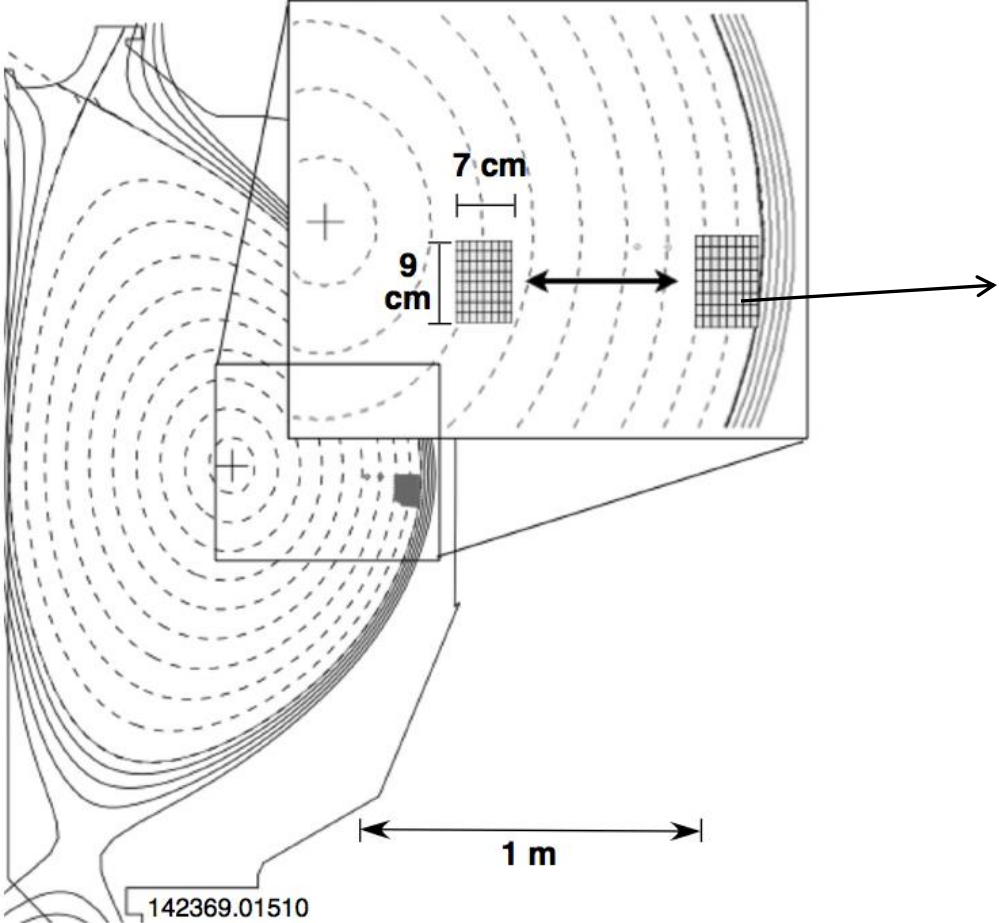
$$\frac{\tilde{I}}{I} \mu \frac{\tilde{n}}{n}$$



# Spectroscopic imaging provides a 2D picture of turbulence in hot tokamak core: cm spatial scales, $\mu\text{s}$ time scales

- Utilize interaction of neutral atoms with charged particles to measure density

DIII-D tokamak (General Atomics)



Movies at: <https://fusion.gat.com/global/BESMovies>

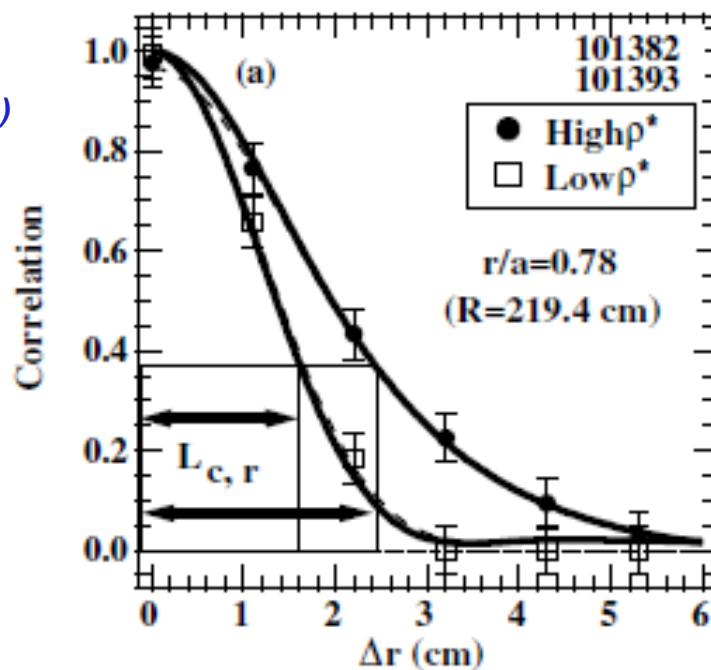
## BES videos

<https://fusion.gat.com/global/BESMovies>

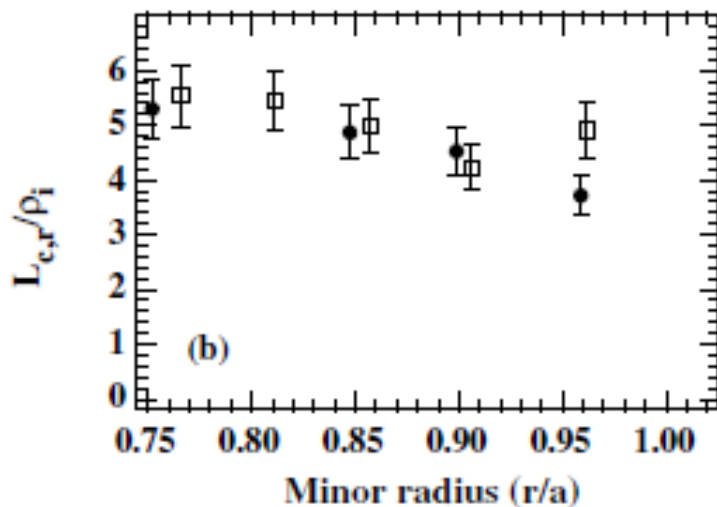
(University of Wisconsin; General Atomics)

# Radial and poloidal correlation lengths scale with $\rho_s$ in core imaging, reinforcing local drift wave nature

DIII-D  
Mckee, Nucl. Fusion (2001)



- Correlation length increases with local gyroradius  $\rho$  ( $\rho_* = \rho/a$ )

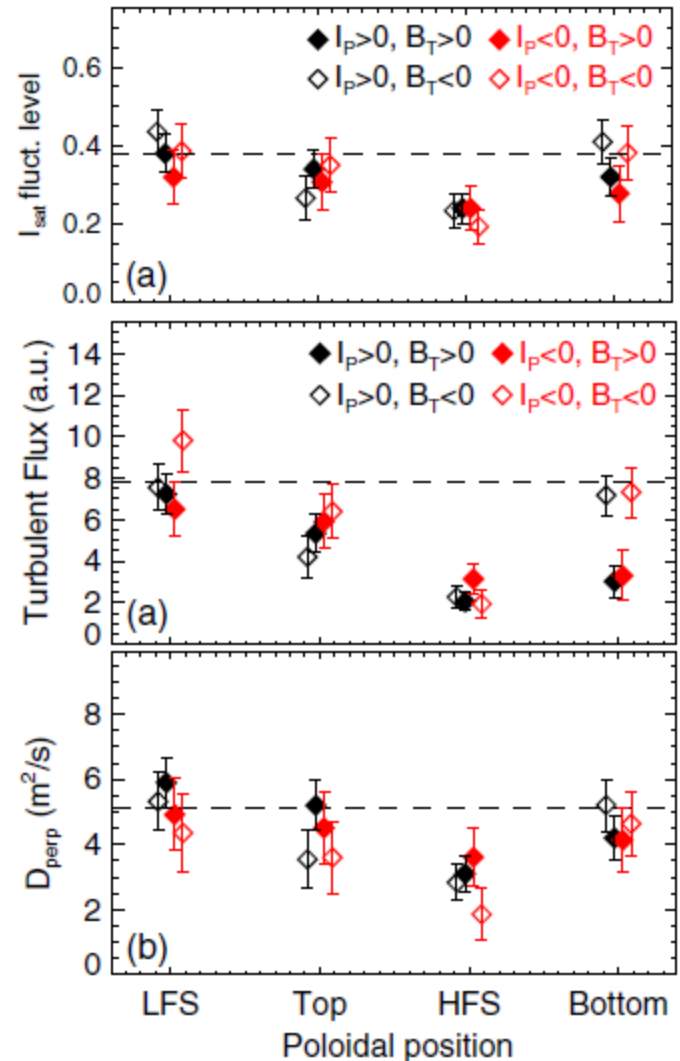
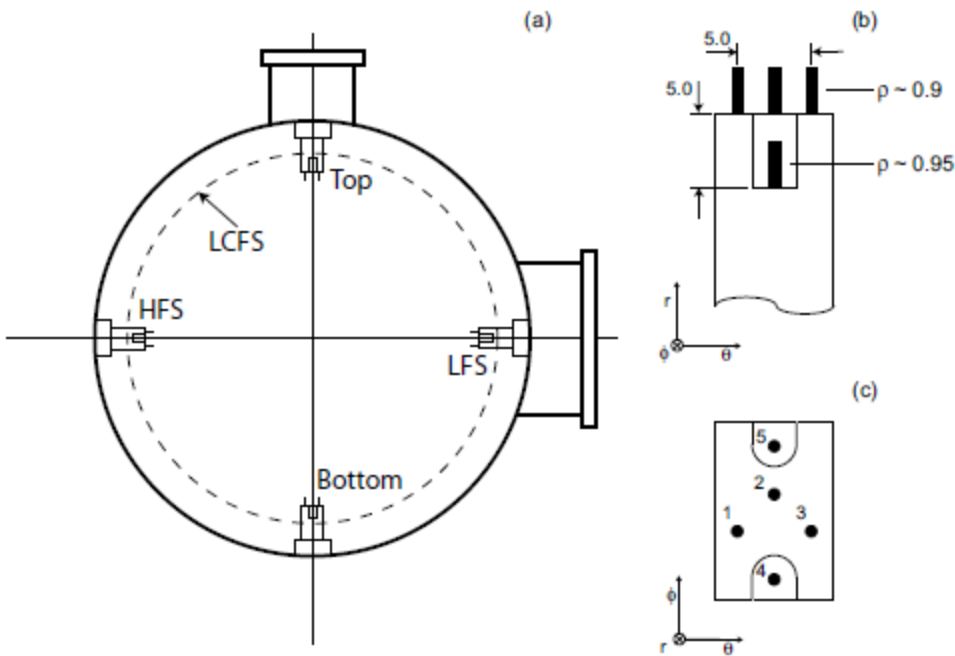


- Ratio of  $L_r/\rho$  relatively constant in radius, for the two different  $\rho_*$  discharges

# Example of stronger turbulence measured on outboard side, “ballooning” in nature

- Consistent with bad curvature drive

ISSTOK [Silva, PPCF (2011)]



## Evidence for quasi-2D ( $L_{\parallel} \gg L_{\perp}$ )

- Assume an exponential or Gaussian correlation function  

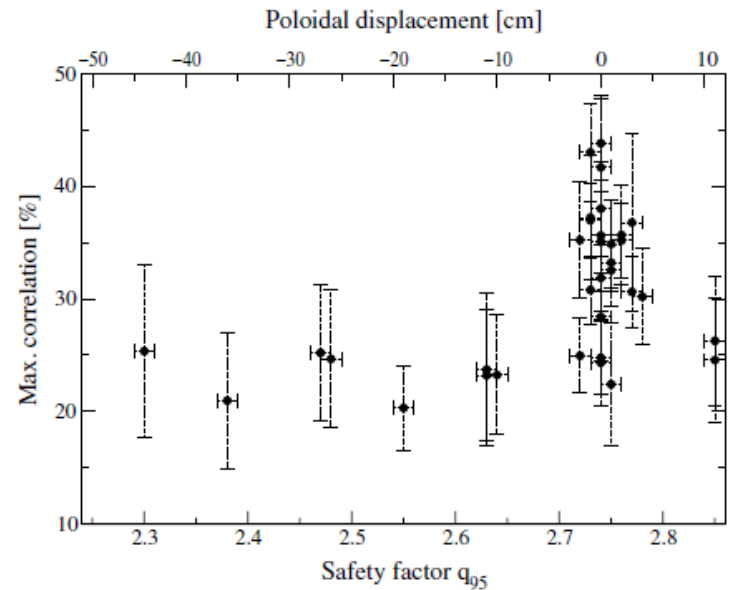
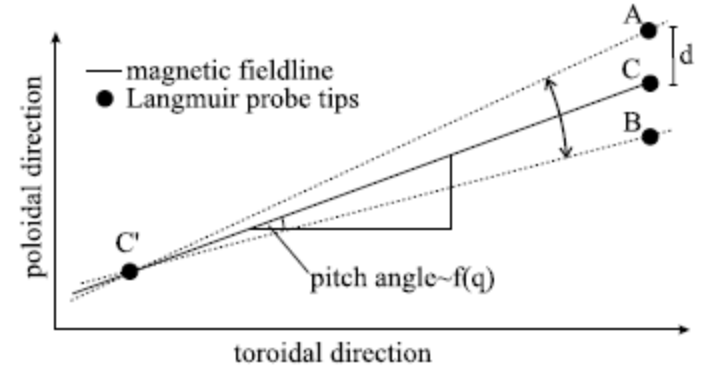
$$C(\Delta_{\perp}, \Delta_{\parallel}) \approx \exp(-\Delta_{\perp} / L_{\perp}) \exp(-\Delta_{\parallel} / L_{\parallel})$$
- Measure correlation between two probes “on the same field line” ( $\Delta_{\perp} \approx 0$ ) separated a large distance  $\Delta_{\parallel} \gg 0$

JET edge plasma

$L_{\parallel} \sim$  many meters

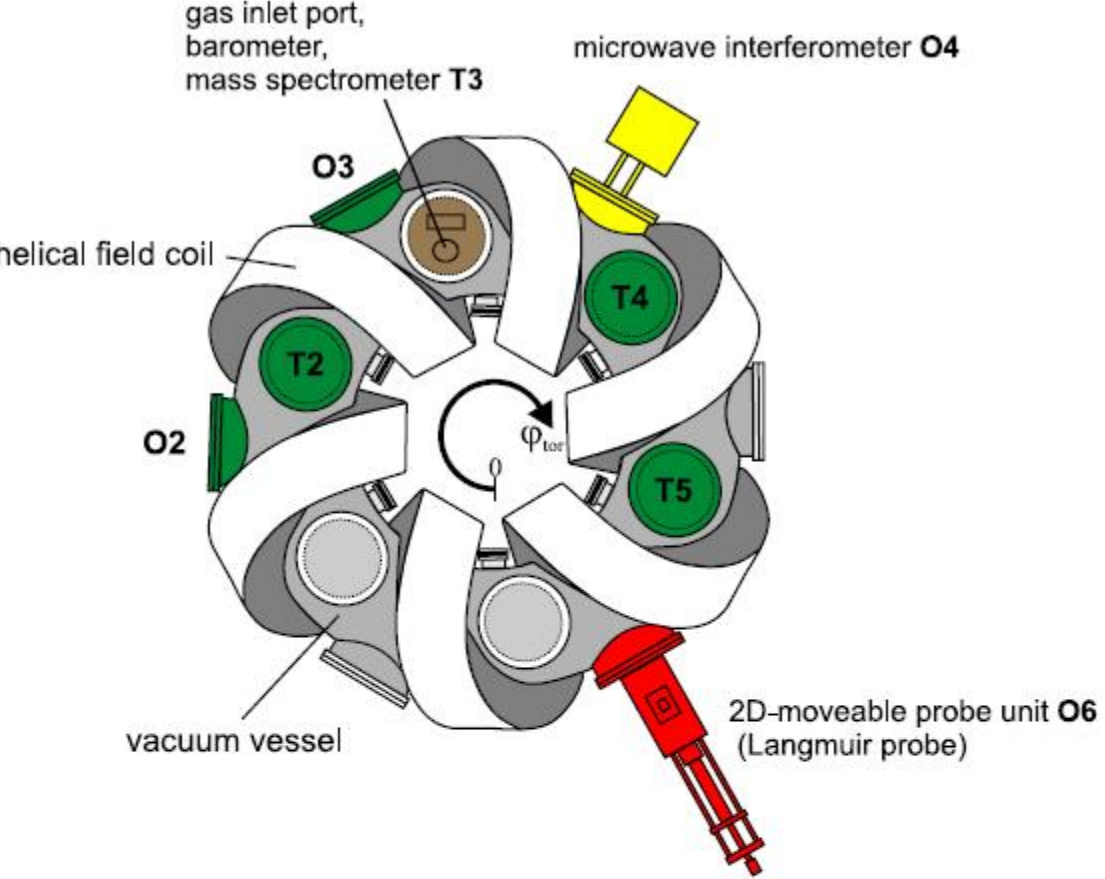
$L_{\perp} \sim$  mm-cm

JET edge [Thomsen, Contrib. Plasma Phys. (2001)]

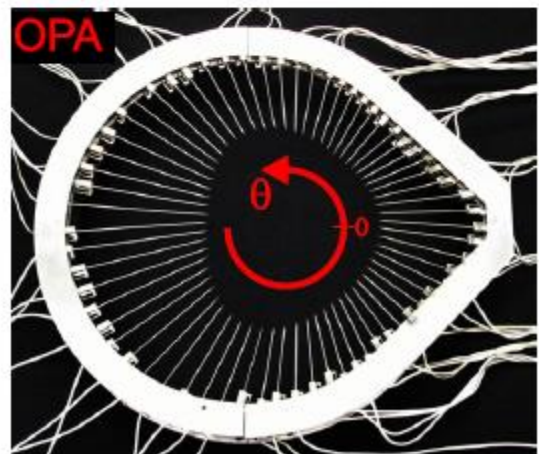
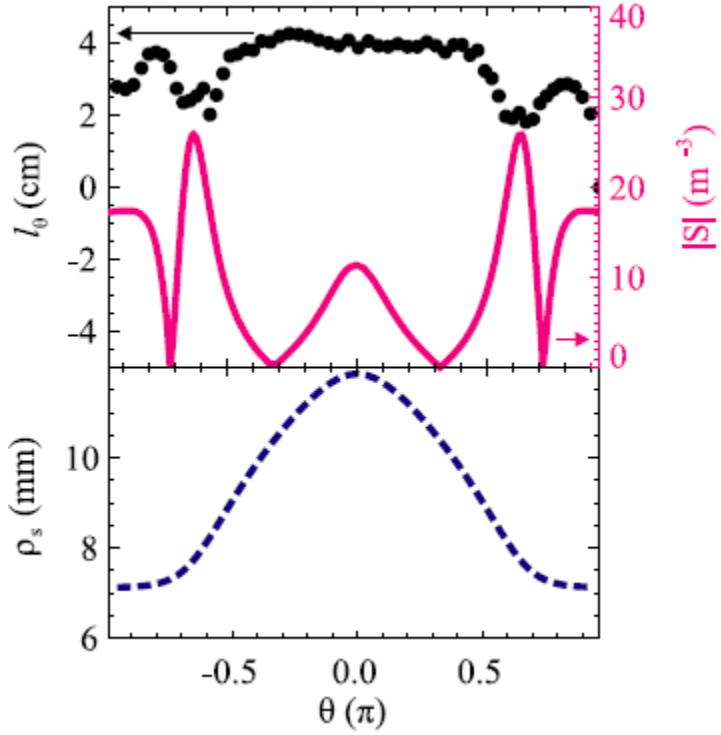
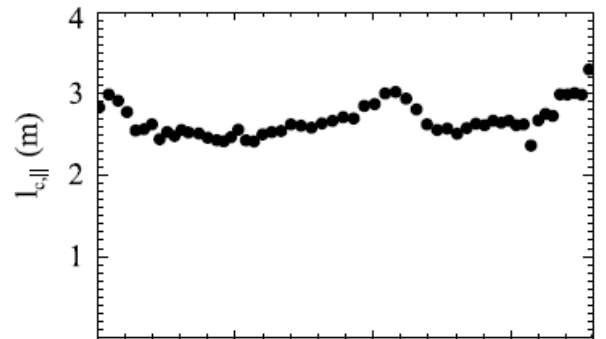




# More direct measurement in TJ-K plasmas



TJ-K [Birkenmeier, PPCF (2012)]



## General turbulence characteristics are useful for testing theory predictions, but we mostly care about transport

- Transport a result of finite average correlation between perturbed drift velocity ( $\delta v$ ) and perturbed fluid moments ( $\delta n$ ,  $\delta T$ ,  $\delta v$ )
  - Particle flux,  $\Gamma = \langle \delta v \delta n \rangle$
  - Heat flux,  $Q = 3/2 n_0 \langle \delta v \delta T \rangle + 3/2 T_0 \langle \delta v \delta n \rangle$
  - Momentum flux,  $\Pi \sim \langle \delta v \delta v \rangle$  (Reynolds stress, just like Navier Stokes)
- Electrostatic turbulence often most relevant  $\rightarrow E \times B$  drift from potential perturbations:  $\delta v_E = B \times \nabla(\delta \phi) / B^2 \sim k_\theta (\delta \phi) / B$
- Can also have magnetic contributions at high beta,  $\delta v_B \sim v_{||} (\delta B_r / B)$  (magnetic “flutter” transport – more later)

# Measuring turbulent particle and heat fluxes using Langmuir probes

- Illustrates that turbulent transport can account for inferred anomalous transport (only possible in edge region)

TEXT, Wooton, PoFB (1990)

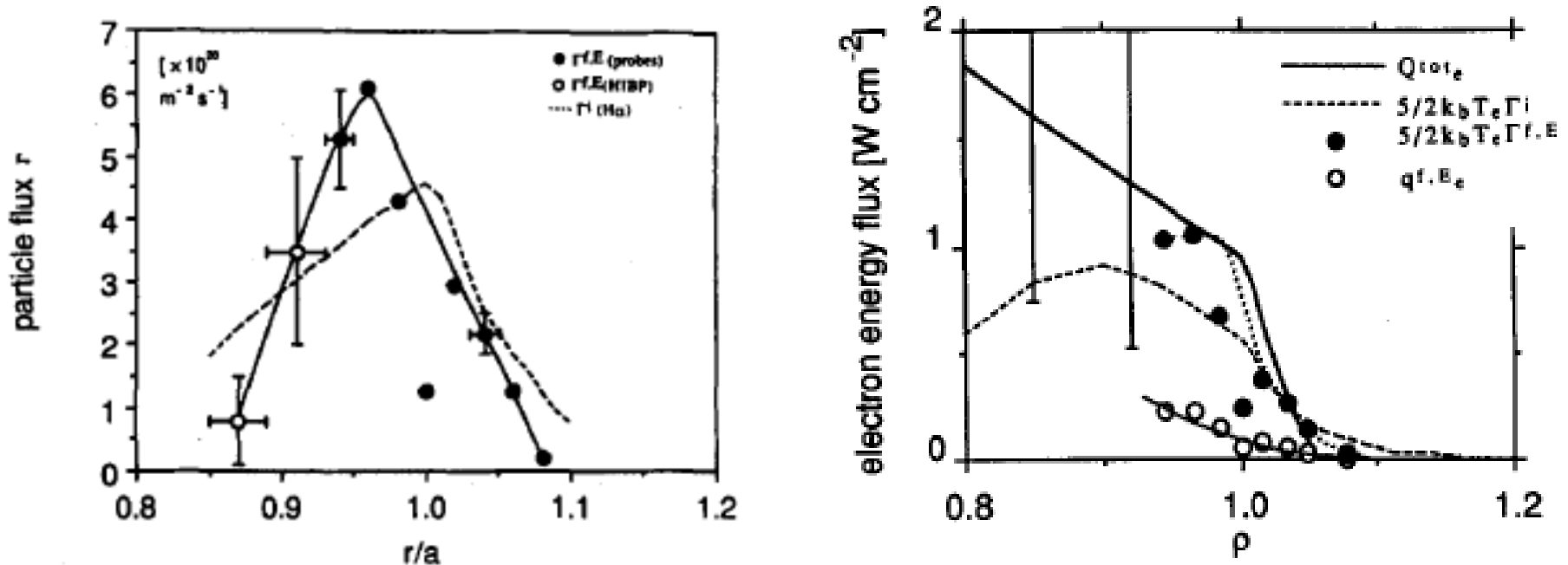


FIG. 3. A comparison of working particle fluxes in TEXT ( $B_\phi = 2 \text{ T}$ ,  $I_p = 200 \text{ kA}$ ,  $\bar{n}_e = 3 \times 10^{19} \text{ m}^{-3}$ ,  $\text{H}^+$ ), the total  $\Gamma^i$  (from  $\text{H}_\alpha$ ), and  $\Gamma^{f,E}$  driven by electrostatic turbulence.  $\Gamma^{f,E}$  is measured with Langmuir probes (solid line, solid points) and the HIBP (open points).

## Useful to Fourier decompose transport contributions, especially for theory comparisons

- E.g. particle flux from electrostatic perturbations:

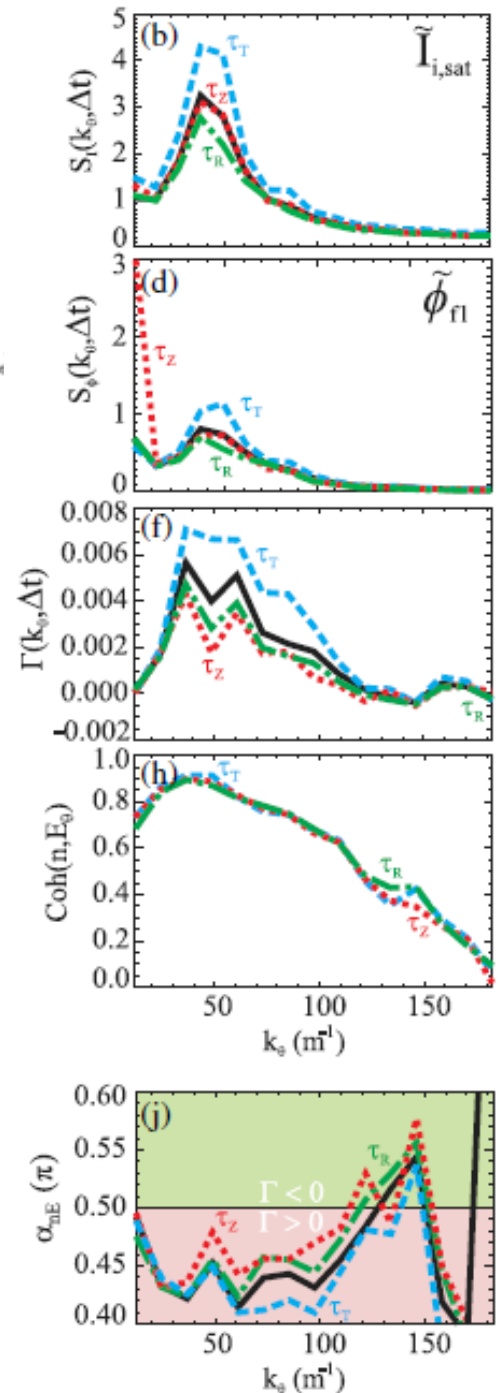
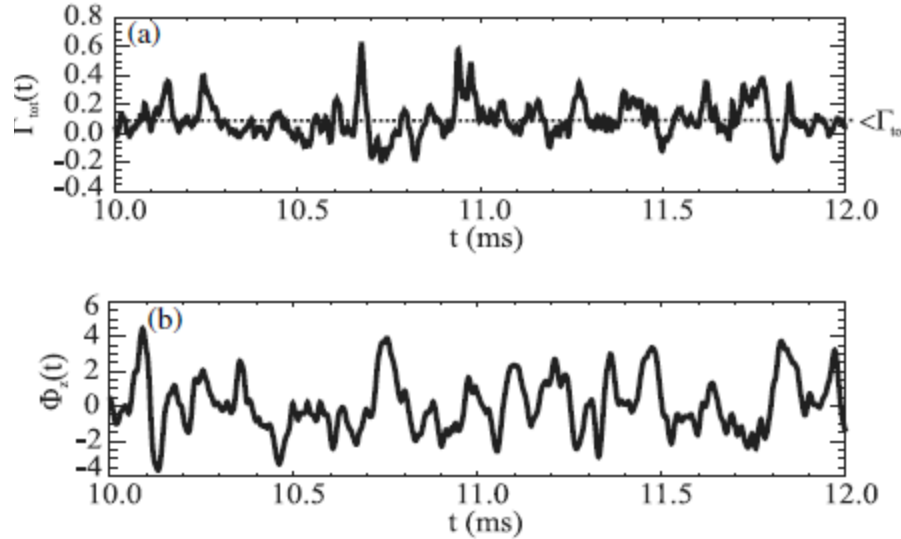
$$\Gamma(k_\theta) = \frac{nT}{B} \sum_{k_\theta} k_\theta \left| \frac{\delta n(k_\theta)}{n_e} \right| \left\| \frac{\delta \varphi(k_\theta)}{T_e} \right\| \gamma_{n\varphi}(k_\theta) \sin \alpha_{n\varphi}(k_\theta)$$

*Amplitude spectra*                      *coherence*                      *Cross phase*

- Everything is a function of wavenumber

# Edge Langmuir probe arrays used to decompose turbulent fluxes in $k_\theta$

TJ-K [Birkenmeier, PPCF (2012)]



- Very rare to measure this comprehensively!
- Useful for challenging theory calculations
- Yet to be done this thoroughly for hot tokamak core, where comprehensive gyrokinetic simulations available for comparison

# Beyond general characteristics, there are many theoretical “flavors” of drift waves possible in tokamak core & edge

- Usually think of drift waves as gradient driven ( $\nabla T_i$ ,  $\nabla T_e$ ,  $\nabla n$ )
  - Often exhibit threshold in one or more of these parameters
- Different theoretical “flavors” exhibit different parametric dependencies, predicted in various limits, depending on gradients,  $T_e/T_i$ ,  $\nu$ ,  $\beta$ , geometry, location in plasma...
  - Electrostatic, ion scale ( $k_\theta \rho_i \leq 1$ )
    - Ion temperature gradient (ITG) – driven by  $\nabla T_i$ , weakened by  $\nabla n$
    - Trapped electron mode (TEM) – driven by  $\nabla T_e$  &  $\nabla n_e$ , weakened by  $\nu_e$
  - Electrostatic, electron scale ( $k_\theta \rho_e \leq 1$ )
    - Electron temperature gradient (ETG) - driven by  $\nabla T_e$ , weakened by  $\nabla n$
  - Electromagnetic, ion scale ( $k_\theta \rho_i \leq 1$ )
    - Kinetic ballooning mode (KBM) - driven by  $\nabla \beta_{pol}$
    - Microtearing mode (MTM) – driven by  $\nabla T_e$ , at sufficient  $\beta_e$

# Challenging to definitively identify a particular theoretical turbulent transport mechanism

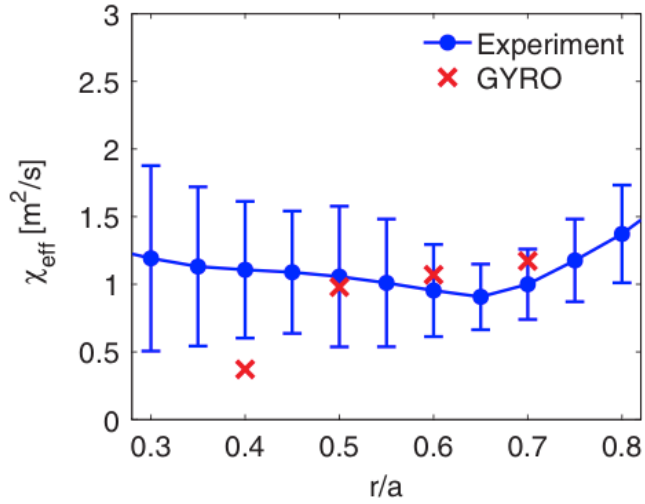
- Best we can do:
  - Measure as many turbulence quantities as possible (amplitude spectra, cross-phases, transport)
  - Compare with theory (simulation) predictions
  - Scaling equilibrium parameters to investigate trends/sensitivities

# CORE ION SCALE TURBULENCE

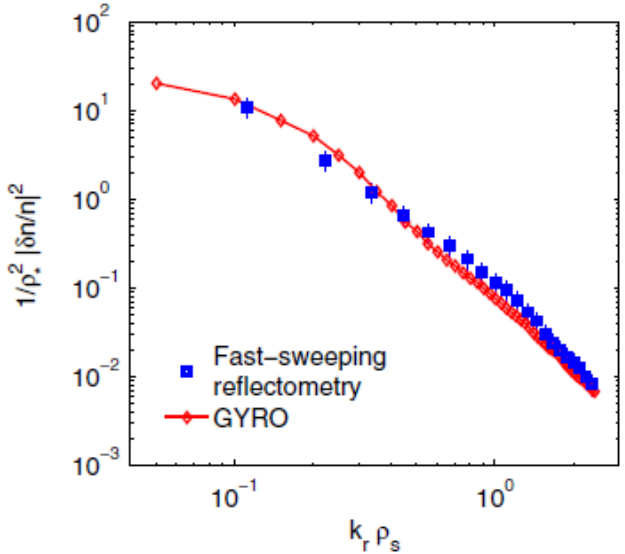
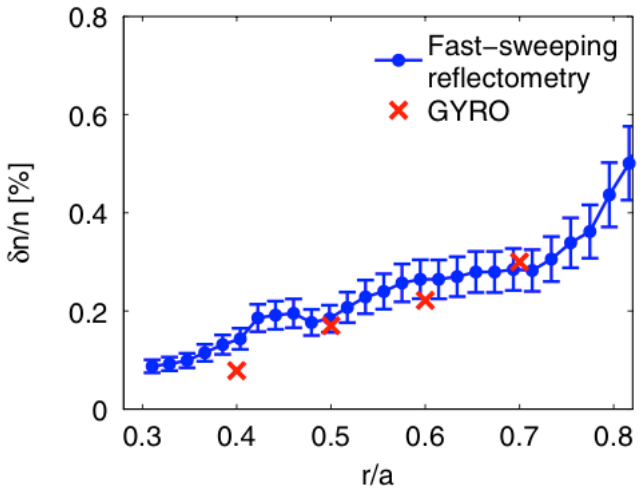
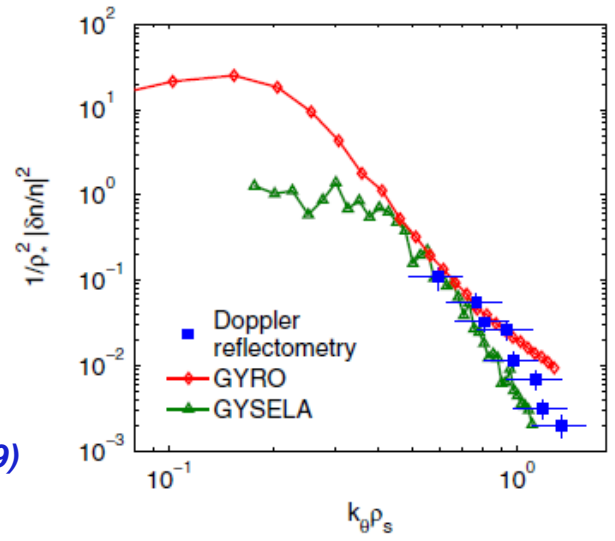


# Transport, density fluctuation amplitude (from reflectometry) and spectral characteristics all consistent with nonlinear ITG simulations in Tore Supra

- Provides confidence in interpretation of transport in conditions when ITG instability/turbulence predicted to be most important

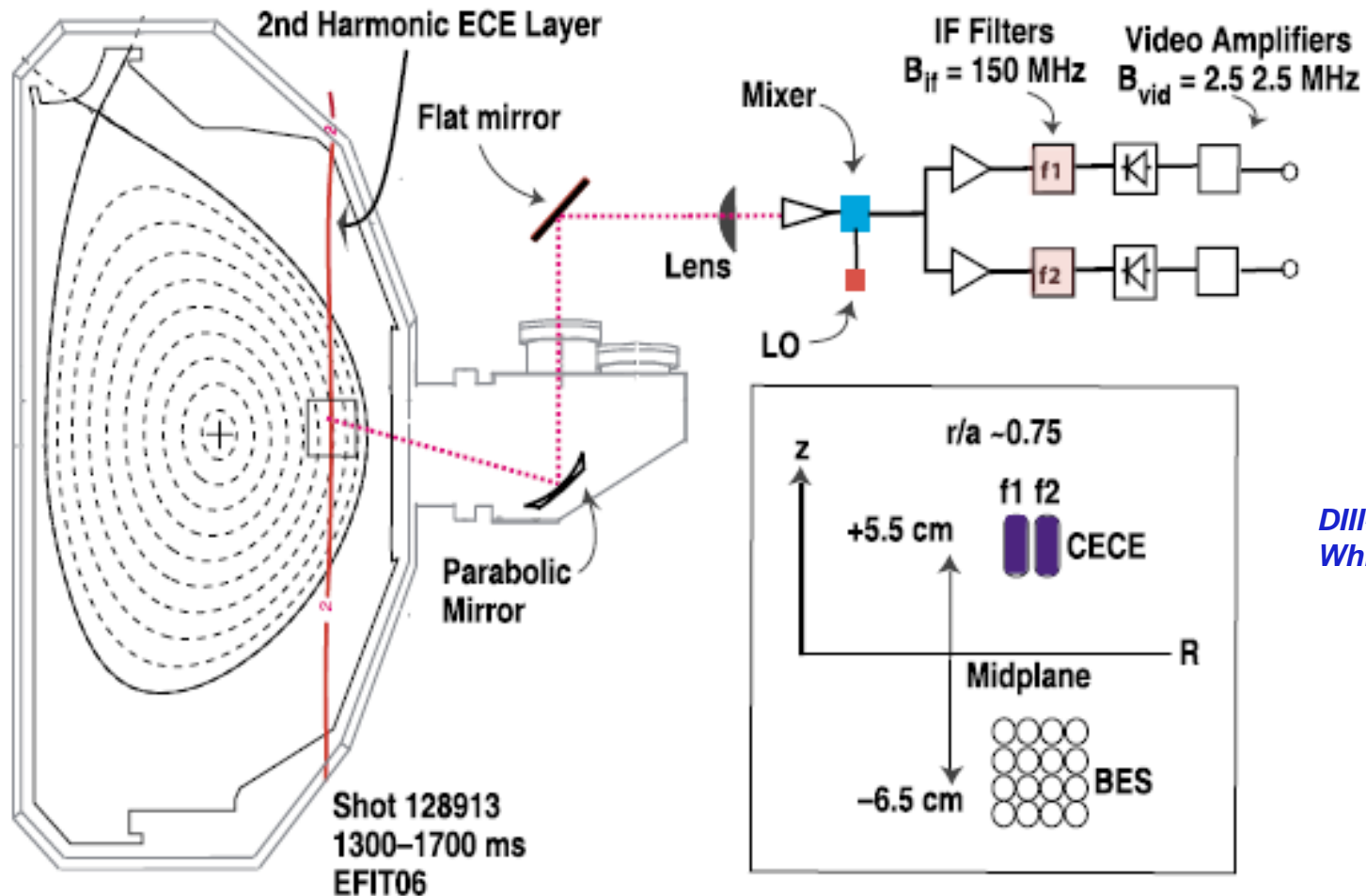


Casati, PRL (2009)



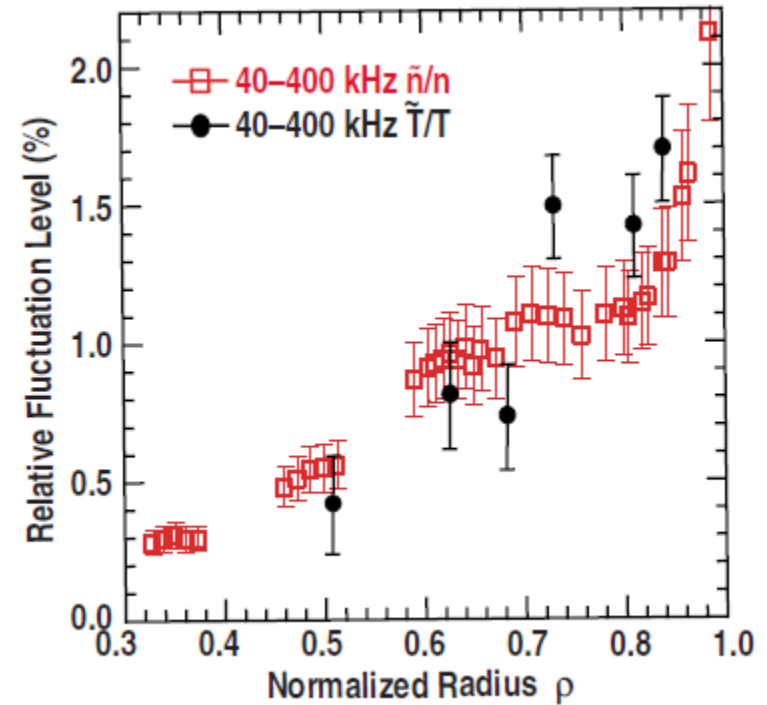
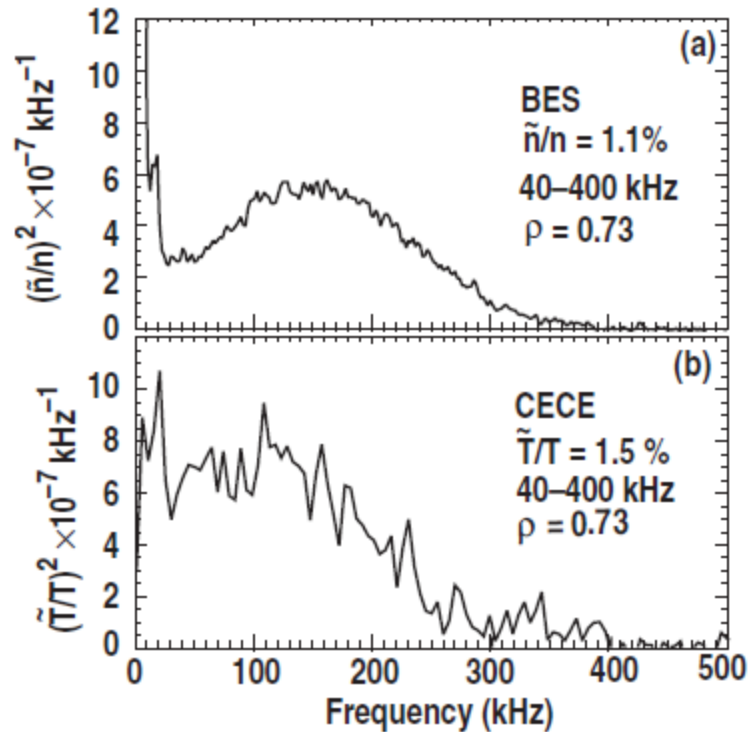
# Measurement of both electron density and temperature fluctuations at overlapping locations (DIII-D)

- Using electron cyclotron emission (ECE) to measure  $\delta T_e$



DIII-D  
White, PoP (2008)

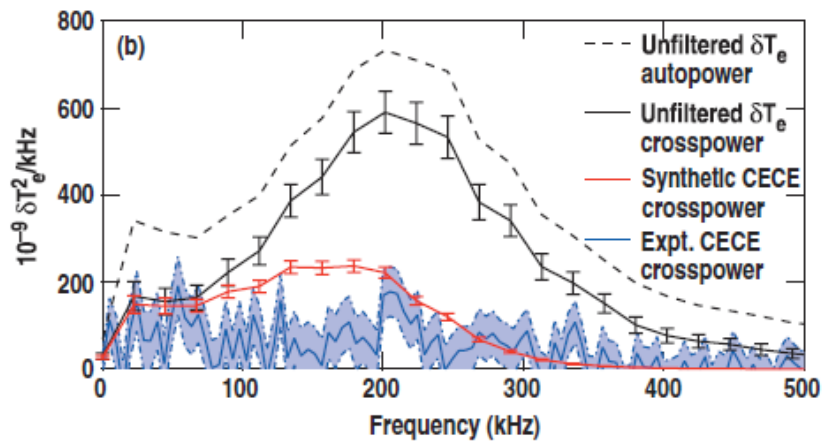
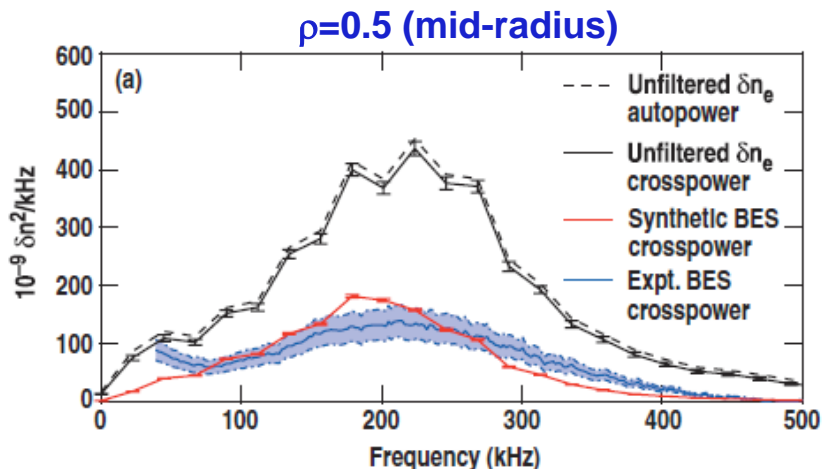
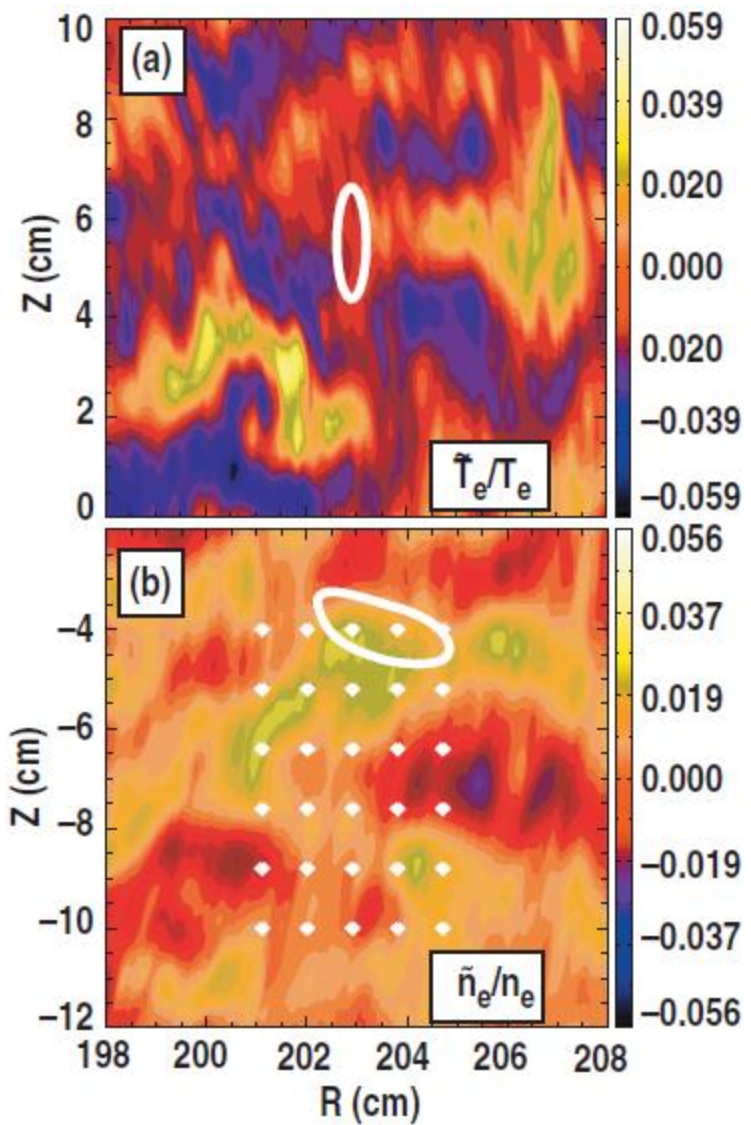
# Normalized density and temperature fluctuations are very similar in amplitude



DIII-D  
White, PoP (2008)

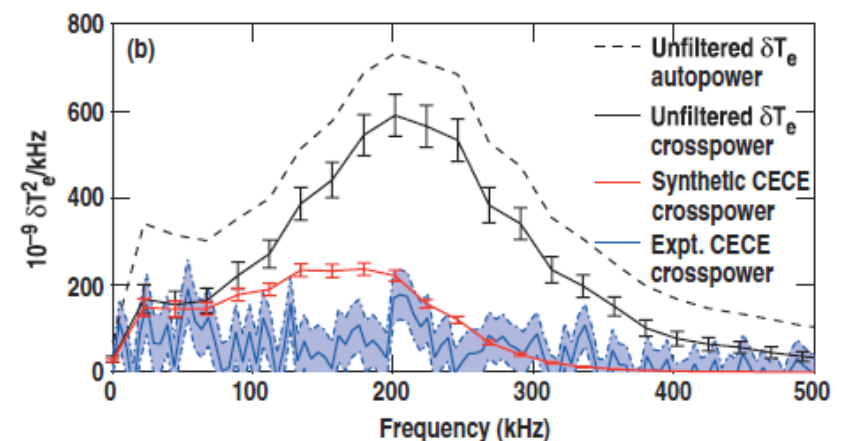
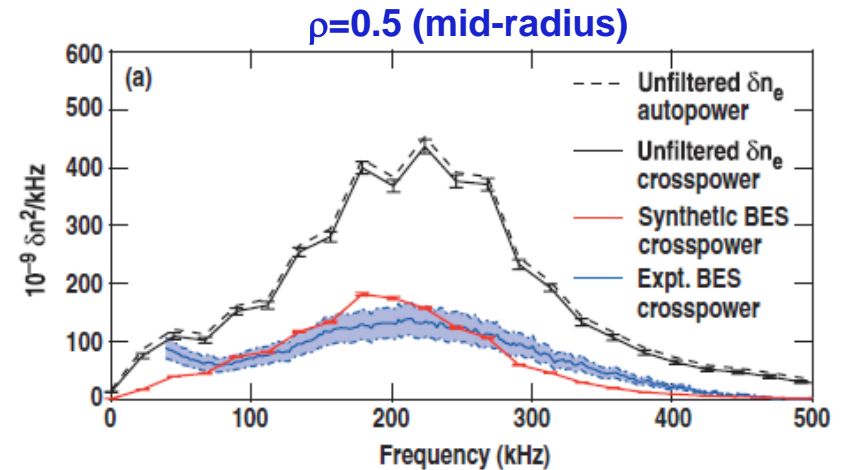
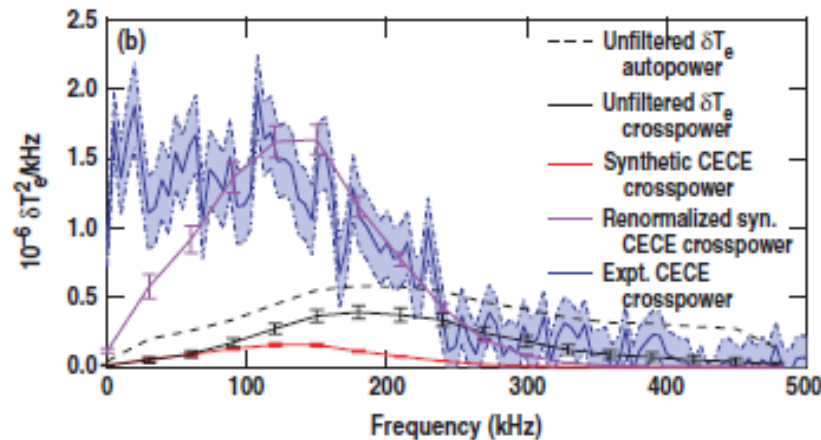
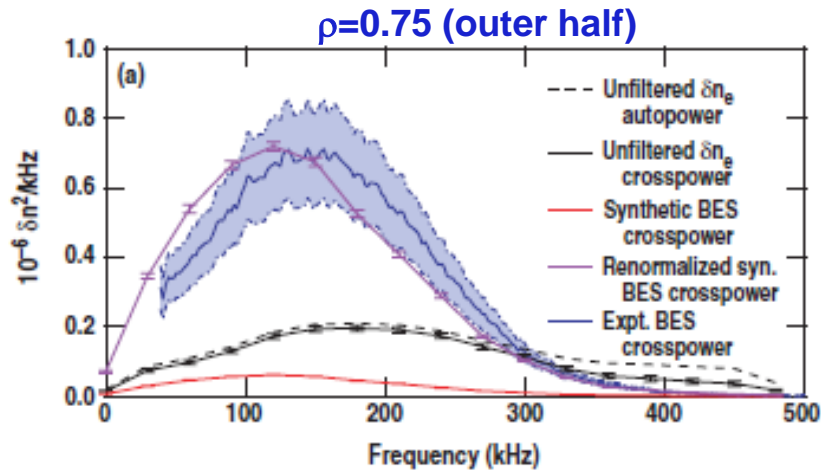
# Comparing $\delta n_e$ , $\delta T_e$ fluctuation spectra with simulations using synthetic diagnostic

- Level of agreement sensitive to accounting for realistic instrument function



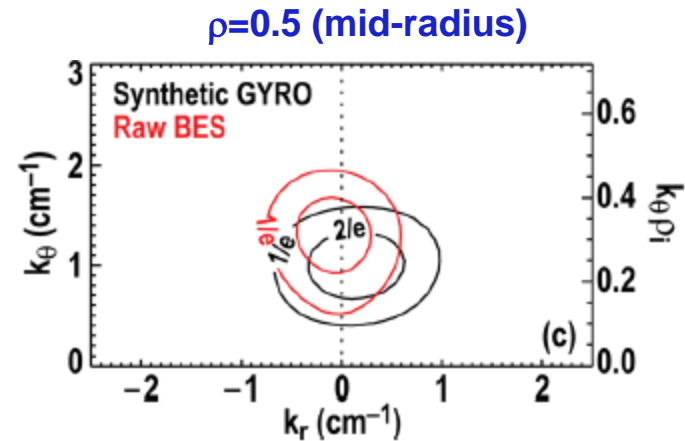
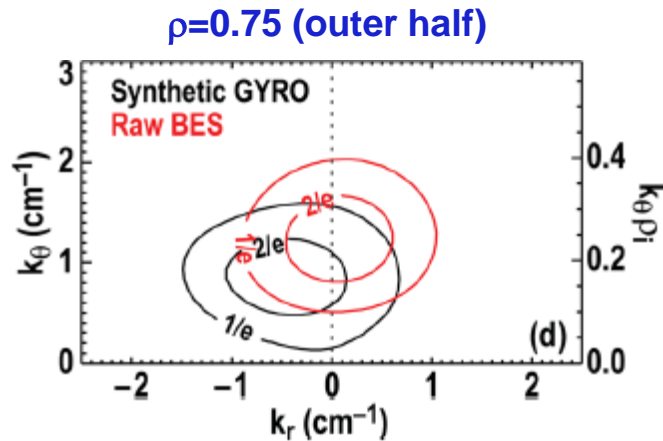
## Agreement worse further out ( $\rho=0.75$ )

- Measured intensity larger than simulations (as is transport), so called **“edge shortfall”** problem challenging gyrokinetic simulations

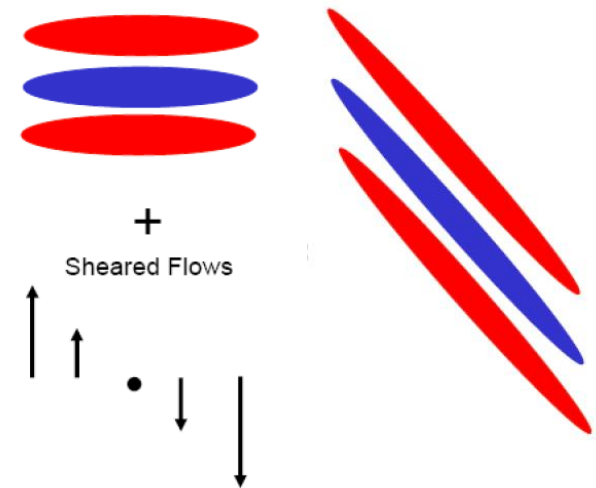


# Can also compare 2D correlation functions for additional validation, try to understand “shortfall” discrepancy

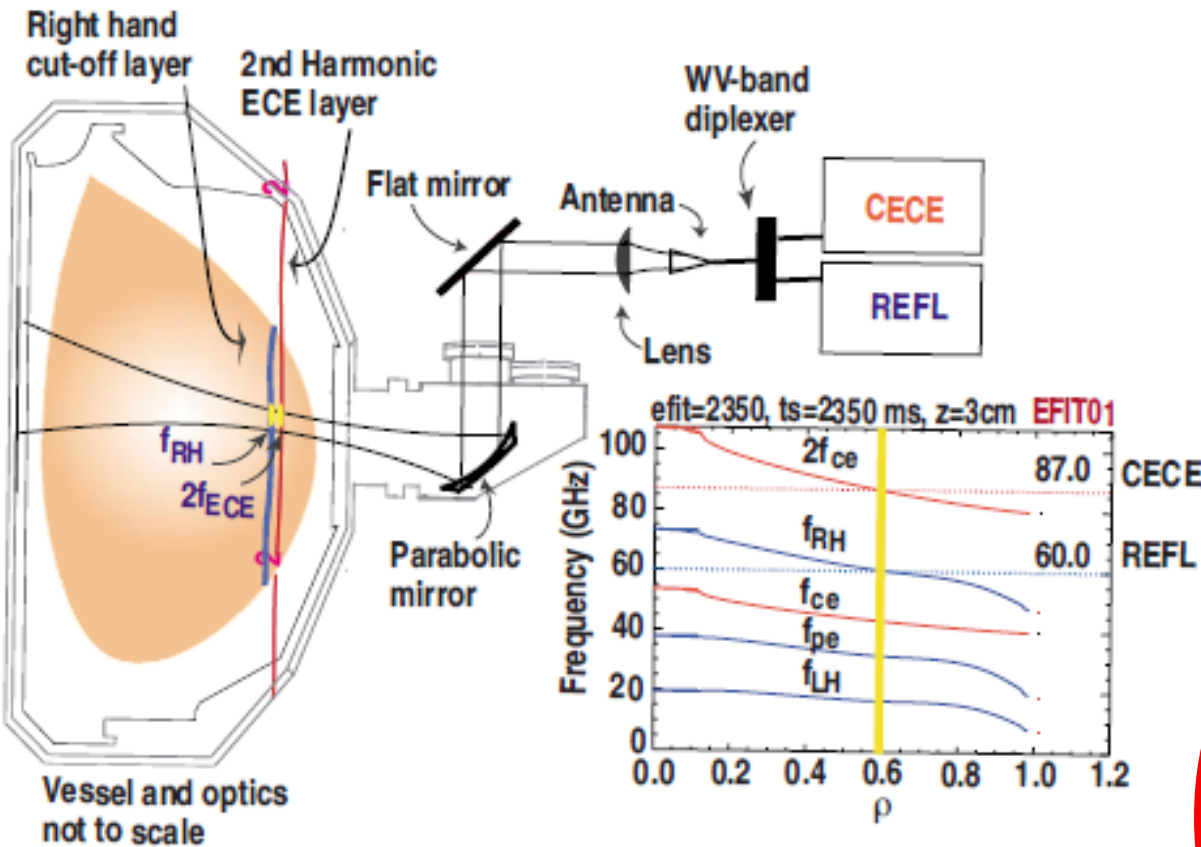
- Comparing 2D correlation/spectra reveals that simulated  $\langle k_r \rangle$  is larger than **experiment** at  $\rho=0.75$



- Larger  $\langle k_r \rangle$  in simulations possibly from tilting due to sheared equilibrium  $E \times B$  flows being too strongly represented  $\rightarrow$  also consistent with small predicted transport (more later)
- Has sparked a huge international code benchmarking & validation effort

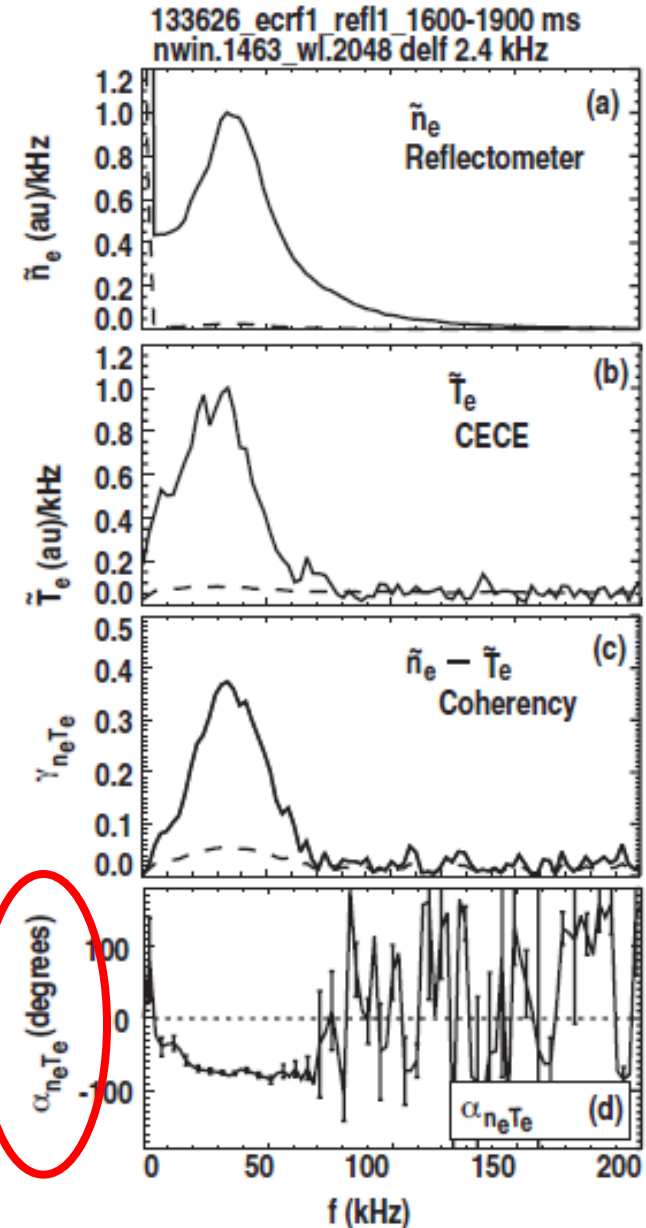


# Simultaneous measurement of $n_e$ and $T_e$ using same beam path allows for cross-phase measurement



$$\gamma_{n_e T_e}(f) = \frac{|\langle S_{\tilde{n}_e}^* S_{\tilde{T}_e} \rangle|}{|\langle S_{\tilde{n}_e} \rangle|^2 |\langle S_{\tilde{T}_e} \rangle|^2}$$

DIII-D  
White, PoP (2010)

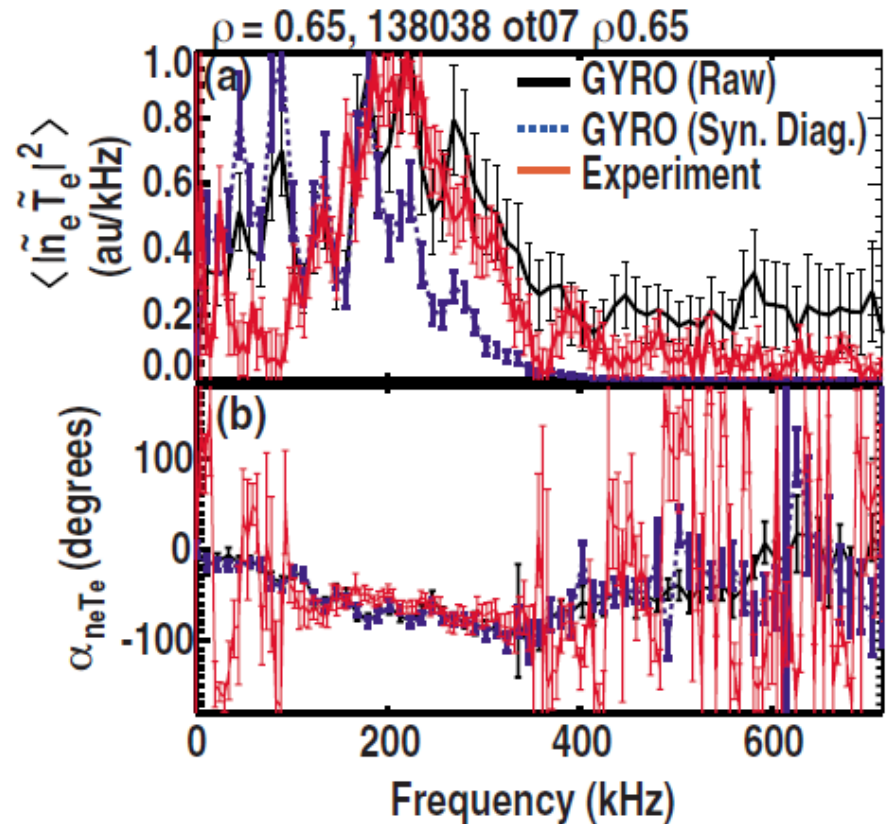


# ne-Te cross phases agree well with simulations

- Amplitude spectra and transport fluxes still off by 2-3

TABLE IV. Postexperiment GYRO simulations from 138 038,  $\rho=0.65$ ,  $t=1525$  ms. Turbulence amplitudes and cross phase are compared with synthetic diagnostic results.

Parameter	GYRO	Experiment
$Q_e$ (MW)	$3.77 \pm 0.06$	$2.43 \pm 0.02$
$Q_i$ (MW)	$0.34 \pm 0.01$	$1.32 \pm 0.02$
$\bar{T}_e/T_e$ (%)	$1.07 \pm 0.10$	$0.95 \pm 0.05$
$\bar{n}/n$ (%)	$0.25 \pm 0.01$	$0.57 \pm 0.06$
$\alpha_{n_e T_e}$ (degrees)	$71 \pm 1$	$61 \pm 12$

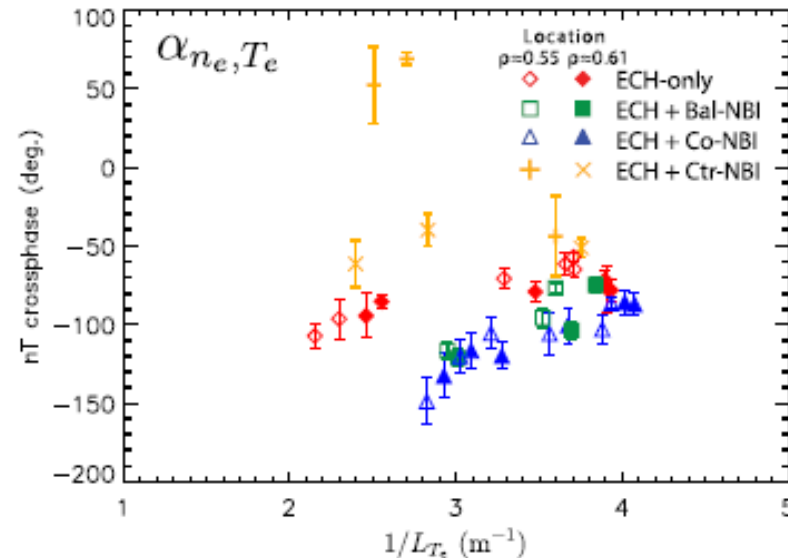
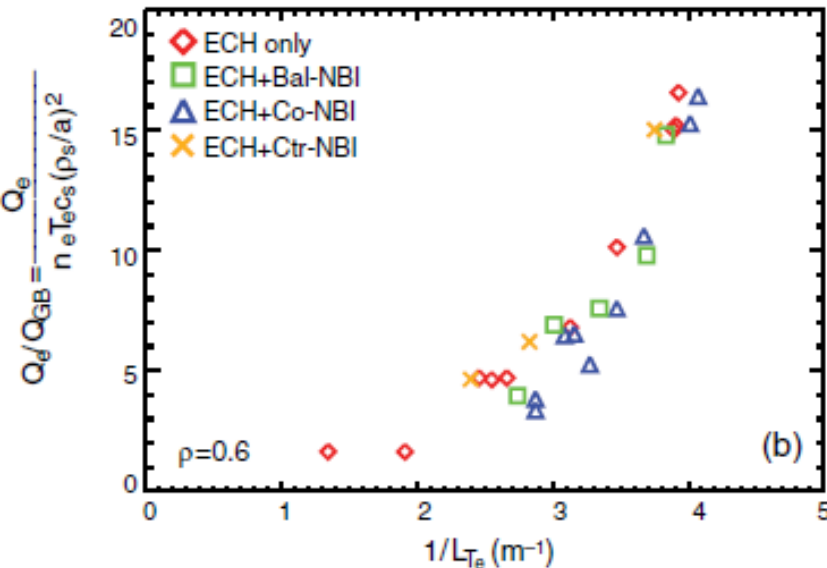
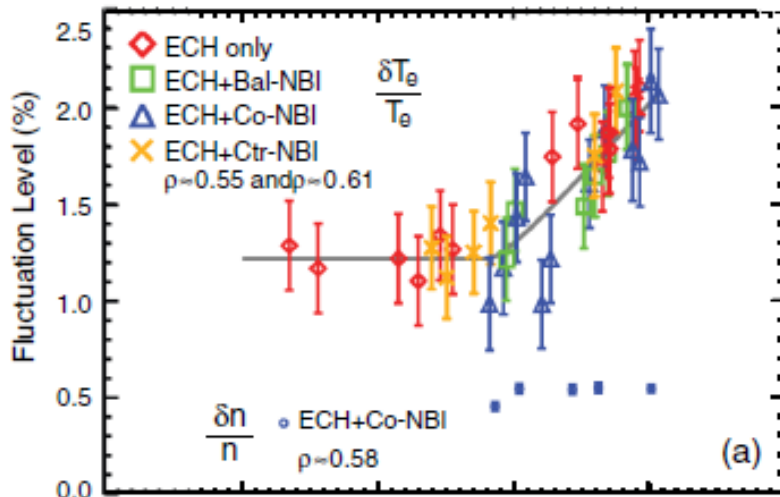




# Measured changes of $\delta T_e$ , $n_e$ - $T_e$ crossphase and transport with increasing $\nabla T_e$ provides constraint for simulations

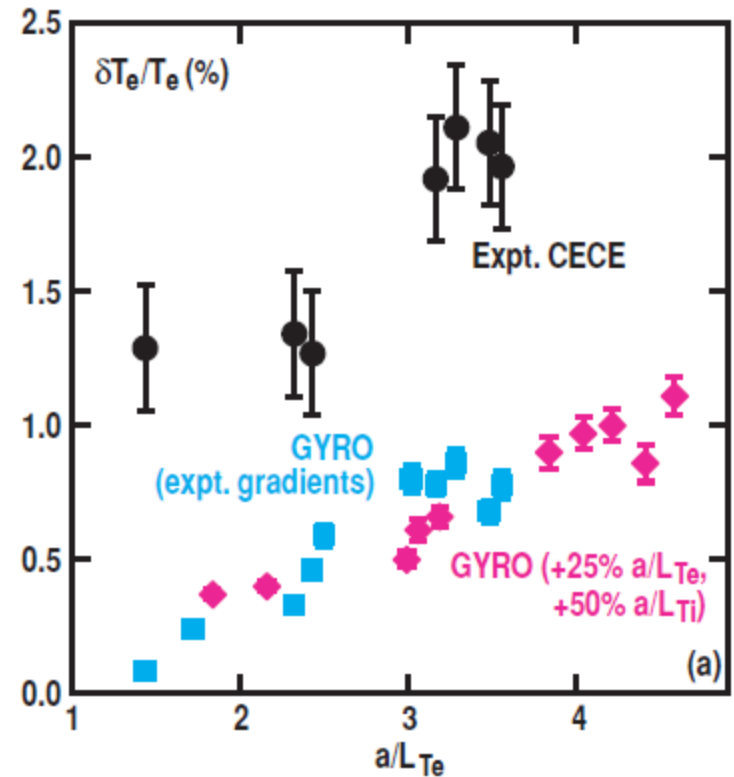
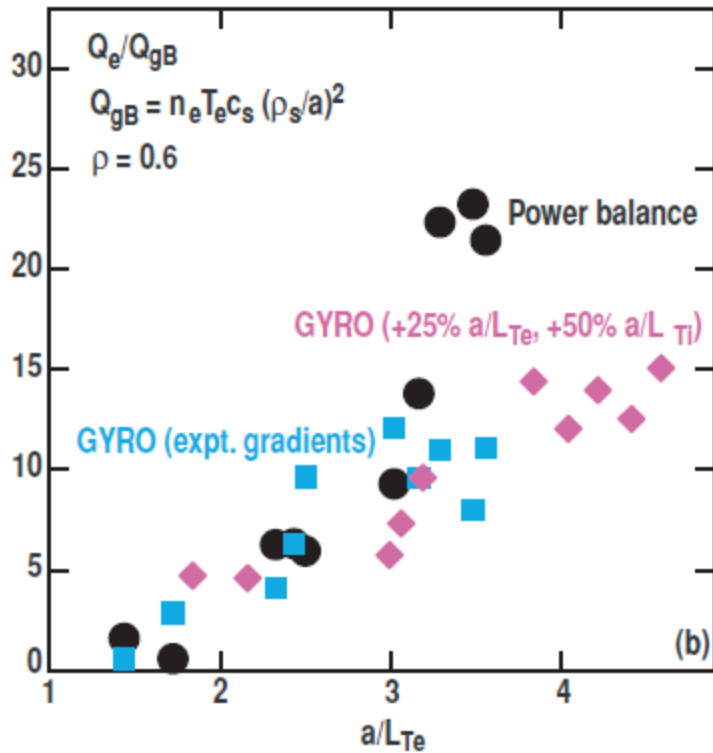
- Increasing fluctuations and transport with  $a/L_{T_e}$  consistent with enhanced TEM turbulence ( $\nabla T_e$  driven TEM)

DIII-D  
Hillesheim, PRL, PoP (2013)



# Simulations can reproduce transport for some observations

- Predicted turbulence levels always too small, even when accounting for sensitivity to  $\nabla T_e$
- Discrepancies point to missing physics in theory/simulation



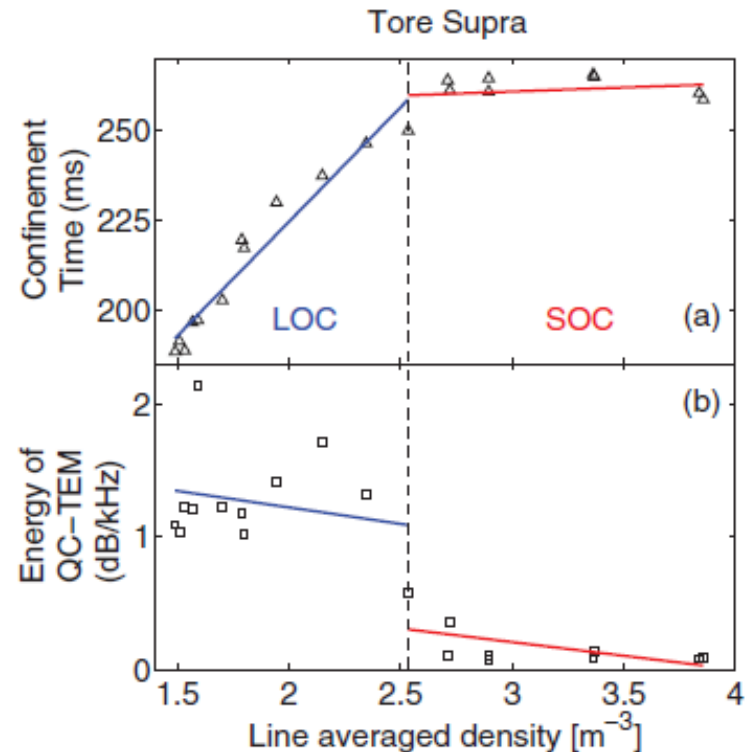
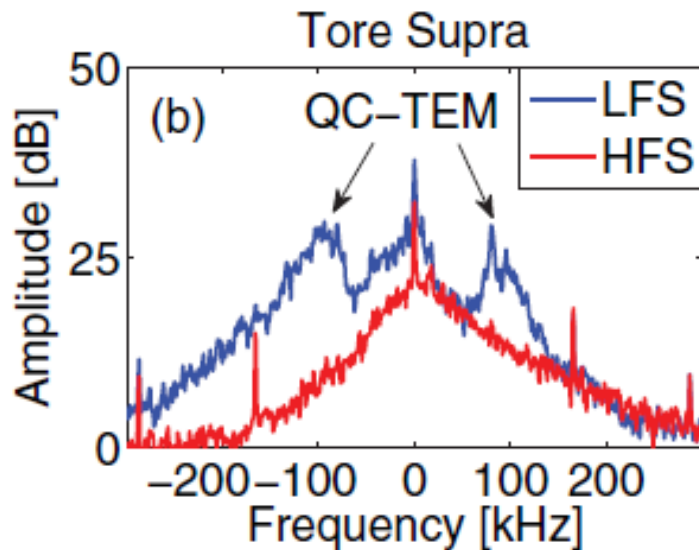
Holland, PoP (2013)

# JET core ITG stiffness results (Mantica, PRL 2011)

# **ADDITIONAL EVIDENCE FOR TRAPPED ELECTRON MODE (TEM) TURBULENCE**

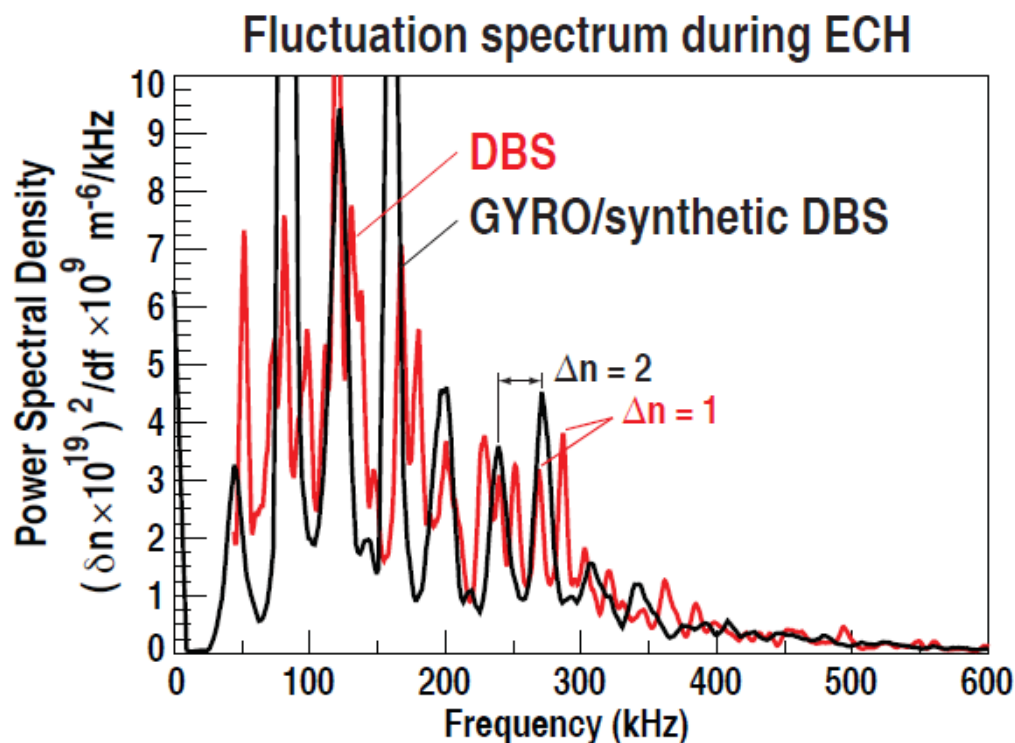
# Quasi-coherent modes observed in the deep core of Tore Supra, TEXTOR and JET tokamaks

- Measured with reflectometers
- Amplitudes large at low collisionality (enhanced TEM growth rates) via low density (below), ECRH heating, ...



# Similar coherent modes observed in the core of ECH heated DIII-D QH-modes, reproduced with nonlinear gyrokinetics

## Nonlinear GYRO Simulations Reproduce New Coherent Fluctuations Seen on DBS, identifying these as TEMs



Now if we do much less frequency smoothing of same data, drilling down ...

- Coherent modes in GYRO correspond to resolution used,  $\Delta n = 2$ 
  - Match every second coherent mode seen on DBS (for which  $\Delta n = 1$ )
- High resolution GYRO simulations in progress with  $\Delta n = 1$
- Similar results for no ECH case

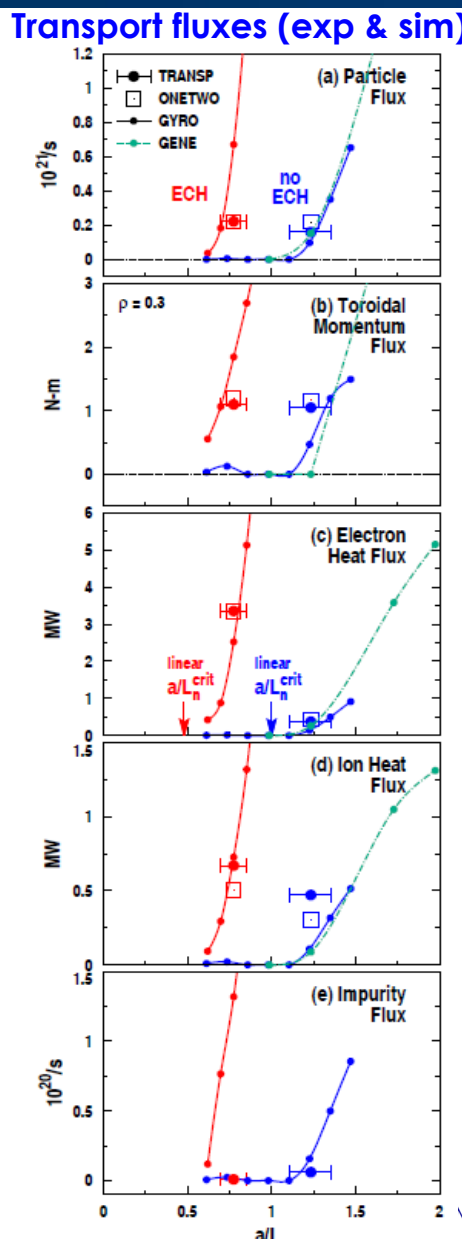
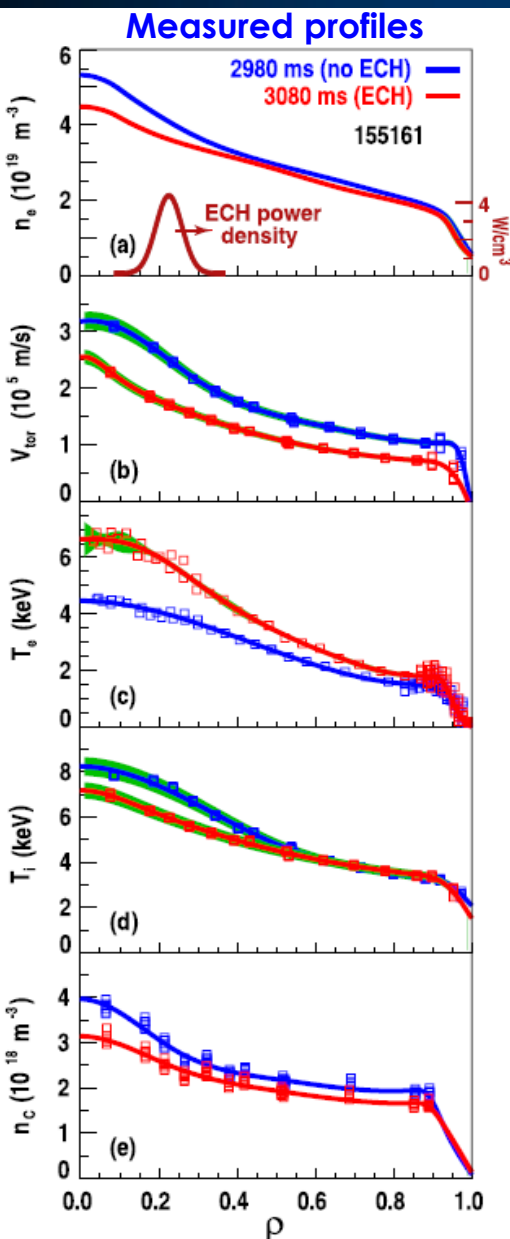


D. R. Ernst/APS-DPP NIS.00003/10:30-11:00 AM Wednesday, November 18

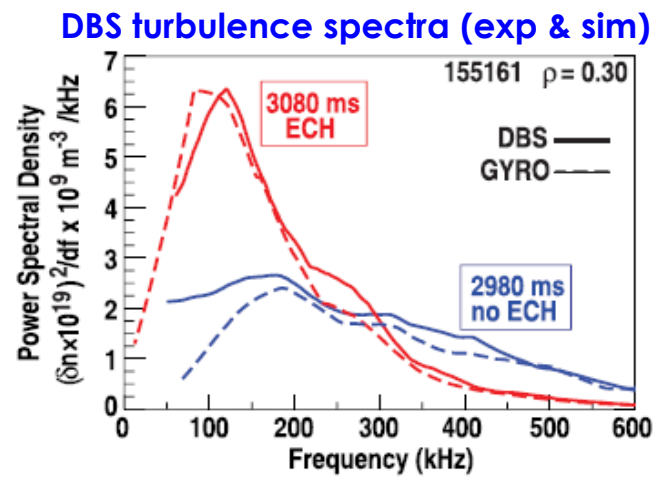
12

Ernst, IAEA (2014), PoP (2016)  
Guttenfelder, APS (2015)

# Nonlinear gyrokinetics of density-gradient driven TEM reproduces change in transport and turbulence with addition of ECH



- Nonlinear GYRO simulations illustrate presence of  $\nabla n$ -driven TEM at  $\rho=0.3$
- Simulations reproduce magnitude of transport ( $Q_i, Q_e, \Gamma_e, \Gamma_c, \Pi_\phi$ ) and DBS spectra using synthetic diagnostic
  - Also reproduces changes of transport and DBS with **addition of ECH**



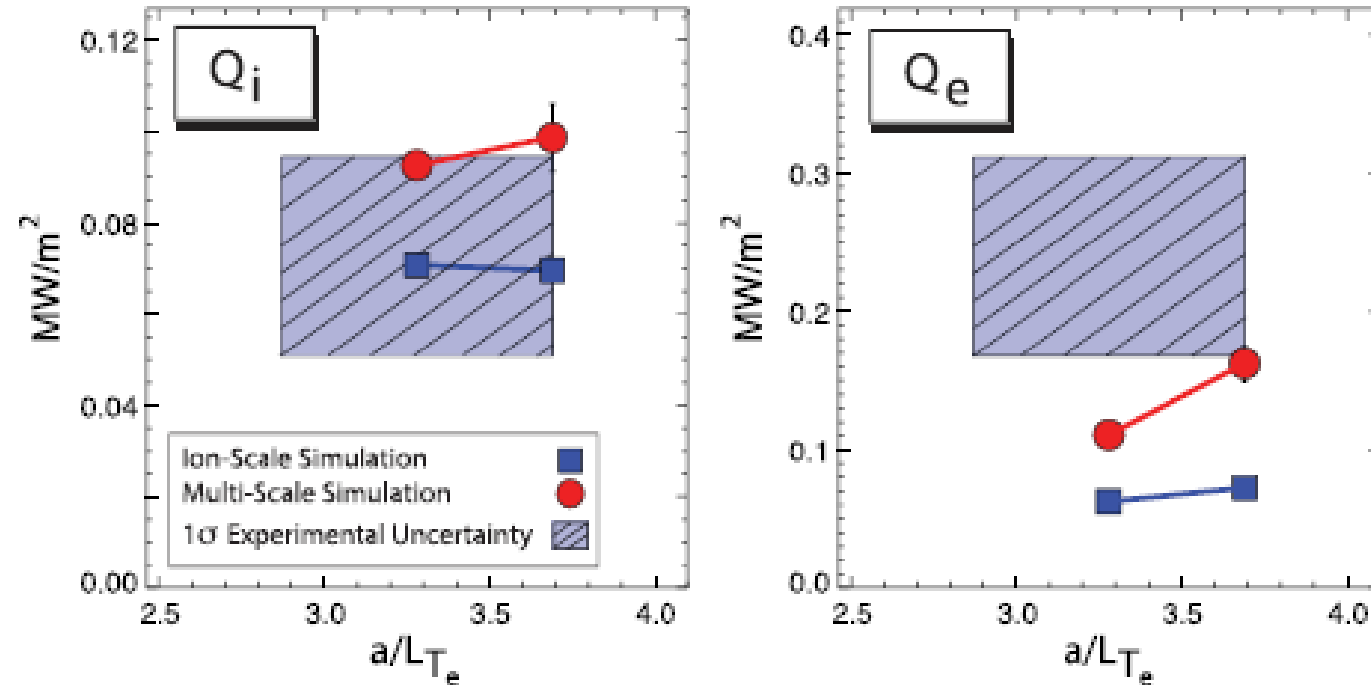
**DND QH-mode 155161 [Ernst, PoP (2016)]**  
 $B_T = 2.05$  T,  $I_p = 1.2$  MA  
 $P_{NBI} = 5.5$  MW (ctr- $I_p$ ),  $P_{ECH} = 3.4$  MW  
 $\beta_N = 1.5$ ,  $q_{95} = 5.2$

Wisla (2017)

# MULTI-SCALE TURBULENCE (FROM $\rho_i$ TO $\rho_e$ SCALES)



# In some instances simulations can account for ion transport, but predicts too small electron transport



*Alcator C-Mod (MIT)*

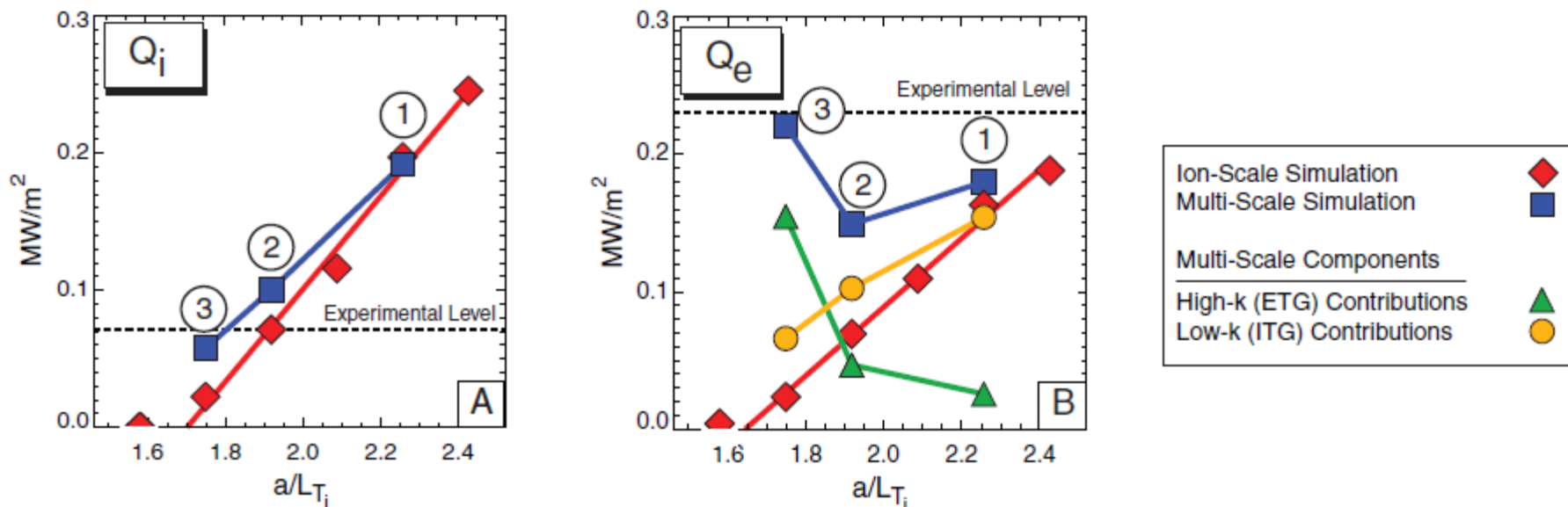
*Howard, PoP (2014)*

- Requires self-consistent multi-scale simulations to account for  $Q_e$  &  $Q_i$  together
- Numerous examples (DIII-D, ITER, C-Mod, NSTX) where this might be important  $\rightarrow$  very expensive computationally  $\sim 20$  M cpu-hrs/sim

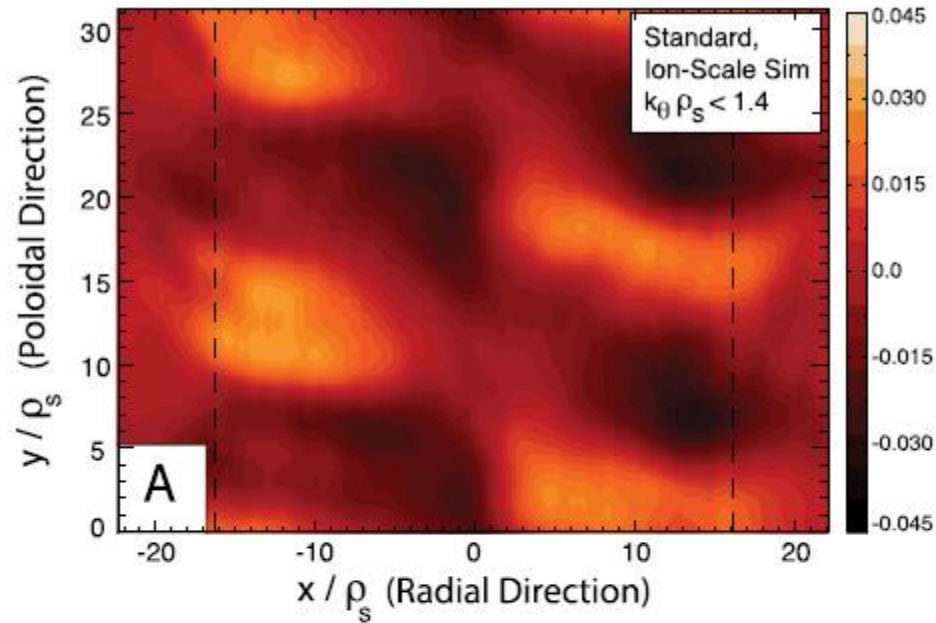
# Non-intuitive change in predicted transport due to cross-scale coupling between $\sim\rho_i$ and $\sim\rho_e$

- As  $a/L_{T_i}$  ( $=-R\nabla T_i/T_i$ ) is reduced towards ITG threshold,  $Q_i$  decreases while electron transport increases due to very small scale ( $k_\theta\rho_i > 1$ ,  $k_\theta\rho_e < 1$ ) turbulence
- can match experiment

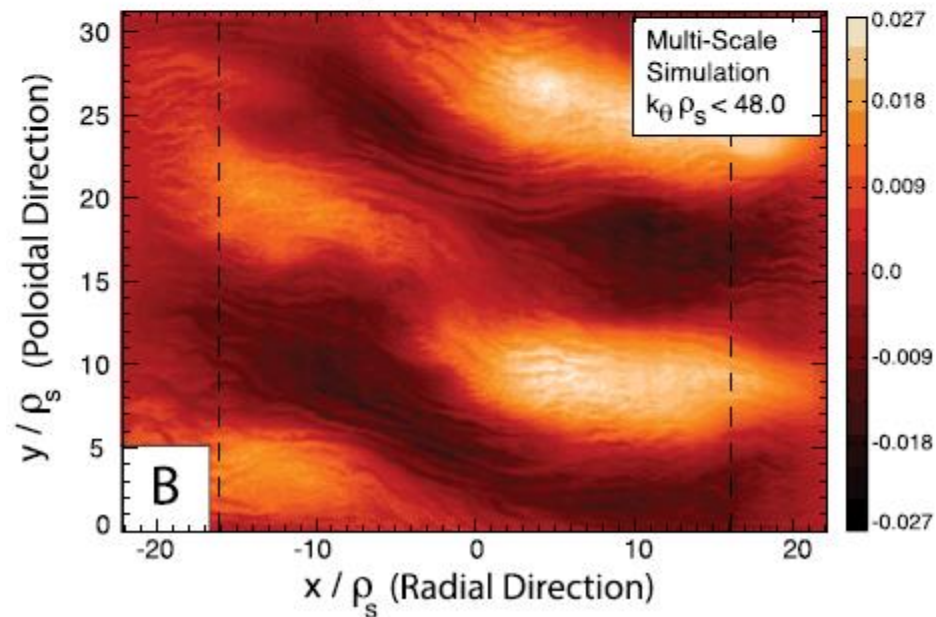
Howard, NF (2016)



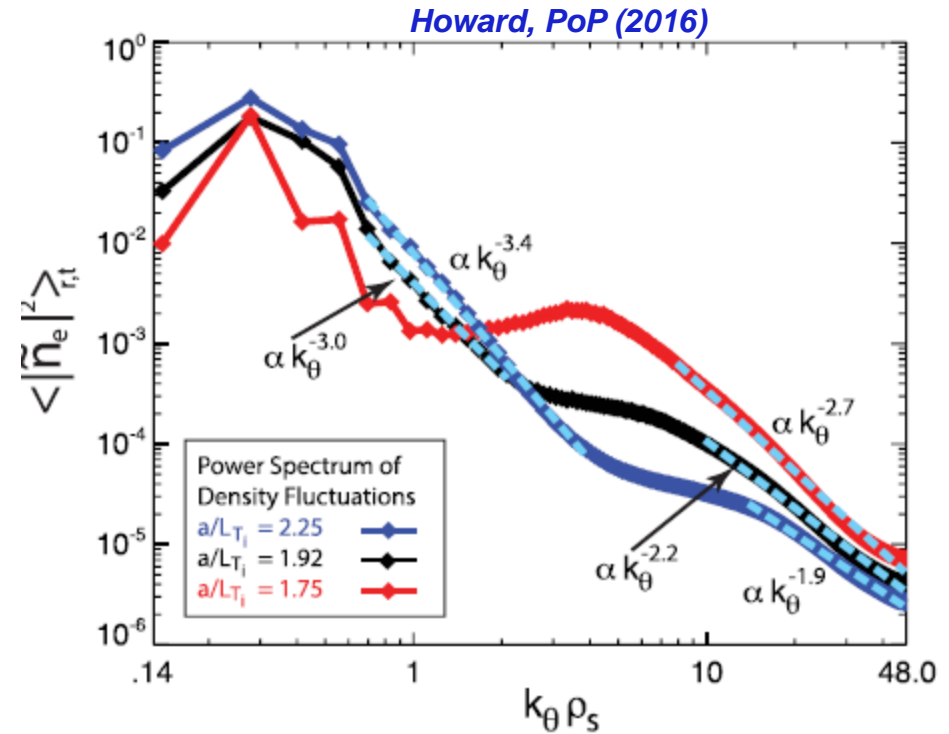
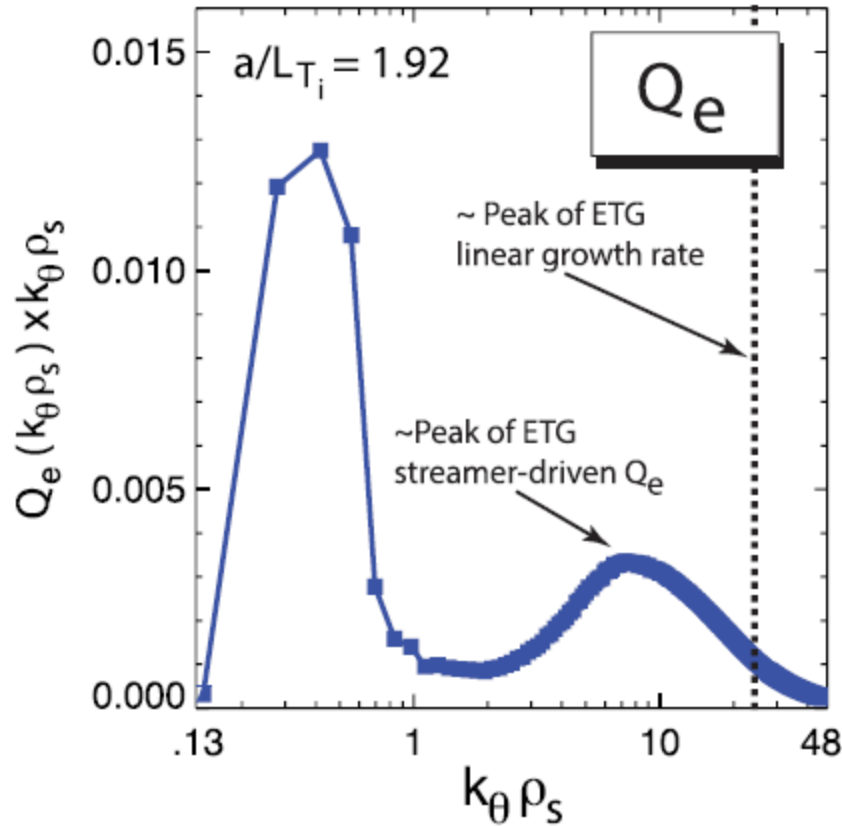
# ETG-like “streamers” predicted to exist on top of ion scale turbulence



*Howard, PoP (2014)*

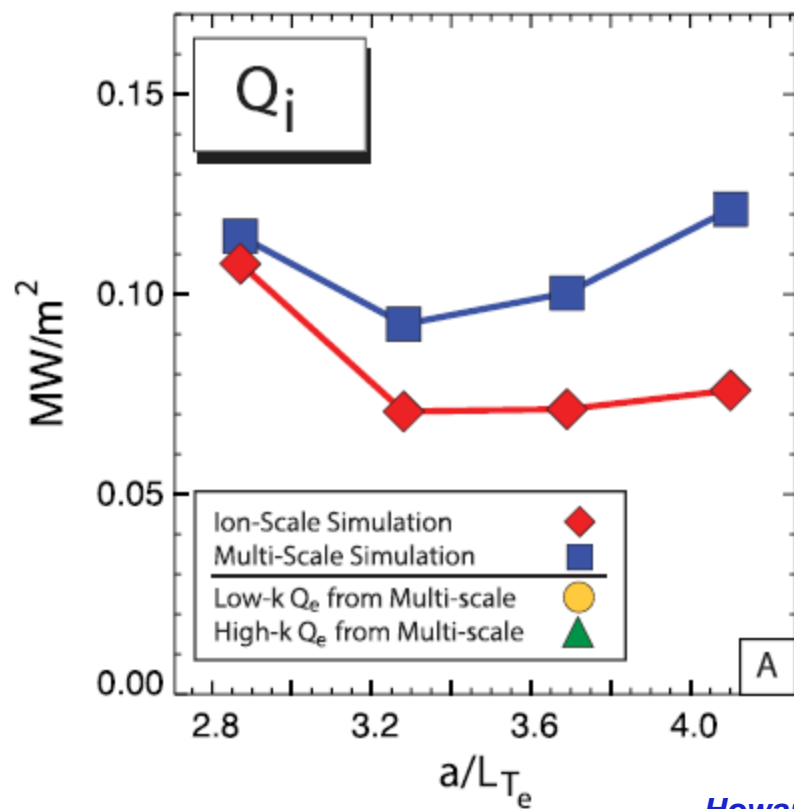


# Hot topic: measure change in turbulence spectrum consistent with multi-scale effects

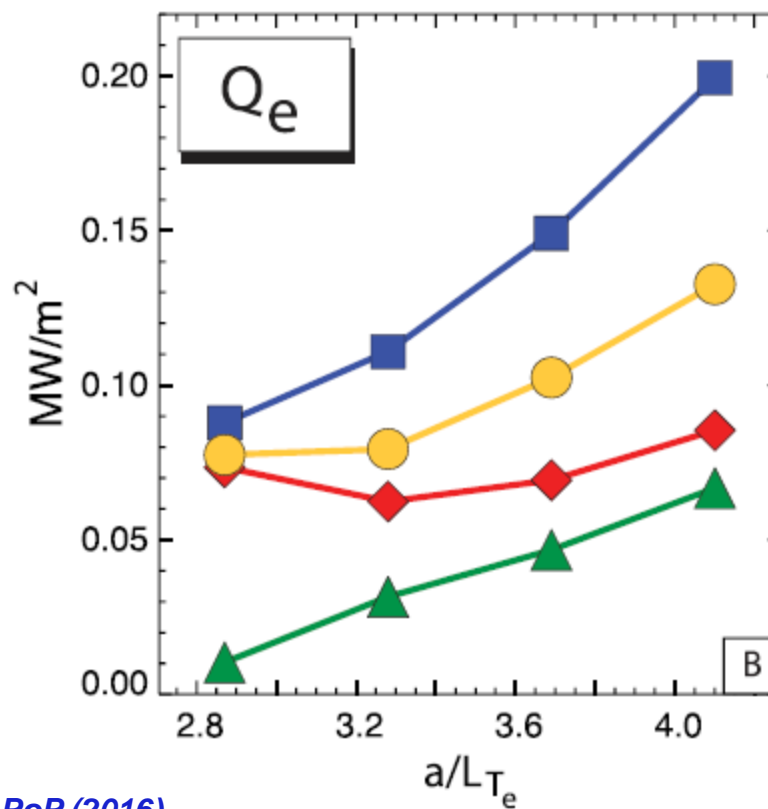


- Proposal to use Phase Contrast Imaging (PCI) on C-Mod (don't think it was done before 2016 end-of-life?)
- Some “multi-scale” turbulence measurements in L. Schmitz, NF (2012)

# Stronger electron stiffness also predicted and consequences observed in experiments



Howard, PoP (2016)



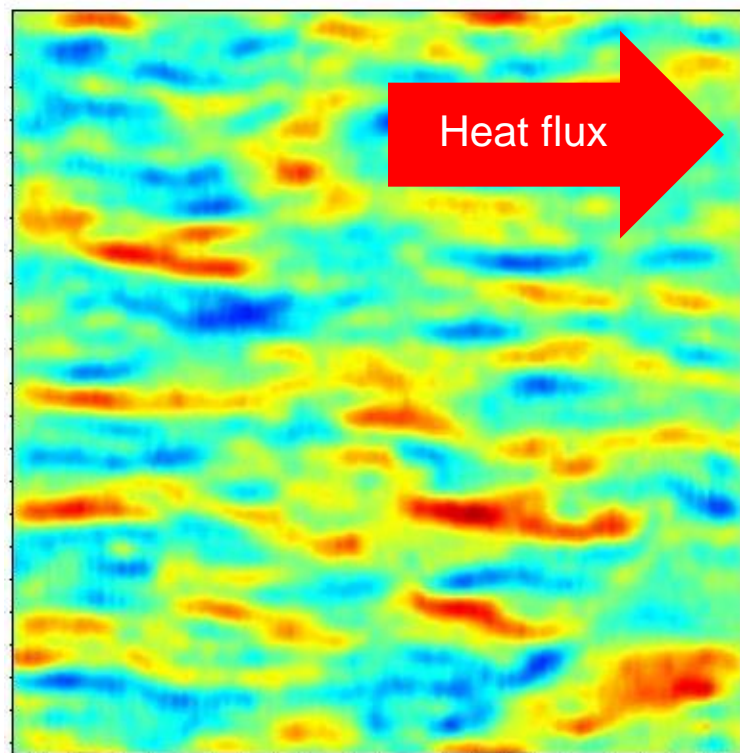
- Transport modeling including above multi-scale effects (Staebler, PoP 2016; Pablo-Fernandez, PRL 2018) reproduces observed fast perturbative transport (e.g. introduce a local cold spot and watch  $T_e$ ,  $\nabla T_e$  propagate, )

# **SUPPRESSION OF ION SCALE TURBULENCE BY SHEARED $E \times B$ FLOWS**

# Large scale sheared flows can tear apart turbulent eddies, reduce turbulence → improve confinement

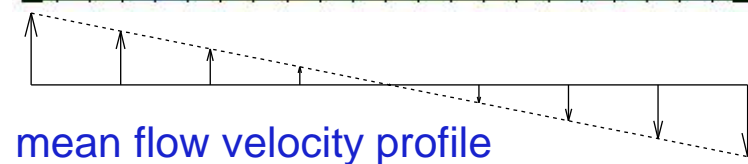
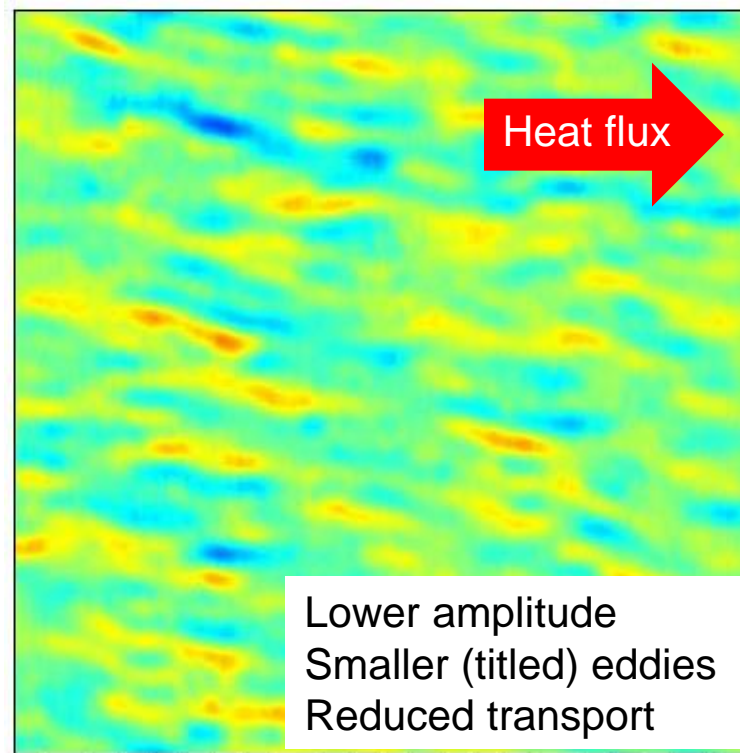
Simulations for NSTX (PPPL) – a low aspect ratio tokamak

Snapshot of density without flow shear

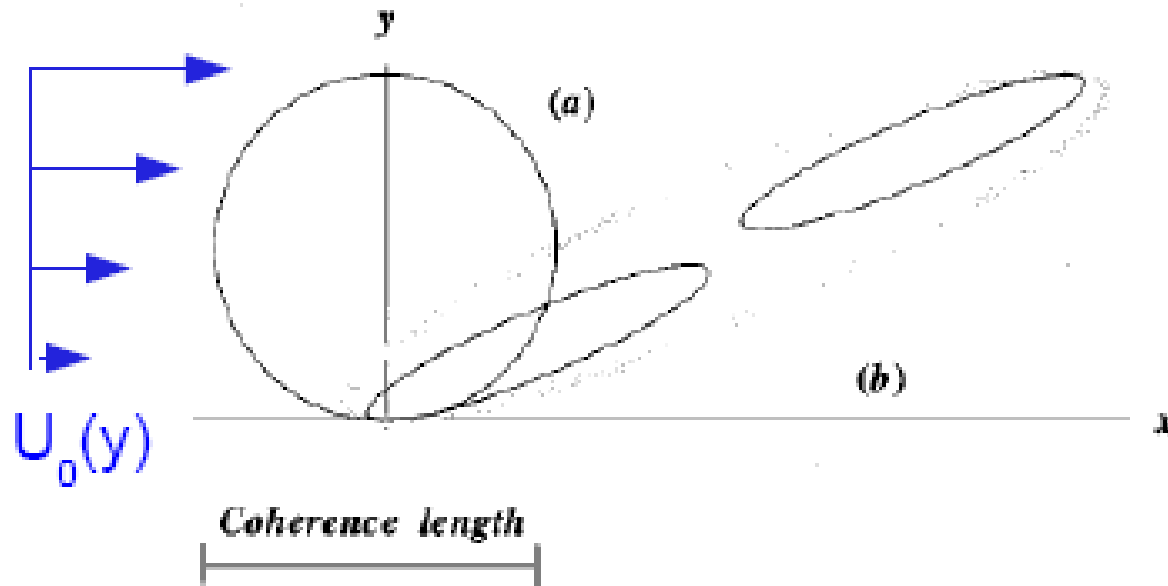


← 100 ion radii  
6,000 electron radii →  
~50 cm

Snapshot of density with flow shear



# Equilibrium background ( $E \times B$ ) flows can suppress turbulence



Loosely need:  
 $dU/dy > \tau_c^{-1}$

- Shear flow in neutral (3D) fluids is a source of free-energy, how does it stabilize turbulence in magnetized plasmas?
- Three conditions for sheared flow suppression of turbulence (Terry, RMP 2000):
  - Shear flow should be stable ( $\rightarrow$  Kelvin-Helmholtz threshold different in 2D)
  - Turbulence must reside in region of shear flow for longer than an eddy-turnover time/decorrelation time ( $\rightarrow$  tokamak is a periodic system)
  - Dynamics should be 2D ( $\rightarrow$  strong guide magnetic field)



# Experimental turbulence and transport measurements of ExB shear suppression

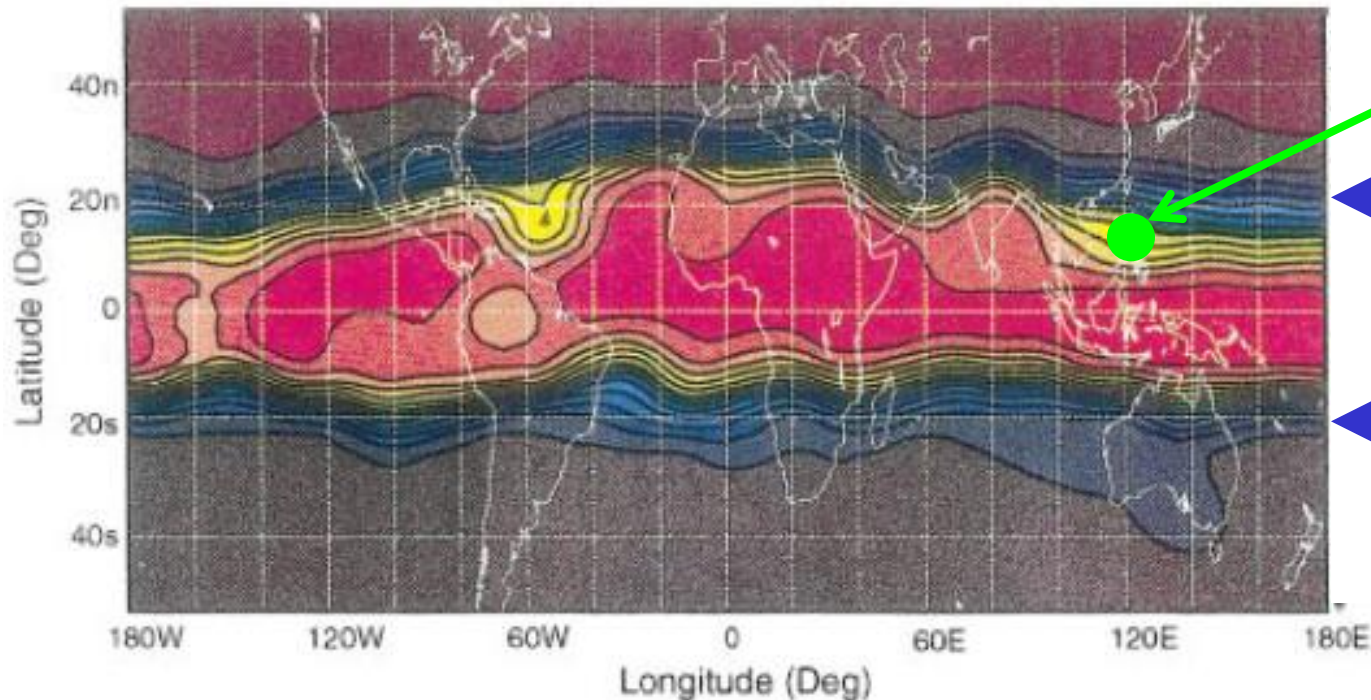
- (I'll show this in section on L-H transition)

# There are also examples of turbulence suppression via sheared flows in neutral fluids

- Thin (quasi-2D) atmosphere in axisymmetric geometry of rotating planets similar to tokamak plasma turbulence
- Stratospheric ash from Mt. Pinatubo eruption (1991) spread rapidly around equator, **but confined in latitude by flow shear**



Aerosol concentration

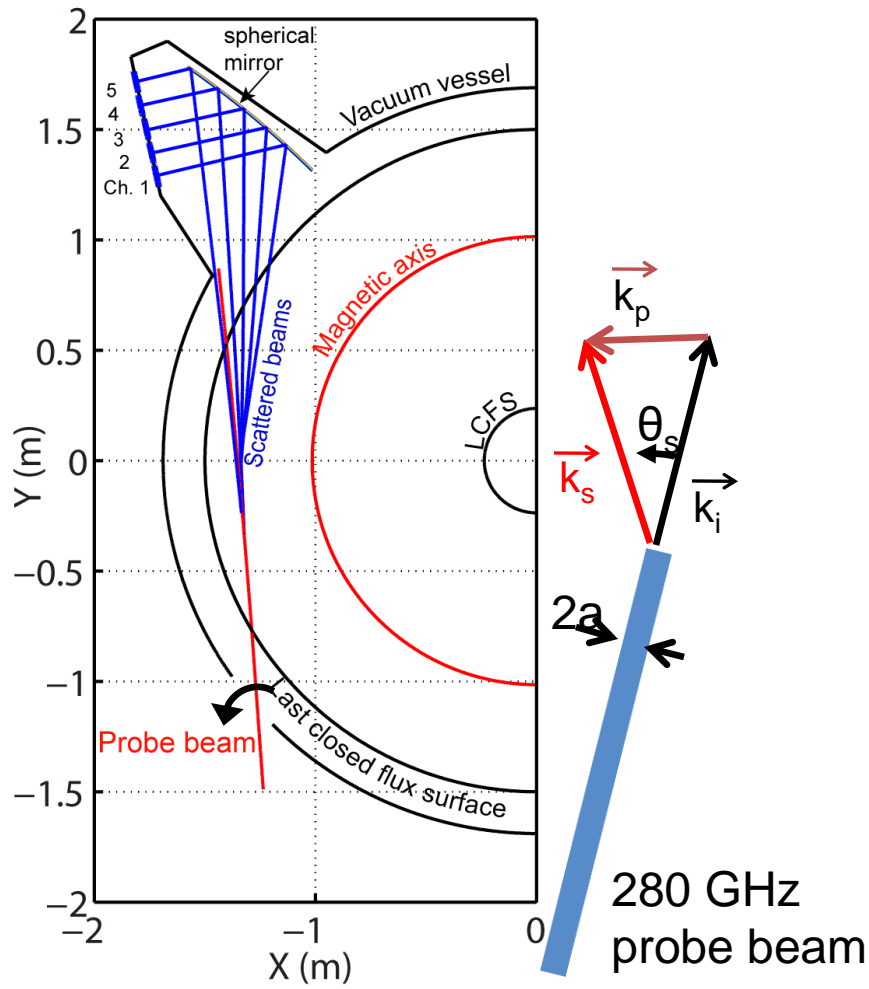


Large shear in stratospheric equatorial jet

(Trepte, 1993)

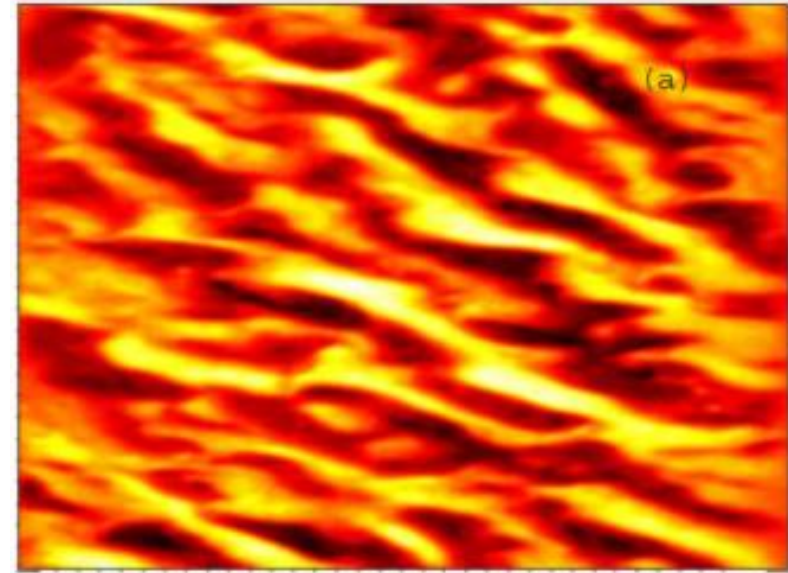
# **“PURE” ELECTRON SCALE TURBULENCE (not multiscale)**

# Microwave scattering used to detect high- $k_{\perp}$ ( $\sim$ mm) fluctuations



Mazzucato, PRL (2008)  
Smith, RSI (2008)

density fluctuations from ETG simulation



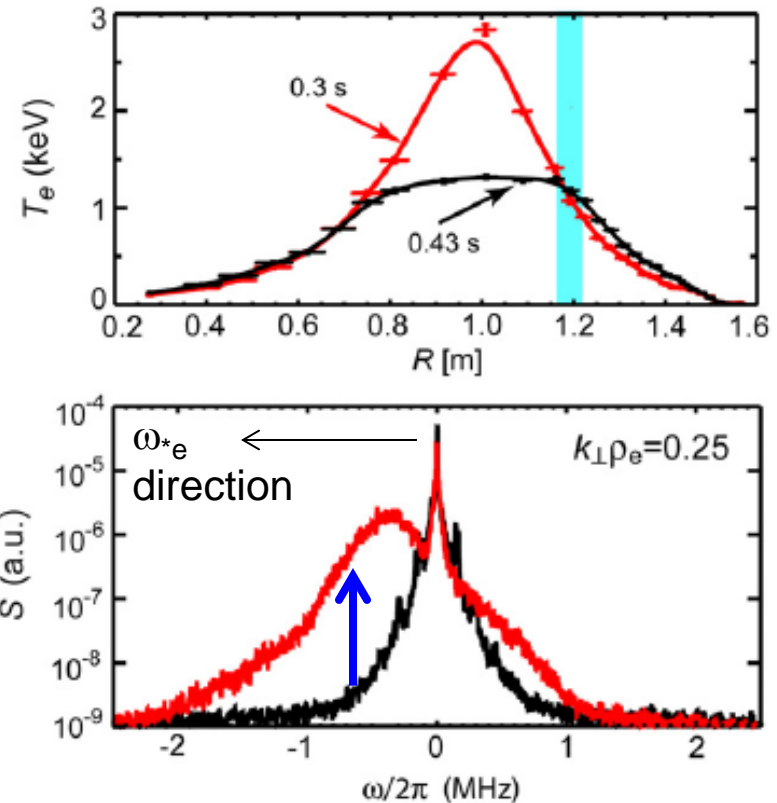
6 ion radii  
360 electron radii  
 $\sim 2$  cm

Guttenfelder, PoP (2011)

**NSTX**

# Correlation observed between high-k scattering fluctuations and $\nabla T_e$

- Applying RF heating to increase  $T_e$
  - Fluctuations increase as expected for ETG turbulence ( $R/L_{Te} > R/L_{Te,crit}$ )
- Other trends measured that are consistent with ETG expectations, e.g. reduction of high-k scattering fluctuations with:
1. Strongly reversed magnetic shear (Yuh, PRL 2011)
    - Simulations predict comparable suppression (Peterson, PoP 2012)
  2. Increasing density gradient (Ren, PRL 2011)
    - Simulations predict comparable trend (Ren, PoP 2012, Guttenfelder NF, 2013, Ruiz PoP 2015)
  3. Sufficiently large  $E \times B$  shear (Smith, PRL 2009)
    - Observed in ETG simulations (Roach, PPCF 2009; Guttenfelder, PoP 2011)



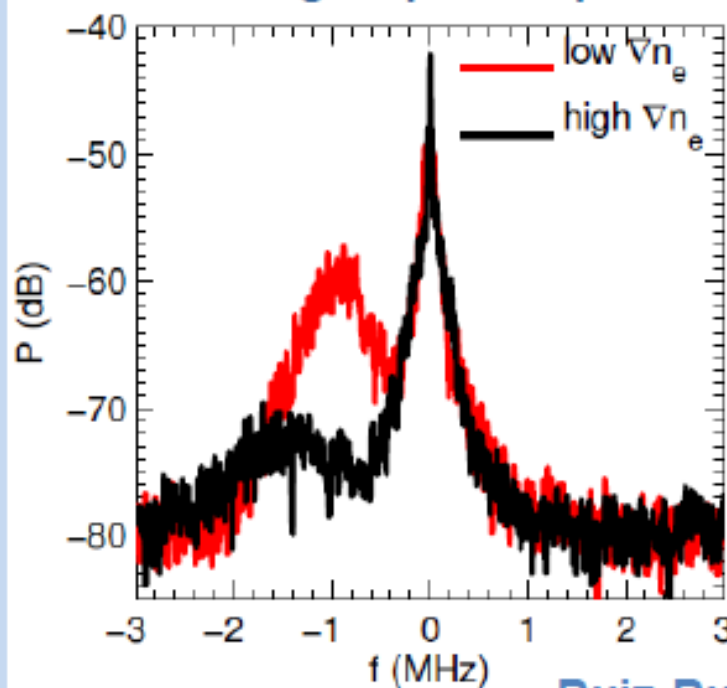
E. Mazzucato et al., NF (2009)

**NSTX**

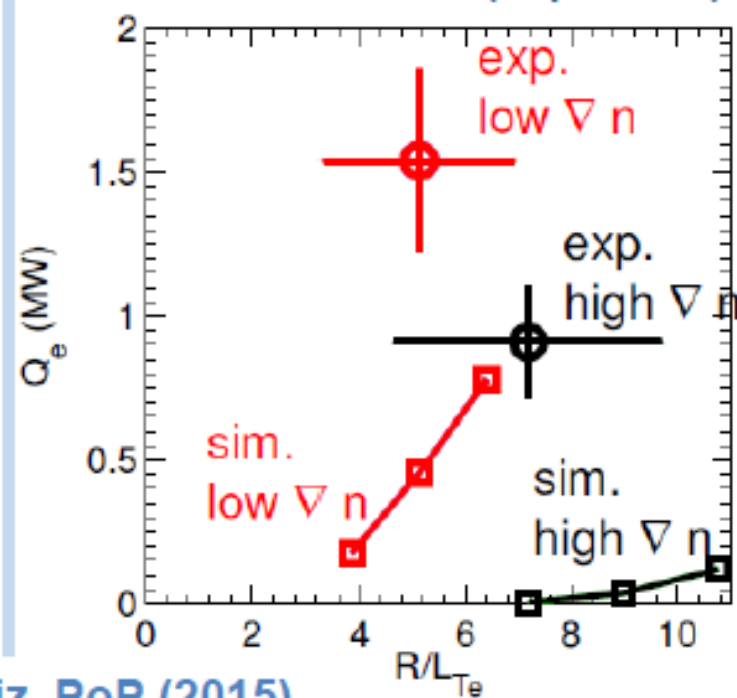
# Many ETG trends observed in NSTX, challenging to correctly predict transport

- BUT majority of nonlinear gyrokinetic ETG simulations predict  $Q_e$  too small to explain experiment

Measured high-k power spectra



Electron heat flux (exp & sim)



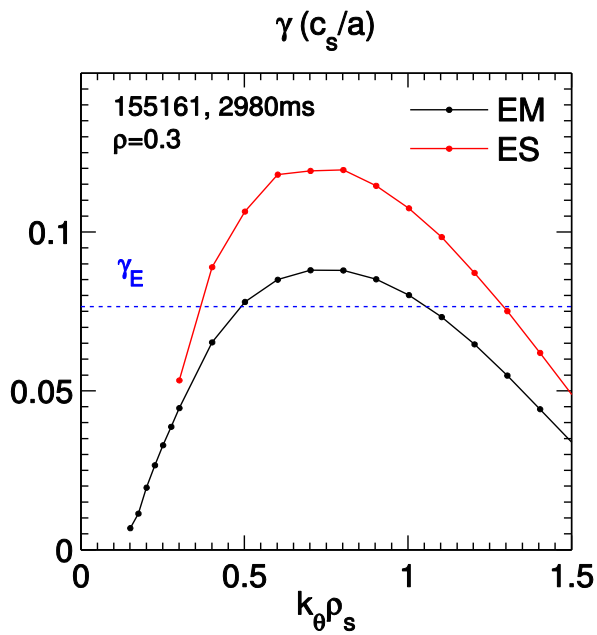
Ruiz-Ruiz, PoP (2015)

*(another potential case for multi-scale simulations)*

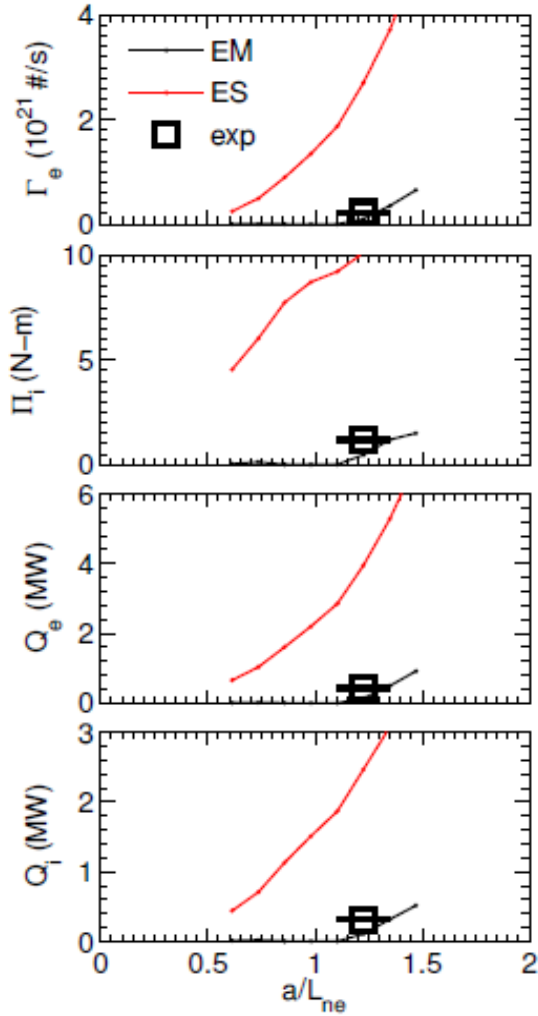
# **ELECTROMAGNETIC EFFECTS ON ITG/TEM TURBULENCE**

# Electromagnetic stabilization at finite $\beta$ predicted to be critical for quantitative agreement in NBI-only scenario

- Good agreement in all transport channels with EM effects ( $\delta B$ )
  - Near marginal
- Transport over-predicted in the electrostatic (ES) limit ( $\delta B \rightarrow 0$ )
  - Downshift of  $\nabla n$  threshold
- Max. growth rates increase  $\sim 35\%$  if electromagnetic effects ignored ( $\delta B \rightarrow 0$ )



nonlinear GYRO simulations  
DIII-D 155161,  $\rho=0.3$ , NBI-only



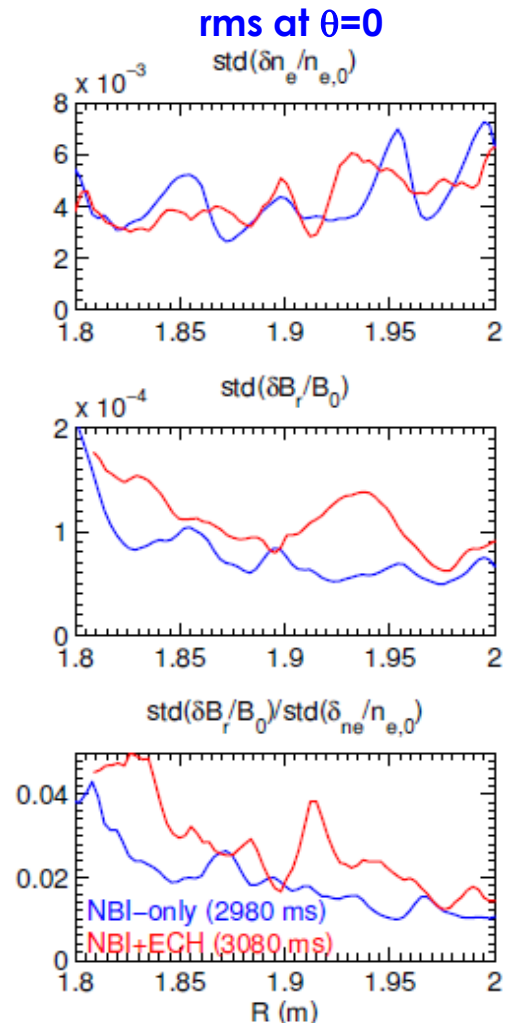
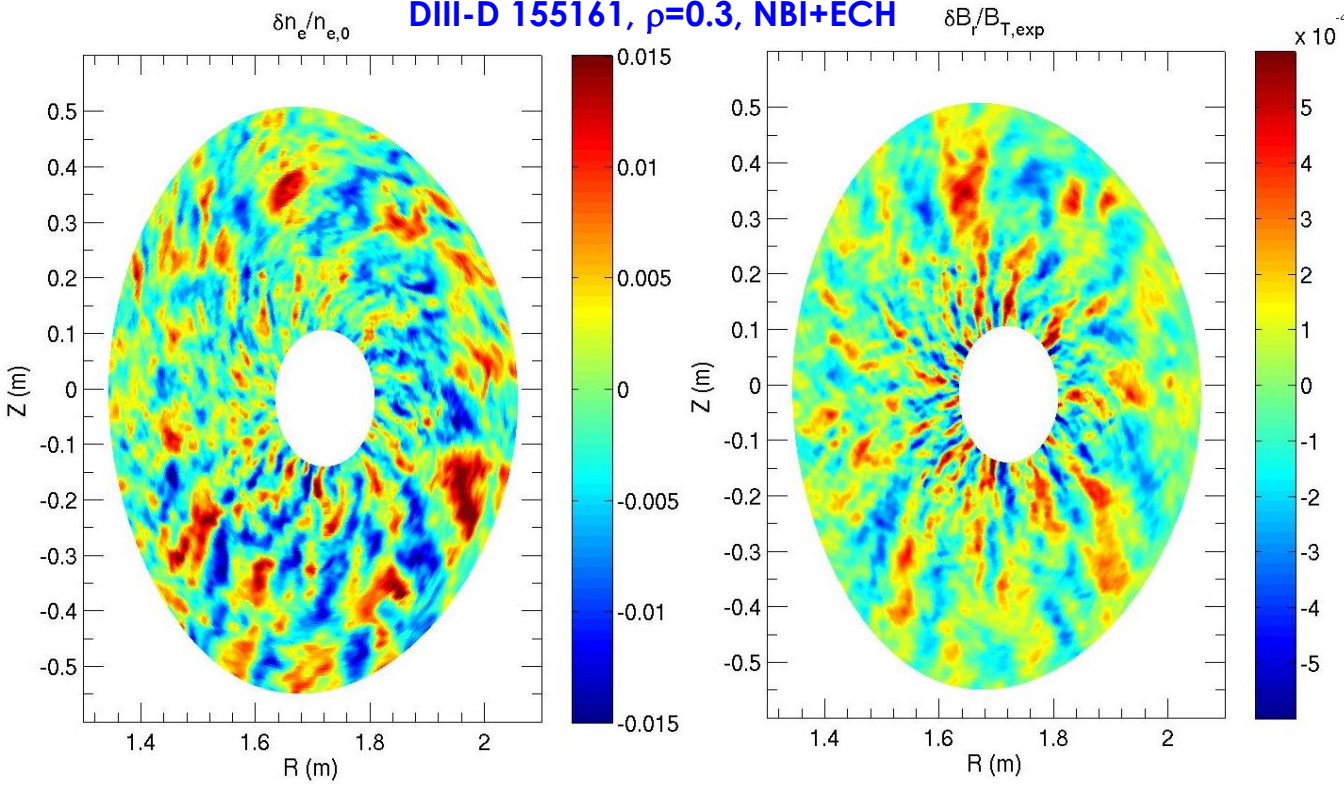
Guttenfelder, APS-DPP (2015)



# Nonlinear gyrokinetic simulations predict $\delta B/B_0 \sim 1-2 \times 10^{-4}$

- $\delta B \sim 3-5$  Gauss
- $(\delta B/B_0) / (\delta n/n_0)$  similar to quasilinear ratio  $\rightarrow$  useful for scoping (next section)

nonlinear GYRO simulations  
 DIII-D 155161,  $\rho=0.3$ , NBI+ECH

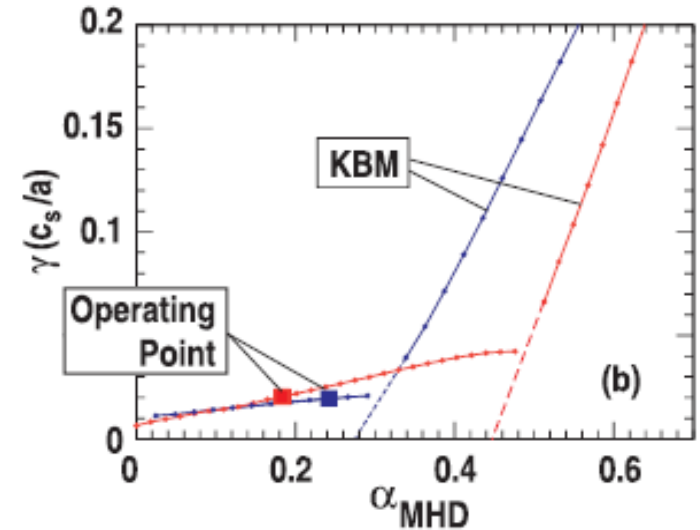
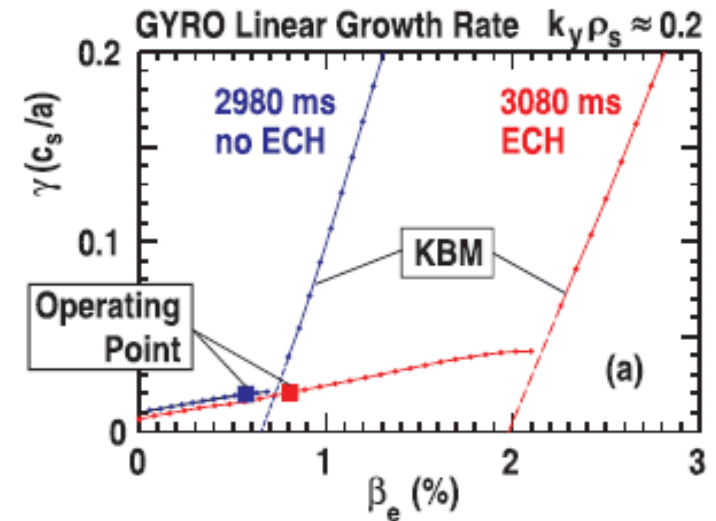


# Strength of EM stabilization consistent with local proximity to KBM threshold

- Theory [7] predicts EM stabilization strengthens as local pressure gradient ( $\alpha = -q^2 \cdot R \nabla P_{\text{tot}} \cdot 2\mu_0 / B^2$ ) approaches the KBM limit ( $\alpha_{\text{crit}}$ )
- In GYRO-normalized units:

$$\alpha_{\text{GYRO}} = q^2 \left( \frac{R}{a} \right) \beta_e \sum_s \left[ \frac{n_s}{n_e} \frac{T_s}{T_e} \left( \frac{a}{L_{\text{ns}}} + \frac{a}{L_{\text{Ts}}} \right) \right]$$

- $\beta_e$  scan used to identify KBM linear threshold
  - Does not account for profile changes
- As a function of  $\alpha$  (including profile changes):
- NBI-only case,  $\alpha$  within  $\sim 15\%$  of  $\alpha_{\text{crit}} \rightarrow$  strong EM stabilization (previous slides)
- ECH case has lower  $\alpha/\alpha_{\text{crit}}$  due to larger  $\alpha_{\text{crit}} \rightarrow$  weak EM stabilization (not shown)

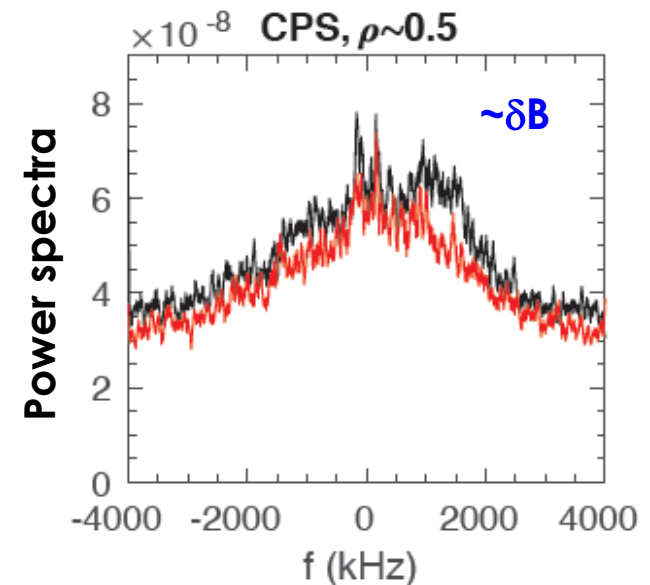
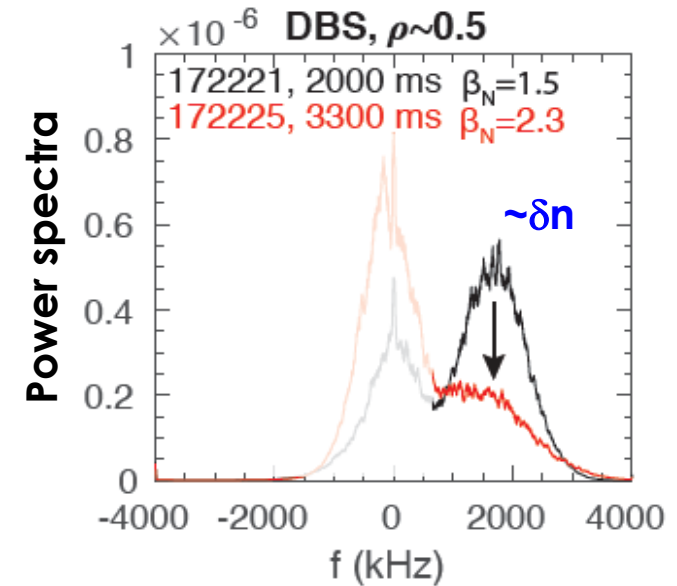
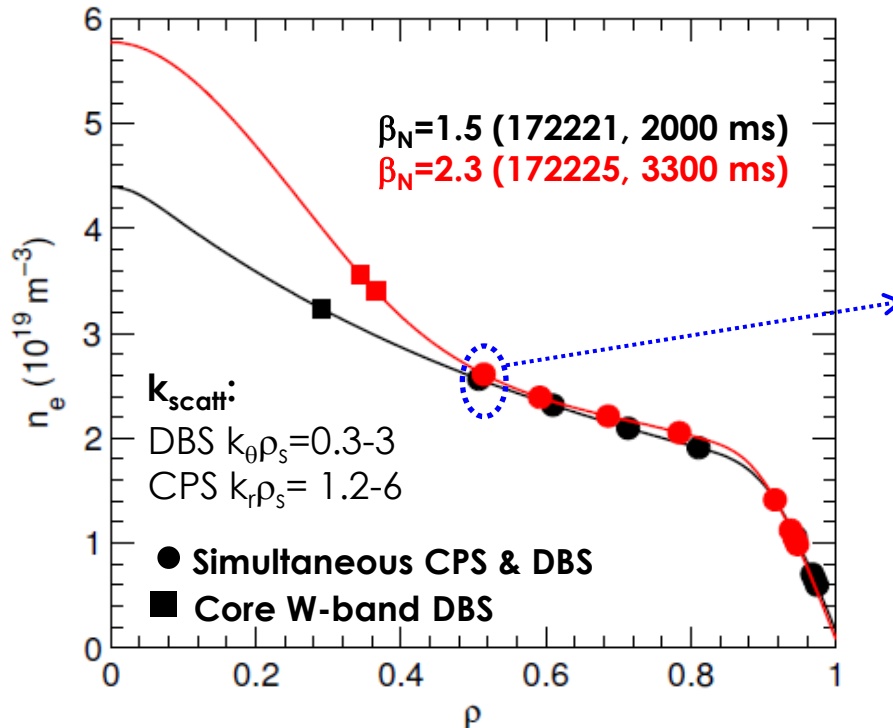


Ernst, PoP (2016)

# Using Doppler backscattering (DBS $\sim \delta n$ ) and cross polarization scattering (CPS $\sim \delta B$ ) to measure core EM turbulence

- Increase of CPS/DBS amplitude ratio ( $\sim \delta B / \delta n$ ) with  $\beta$  consistent with expectations

$\Rightarrow$  Requires ray tracing, gyrokinetic simulations + synthetic diagnostics to thoroughly validate

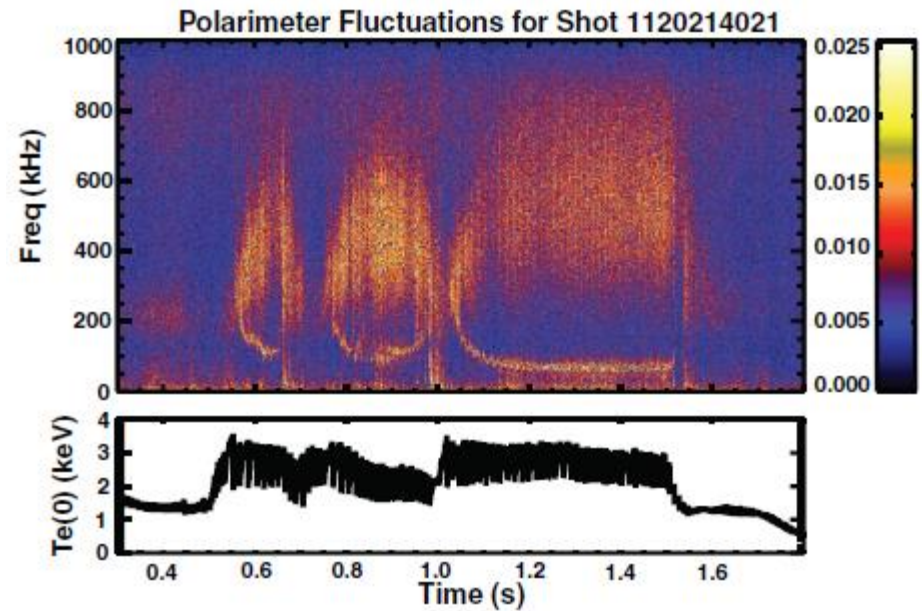
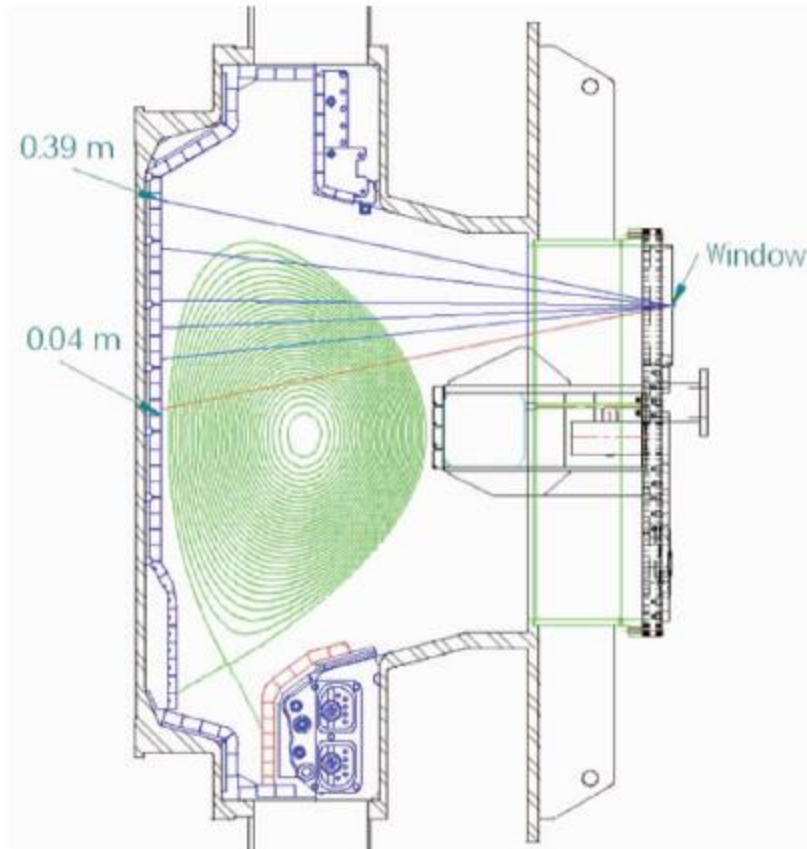


## Stabilization of ITG from coupling to dB at high beta + FI

- Proximity of local profiles to KBM/BAE stability limit
- Provides increase in predicted  $T_i$
- Potentially beneficial for deep core of burning plasmas
- CPS stuff here (and Barada, Rhodes)

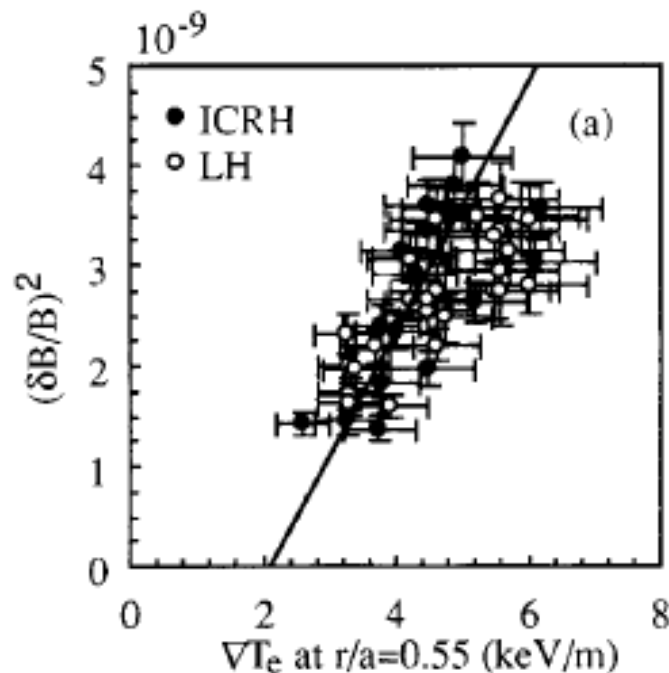
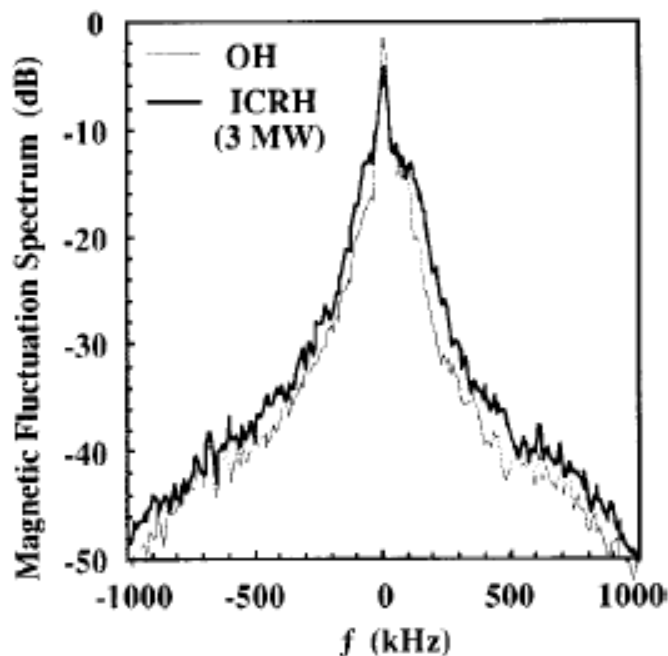
# Polarimetry on C-Mod has observed broadband high frequency polarization fluctuations

- Requires careful interpretation to separate  $\delta n_e$  and  $\delta B$  influence



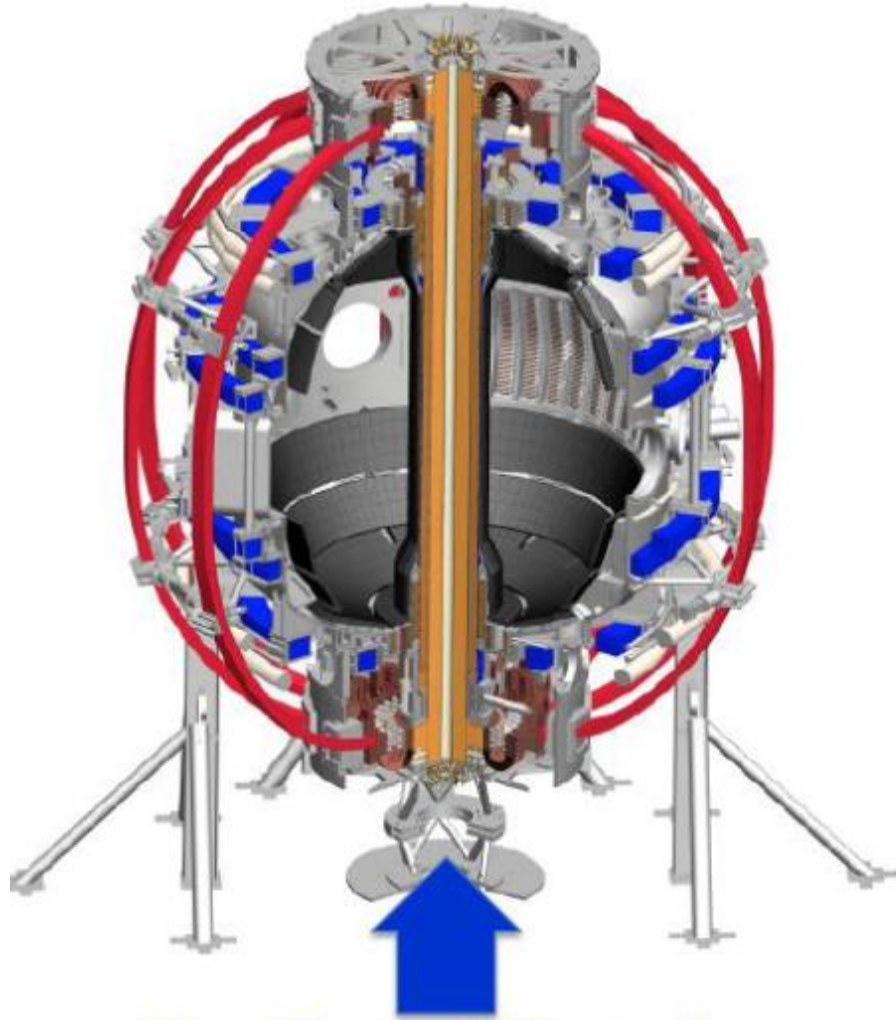
# Cross polarization scattering used on Tore Supra to measure internal magnetic fluctuations

- Broad  $\delta B$  frequency spectra
- Correlation between  $\delta B/B$  increasing with local  $\nabla T_e$
- However, require additional measurements/simulations to determine whether  $\delta B$  due to
  - $j_{\parallel}$  from predominantly electrostatic turbulence (Callen PRL 1977)
  - fundamentally different turbulence (e.g. microtearing)



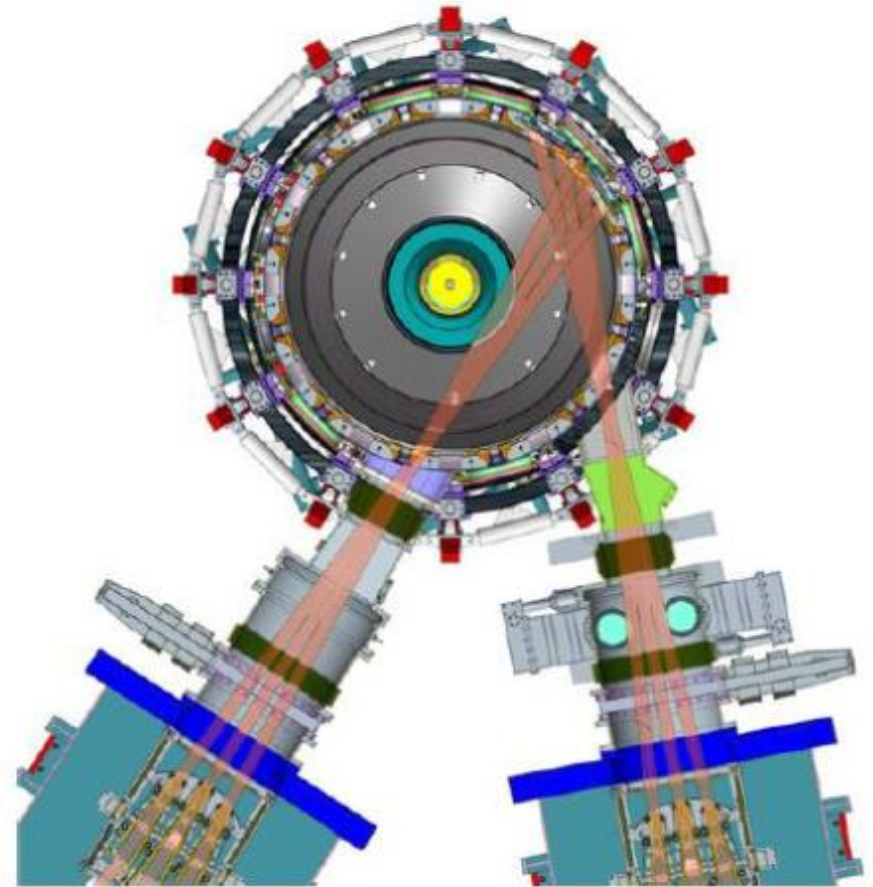
**“PURE”  
ELECTROMAGNETIC  
TURBULENCE**

# But first, an aside on low aspect ratio “spherical” tokamaks, like NSTX-U at PPPL



**New Central Magnet**

1 Tesla at plasma center,  $I_p = 2\text{MA}$ , 5s



**Original NBI**

( $R_{\text{TAN}} = 50, 60, 70\text{cm}$ )  
5MW, 5s, 80keV

**New 2<sup>nd</sup> NBI**

( $R_{\text{TAN}} = 110, 120, 130\text{cm}$ )  
5MW, 5s, 80keV

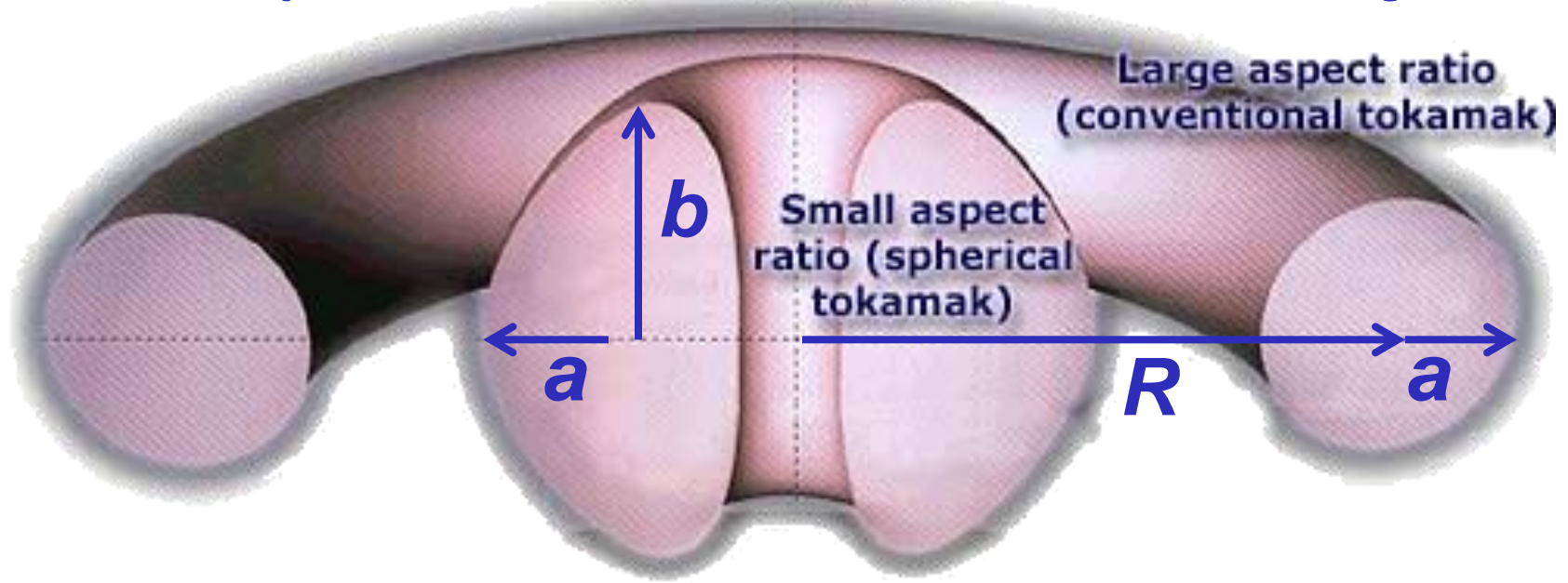


# Aspect ratio is an important free parameter, enables higher beta, more compact devices

*Aspect ratio  $A = R / a$*

*Elongation  $\kappa = b / a$*

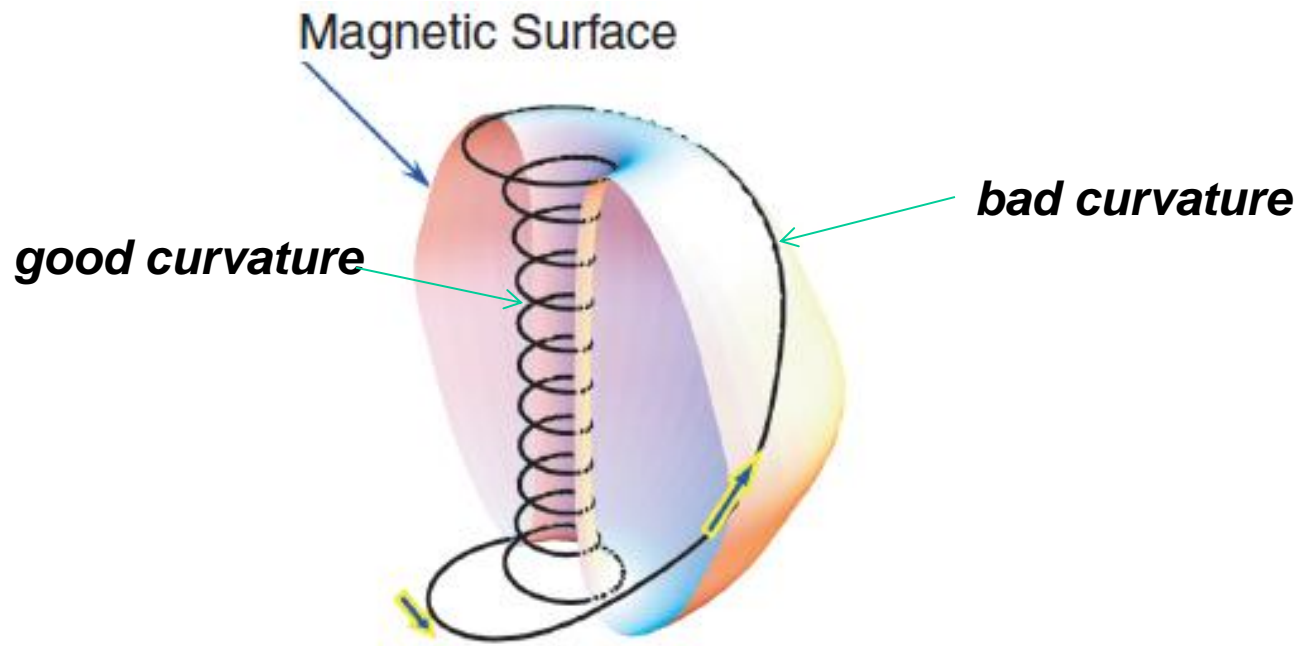
*$R =$  major radius,  $a =$  minor radius,  $b =$  vertical  $\frac{1}{2}$  height*



*But smaller  $R =$  larger curvature,  $\nabla B (\sim 1/R)$  -- isn't this terrible for "bad curvature" driven instabilities?!?!?!*

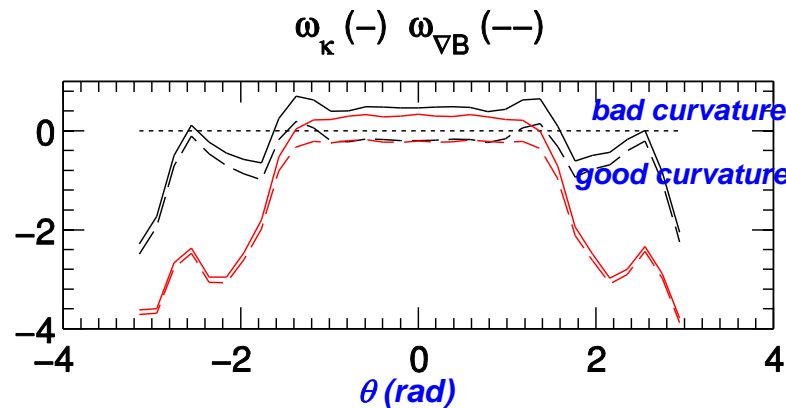
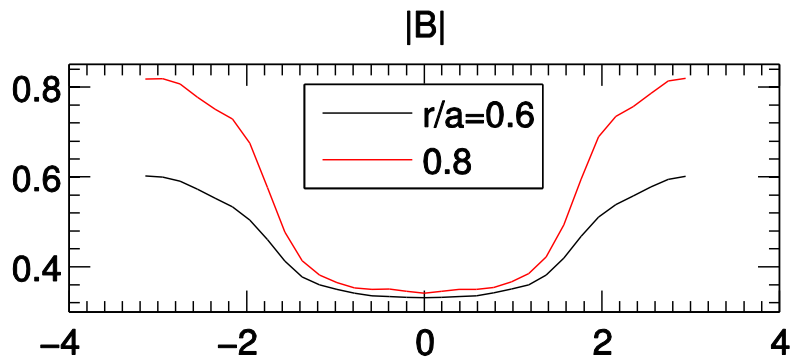
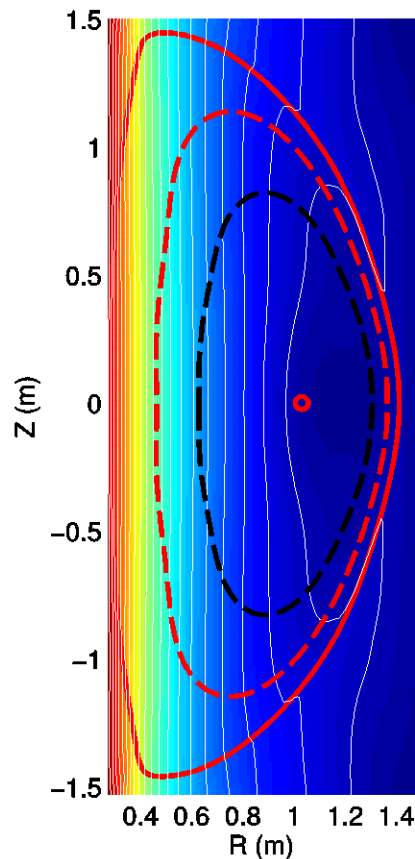
# Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

- Short connection length → **smaller average bad curvature**



# Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

- Short connection length → **smaller average bad curvature**
- Quasi-isodynamic ( $\sim$ constant B) at high  $\beta$  → **grad-B drifts stabilizing [Peng & Strickler, NF 1986]**

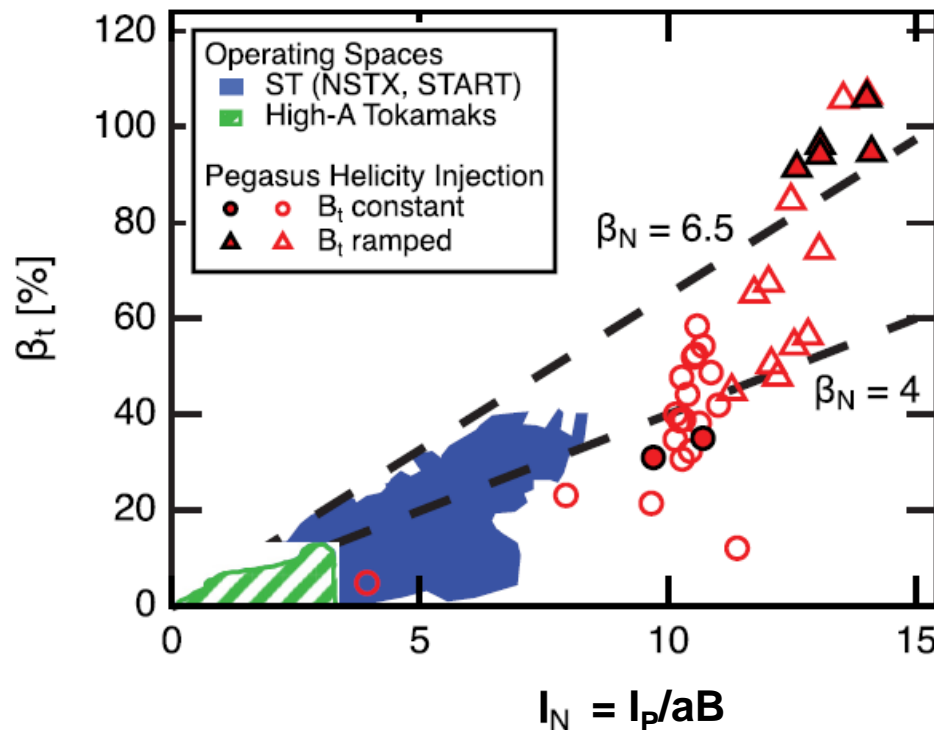


$$\vec{v}_\kappa = mv_\parallel^2 \frac{\hat{b} \times \vec{\kappa}}{qB}$$

$$\vec{v}_{\nabla B} = \frac{mv_\perp^2}{2} \frac{\hat{b} \times \nabla B / B}{qB}$$

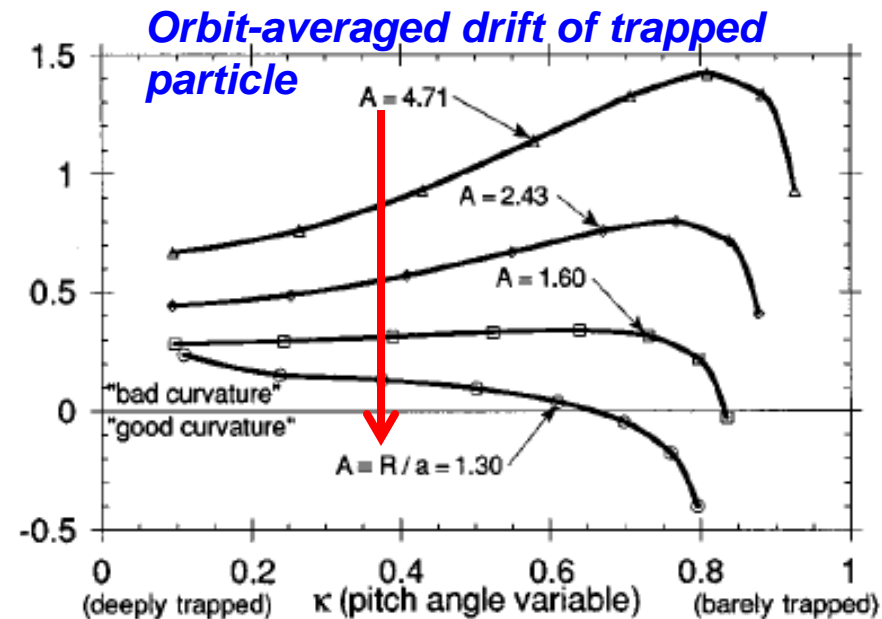
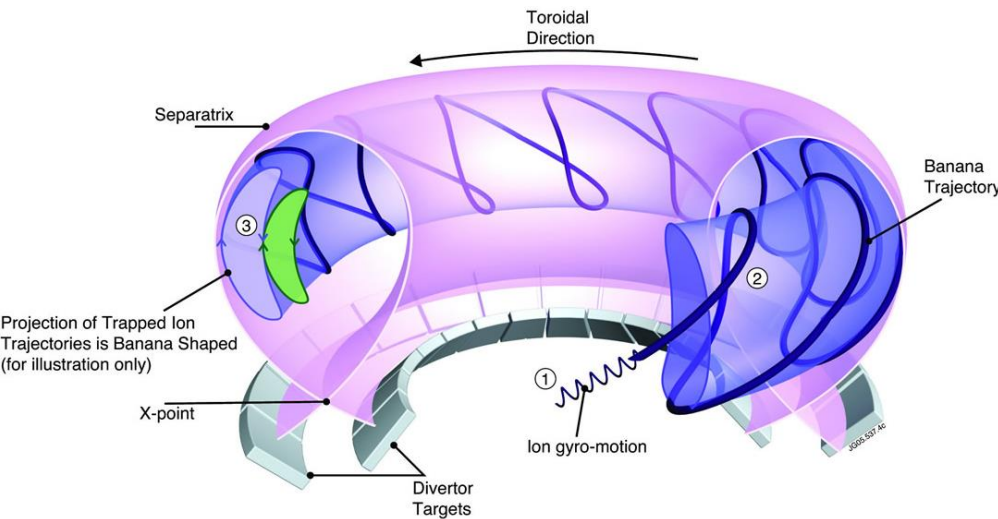
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- **These same features stabilize macroinstabilities (MHD), allowing for very high  $\beta$  equilibrium:  $\sim 40\%$  on NSTX,  $\sim 100\%$  on Pegasus (U-Wisc)**



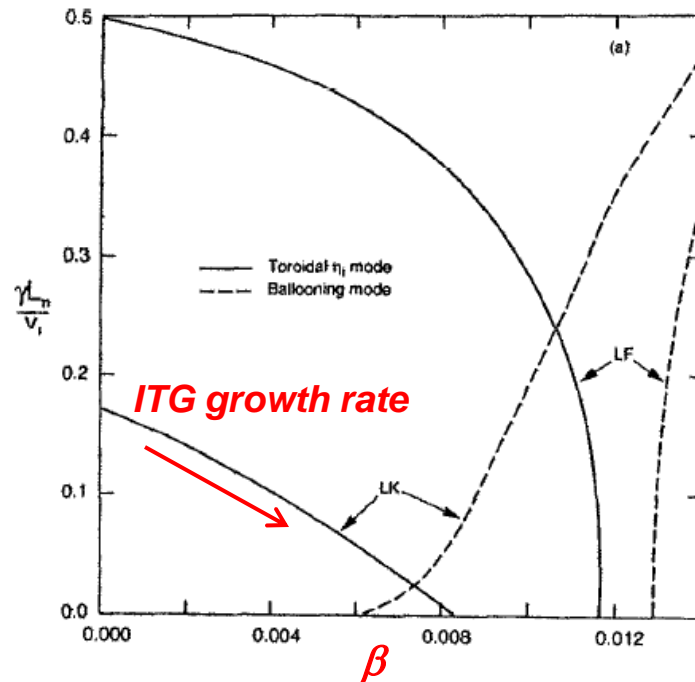
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- Large fraction of trapped electrons, BUT precession weaker at low A → **reduced TEM drive [Rewoldt, Phys. Plasmas 1996]**



# Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

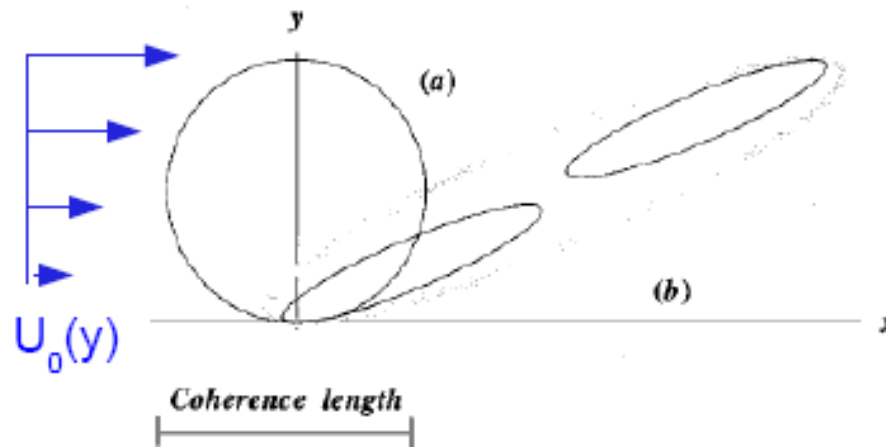
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- Strong coupling to  $\delta B_{\perp} \sim \delta A_{\parallel}$  at high  $\beta$  → **stabilizing to ES-ITG**



Kim, Horton, Dong, PoFB (1993)

# Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

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- Small inertia ( $nm\bar{R}^2$ ) with uni-directional NBI heating gives strong toroidal flow & flow shear →  **$\mathbf{E} \times \mathbf{B}$  shear stabilization ( $dv_{\perp}/dr$ )**



*Biglari, Diamond, Terry, PoFB  
(1990)*

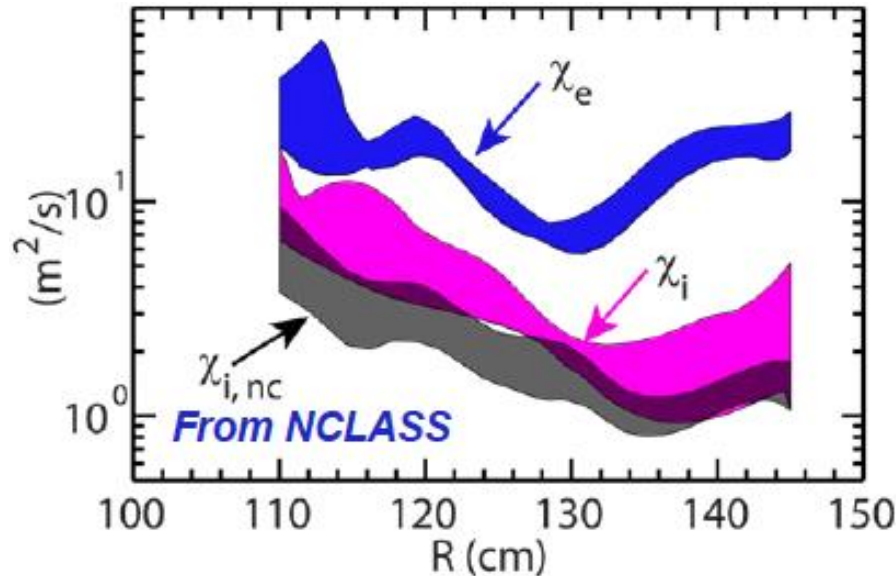
# Many elements of ST are stabilizing to toroidal, electrostatic ITG/TEM drift waves

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  - Small inertia ( $nmR^2$ ) with uni-directional NBI heating gives strong toroidal flow & flow shear →  **$E \times B$  shear stabilization ( $dv_{\perp}/dr$ )**
- ⇒ **Not expecting strong ES ITG/TEM instability (much higher thresholds)**

- BUT High beta drives EM instabilities:
  - **microtearing modes (MTM)**  $\sim \beta_e \cdot \nabla T_e$
  - **kinetic ballooning modes/energetic particle modes (KBM/EPM)**  $\sim \alpha_{\text{MHD}} \sim q^2 \nabla P / B^2$  &  $\nabla P_{\text{fast}}$
- Large shear in parallel velocity can drive **Kelvin-Helmholtz-like instability**  $\sim dv_{\parallel}/dr$



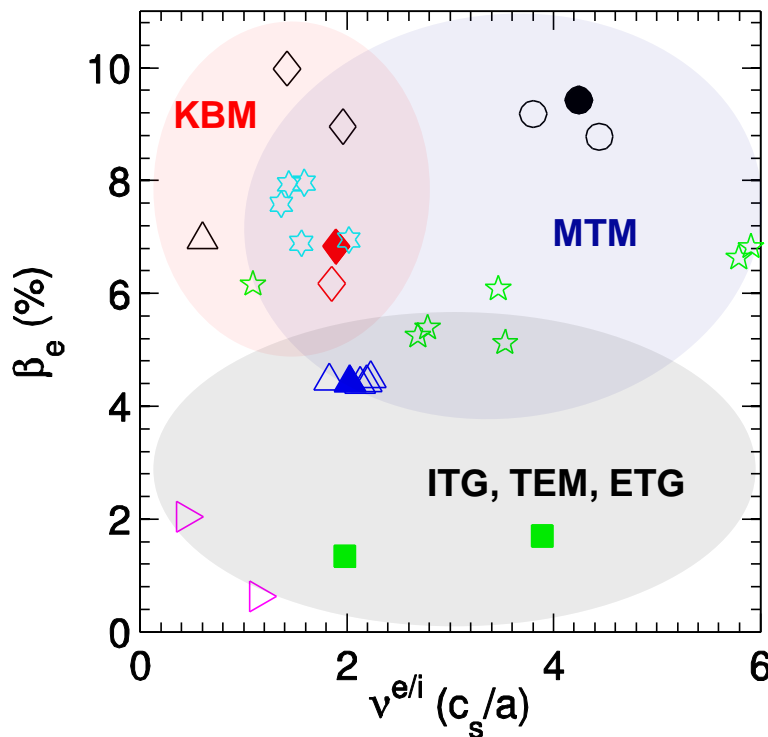
# Ion thermal transport in ST H-modes (higher beta) usually very close to collisional (neoclassical) transport theory



- Consistent with ITG/TEM stabilization by equilibrium configuration & strong  $E \times B$  flow shear
  - Impurity transport (intrinsic carbon, injected Ne, ...) also usually well described by neoclassical theory [Delgado-Aparicio, NF 2009 & 2011 ; Scotti, NF 2013]
- **Electron energy transport always anomalous**
  - Toroidal angular momentum transport also anomalous (Kaye, NF 2009)

# Predicted dominant core-gradient instability correlated with local beta and collisionality

- For sufficiently small  $\beta$ , ES instabilities can still exist (ITG, TEM, ETG)
- At increasing  $\beta$ , MTM and KBM are predicted  $\rightarrow$  depending on  $\nu$ 
  - Various instabilities often predicted in the same discharge – global, nonlinear EM theory & predictions will hopefully simplify interpretation (*under development*)



## NSTX

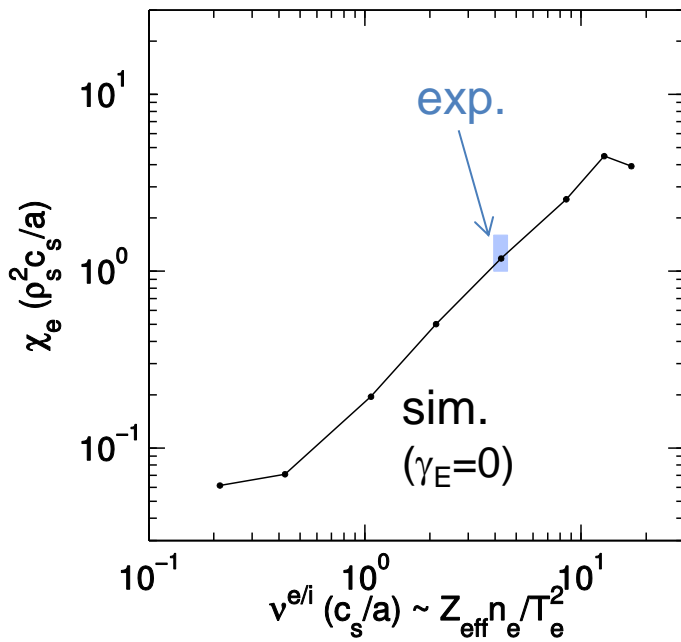
Local gyrokinetic analyses at  $\sim 2/3$  radius

Guttenfelder, NF (2013)

# Simulations of core microtearing mode (MTM) turbulence predict significant transport at high $\beta$ & $\nu$

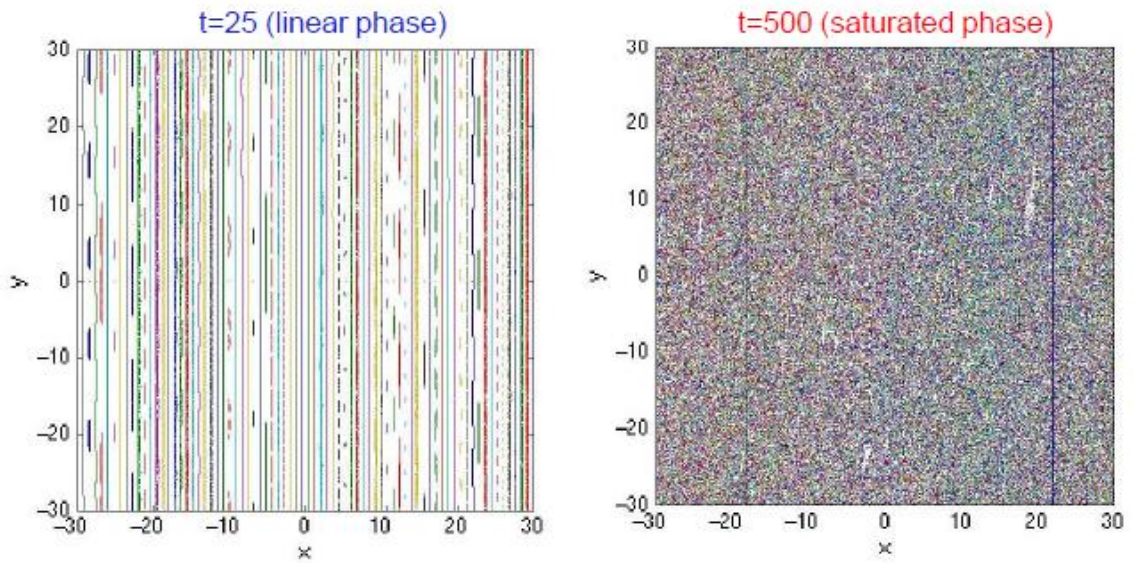
- Collisionality scaling ( $\chi_{e,MTM} \sim \nu_e$ ) consistent with global confinement ( $\tau_E \sim 1/\nu$ ), follows linear stability trends:
  - In the core, driven by  $\nabla T_e$  with time-dependent thermal force (e.g. Hassam, 1980)
  - *Requires collisionality*  $\rightarrow$  **not explicitly driven by bad-curvature**
- $\delta B$  leads to flutter transport ( $\sim \nu_{||} \cdot \delta B^2$ ) consistent with stochastic transport

Predicted transport



Guttenfelder, PRL (2011), PoP (2012)

Poincare plots of flux-tube surfaces

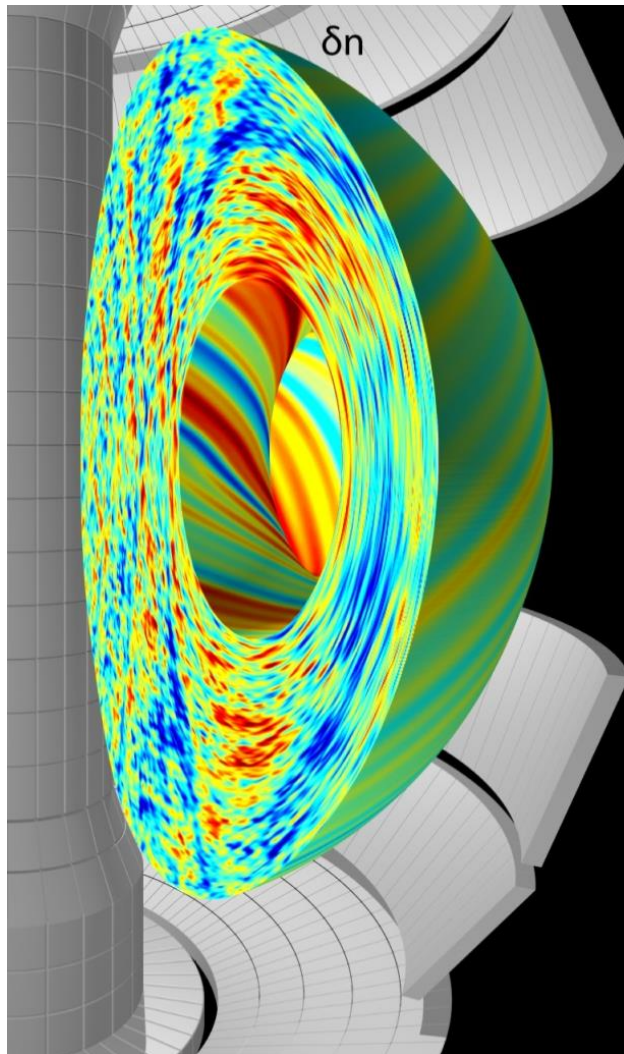


**NSTX**

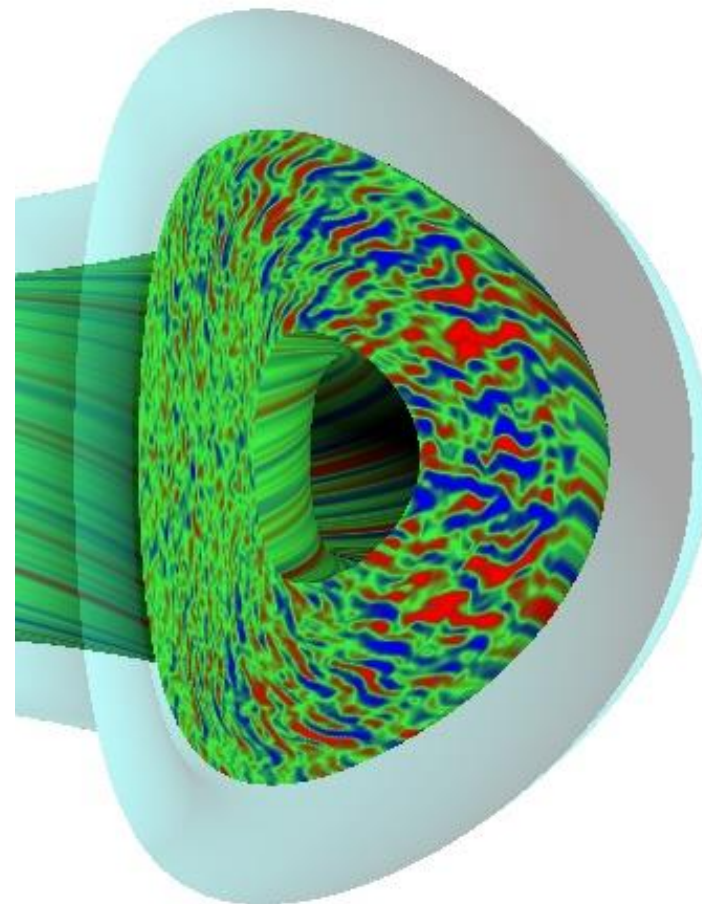
E. Wang, PoP (2011)

# MTM density fluctuations distinct from ballooning modes like ITG (simulations)

**NSTX MTM turbulence**

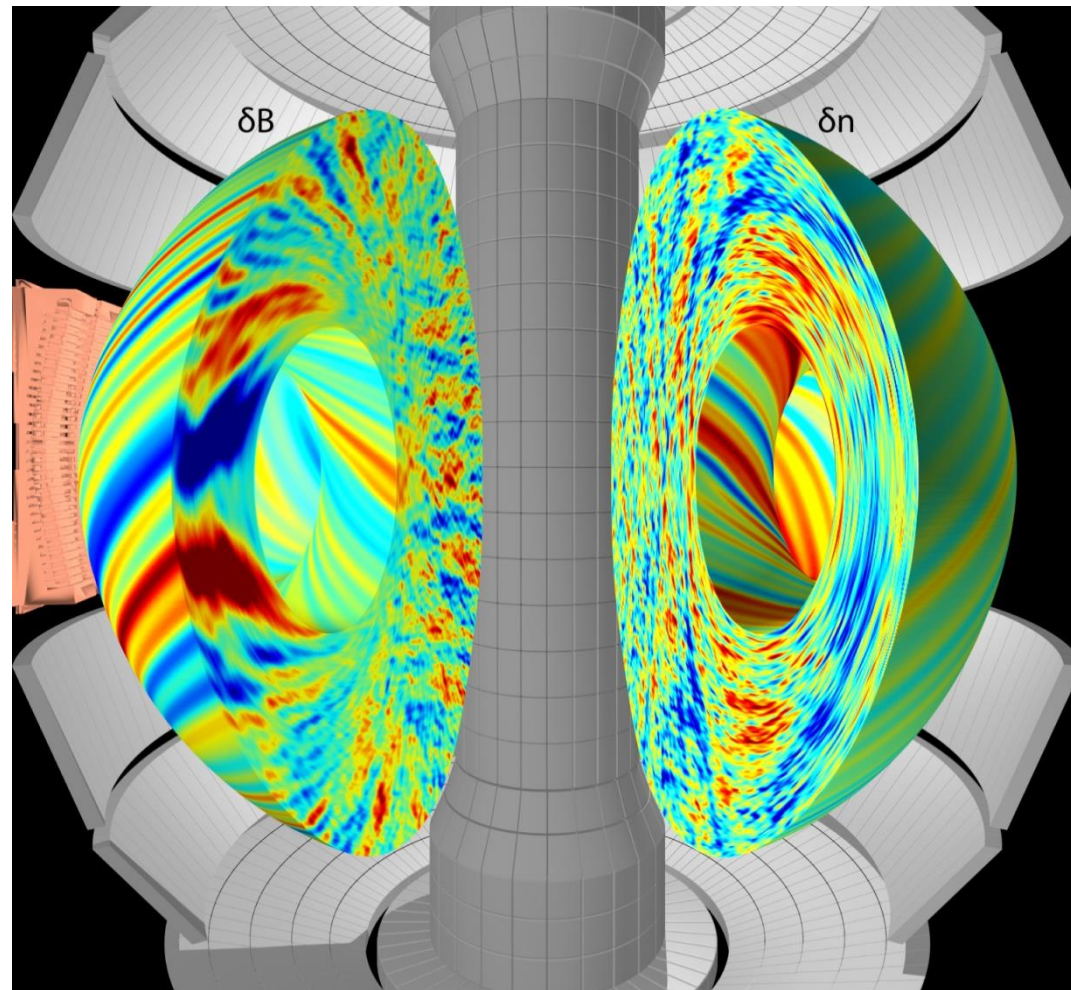


**DIII-D ITG turbulence**



# MTM structure distinct from ballooning modes

## Predictions from MTM simulation



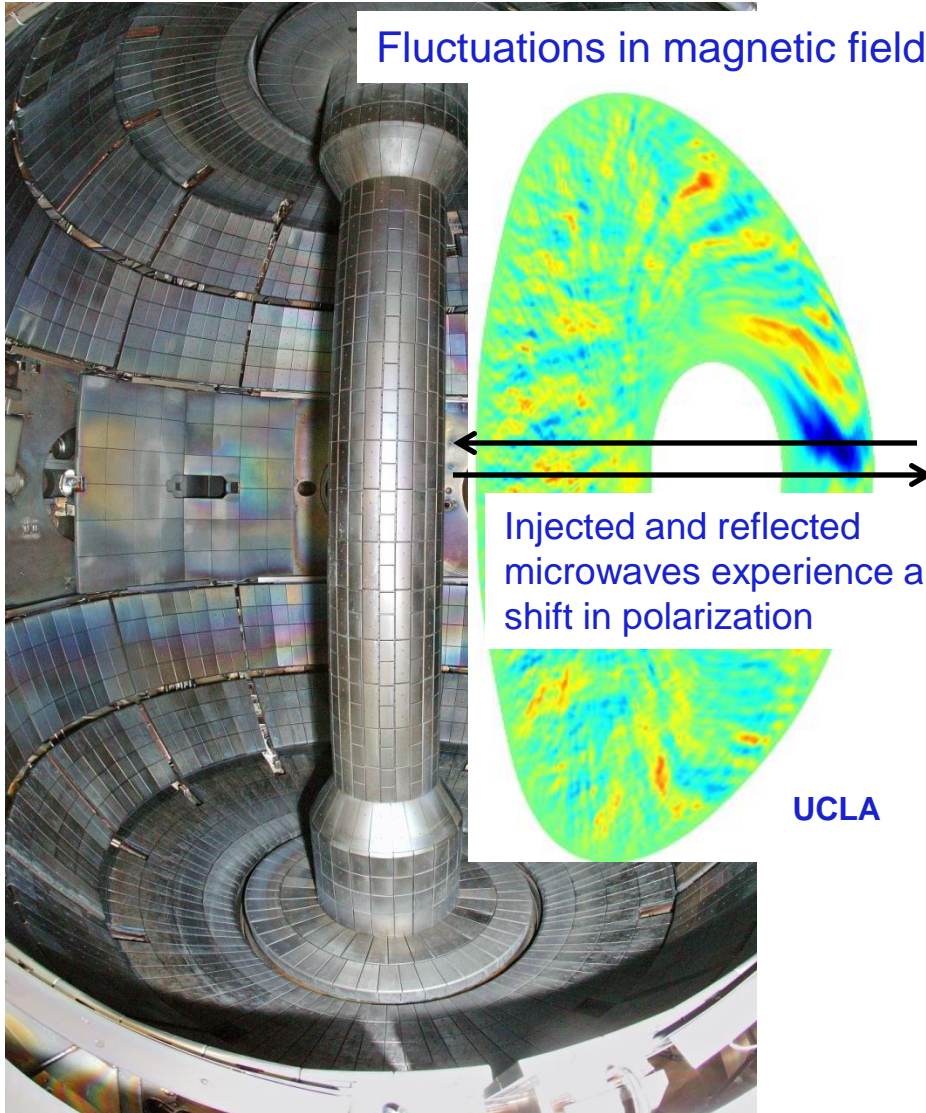
- Narrow density perturbations due to high- $m$  tearing mode around rational surfaces  $q=m/n$ 
  - Potential to validate with beam emission spectroscopy (BES) imaging [Smith, RSI (2012)]
- Large  $\delta B/B \sim 10^{-3}$ 
  - Potential for internal  $\delta B$  measurements via Cross Polarization Scattering, CPS (UCLA collaboration)  $\Rightarrow$  **focus of a 2017 DIII-D National Campaign experiment**

Visualization courtesy F. Scotti (LLNL)

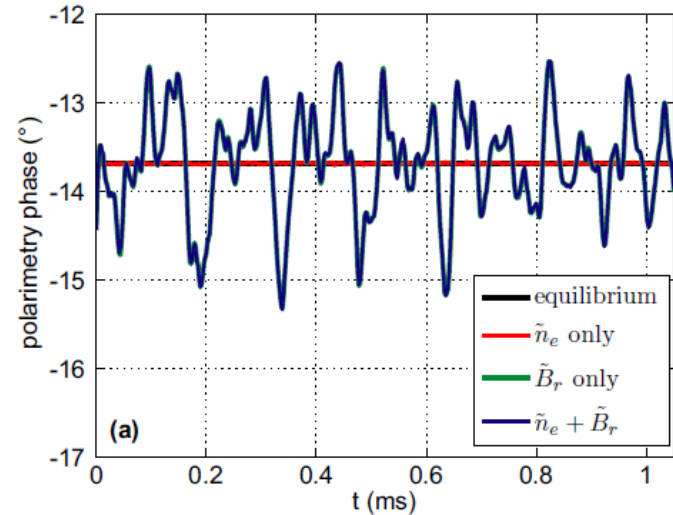
# Very challenging to measure internal magnetic fluctuations

NSTX (PPPL)

Fluctuations in magnetic field



- Synthetic diagnostic calculations predict polarimetry could be sensitive

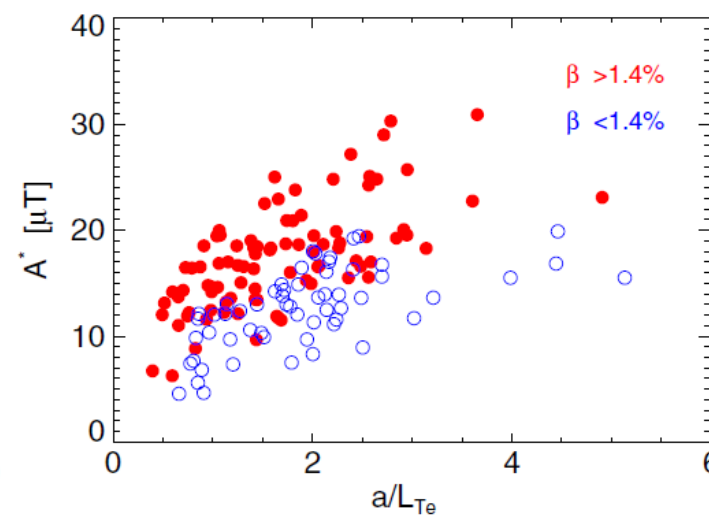
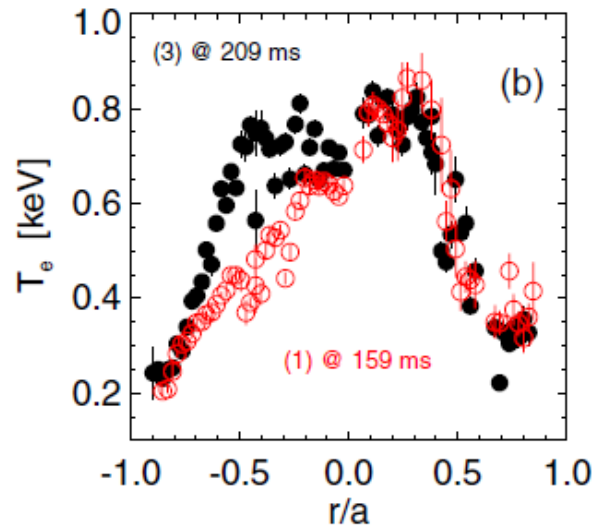
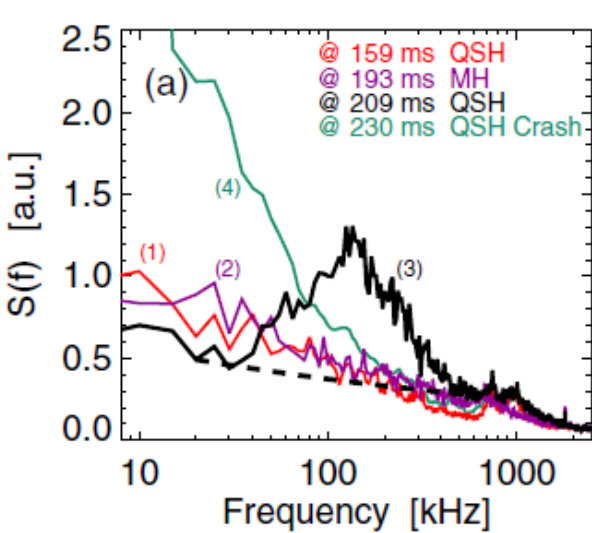


Zhang, PPCF (2013)  
Guttenfelder, PRL (2011)

- Will try to validate using CPS (UCLA) on NSTX-U

# Inference of microtearing turbulence via magnetic probes in RFX reversed field pinch (Zuin, PRL 2013)

- Used internal array of closely spaced ( $\sim$ wavenumber resolved) high frequency Mirnov coils ( $\sim$ dB/dt) mounted near vacuum vessel wall
- Confinement and  $T_e$  increase during “quasi-single helicity” (QSH) state  $\rightarrow$  broadband  $\delta B$  measured (3 below left)
- $\delta B$  amplitude increases with  $a/L_{Te}$  &  $\beta$  (expected for MTM)
- Measured frequency and mode numbers ( $n,m$ ) align with linear gyrokinetic predictions of MTM



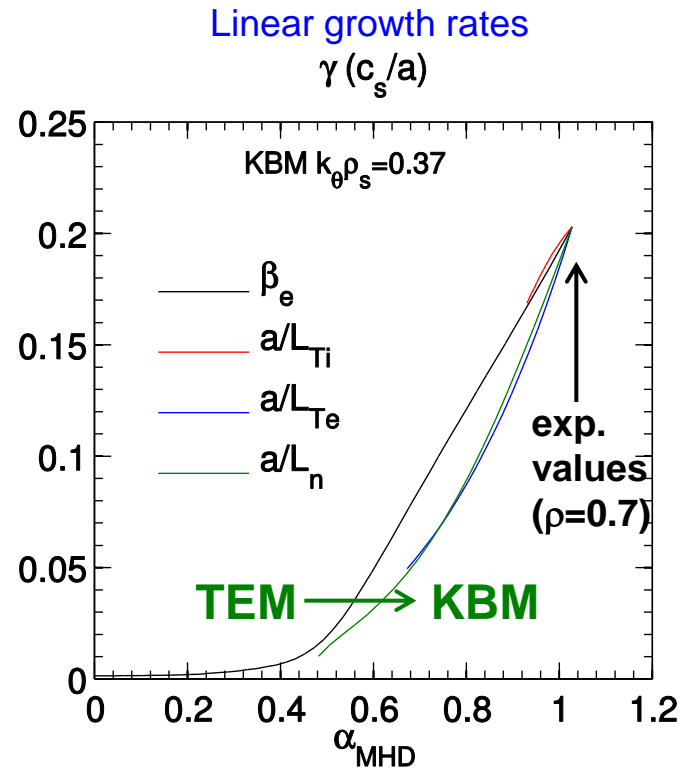
- Additional MTM inferences using novel heavy ion beam probe technique (internal, non-perturbative) in JIPPT-IIU tokamak (Hamada, NF 2015)

# Core KBM (NSTX, high beta<sub>pol</sub>)



# At high $\beta$ & lower $\nu$ , KBM modes predicted; Sensitive to compressional magnetic ( $B_{\parallel}$ ) perturbations

- Kinetic analogue of MHD high-n ballooning mode, driven by total  $\nabla P$  ( $\alpha_{\text{MHD}}$ )
- Smooth transition from ITG/TEM at reduced  $\nabla P$
- Transport has significant compressional component ( $\sim \delta B_{\parallel}$ )



**NSTX**

$$\alpha_{\text{MHD}} = -q^2 R \cdot 2\mu_0 \nabla P / B^2$$

Guttenfelder, NF (2013)

# ZONAL FLOWS, GAMs

(important elements 2D turbulence nonlinear saturation)

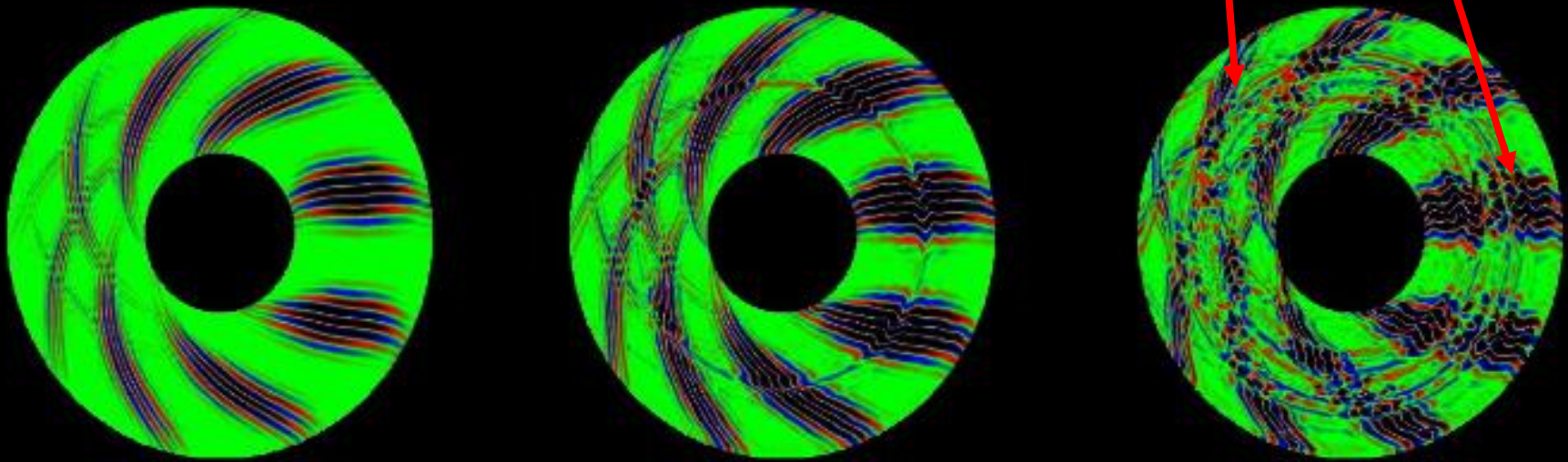
# Self-generated “zonal flows” impact saturation of turbulence and overall transport (roughly analogous to jet stream)

- Potential perturbations uniform on flux surfaces, near zero frequency ( $f \sim 0$ )
- Predator-prey like behavior: turbulence drives ZF, which regulates/clamps turbulence; if turbulence drops enough, ZF drive drops, allows turbulence to grow again...

Linear instability stage demonstrates structure of fastest growing modes

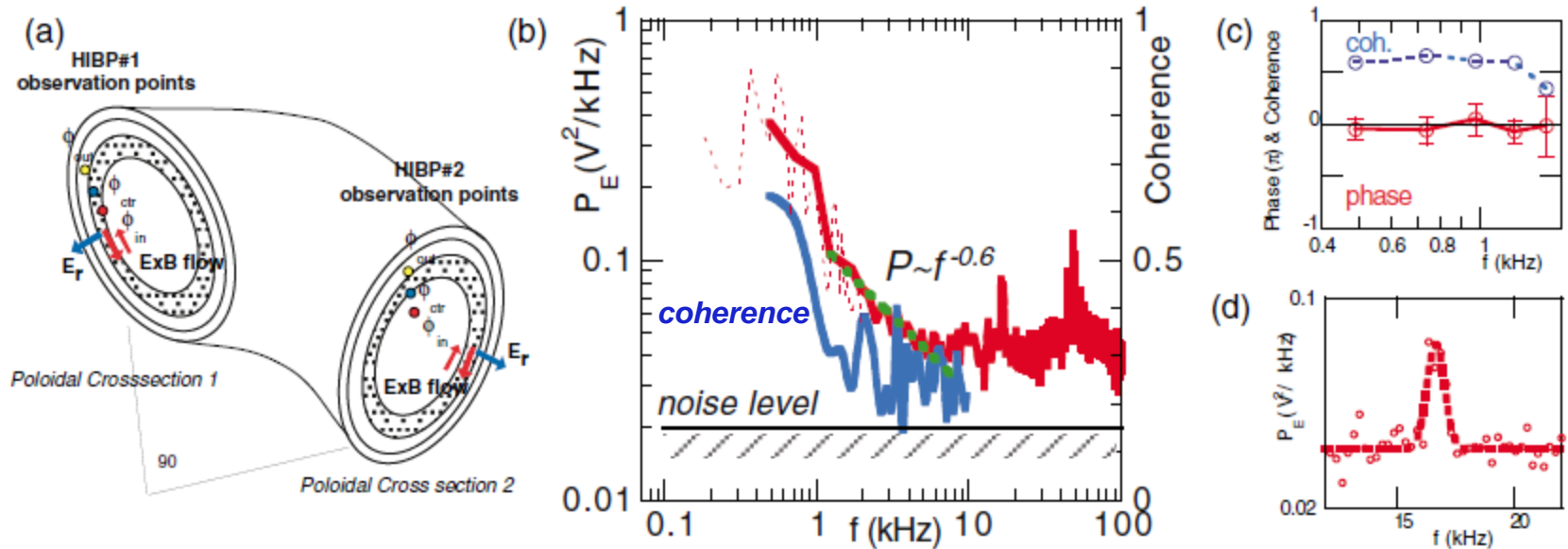
Large flow shear from instability cause perpendicular “zonal flows”

Zonal flows help moderate the turbulence!!!



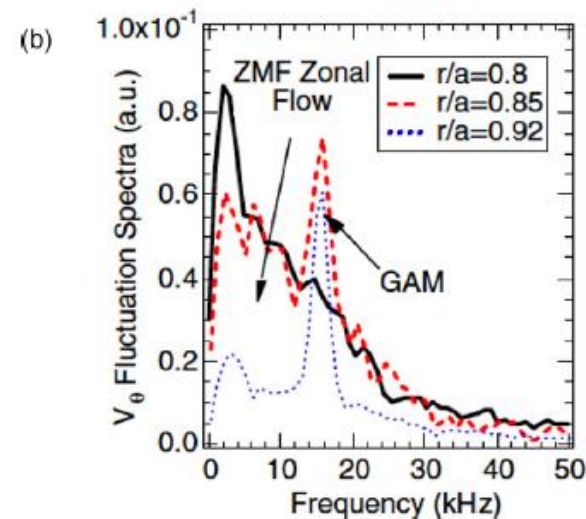
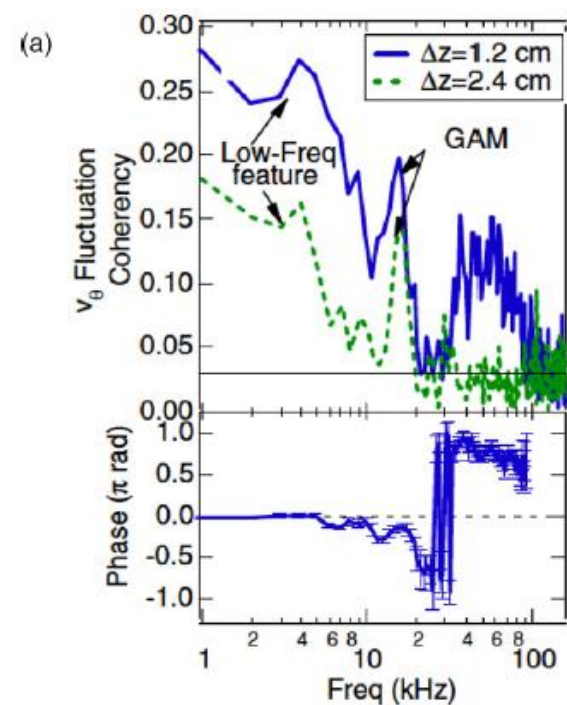
# Evidence of zonal flows from measuring potential on same flux surface at two different toroidal locations

- High coherency at very low frequency with zero phase shift suggests uniform zonal perturbation
- Also evidence of a coherent mode around 17 kHz - geodesic acoustic mode ( $\omega_{\text{GAM}} \approx c_s/R$ ) from associated  $n=0, m=1$  pressure perturbation



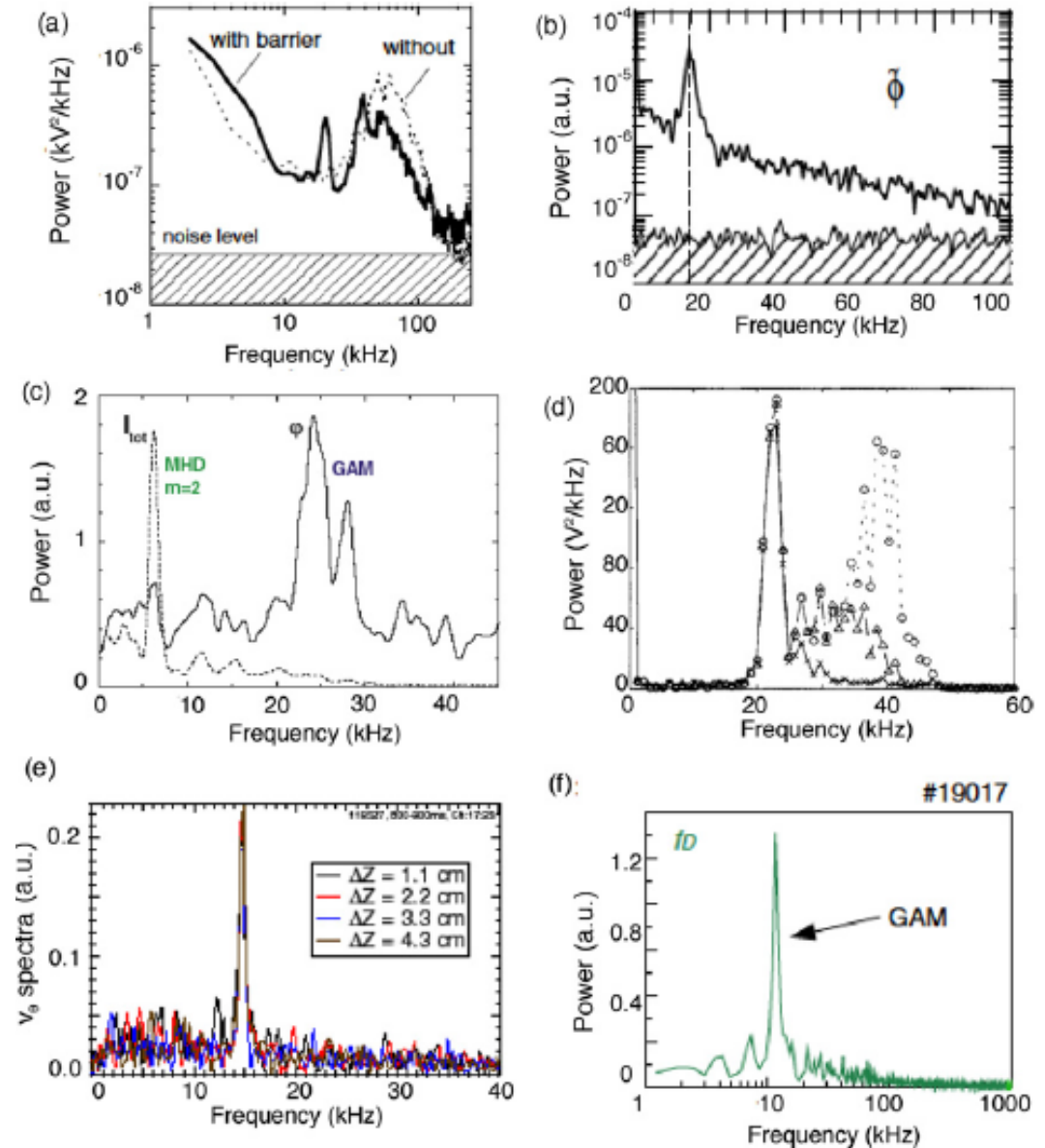
# Also found using poloidal flow measurements from BES on DIII-D

- Poloidal flow determined from time delay estimation of poloidally separated BES channels
- High coherency at low frequency, zero phase shift
- Evidence of GAM oscillation
- Relative strength of each varies with radius

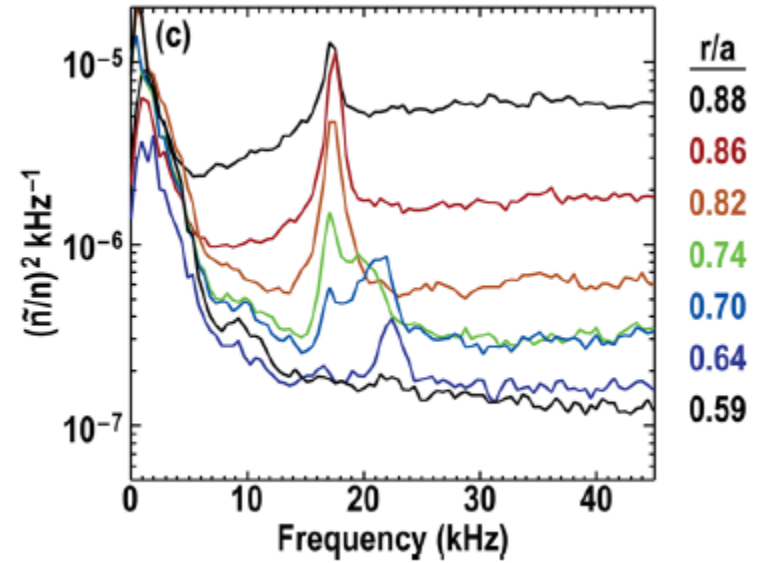
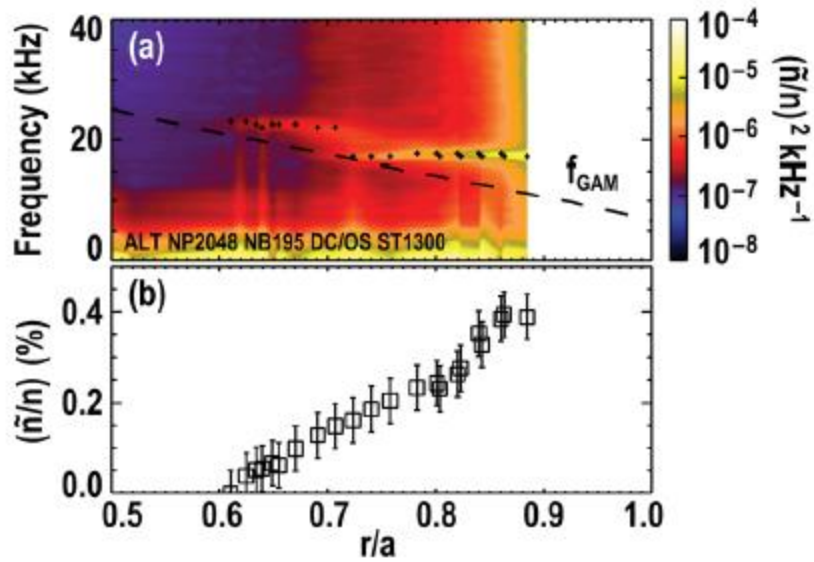


# GAM seen on numerous devices using different measurement techniques

- Seems to be in nearly all machines, if looked for
- See Fig. 11 of Fujisawa, Nuclear Fusion (2009) for legend

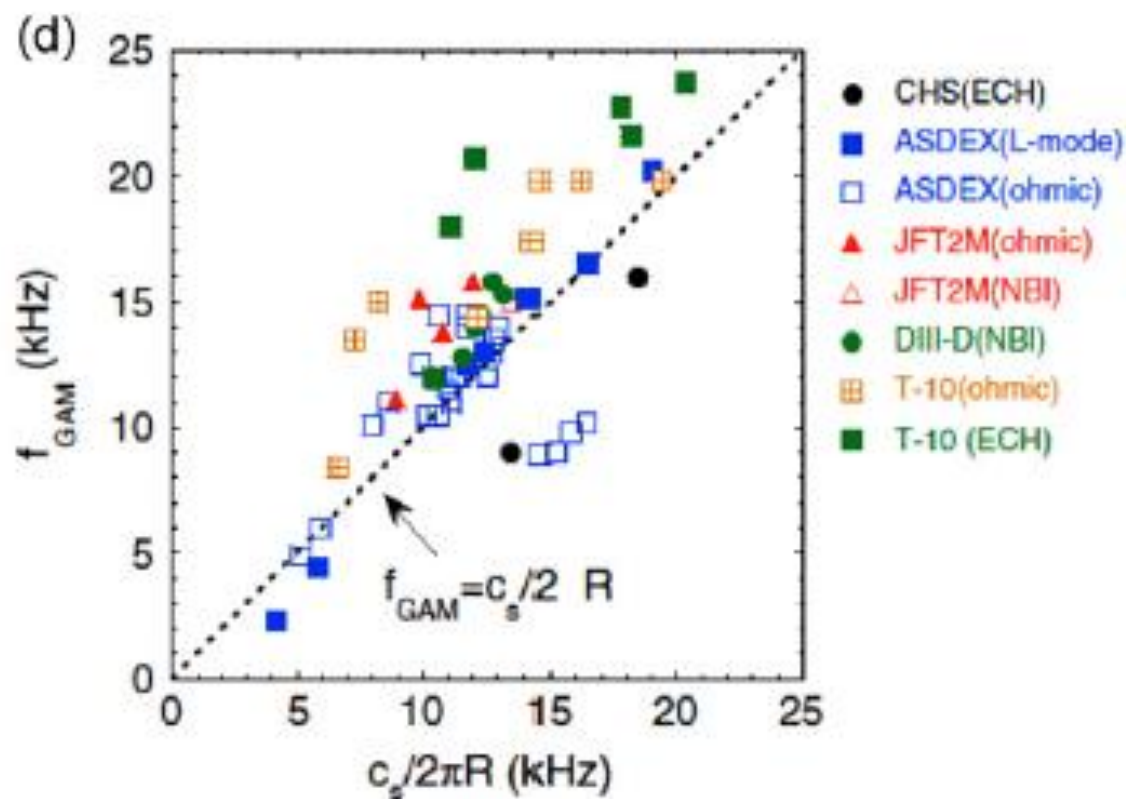


# Shafer (L-mode)



# Broad cross-machine agreement of GAM frequency with theory

- Discrepancies have spurred additional theory developments to refine gam frequency and damping rates (due to geometry, nonlinear effects, ...)



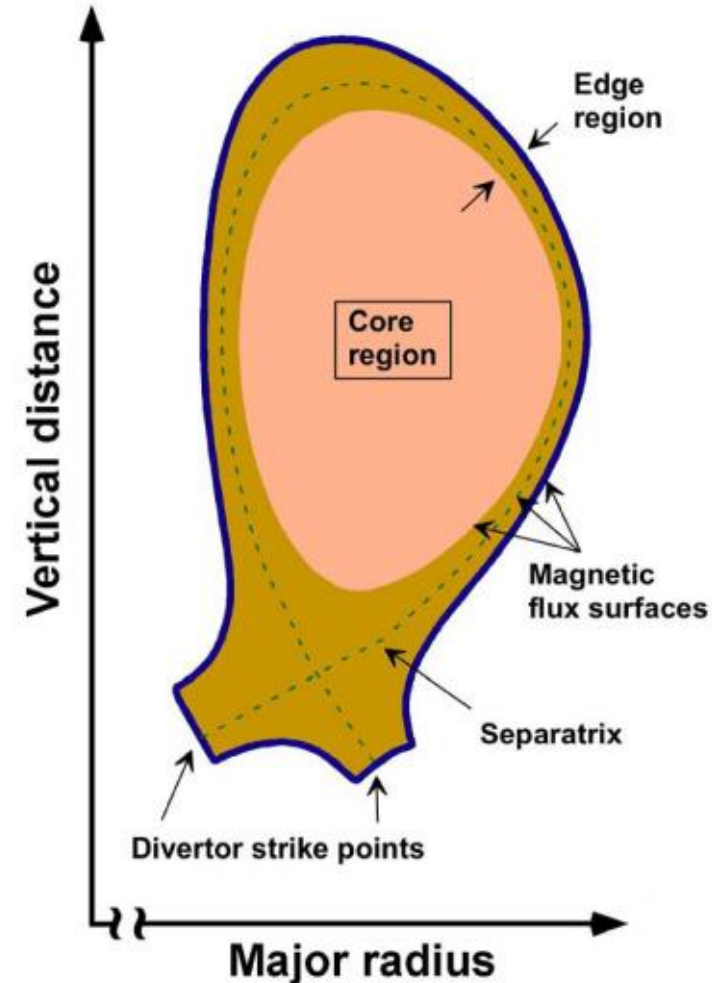
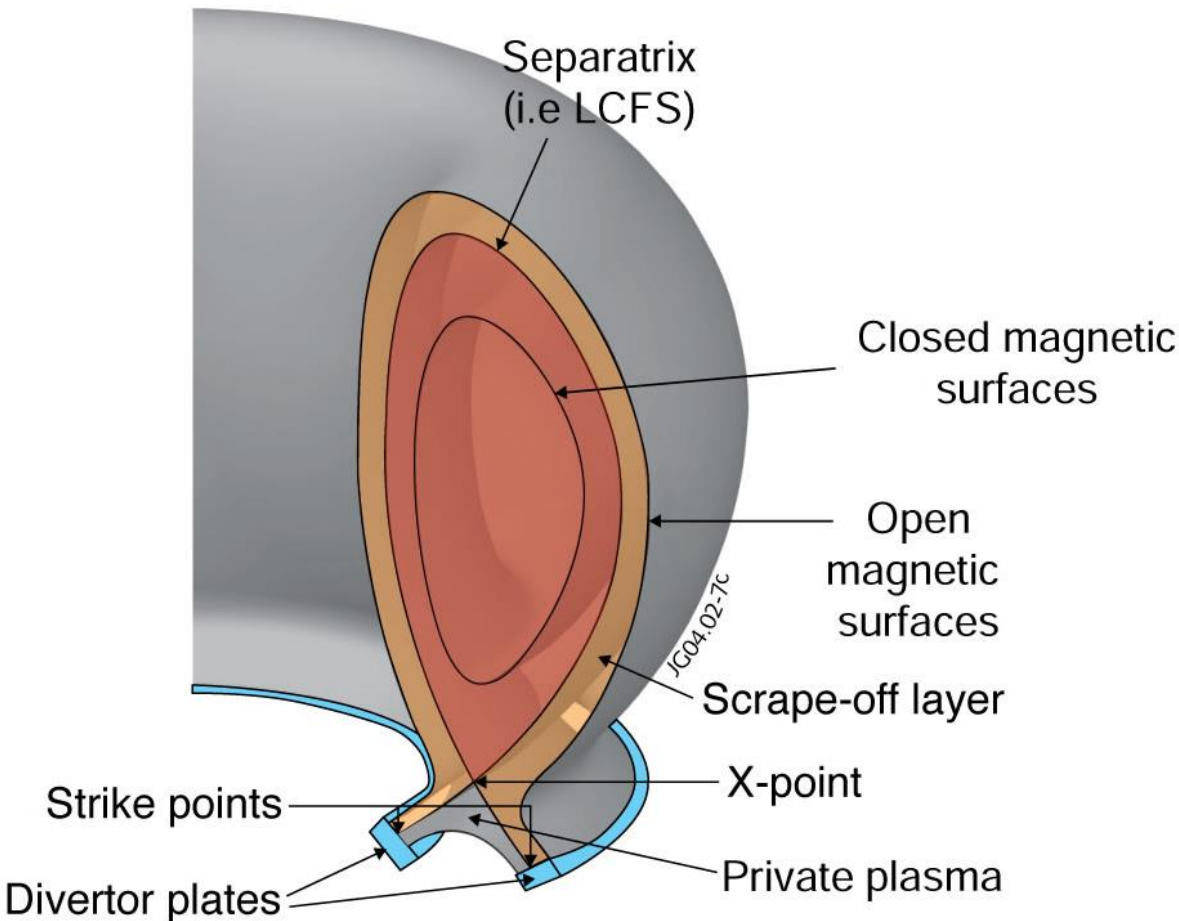


# Three-wave coupling, cascades, bispectrum measurements

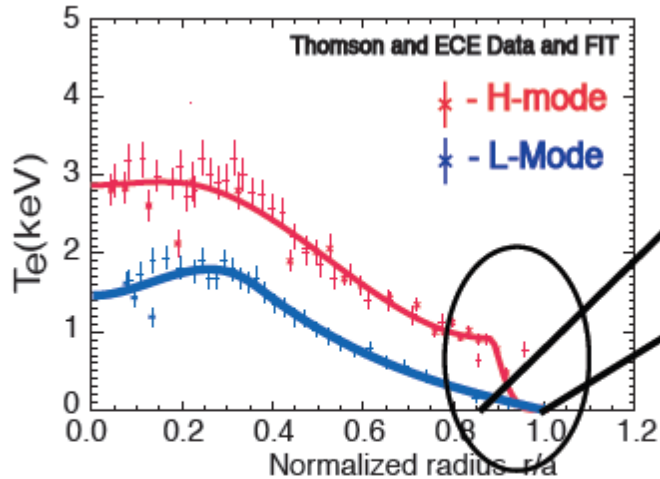
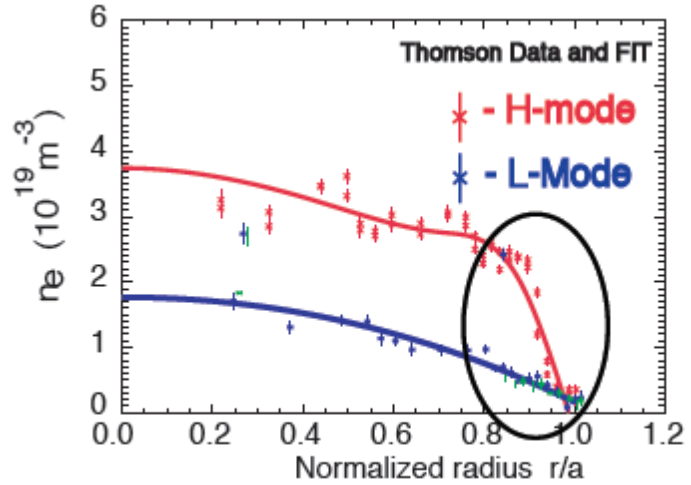
# **EDGE TURBULENCE L-H TRANSITION**

# Going to refer to different spatial regions in the tokamaks

- Especially **core**, **edge** (just inside separatrix), and **scrape-off layer** (SOL, just outside separatrix)

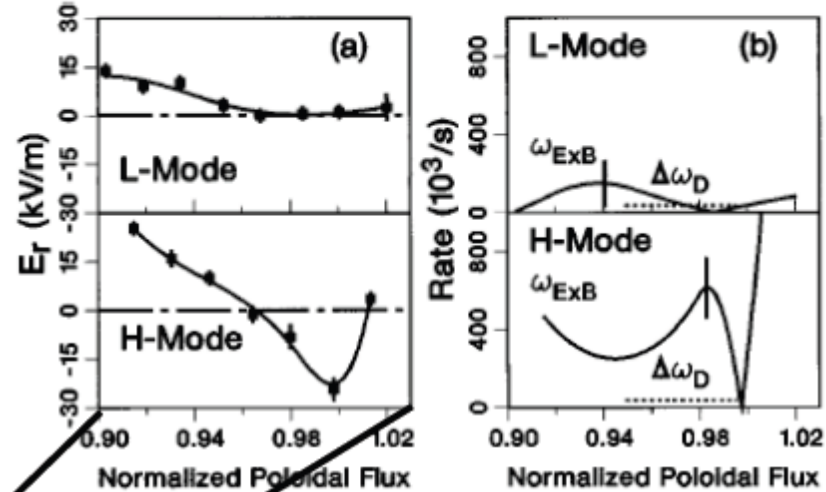


# Spontaneous “H-mode” edge transport barrier can form with sufficient heating power $\rightarrow$ improved confinement



Data from DIII-D

(from Carter, 2013)

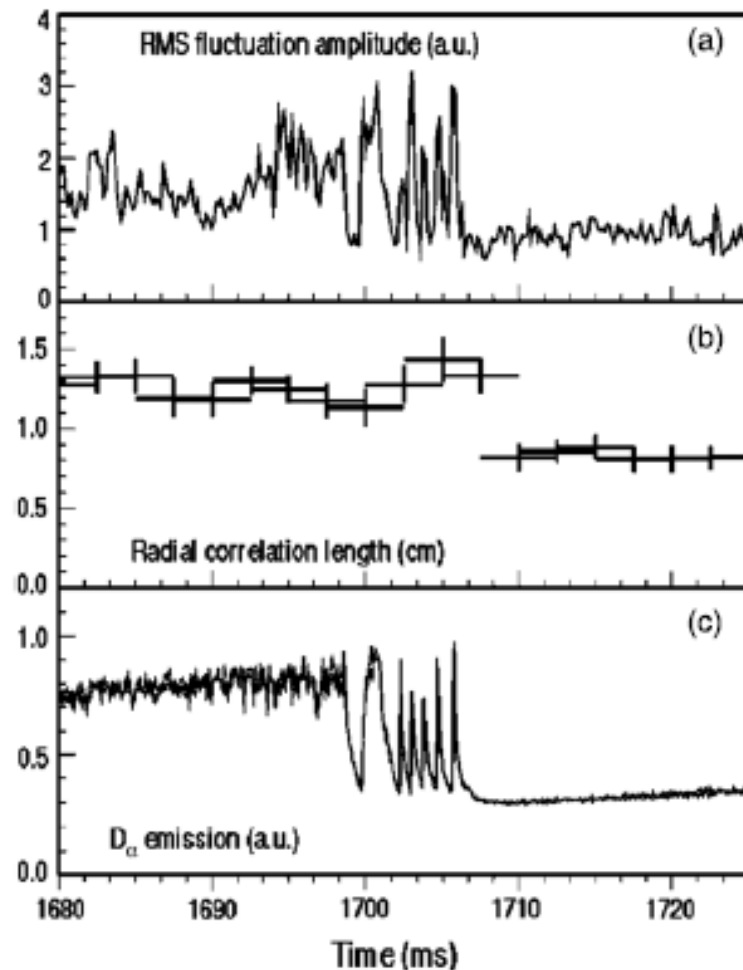


Burrell 1997

- Correlated with strong shear in equilibrium radial electric field ( $E_r$ )
- Suppression of turbulence predicted when equilibrium shearing rate ( $\omega_{E \times B}$ )  $>$  turbulence decorrelation rate ( $\Delta\omega_D$ ) [Biglari, 1990; Hahm, 1994]

# Transition from L→H correlated with drop in turbulence amplitude, reduction in radial correlation length

- Consistent with  $E \times B$  shear suppression
- However, there is still no clear understanding regarding what *initiates* the transition and the dynamics involved
- Practically important for understanding how much power required to reach H-mode ( $\rightarrow$  *almost all reactor designs assume H-mode*)

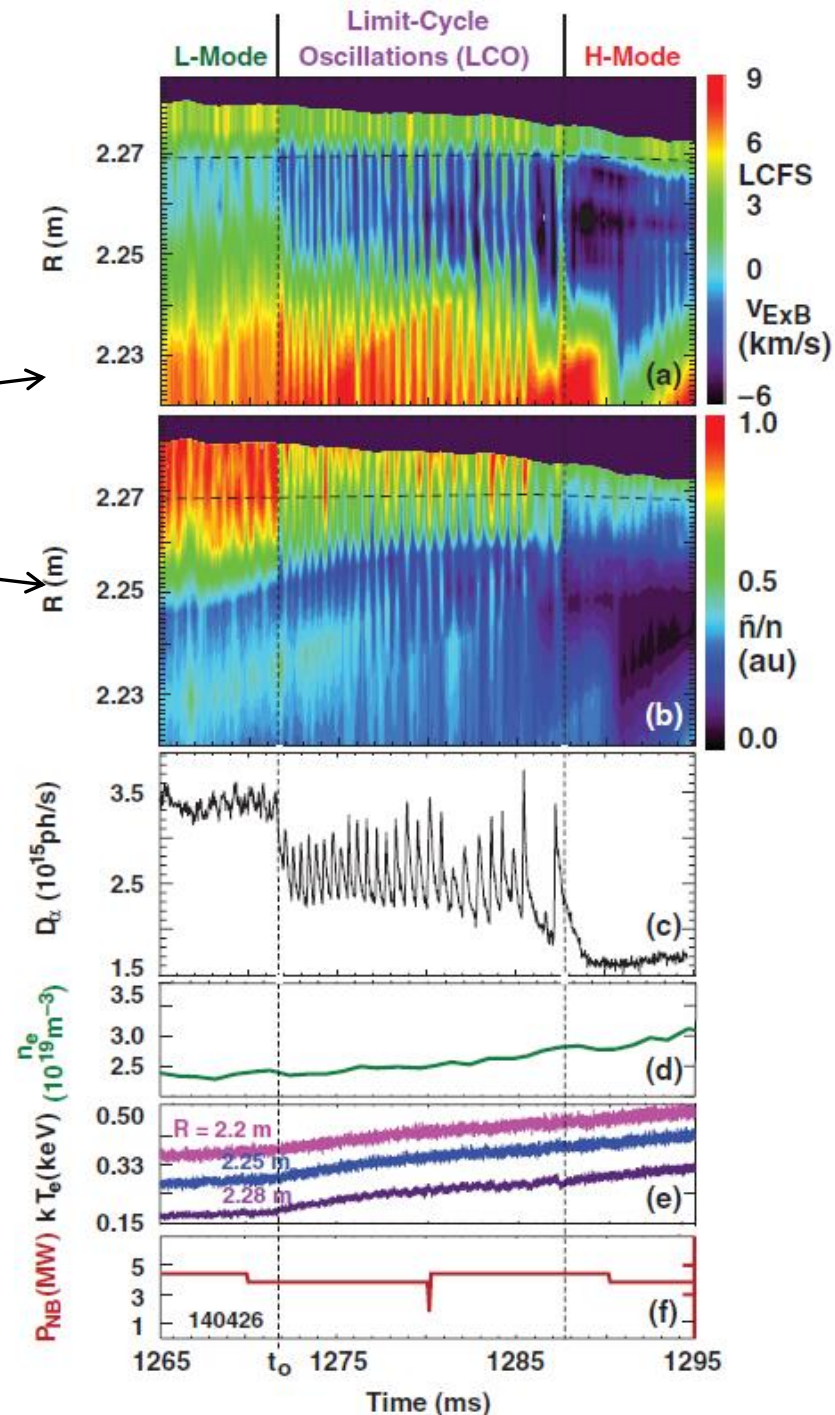


Burrell, PoP (1997)  
Coda, Phys. Lett. A (2000)

# Multiple doppler backscattering diagnostics provide $\delta n$ , $\delta v_{E \times B}$ at multiple radii simultaneously

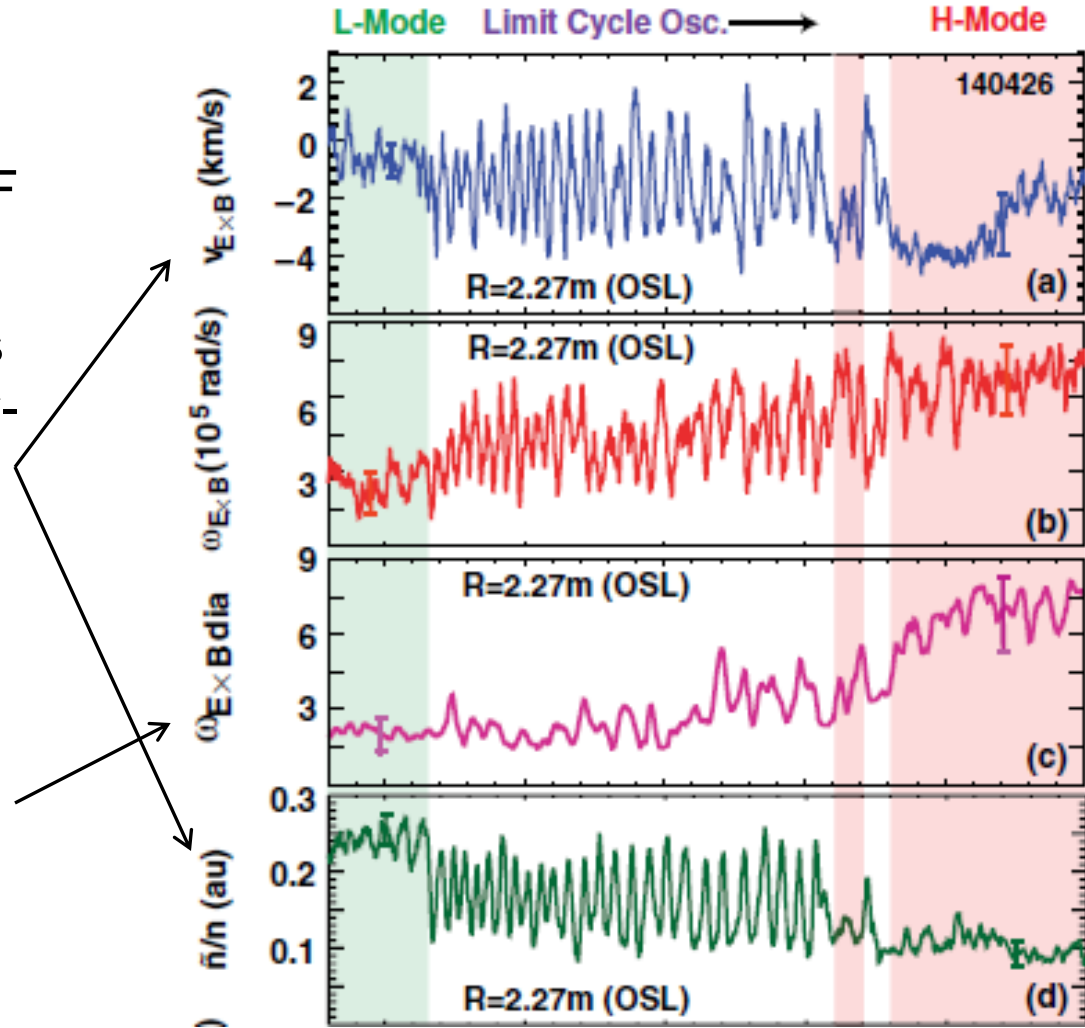
- During dithering L-H phase (identified by  $D_\alpha$  signal),  $\delta v_{E \times B}$  and  $\delta n$  start to oscillate
- Equilibrium  $n_e$ ,  $T_e$  begin to increase
- Eventually strong equilibrium flow shear locks in, fluctuations drop permanently, and pedestal finishes forming

DIII-D, Schmitz, PRL (2012)



# Dynamics consistent with two-predator – prey model (Kim, PRL 2003)

- In L-mode, increasing turbulence drives stronger ZF
- Eventually starts to suppress turbulence, leads to predator-prey limit cycle oscillation between ZF and turbulence
- As confinement (and gradients) increases, equilibrium  $E_r$  driven by  $\nabla \Pi$  increases, until it is strong enough to maintain suppression



DIII-D, Schmitz, PRL (2012)

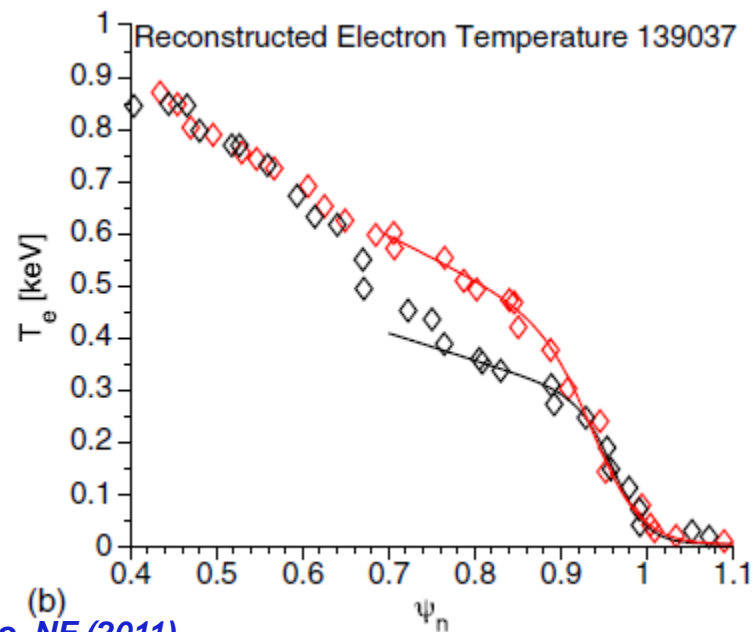
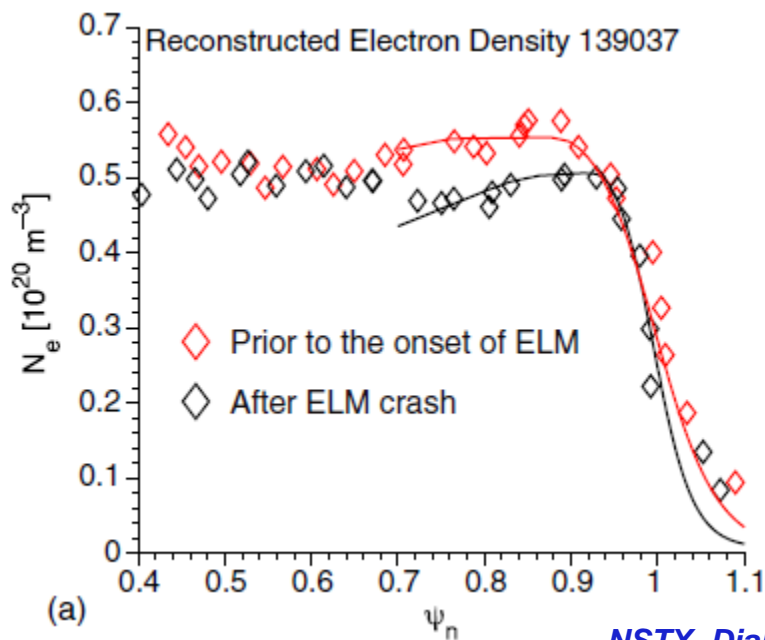
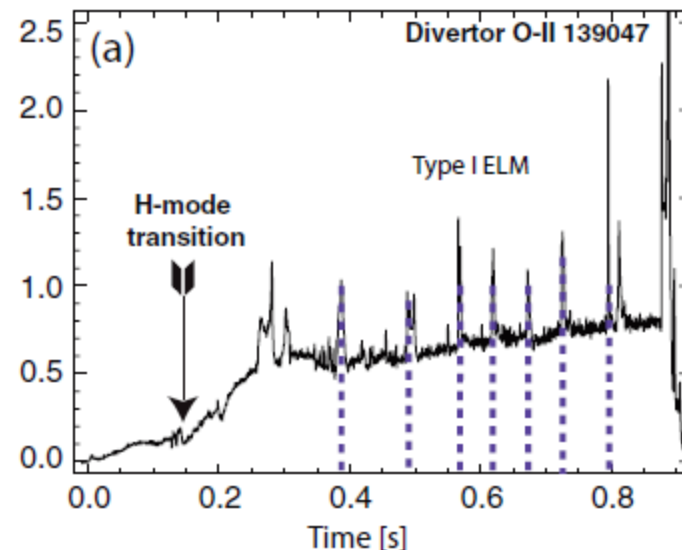
# **EDGE TURBULENCE**

## **H-mode pedestal**

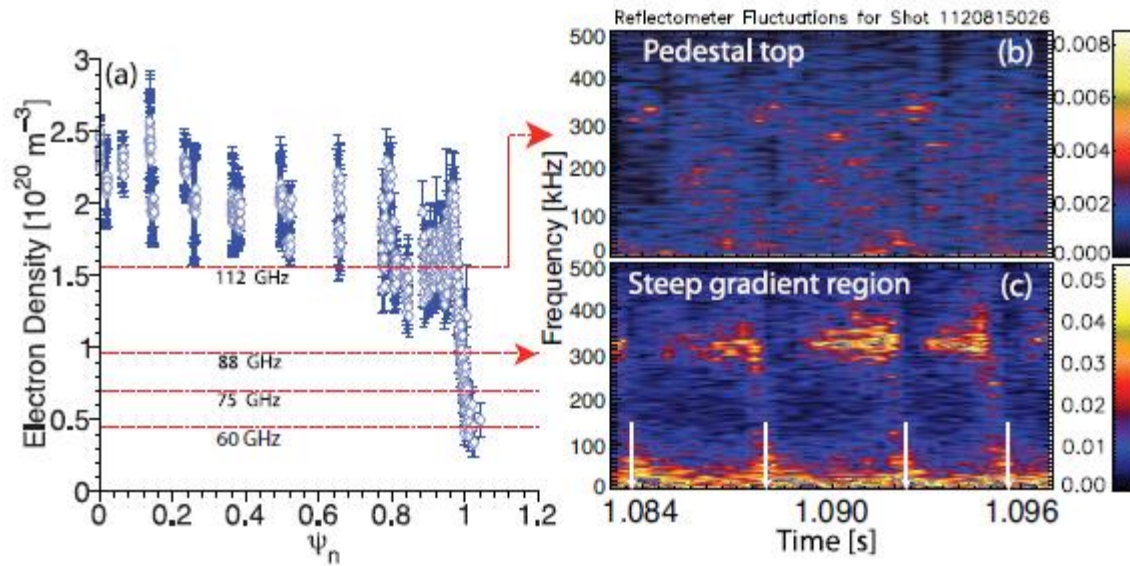


# In established H-modes, periodic MHD instabilities (Edge Localized Modes, ELMs) often occur

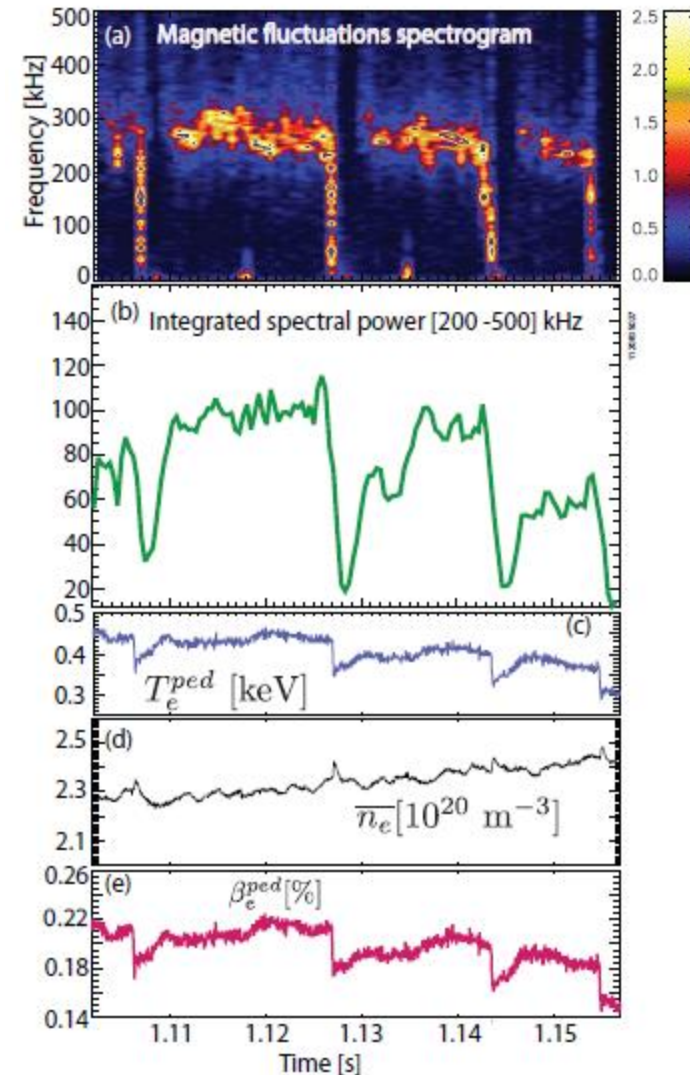
- Rapidly expels energy
- Profiles drop after ELM, recover between ELMs
- General question of what transport mechanism limits H-mode pedestal & post-ELM recovery



# Local density and magnetic fluctuations measured inter-ELM - possible importance of EM turbulence



- Density from reflectometry (& Gas Puff Imaging)
- Magnetic probes inserted 2 cm from separatrix (measures same  $k_\theta$  as density)
- Evidence for importance of EM turbulence?
- Leading theory posits KBM (EM drift wave) as a key contributor setting H-mode pedestal (Snyder, NF, 2011)

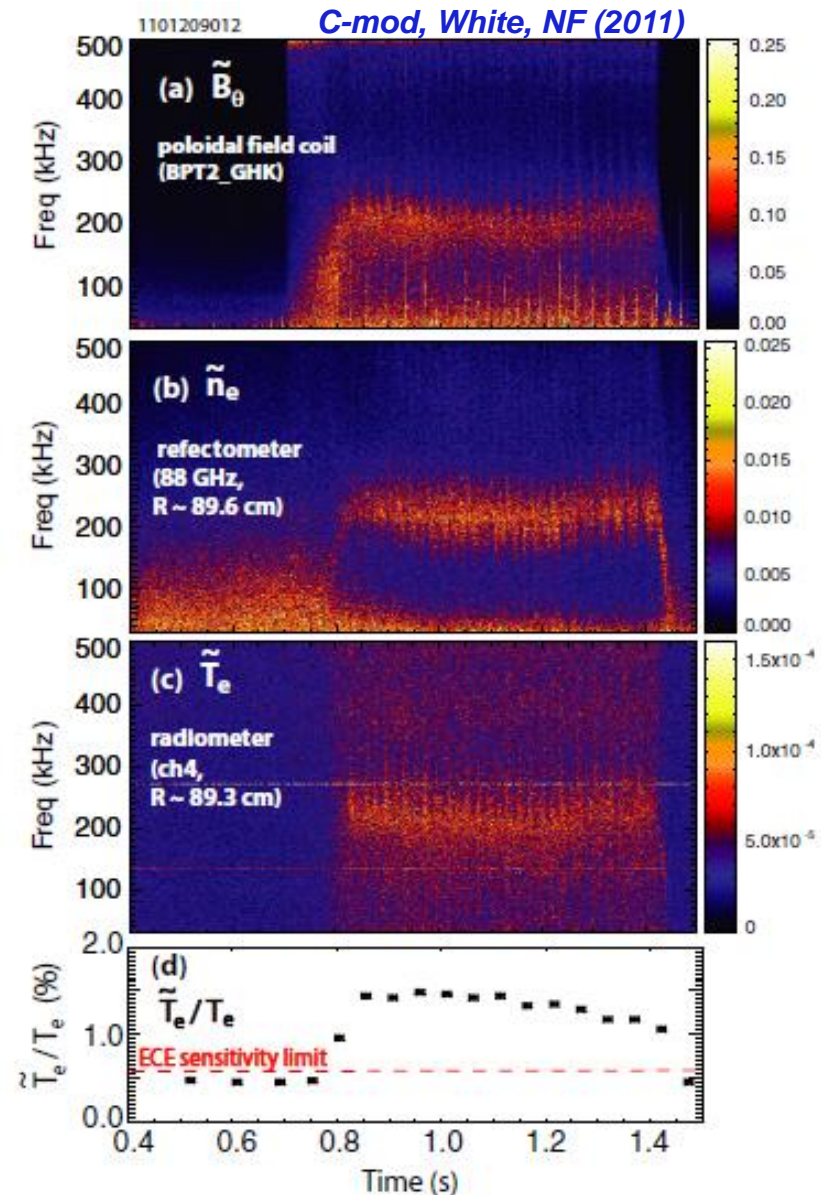


- SLIDE ON KBM CONSTRAINT FROM EPED???
- NONLOCAL EFFECTS?
- LOW HANGING FRUIT – KBM/EPM LINEAR THRESHOLDS IN NSTX/NSTX-U

# DIII-D BES measurements of KBM???

# Various fluctuations observed in ELM free pedestal regions – Weakly Coherent Mode in C-mod I-mode

- I-mode in C-mod similar to H-mode except temperature pedestal only
- Evidence for weakly coherent density, temperature & magnetic fluctuations associated with increased particle transport preventing density pedestal
- Other examples exist in ELM-free H-modes (EHO in DIII-D; QCM in C-Mod)



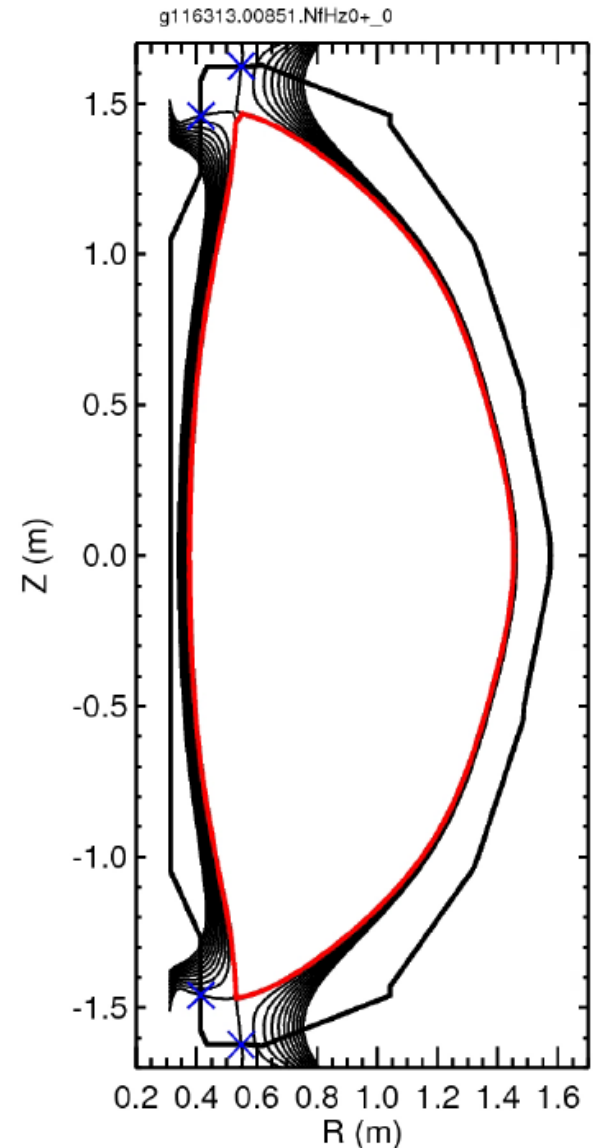
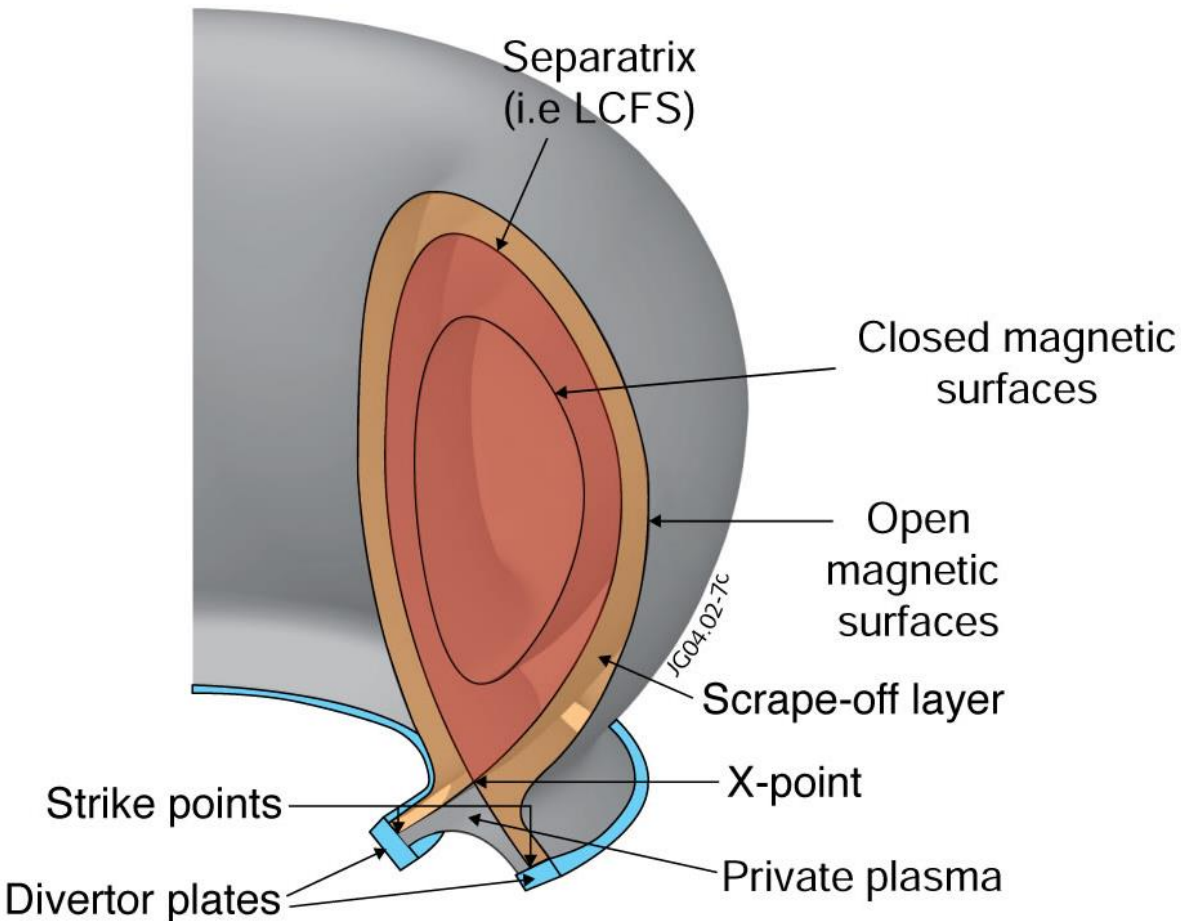
# Theory pedestal calcs for pedestal

- D.R. Hatch, Mike K.
- MTM + ETG + NC
- ETG at bottom, high-k measurements (Canik)
- AUG inter-ELM examples
- DIII-D inter-ELM examples
- MAST/DIII-D edge CPS

# SCRAPE OFF LAYER TURBULENCE

# Going to refer to different spatial regions in the tokamaks

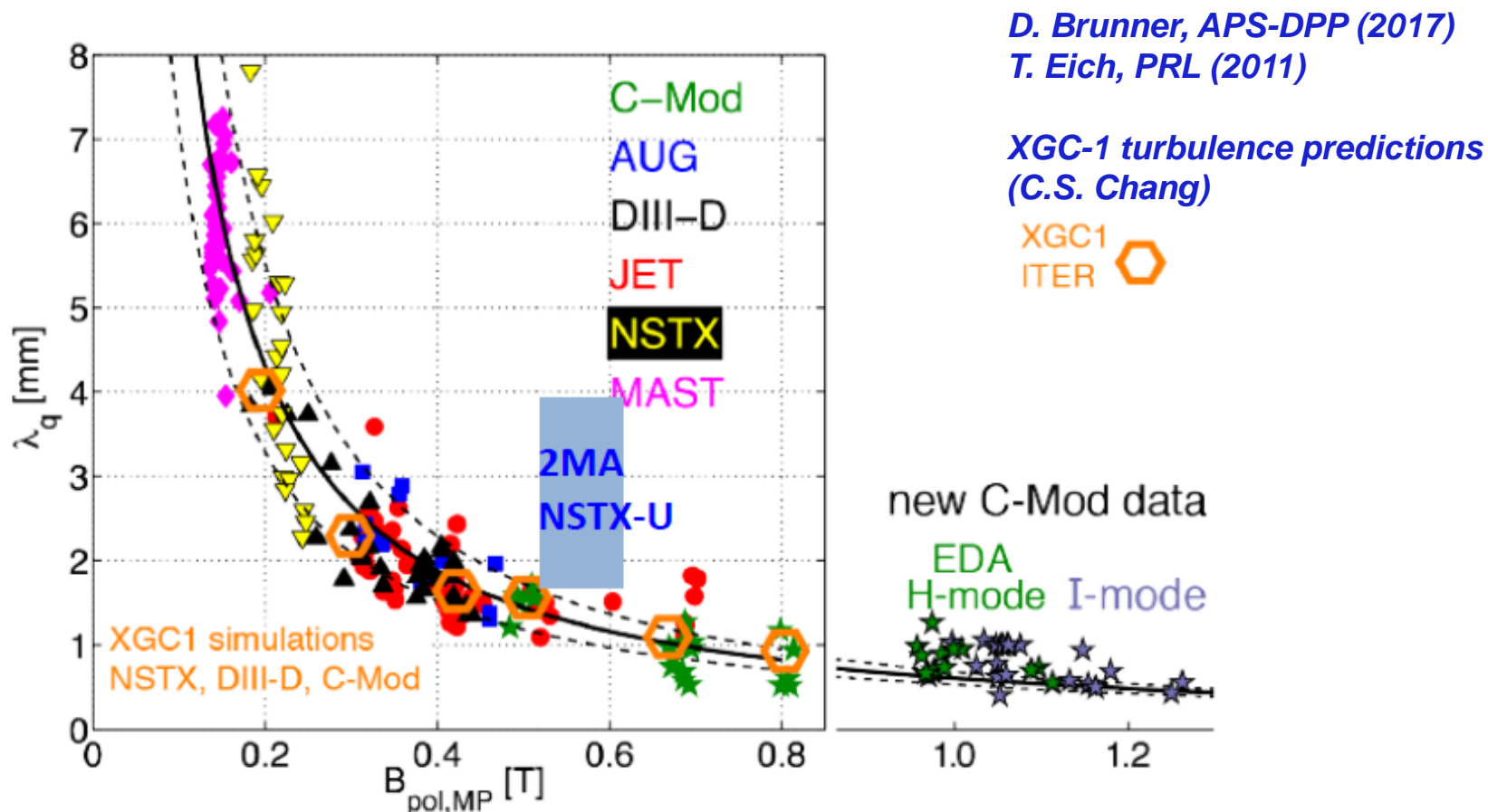
- Especially **core**, **edge** (just inside separatrix), and **scrape-off layer** (SOL, just outside separatrix)





# Understanding scrape-off-layer (SOL) heat-flux width extremely important under reactor conditions

- Narrow SOL heat flux width  $\lambda_q$  leads to huge ( $>10 \text{ MW/m}^2$ ) heat flux density on the divertor plasma facing components (PFCs)  $\rightarrow$  significant concern for sputtering and erosion
- Empirical scaling ( $\lambda_q \sim 1/B_{\text{pol,MP}}$ ) very unfavorable for reactors
- **Recent turbulence simulations suggest a possible break from this scaling**



# Many options being considered for divertor/SOL magnetic geometry

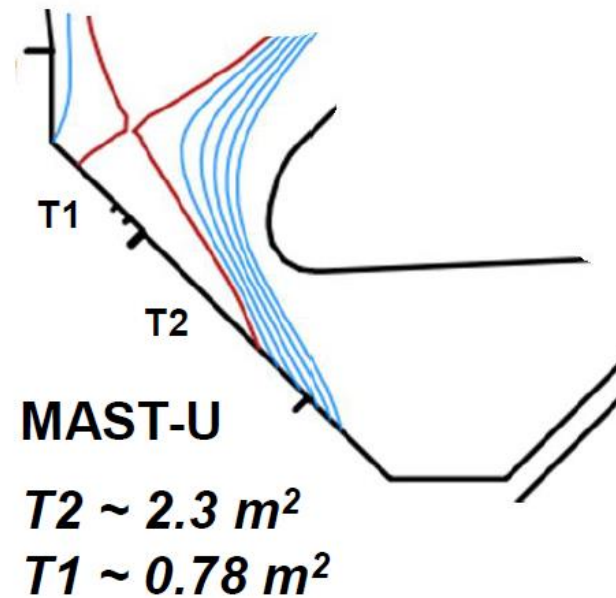
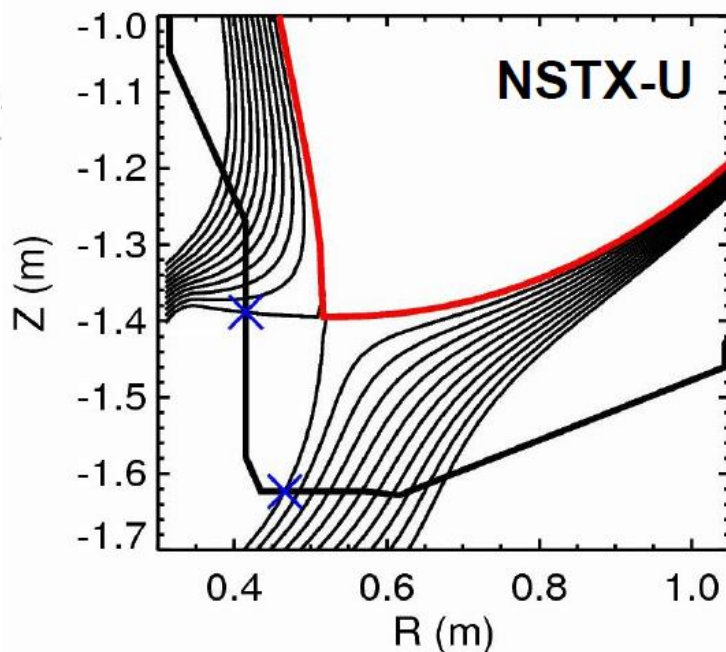
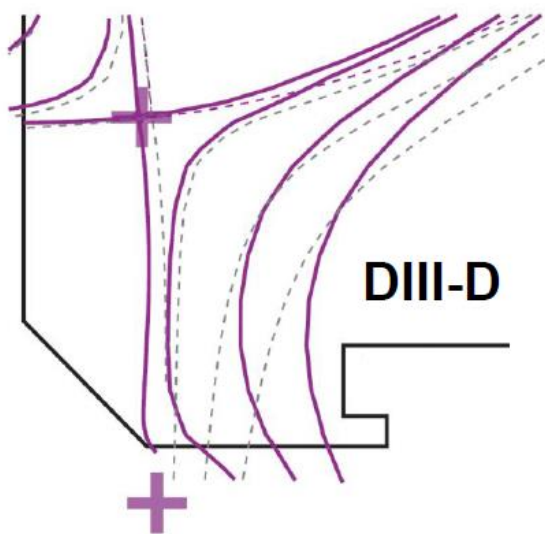
- Requires additional complexity in poloidal field coils and controllability
- Generally will also required impurity seeding in core/edge plasma to radiate much of the power
- **Spreading (from turbulence) could reduce heat flux density**

## X divertor

## Snowflake divertor

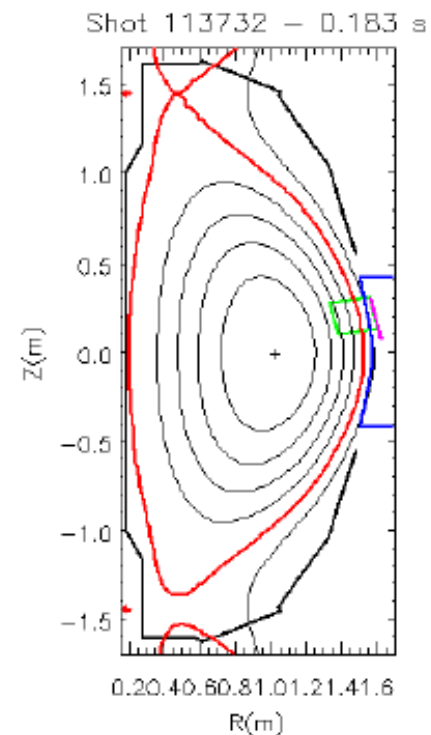
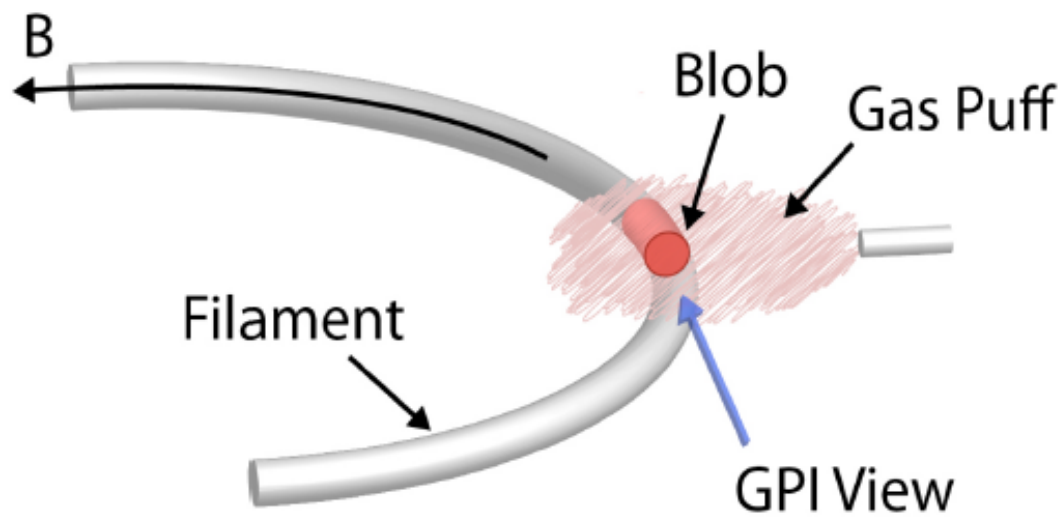
## Super-X divertor

### X Divertor



# Edge Turbulence Measurements in NSTX

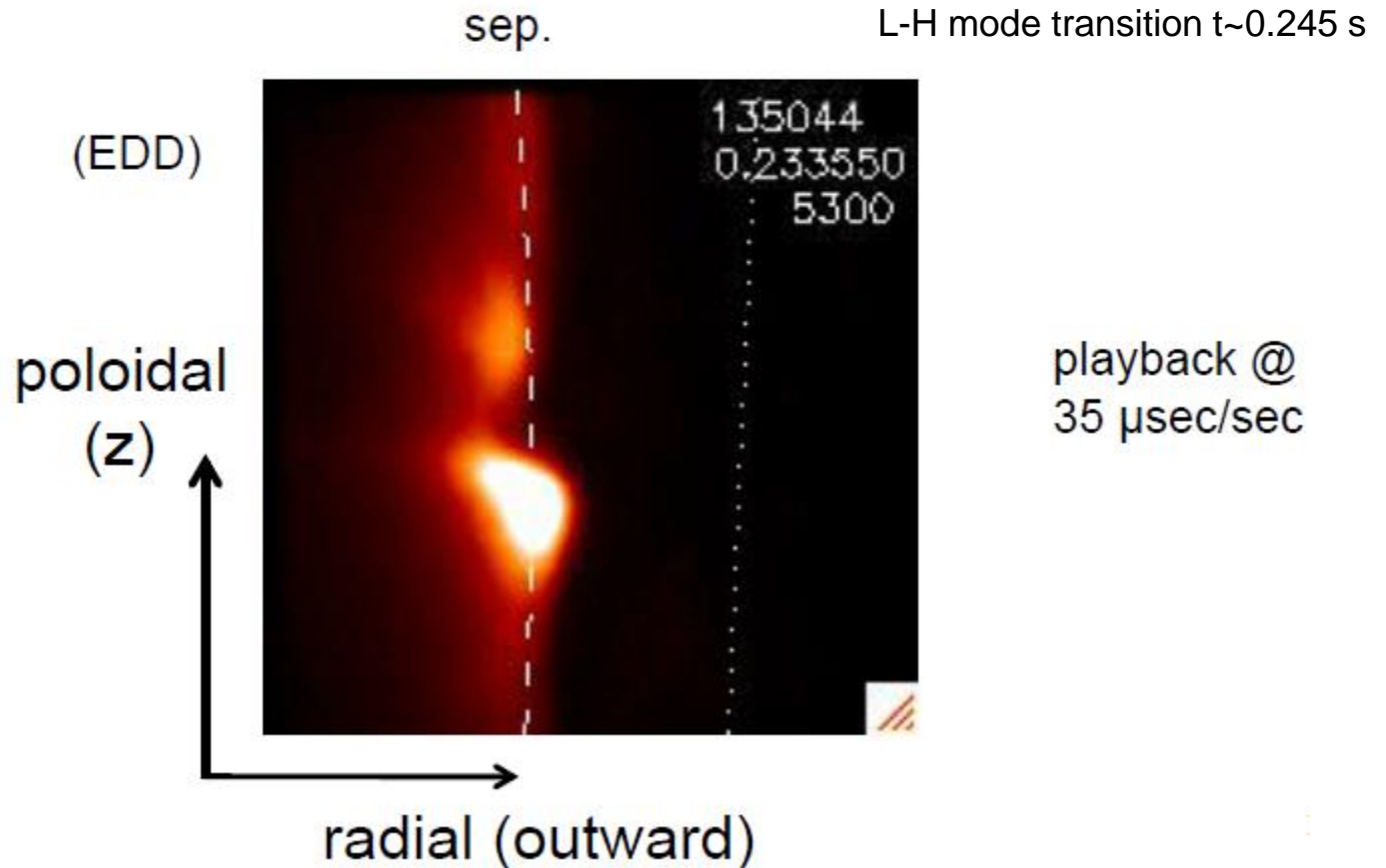
- High speed cameras make images of edge turbulence
- 3-D 'filaments' localized to 2-D by gas puff imaging (GPI)



Zweben et al, Nuclear Fusion 44 (2004), R. Maqueda et al, Nucl. Fusion 50 (2010)

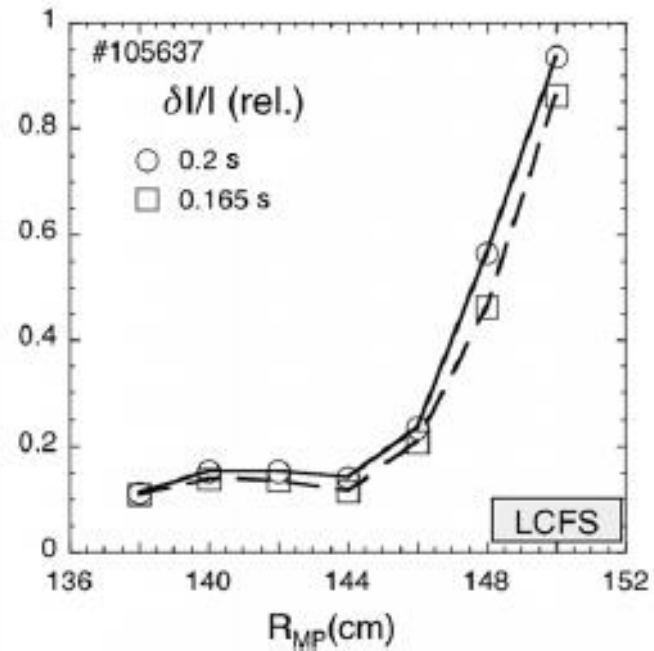
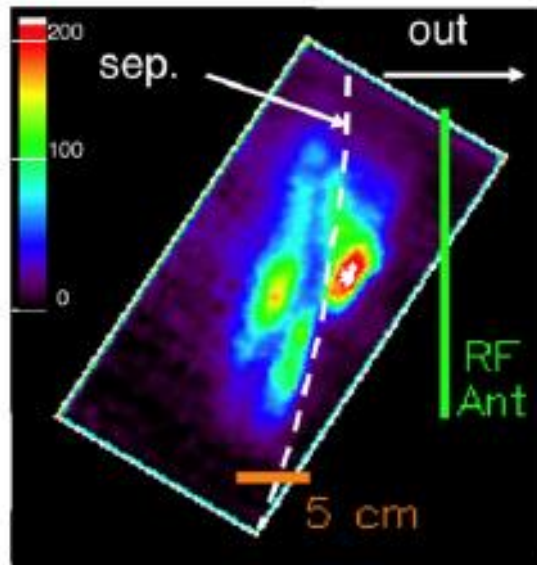
# Lots of videos via Stewart Zweben: <http://w3.pppl.gov/~szweben/>

- This movie 285,000 frames/sec for ~ 1.4 msec
- Viewing area ~ 25 cm radially x 25 cm poloidally



# Outside separatrix, blobs can be ejected and self-propagate to vessel wall

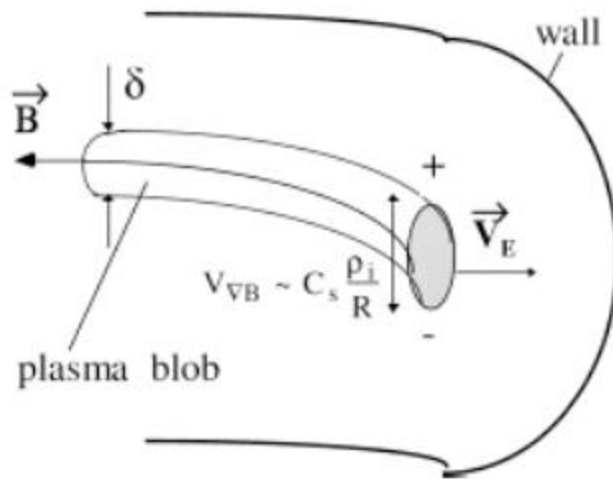
- Plasma is much less dense farther out in scrape-off layer
- Relative intensity of blob becomes large ( $\delta I/I$ )



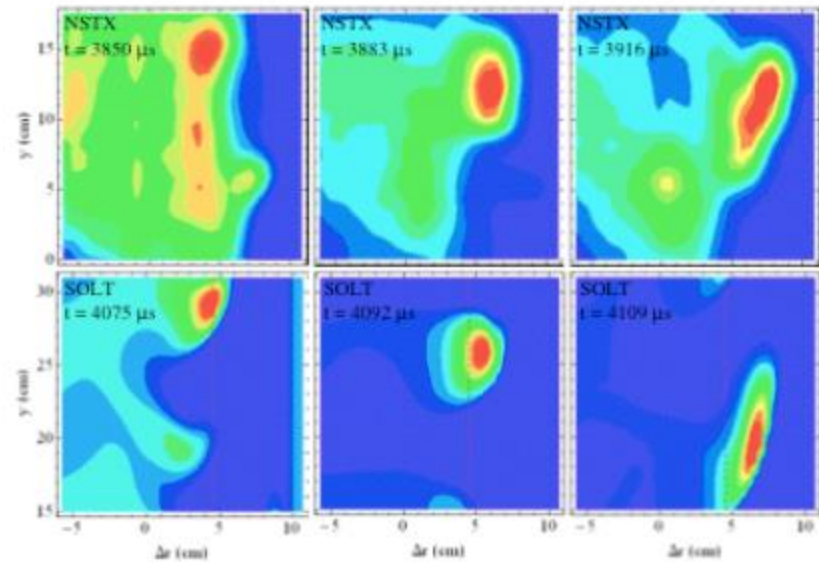
# Theories and simulations exist that predict blob characteristics: size, density, velocity

- Simulations further out in edge become progressively more challenging, more effects to deal with (neutrals, open field lines to conducting walls, dust, ...)

simple 'blob' model (Krash. 2001)

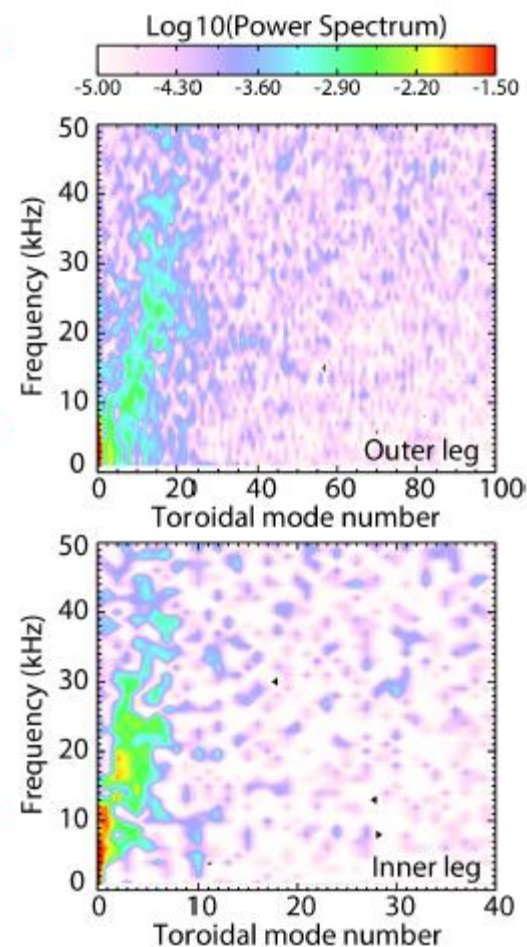
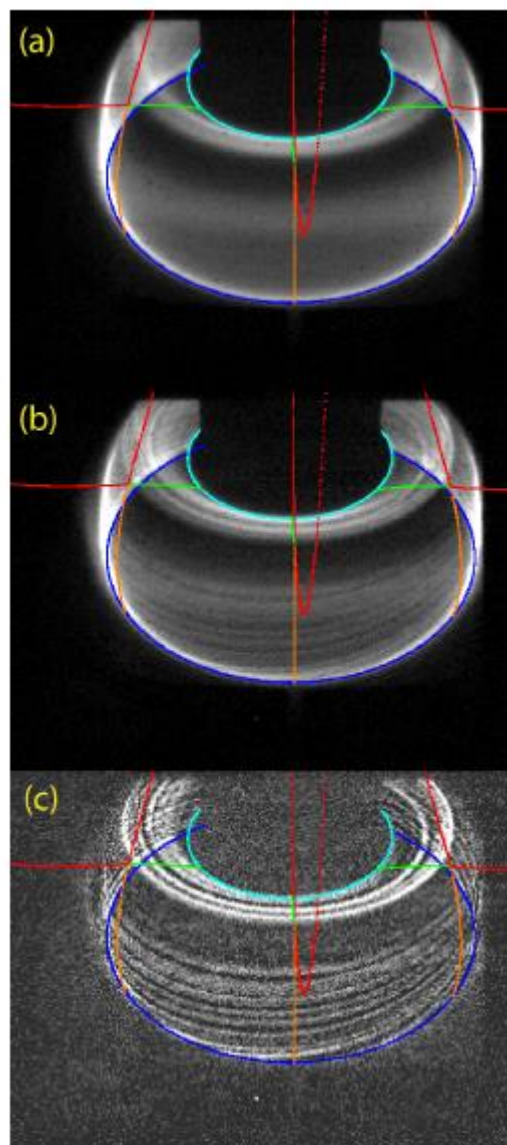
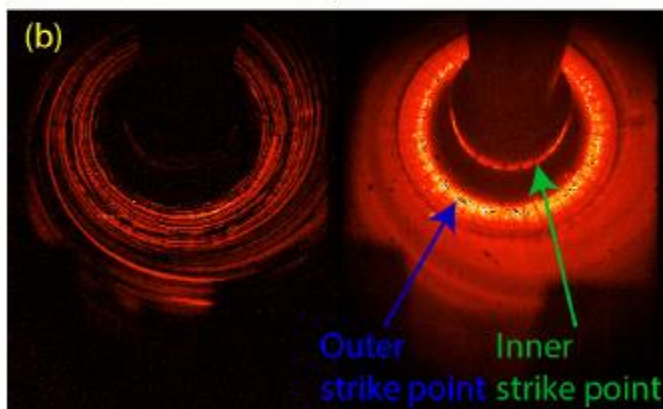
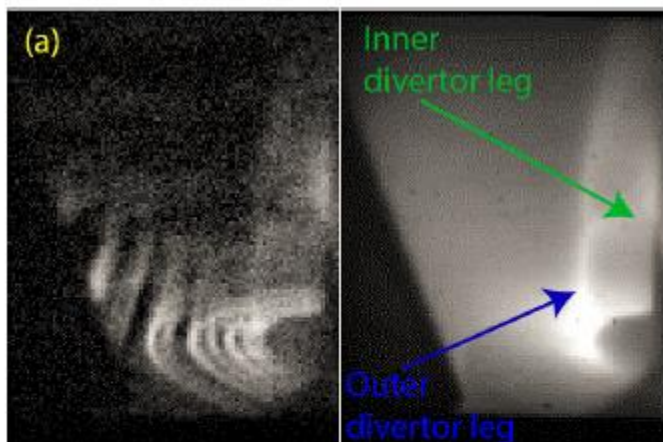


2D turbulence model (D'Ippolito 2008)



# Blob filaments seen to propagate down to divertor, but also can exist in isolation, be driven near X-point (not traditional outboard midplane “bad curvature” region)

- Scotti (2017/2018)
- Imaging techniques



## Intermittency, skewed PDFs

- Much larger  $dn/n_0 \sim 10-100\%$  (compared to core,  $<1\%$ )



# STELLARATOR TURBULENCE

- No direction of symmetry
- Parallel connection between good/bad curvature and varying local magnetic shear complicates dynamics (and theory), BUT opens the door for optimization

## SUMMARY

- Many experiments and diagnostics developed to measure fluctuation amplitudes, spectra, cross-phases, transport, etc... in various regions of magnetically confined plasmas
- Have seen progress in comparing theory/simulation & measurements, with agreement improving from order-of-magnitude to factor of 2-3 or better in limited cases
- Improves confidence (in some regimes) in our physics understanding, which improves our predictive ability (not really addressed here)
- Plenty more to do

# Hot topics, low hanging fruit