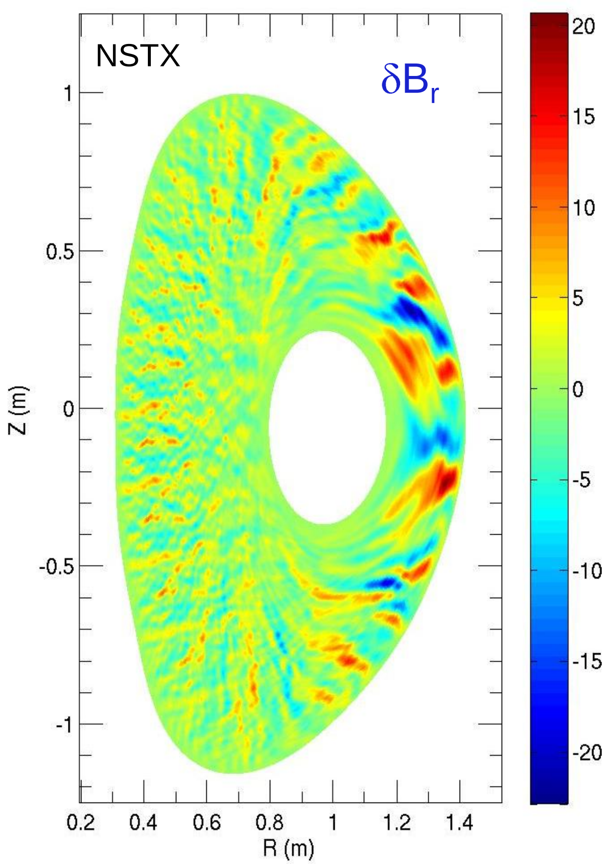
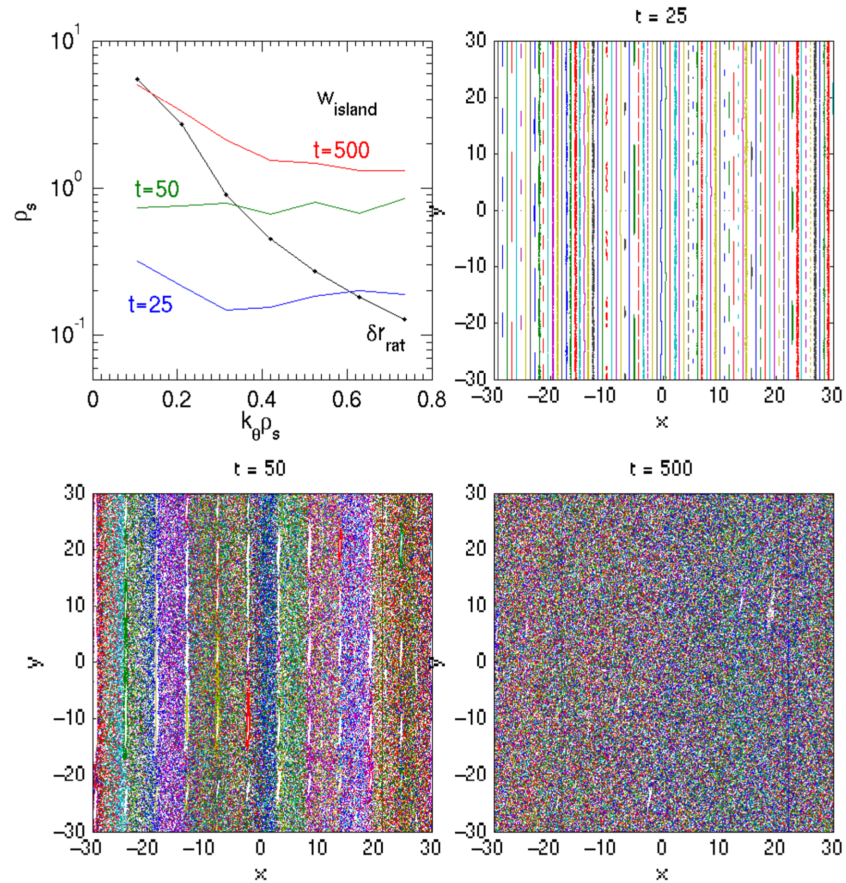
NSTX-U Supported by U.S. DEPARTMENT OF ENERGY Office of Science

Microtearing modes: some physics and some unanswered questions

Walter Guttenfelder

PPPL Graduate Student Seminar March 25, 2013





NSTX microtearing instability exhibits thresholds in electron temperature gradient and beta, collisionality important

(1) Apparent threshold in ∇T_e, $(a/L_{Te})_{crit} \approx 1.3\text{-}1.5$ $(a/L_{Te,exp}=2.7)$

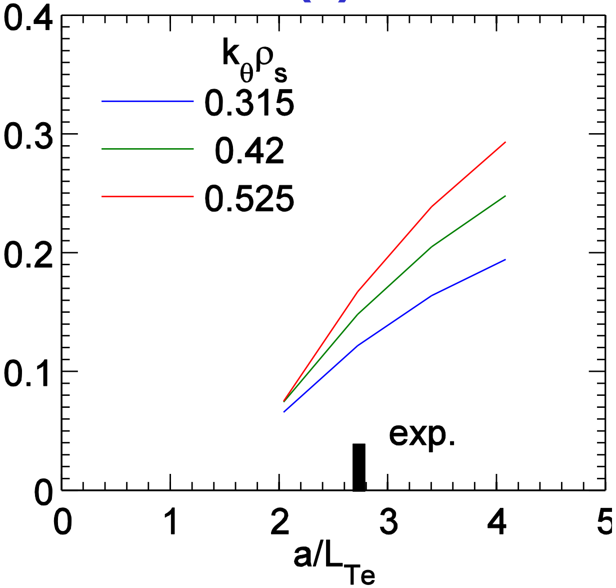
(2) Growth rates depend on ν_e non-monotonically

(3) Lowering beta stabilizes microtearing

All three ∇T_e, β_e, ν_e appear to be important

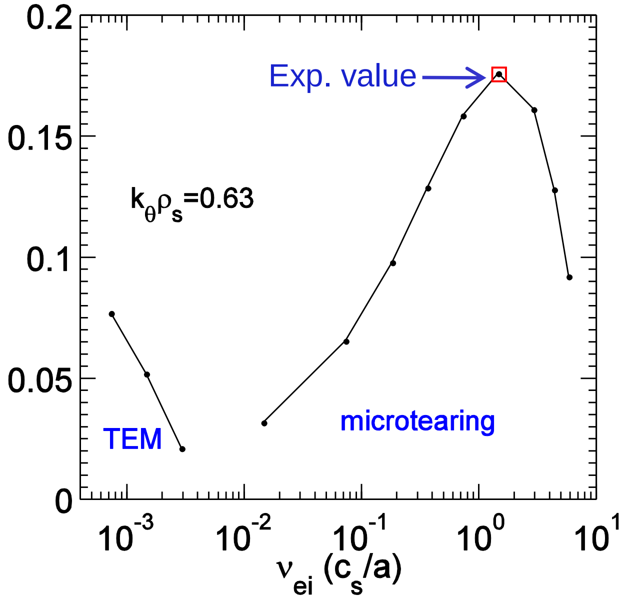
Linear growth rates ($\gamma \cdot a/c_s$) from gyrokinetic simulations for NSTX 120968 t=0.56 s r/a=0.6

(1)

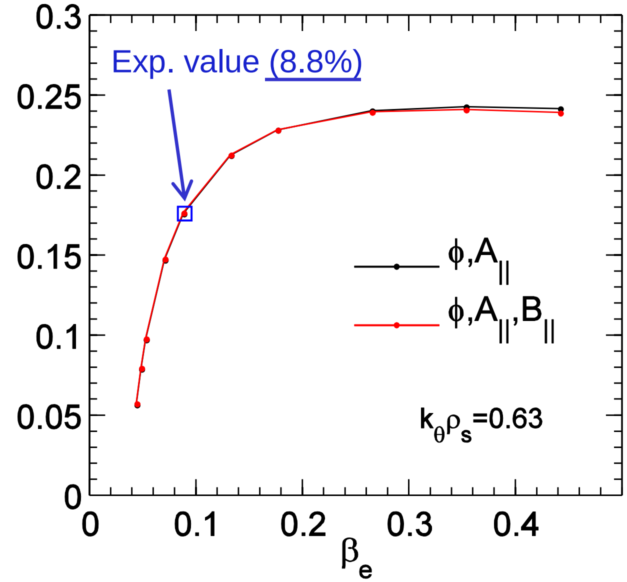


$$\frac{a}{L_{Te}} = -\frac{a}{T_e}\nabla T_e$$

(2)



(3)



Experimental motivation

- Microtearing modes predicted to be linearly unstable in many devices:
  - high beta spherical tokamaks (NSTX, MAST)
  - conventional tokamaks (ASDEX-UG, DIII-D, JET, possibly ITER edge)
  - reversed field pinches (RFX, MST)

- Important to determine:

  (1) whether they cause significant transport

  (2) whether they matter for next generation devices (NSTX-U, ST-FNSF, ITER)

$\Rightarrow$ Want to better understand:

(1) Linear stability

(2) Nonlinear saturation

Overview

- MHD & resistive tearing instability
- Schematic of magnetic drift wave & linear microtearing instability due to time-dependent thermal force
- Examples from linear gyrokinetic simulations
- Transport and stochasticity in nonlinear simulations

Tokamak review: Ideal MHD equilibrium ($J \times B = \nabla P$) gives nested flux surfaces with helical field lines

- External coils establish B_φ
- Plasma current gives B_θ
- Helical pitch characterized by safety factor, q

$$q = \frac{\text{toroidal transits}}{\text{poloidal transits}}$$

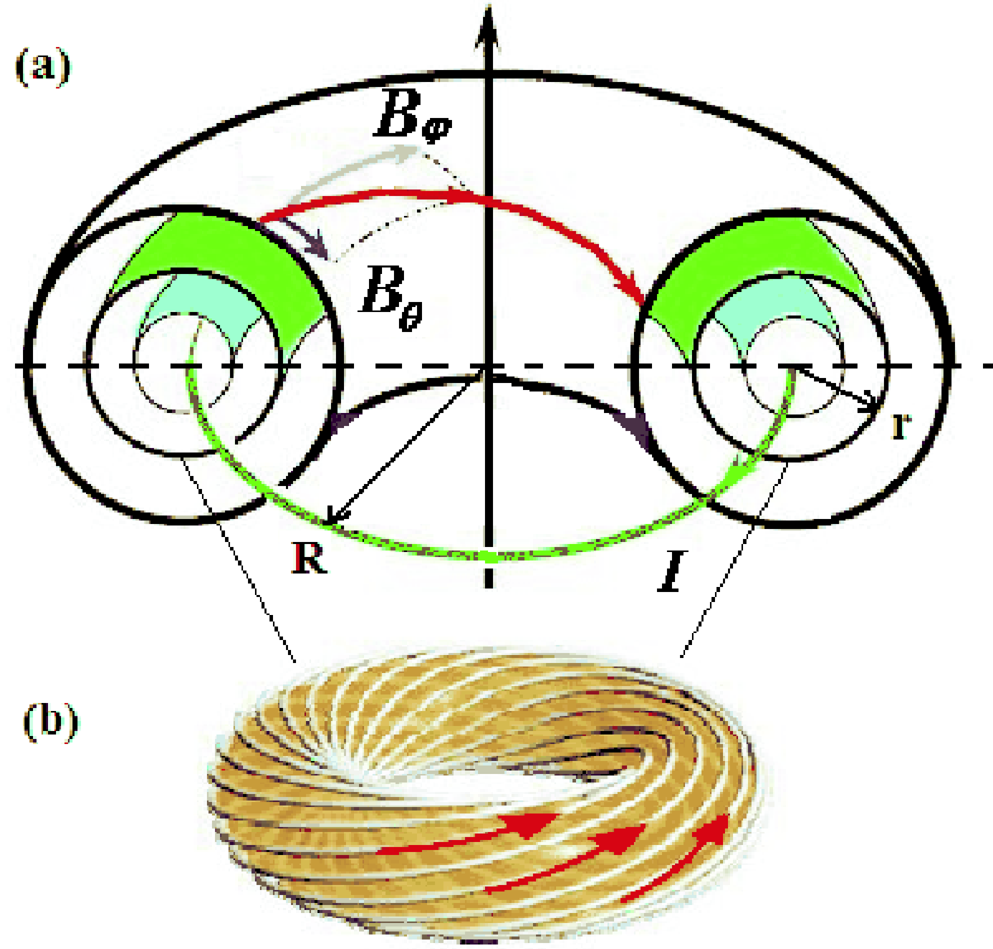
- Low beta, high aspect ratio (R/a) limit:

$$q = \frac{rB_\varphi}{RB_\theta}$$

- We're interested in perturbations with toroidal, poloidal mode numbers (n,m)

$$A = A(r)\cos(m\theta - n\varphi - \omega t)$$

$$A = A(r)\exp(im\theta - in\varphi - i\omega t)$$



Often interested in behavior near rational surfaces (where q=m/n)

With infinite conductivity plasma topology remains unchanged

- Ideal Ohms law leads to "frozen flux"
- Perturbations can occur but field lines tied to plasma

$$\left.\begin{array}{l} E + v\times B = 0 \\ \dfrac{\partial B}{\partial t} = -\nabla\times E \end{array}\right\} \rightarrow \frac{\partial B}{\partial t} = \nabla\times(v\times B)$$

- Finite resistivity allows field to diffuse

$$\left.\begin{array}{l} E + v\times B = \eta J \\ \dfrac{\partial B}{\partial t} = -\nabla\times E \\ \mu_0 J = \nabla\times B \end{array}\right\} \rightarrow \frac{\partial B}{\partial t} = \nabla\times(v\times B) + \frac{\eta}{\mu_0}\nabla^2 B$$

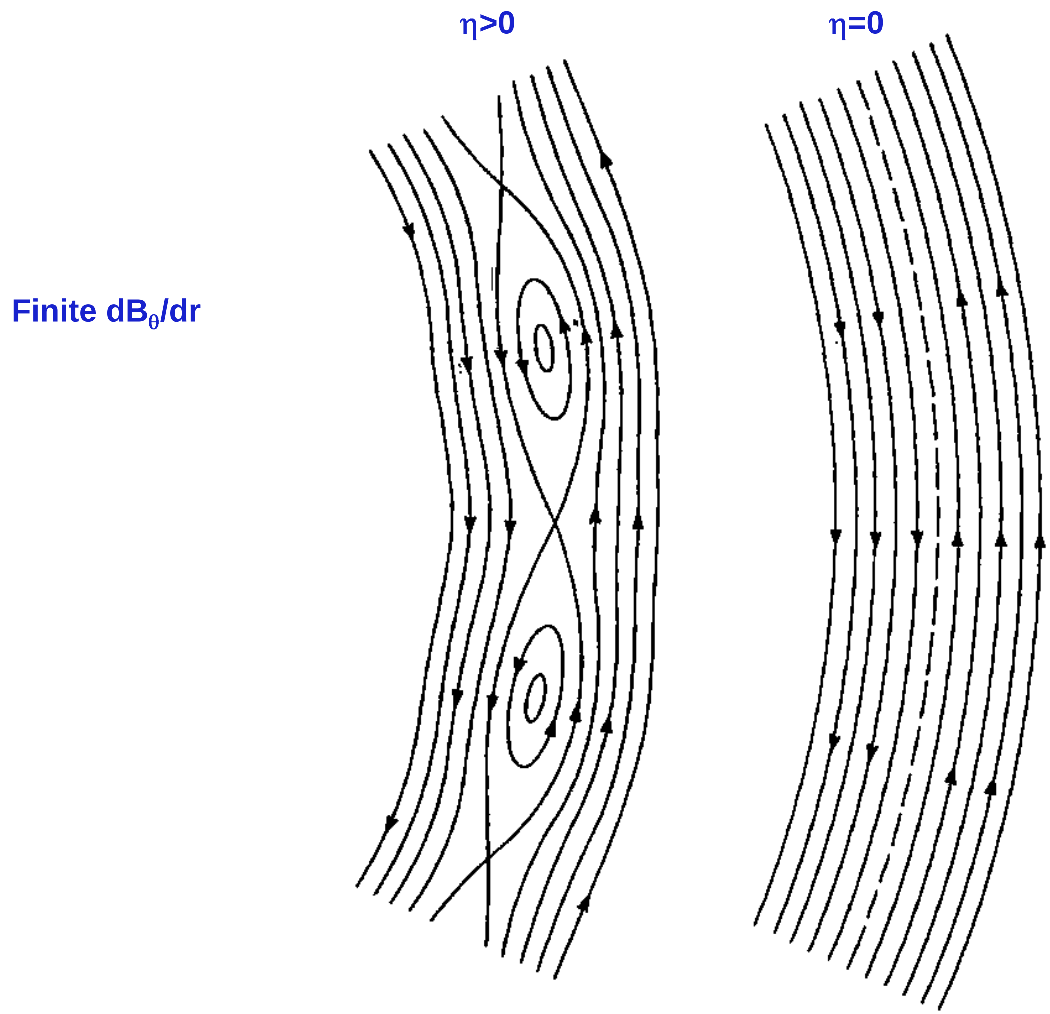
- Over equilibrium scales (∇~1/L) diffusion is very slow, characterized by magnetic Reynolds numbers

$$R_M = \frac{\nabla\times(v\times B)}{\frac{\eta}{\mu_0}\nabla^2 B} \sim \frac{vL\mu_0}{\eta} \sim 10^6 - 10^8$$

- Resistive effects can be much faster if they occur over much shorter scale, ∇ << L

Tearing/reconnection of field lines can occur with sheared magnetic field and finite resistivity

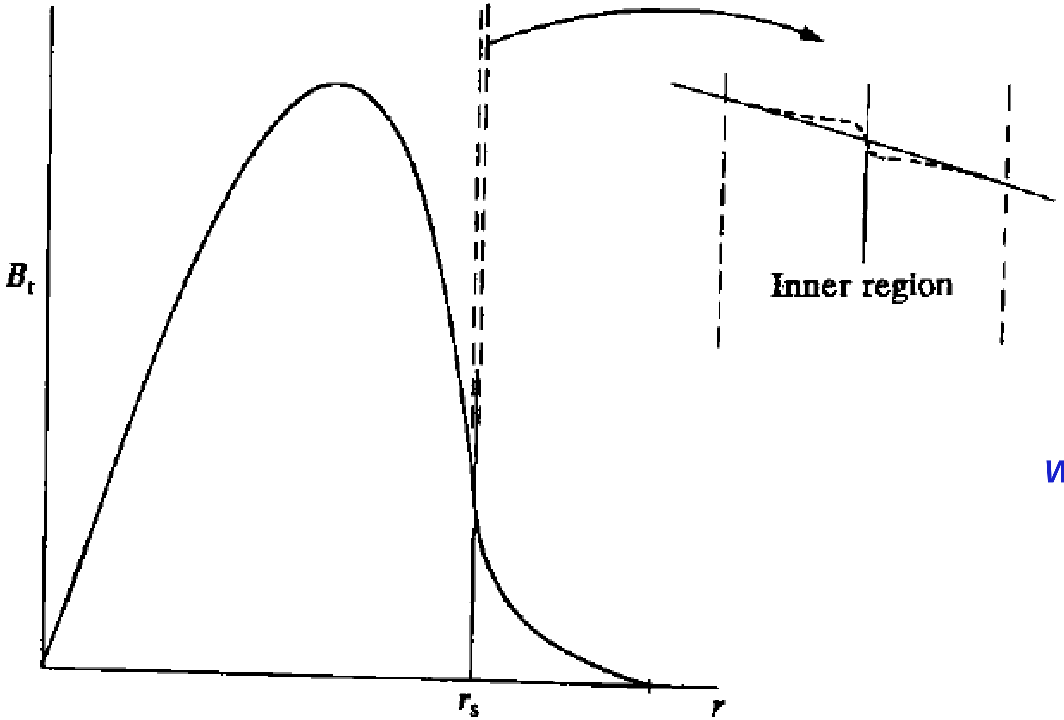
- Islands can form which flatten T_e profile (from fast parallel conduction), degrade confinement, cause other bad things to happen (neoclassical tearing modes)



Let's consider a resistive tearing instability for chosen (n,m) [Wesson, 6.8; Goldston Ch. 20]

- Resistive instability can be driven by equilibrium current gradient (∇J_0)
- Inner region near the rational surface $q(r_s)=m/n$ must be solved including resistive Ohm's law ($\eta/\mu_0 \nabla^2 B$)
- Ideal equations are sufficient "far enough" outside the inner layer

$\Rightarrow$ Two regions solved separately, then matched at boundaries to solve perturbation structure and growth rate



Wesson 6.8

Perturbed outer region determined entirely by ideal force balance J×B=∇P

$$\mathbf{j} \times \mathbf{B} = -\nabla p, \longrightarrow \nabla \times \mathbf{j} \times \mathbf{B} = 0. \longrightarrow \mathbf{B} \cdot \nabla \mathbf{j} - \mathbf{j} \cdot \nabla \mathbf{B} = 0.$$

Linearize and assume low β, high R/a

$$B_{\phi 1} \sim \varepsilon B_{r1} \sim \varepsilon B_{\theta 1} \qquad B_\theta \sim \frac{1}{r}\frac{\mathrm{d}B_\phi}{\mathrm{d}r} \sim \varepsilon B_\phi$$

$$j_{r1} \sim j_{\theta 1} \sim \varepsilon j_{\phi 1}, \qquad j_\theta \sim \varepsilon j_\phi$$

$$(\mathbf{B} \cdot \nabla j_\phi)_1 = 0.$$

$$\boxed{\mathbf{B} \cdot \nabla j_{\phi 1} + \mathbf{B}_1 \cdot \nabla j_\phi = 0}$$

Equation for ideal outer region

Stability determined by equilibrium current gradient, ∇J_0

Expand equilibrium gradient of perturbed toroidal current

$$\boldsymbol{B}\cdot\nabla j_{\phi 1}+\boldsymbol{B}_1\cdot\nabla j_{\phi}=0$$

Equation for ideal outer region

$$\left.\begin{aligned} B &= B_\varphi \hat{e}_\varphi + B_\theta \hat{e}_\theta \approx B_\varphi\left(\hat{e}_\varphi+\frac{r}{qR}\hat{e}_\theta\right) \\ \nabla &= \frac{1}{R}\frac{\partial}{\partial\varphi}\hat{e}_\varphi+\frac{1}{r}\frac{\partial}{\partial\theta}\hat{e}_\theta=\frac{-n}{R}\hat{e}_\varphi+\frac{m}{r}\hat{e}_\theta \end{aligned}\right\}\rightarrow B\cdot\nabla\tilde{j}_\varphi \approx B_\varphi\left(\frac{m-nq(r)}{qR}\right)\tilde{j}_\varphi$$

Disappears at rational surface, $q(r_s)=m/n$

$$\mu_0 j_{||}=(\nabla\times B)_{||}=(\nabla\times\nabla\times A)_{||}=-\nabla_\perp^2 A_{||}=\left(-\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{m^2}{r^2}\right)A_{||}$$

Ampere's law

$$\boxed{B\cdot\nabla\tilde{j}_\varphi \approx B_\varphi\left(\frac{m-nq(r)}{qR}\right)\left(-\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{m^2}{r^2}\right)A_{||}}$$

First term

Expand perturbed gradient of equilibrium toroidal current

$$B \cdot \nabla j_{\phi 1} + B_1 \cdot \nabla j_{\phi} = 0$$

Equation for ideal outer region

↓

$$\tilde{B}_r \cdot \nabla J_0 = \left(\frac{1}{r} \frac{\partial}{\partial \theta} \tilde{A}_{\|} \right) \nabla J_0 = \left(\frac{m}{r} \tilde{A}_{\|} \right) \nabla J_0$$

Equilibrium current in the ideal outer region determines instability, given by Δ'

$$\boldsymbol{B} \cdot \nabla j_{\phi 1} + \boldsymbol{B}_1 \cdot \nabla j_{\phi} = 0$$

Equation for ideal outer region

$$B_\varphi \left(\frac{m - nq(r)}{qR} \right) \left(-\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{m^2}{r^2} \right) \tilde{A}_{||} + \left(\frac{m}{r} \tilde{A}_{||} \right) \nabla J_0 = 0$$

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} \tilde{A}_{||} = \frac{m^2}{r^2} \tilde{A}_{||} + \frac{\nabla J_0}{\frac{B_\theta}{\mu_0} \left(1 - \frac{nq(r)}{m} \right)} \tilde{A}_{||}$$

Competition between destabilization from current gradient and stabilization from field line bending

We care about the solution of B_r ($\sim dA_{||}/dr$) as we approach the rational surface from either side, usually given in the form of the tearing parameter, Δ'

$$\Delta' = \frac{A'_{||}\big|_{r_s+\delta} - A'_{||}\big|_{r_s-\delta}}{A_{||}}$$

Solution to inner layer provides relation between growth rate and Δ'

- Must include resistivity in Ohm's law and inertia in momentum equation
- Also assuming radial scale length much smaller than poloidal ($d^2/dr^2 >> m^2/r^2$)

$$\left.\begin{array}{c} \dfrac{\partial B}{\partial t} = \nabla\times(v\times B) + \dfrac{\eta}{\mu_0}\nabla^2 B \\ \nabla\times\left(\rho\dfrac{dv}{dt} = j\times B - \nabla p\right) \end{array}\right\} \rightarrow \boxed{\gamma = 0.55\,\frac{(a\Delta')^{4/5}}{\tau_R^{3/5}\tau_A^{2/5}}\left(n\,\frac{a}{R}\,\frac{aq'}{q}\right)^{2/5}}$$

- $\Delta' > 0$ necessary for instability
- Also requires positive magnetic shear q'>0
- Grows on a hybrid time scale between resistive and Alfvenic

$$\tau_R = \frac{a^2}{\eta/\mu_0}$$

$$\tau_A = \frac{a}{V_A} = \frac{a}{B_\varphi/(\mu_0\rho)^{1/2}}$$

Stabilization from field line bending dominates microscopic perturbations (large n,m)

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\tilde{A}_{||} = \frac{m^2}{r^2}\tilde{A}_{||} + \frac{\nabla J_0}{\frac{B_\theta}{\mu_0}\left(1-\frac{nq(r)}{m}\right)}\tilde{A}_{||}$$

- For large m, solutions go like $A_{||}\sim r^{+m}$ ($r<r_s$) and $A_{||}\sim r^{-m}$ ($r>r_s$)

$$\Delta' = \frac{-2m}{r} \quad \text{(limit of high m)}$$

$\Rightarrow$ Resistive tearing modes always stable in high m,n limit

- Some other mechanism(s) required to drive microtearing instabilities…

- Parallel current is important, let's consider parallel electron momentum equation in the Braginskii (fluid) limit, $\omega \ll (k_{||}v_{Te})^2/\nu_e$, $k_{||}\lambda_e \ll 1$

Fluid limit of parallel electron momentum

$$n_e m_e \frac{dv_e}{dt} = -\nabla_{||} p_e - n_e e E_{||} + (n_e e)^2 \eta (v_i - v_e) - \alpha_T n_e \nabla_{||} T_e$$

Inertia | pressure | electric field | resistivity | thermal force

Parallel electron momentum
Braginskii
(1965)

Fluid limit of parallel electron momentum

$$n_e m_e \frac{dv_e}{dt} = -\nabla_{||} p_e - n_e e E_{||} + (n_e e)^2 \eta (v_i - v_e) - \alpha_T n_e \nabla_{||} T_e$$

Inertia pressure electric field resistivity thermal force

Parallel electron momentum Braginskii (1965)

- Assuming (n,m) perturbations around surfaces with rational q
- Linearize parallel gradients allowing for magnetic perturbations, $B_r = ik_y A_{||}$, e.g. $\nabla_{||} p_e = \nabla_{||,0} \tilde{p}_e + \tilde{\nabla}_{||} p_{e,0}$

$$\nabla_{||,0} = \frac{B_0 \cdot \nabla}{B_0} \approx \left(\frac{m - nq(r)}{qR} \right) \equiv ik_{||}(r)$$

$$\tilde{\nabla}_{||} = \frac{\tilde{B}_r \cdot \nabla}{B_0} \approx \frac{ik_y \tilde{A}_{||}}{B_0} \nabla_r$$

Fluid limit of parallel electron momentum

$$n_e m_e \frac{dv_e}{dt} = -\nabla_{||} p_e - n_e e E_{||} + (n_e e)^2 \eta (v_i - v_e) - \alpha_T n_e \nabla_{||} T_e$$

Inertia — pressure — electric field — resistivity — thermal force

Parallel electron momentum
Braginskii
(1965)

- Assuming (n,m) perturbations around surfaces with rational q
- Linearize parallel gradients allowing for magnetic perturbations, $B_r = ik_y A_{||}$, e.g. $\nabla_{||} p_e = \nabla_{||,0} \tilde{p}_e + \tilde{\nabla}_{||} p_{e,0}$

$$\nabla_{||,0} = \frac{B_0 \cdot \nabla}{B_0} \approx \left(\frac{m - nq(r)}{qR} \right) \equiv ik_{||}(r) \qquad \tilde{\nabla}_{||} = \frac{\tilde{B}_r \cdot \nabla}{B_0} \approx \frac{ik_y \tilde{A}_{||}}{B_0} \nabla_r$$

- Do same for T_e and $\nabla\varphi$ in $E_{||}$ $\quad E_{||} = -\frac{\partial}{\partial t} A_{||} - \nabla_{||}\varphi = i\omega A_{||} - ik_{||}\varphi$

- In simplest limit, ignore electron inertia and resistivity (m_e/m_i smaller)

Very near the rational surface the equilibrium parallel gradient vanishes, $k_{||}(r)\rightarrow 0$

$$0 = -ik_{||}\tilde{p}_e - \frac{ik_y\tilde{A}_{||}}{B}\nabla_r p_{e,0} - n_e e(i\omega A_{||} - ik_{||}\varphi) - \alpha_T n_e\left(ik_{||}\tilde{T}_e + \frac{ik_y\tilde{A}_{||}}{B}\nabla_r T_{e,0}\right)$$

$k_{||}(r)\rightarrow 0$ as $q(r)\rightarrow m/n$

$$0 = -\frac{ik_y\tilde{A}_{||}}{B}\nabla_r p_{e,0} - n_e e i\omega\tilde{A}_{||} - \alpha_T n_e\frac{ik_y\tilde{A}_{||}}{B}\nabla_r T_{e,0}$$

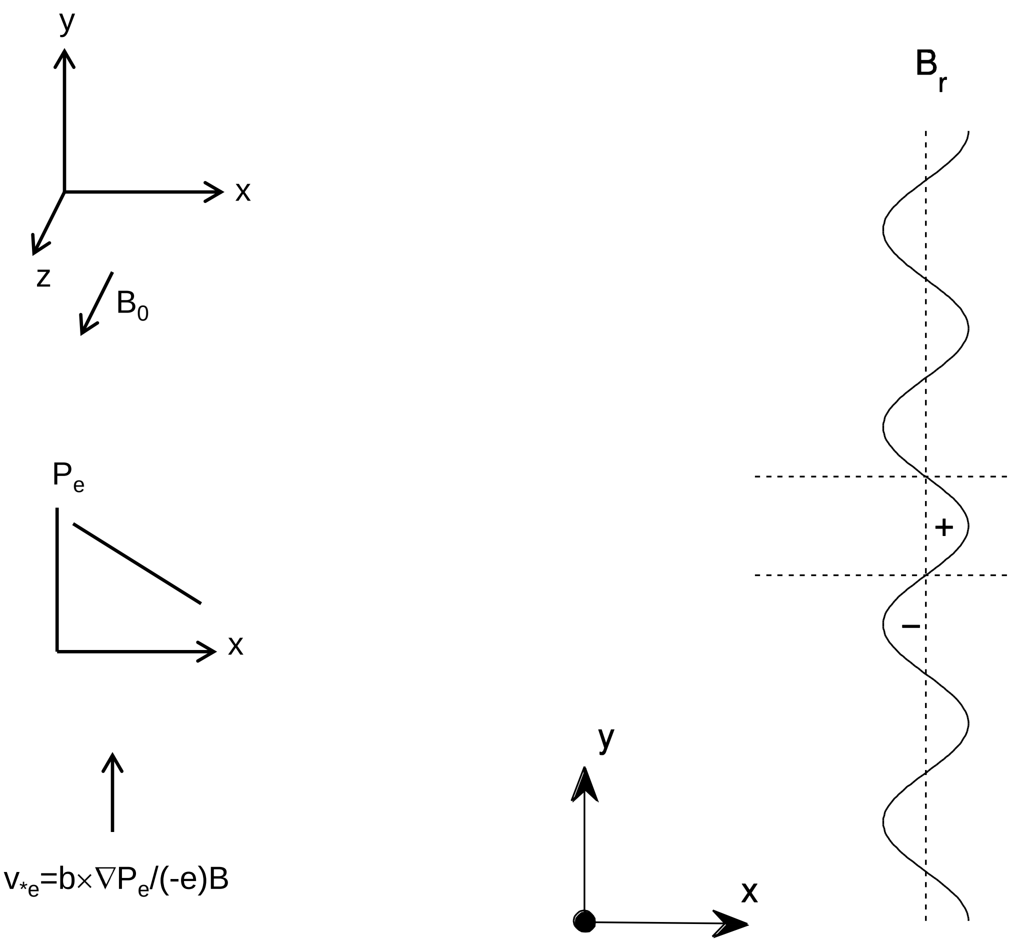
$$\boxed{0 = (\omega - k_y v_{*p} - k_y v_{*T})\tilde{A}_{||}}$$

$$v_{*p} = \frac{-\nabla p_{e,0}}{n_e eB}$$

$$v_{*T} = \frac{-\nabla T_{e,0}}{eB}$$

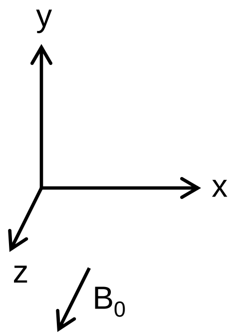
- Ignoring radial variations around the rational surface, parallel pressure and thermal force simply balanced by inductive electric field ($E_{||}=-dA_{||}/dt$), establishing a finite frequency, drift wave like magnetic perturbation

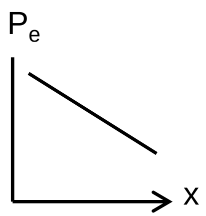
Schematic of magnetic drift wave propagation – imagine an infinitesimal magnetic perturbation at a resonant q=m/n

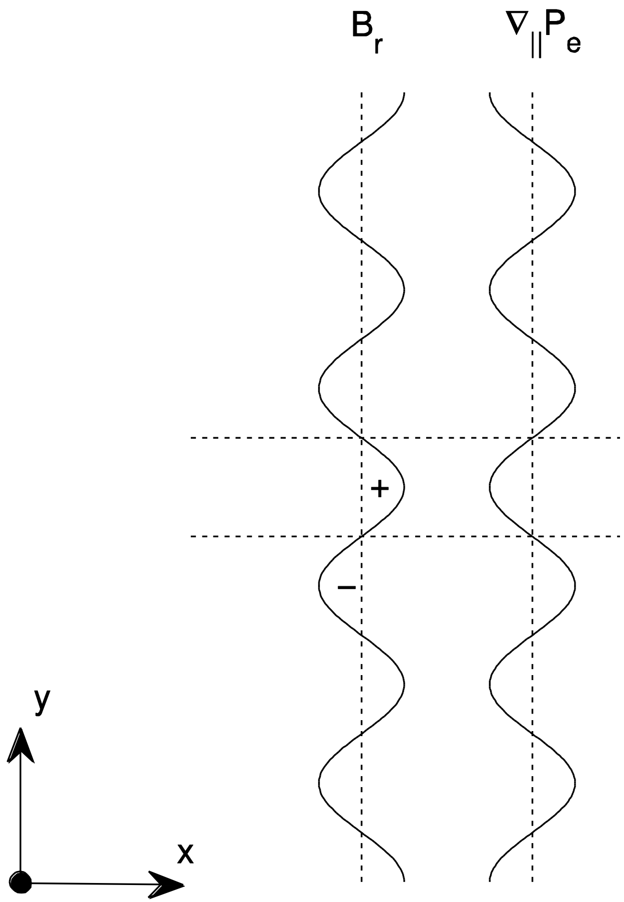


A parallel pressure gradient occurs in the presence of an equilibrium radial pressure gradient

$$\tilde{\nabla}_{\|} p_{e0} = \frac{\tilde{B}_r \cdot \nabla p_{e0}}{B_0} \approx \frac{i k_y \tilde{A}_{\|}}{B_0} \nabla_r p_{e0}$$

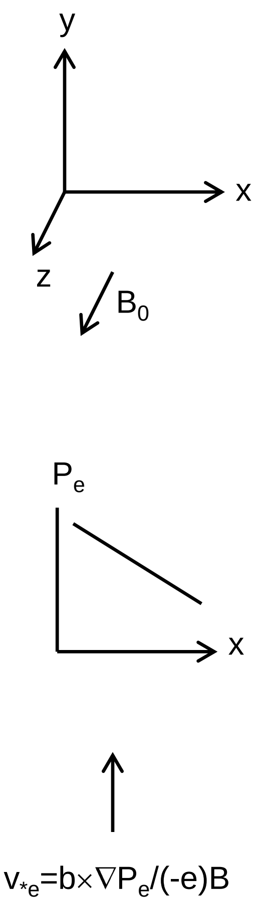


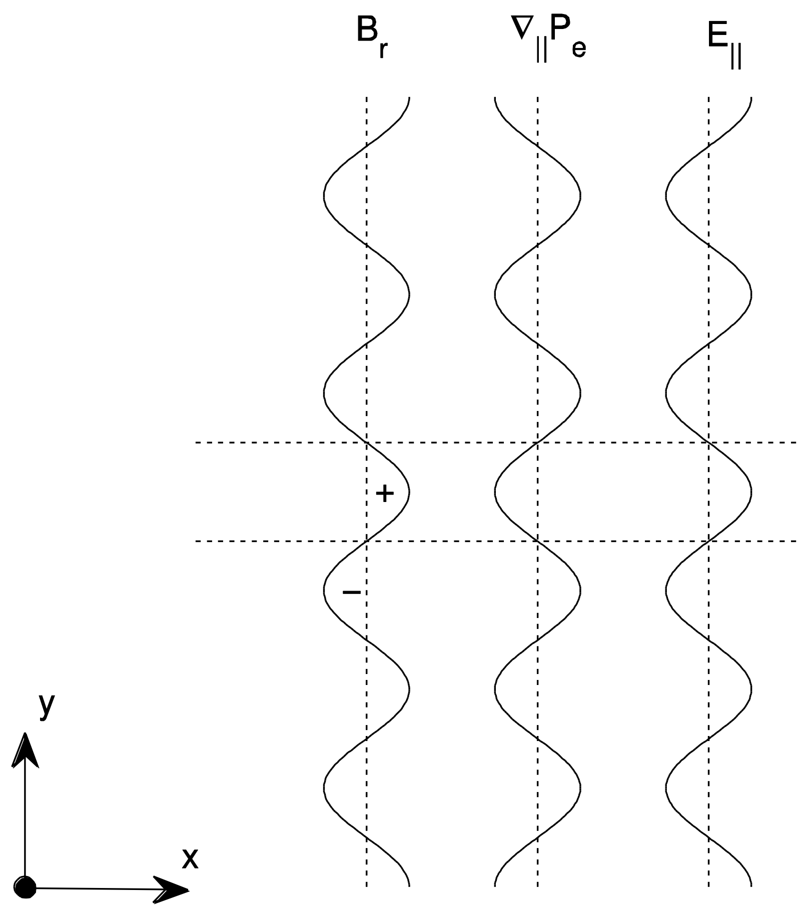




A parallel electric field is rapidly established to balance the pressure gradient force

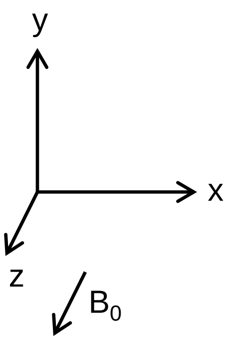
$$n_e e E_{||} = -\tilde{\nabla}_{||} p_{e0}$$

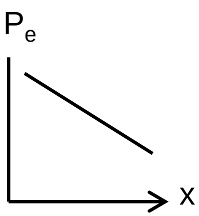




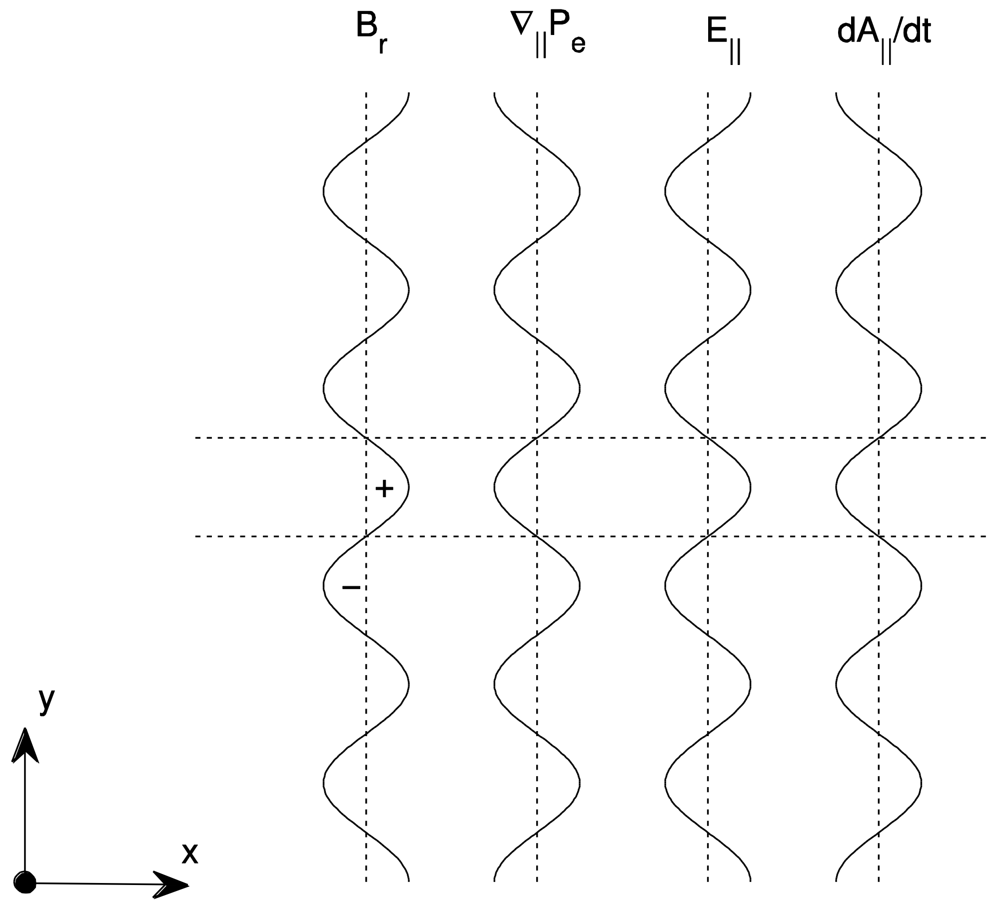
Only an inductive electric field as we're not allowing for electrostatic perturbations

$$\frac{dA_{||}}{dt} = -E_{||}$$





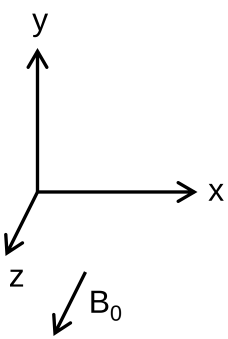
$v_{*e}=b\times\nabla P_e/(-e)B$

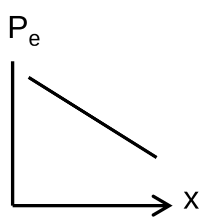


Inductive field causes dB/dt out of phase with B which propagates perturbation in electron drift direction

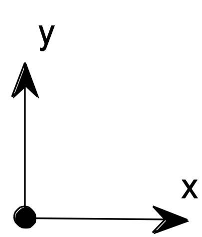
$$\frac{dB_r}{dt} = \frac{d}{dt}\left(\frac{dA_{||}}{dy}\right)$$

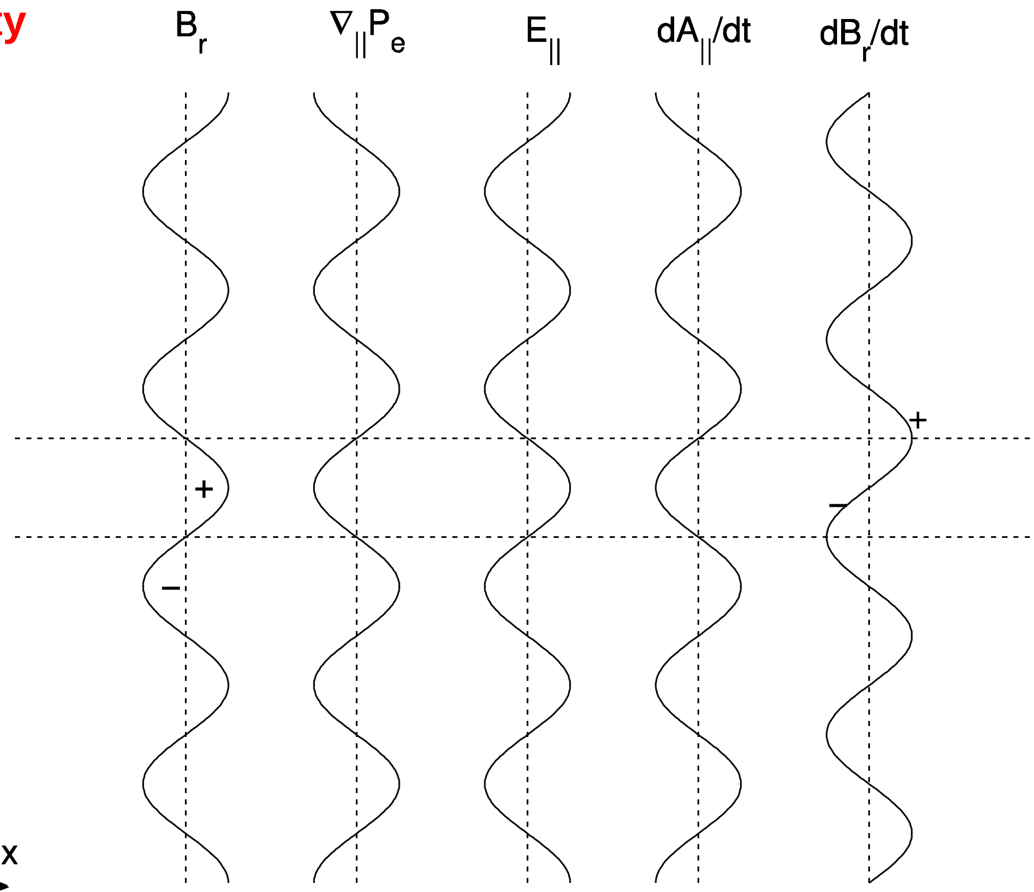
Establishes a propagating wave, but still no instability





$v_{*e}=b\times\nabla P_e/(-e)B$





Accounting for finite perturbation width introduces corrections from $k_{||}(r)$ and stabilization from Δ'

$$n_e m_e \frac{dv_e}{dt} = -ik_{||}\tilde{p}_e - \frac{ik_y \tilde{A}_{||}}{B}\nabla_r p_{e,0} - n_e e(i\omega A_{||} - ik_{||}\varphi) + n_e e\eta j_{||} - \alpha_T n_e \left(ik_{||}\tilde{T}_e + \frac{ik_y \tilde{A}_{||}}{B}\nabla_r T_{e,0} \right)$$

$$n_e m_e \frac{dv_e}{dt} - n_e e\eta j_{||} \sim (i\omega - \nu_e)\frac{m_e}{m_i}\left(\frac{\partial^2}{\partial x^2} - k_y^2\right)A_{||}$$

$\leftarrow$Ampere's law

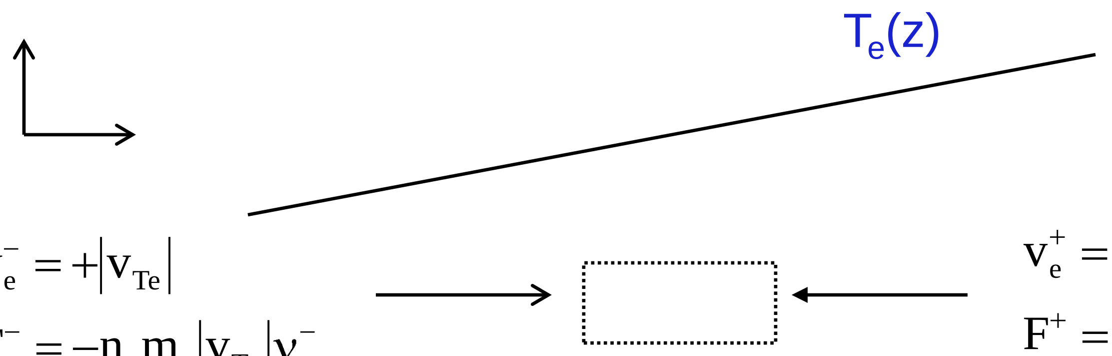
$$\mu_0 j_{||} = -\nabla_\perp^2 A_{||} = -\left(\frac{\partial^2}{\partial x^2} - k_y^2\right)A_{||}$$

$\Delta' \rightarrow -2m/r = -2k_y$

- Accounting for inertia and resistivity requires solving for radial structure and matching to ideal outer region solution which gives stabilizing influence
- So what causes instability?

Thermal force (Braginskii, 1965)

- Braginskii limit, collisional/fluid-like, very slow perturbations, $\omega << k_{||}^2 v_{Te}^2 / \nu_e$
- Electrons experience drag from ions $F = -n_e m_e (v_e - v_i)\nu_e$
- If T_e gradient exists, drag varies spatially



$$k_{||}\lambda_{mfp} << 1$$

$$v_e^- = +|v_{Te}|$$

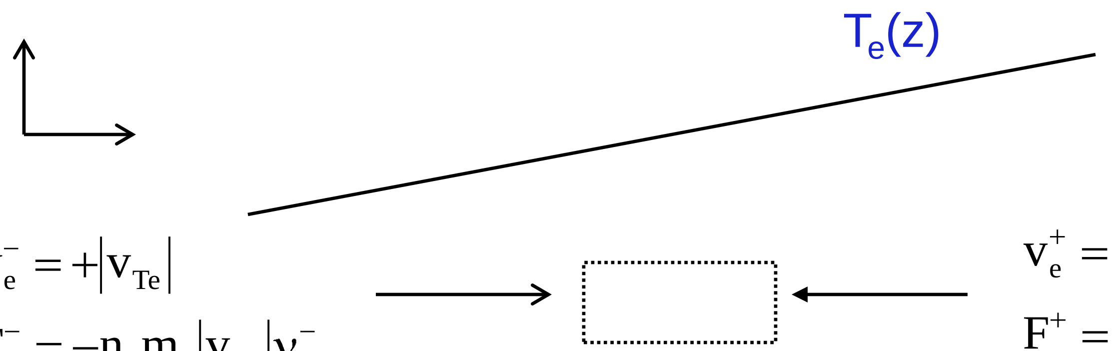
$$F^- = -n_e m_e |v_{Te}| \nu_e^-$$

$$v_e^+ = -|v_{Te}|$$

$$F^+ = n_e m_e |v_{Te}| \nu_e^+$$

Thermal force (Braginskii, 1965)

- Braginskii limit, collisional/fluid-like, very slow perturbations, $\omega \ll k_{||}^2 v_{Te}^2/\nu_e$
- Electrons experience drag from ions $F = -n_e m_e (v_e - v_i)\nu_e$
- If T_e gradient exists, drag varies spatially



$$k_{||}\lambda_{mfp} \ll 1$$

$$v_e^- = +|v_{Te}|$$
$$F^- = -n_e m_e |v_{Te}| \nu_e^-$$

$$v_e^+ = -|v_{Te}|$$
$$F^+ = n_e m_e |v_{Te}| \nu_e^+$$

$$R_{T||} = F^+ + F^- = n_e m_e v_{Te} (\nu_e^+ - \nu_e^-)$$

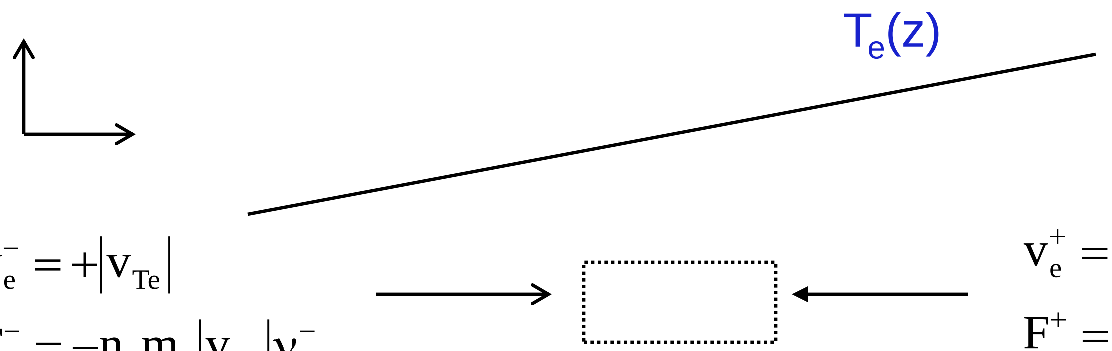
$$(\nu_e^+ - \nu_e^-) \approx \int dz \left(\frac{d\upsilon_e}{dT_e}\right)\frac{dT_e}{dz} = \int dz \left(-\frac{3}{2}\frac{\nu_e}{T_e}\right)\nabla_{||} T_e$$

$$\int dz \approx \lambda_{mfp} = \frac{v_{Te}}{\nu_e}$$

$$R_{T||} \approx n_e m_e v_{Te} \frac{v_{Te}}{\nu_e}\left(-\frac{3}{2}\frac{\nu_e}{T_e}\right)\nabla_{||} T_e = -1.5 n_e \nabla_{||} T_e$$

Thermal force (Braginskii, 1965)

- Braginskii limit, collisional/fluid-like, very slow perturbations, $\omega << k_{||}^2 v_{Te}^2/\nu_e$
- Electrons experience drag from ions $F = -n_e m_e (v_e - v_i)\nu_e$
- If T_e gradient exists, drag varies spatially



$$k_{||}\lambda_{mfp} << 1$$

$$v_e^- = +|v_{Te}| \qquad v_e^+ = -|v_{Te}|$$

$$F^- = -n_e m_e |v_{Te}| \nu_e^- \qquad F^+ = n_e m_e |v_{Te}| \nu_e^+$$

$$R_{T||} = F^+ + F^- = n_e m_e v_{Te} (\nu_e^+ - \nu_e^-)$$

$$(\nu_e^+ - \nu_e^-) \approx \int dz \left(\frac{d\upsilon_e}{dT_e}\right)\frac{dT_e}{dz} = \int dz \left(-\frac{3}{2}\frac{\nu_e}{T_e}\right)\nabla_{||}T_e$$

$$\int dz \approx \lambda_{mfp} = \frac{v_{Te}}{\nu_e}$$

$$R_{T||} \approx n_e m_e v_{Te} \frac{v_{Te}}{\nu_e}\left(-\frac{3}{2}\frac{\nu_e}{T_e}\right)\nabla_{||}T_e = -1.5 n_e \nabla_{||} T_e \quad \Rightarrow$$

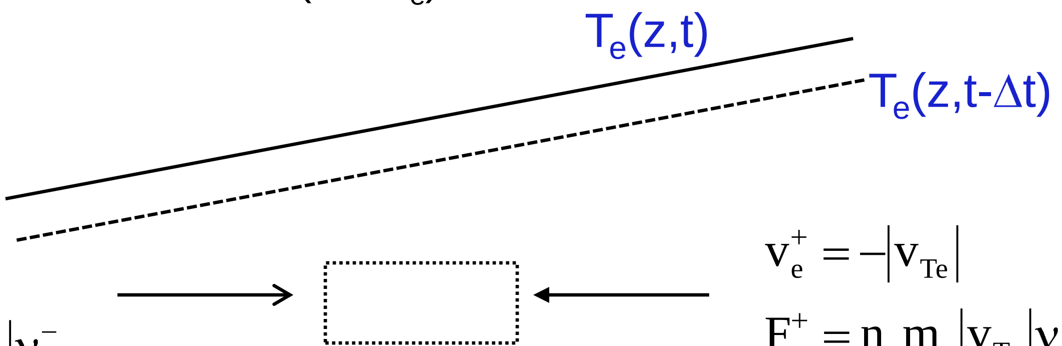
- $R_{T||}$ independent of collision rate although it depends explicitly on e-i collisions
- e-e collisions modify α_T

$$R_{T||} = -\alpha_T n_e \nabla T_e$$

$$\alpha_T = 0.71 \rightarrow 1.5 \quad \text{for} \quad Z_i = 1 \rightarrow \infty$$

Time-dependent thermal force (as described by A. Hassam, 1980)

- Slightly less restrictive constraint, $\omega \ll \nu_e$, 2nd order Chapman-Enskog expansion
- While electrons equilibrate on a time scale (ν_e^{-1}) faster than T_e varies (ω^{-1}), a small fraction will lag behind, adding a small correction ($\sim i\omega/\nu_e$)

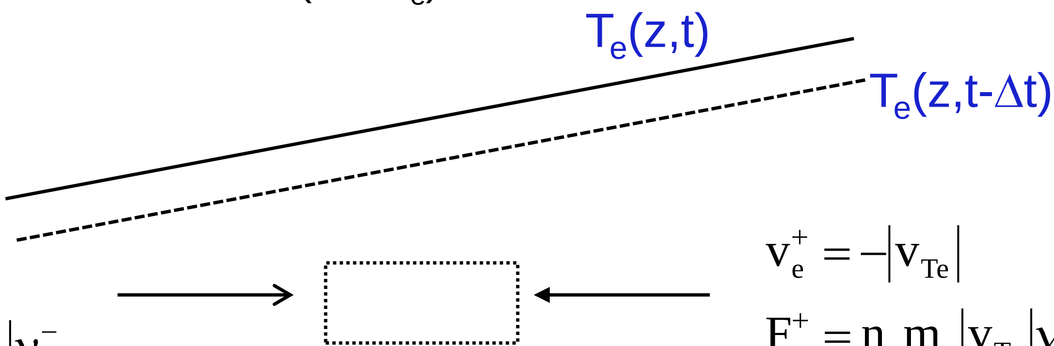


$$v_e^- = +|v_{Te}|$$
$$F^- = -n_e m_e |v_{Te}| \nu_e^-$$

$$v_e^+ = -|v_{Te}|$$
$$F^+ = n_e m_e |v_{Te}| \nu_e^+$$

Time-dependent thermal force (as described by A. Hassam, 1980)

- Slightly less restrictive constraint, $\omega \ll \nu_e$, 2nd order Chapman-Enskog expansion
- While electrons equilibrate on a time scale (ν_e^{-1}) faster than T_e varies (ω^{-1}), a small fraction will lag behind, adding a small correction ($\sim i\omega/\nu_e$)



$$v_e^- = +|v_{Te}|$$
$$F^- = -n_e m_e |v_{Te}| \nu_e^-$$

$$v_e^+ = -|v_{Te}|$$
$$F^+ = n_e m_e |v_{Te}| \nu_e^+$$

$$R_{T\|} \approx n_e m_e v_{Te} \frac{v_{Te}}{\nu_e}\left(-\frac{3}{2}\frac{\nu_e}{T_e}\right)\nabla_{\|} T_e$$

$$\frac{\partial}{\partial t}(R_{T\|})\Delta t = \frac{\partial R}{\partial T}\frac{\partial T}{\partial t}\nu_e^{-1} + \frac{\partial R}{\partial \nabla T}\frac{\partial \nabla T}{\partial t}\nu_e^{-1} \sim \frac{i\omega}{\upsilon_e} R_{T\|}$$

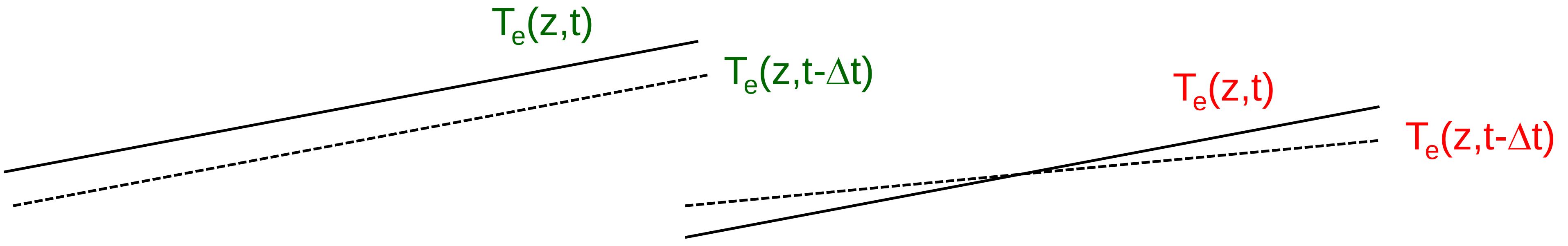
$$\Rightarrow R_{T\|} = -\alpha_T n_e \nabla T_e \left(1 + i\frac{\omega}{\upsilon_e}\alpha_{TD}\right)$$

Finite frequency of mode $\omega \approx \omega_{*e}$ is important

Similar effect occurs due to changing gradient

$$\alpha'' \frac{\partial}{\partial t}(nmu_{\parallel}) = -neE_{\parallel} - \nabla_{\parallel} p_e - (ne)^2 \eta u_{\parallel}\left[1 + \frac{3}{2}\alpha''\left(\frac{m}{\eta n e^2}\right)\frac{\partial}{\partial t}\ln T_e\right] - \alpha n \nabla_{\parallel} T_e\left(1 - \frac{3\alpha'}{\nu_e}\frac{\partial}{\partial t}\ln T_e - \frac{\alpha'}{\nu_e}\frac{\partial}{\partial t}\ln(\nabla_{\parallel} T_e)\right). \qquad (3)$$

(Hassam, 1980)



- Time lag corrections also in inertia and resistive terms

Time lag of thermal force now allows for instability

- $k_{||}(r) \to 0$ as $q(r) \to m/n$

$$0 = -\frac{ik_y \tilde{A}_{||}}{B}\nabla_r p_{e,0} - n_e e i\omega \tilde{A}_{||} - \alpha_T n_e \frac{ik_y \tilde{A}_{||}}{B}\nabla_r T_{e,0}\left(1 + \frac{i\omega}{\upsilon_e}\right)$$

$$0 = \left(\omega - \omega_{*p} - \omega_{*T} - i\omega_{*T}\frac{\omega}{\nu_e}\right)\tilde{A}_{||}$$

$$\boxed{\gamma \approx \omega_{*T}\frac{\omega}{\nu_e}}$$

Instability requires:
$\nu^{e/i}$ (TF)
finite ω_r (TDTF)
finite ∇T_e (TF)
finite β_e (couple to δB_r)

Δ' (field line bending) still provides stabilizing influence

$$n_e m_e \frac{dv_e}{dt} = -ik_{\|}\tilde{p}_e - \frac{ik_y \tilde{A}_{\|}}{B}\nabla_r p_{e,0} - n_e e(i\omega A_{\|} - ik_{\|}\varphi) + n_e e\eta j_{\|} - \alpha_T n_e\left(ik_{\|}\tilde{T}_e + \frac{ik_y \tilde{A}_{\|}}{B}\nabla_r T_{e,0}\left(1 + \frac{i\omega}{\nu_e}\right)\right)$$

$$n_e m_e \frac{dv_e}{dt} - n_e e\eta j_{\|} \sim (i\omega - \nu_e)\frac{m_e}{m_i}\left(\frac{\partial^2}{\partial x^2} - k_y^2\right)A_{\|}$$

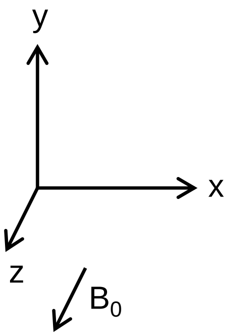
$\Delta' \rightarrow$-2m/r=-2k_y

$$\gamma \approx \omega_{*T}\frac{\omega}{\nu_e} + \Delta' \cdot (k_y s \cdots)$$

Finite threshold for instability

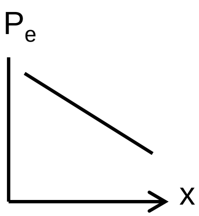
Depends on magnetic shear and stuff

$\nabla_{\|}T_e$ part gives wave propagation like $\nabla_{\|}P_e$

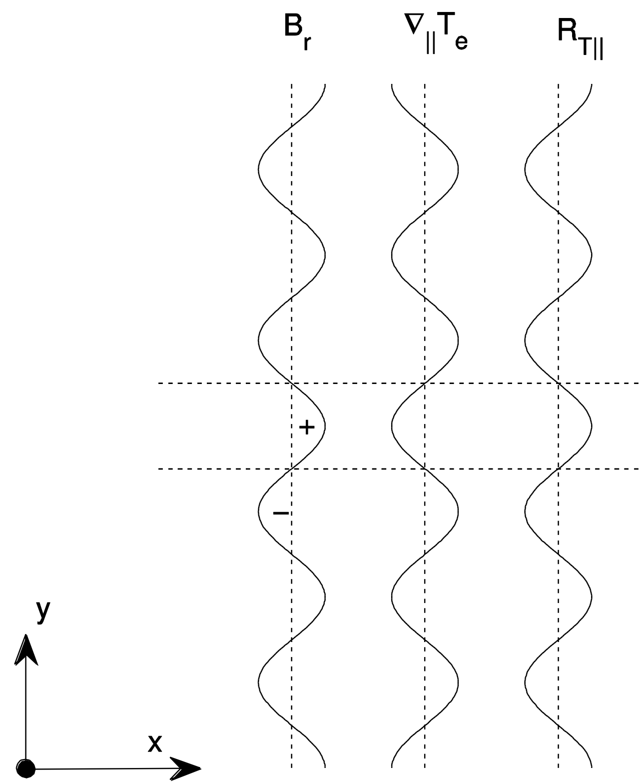


$$n_e e E_{\|} = R_{T,\|}$$

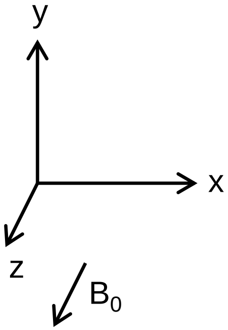
→wave propagation

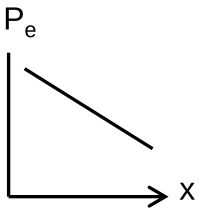


$v_{*e}=b\times\nabla P_e/(-e)B$

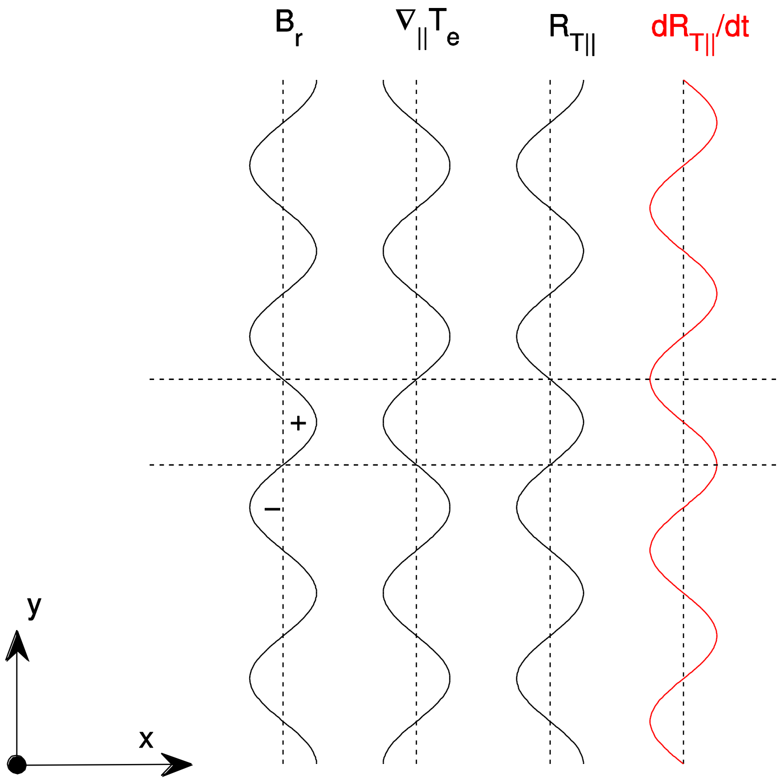


Time dependent part gives a lag contribution



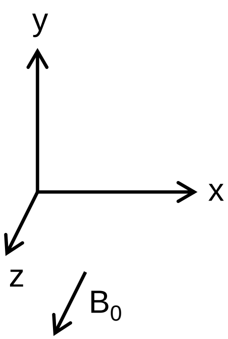


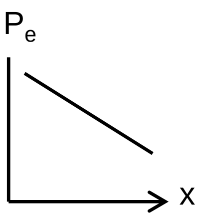
$v_{*e}=b\times\nabla P_e/(-e)B$



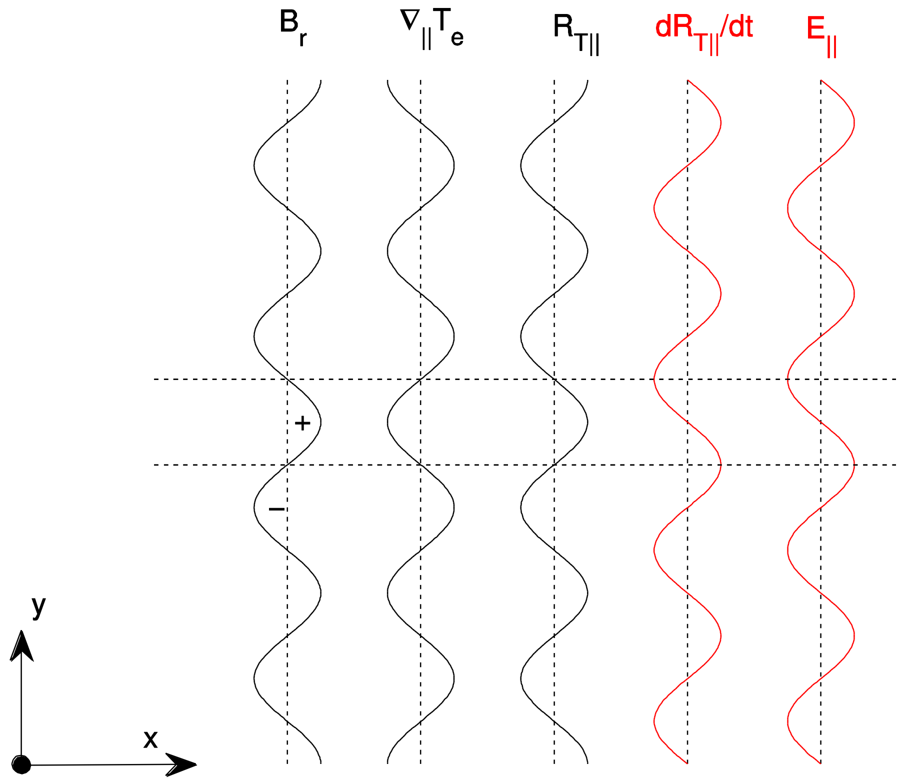
Time dependent part gives a lag contribution

$$n_e e E_{\|} = \frac{dR_{T,\|}}{dt} \sim \frac{i\omega}{\nu_e}$$





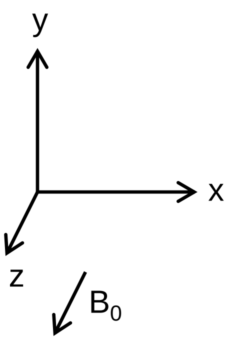
$v_{*e} = b \times \nabla P_e / (-e)B$

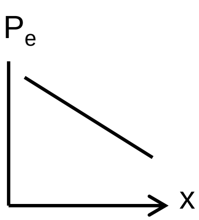


Inductive field dB/dt from time lag thermal force adds in-phase to initial perturbation → linear growth

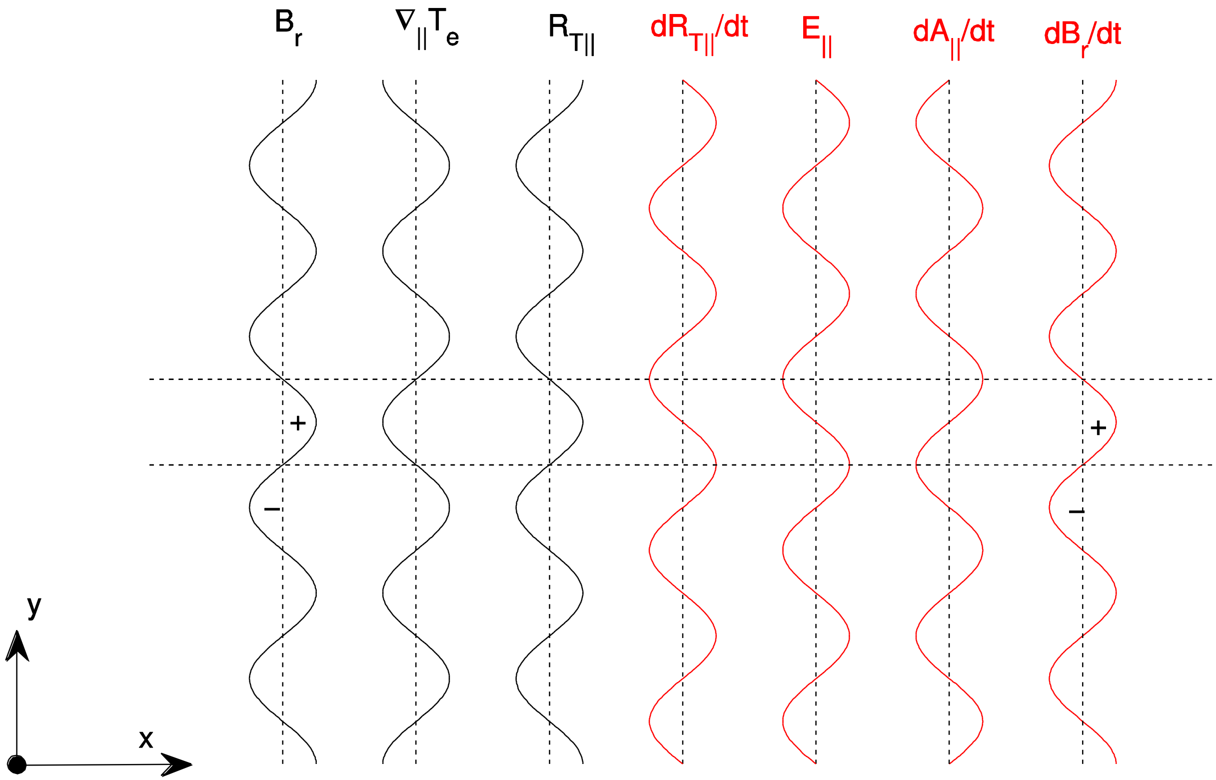
$$\frac{dA_{||}}{dt} = -E_{||}$$

$$\frac{dB_r}{dt} = \frac{d}{dt}\left(\frac{dA_{||}}{dy}\right)$$



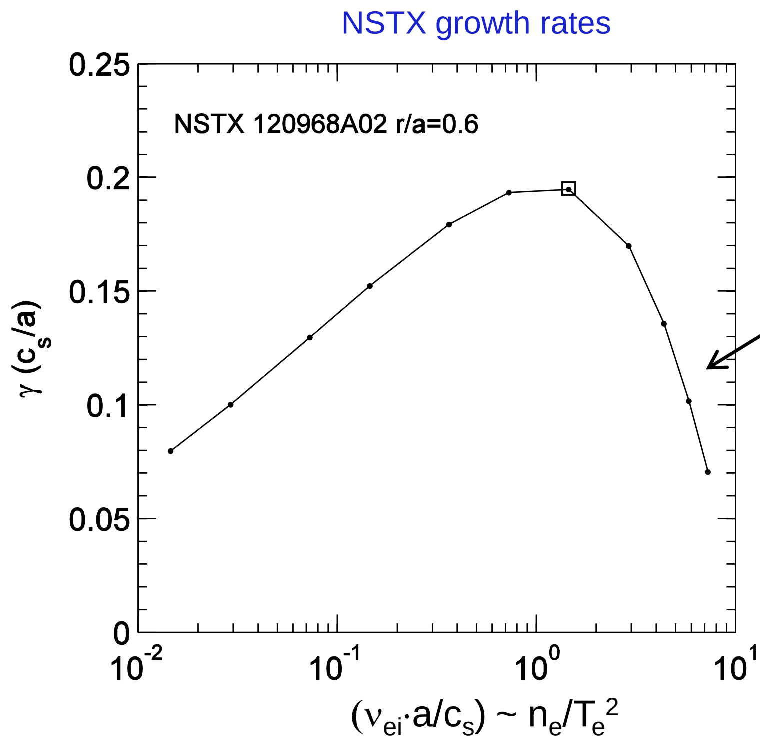


$v_{*e}=b\times\nabla P_e/(-e)B$



Collisionality dependence from TDTF predicted at high collisionality

- Peak γ occurs for $\nu^{e/i}/\omega$ ~ 1-6
- Experimental values often $\nu^{e/i}/\omega$ <=1,



"semi-collisional" regime of Drake & Lee (1997), $\nu^{e/i}$>>ω (& Δ_j<ρ_s)

γ~$\omega/\nu^{e/I}$

Hassam (1980) fluid approach

Experimental values often more weakly-collisional

- Peak γ occurs for $\nu^{e/i}/\omega$ ~ 1-6
- Experimental values often $\nu^{e/i}/\omega$ <=1,

weakly-collisional regime, $\nu^{e/i} \leq \omega$

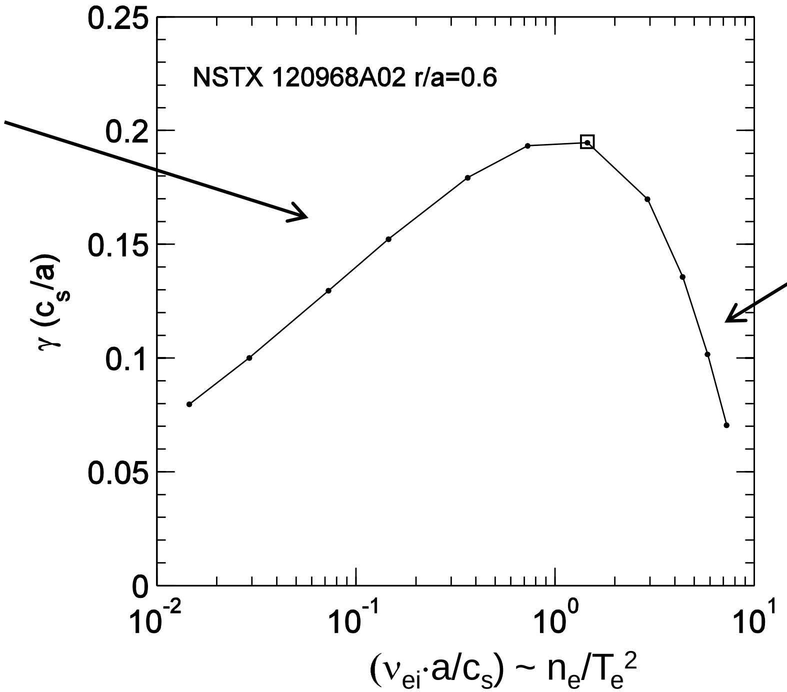
$\gamma \sim \nu^{e/i}/\omega$

Requires improved (numerical) kinetic treatment:

<u>Slab examples:</u>
Hazeltine et al. (1975)
Gladd et al. (1980)
D'Ippolito et al. (1980)
Rosenberg et al. (1980)

<u>Gyrokinetic torus examples:</u>
Redi et al. (2003-2005)
Roach et al. (2005)
Applegate et al. (2006/2007)
Wong et al. (2007/2008)
Told et al. (2008)
Predebon et al. (2010)
...

NSTX growth rates



"semi-collisional" regime of Drake & Lee (1997), $\nu^{e/i} >> \omega$ (& $\Delta_j < \rho_s$)

$\gamma \sim \omega/\nu^{e/I}$

Hassam (1980) fluid approach

Non-monotonic ν_e scaling consistent with time-dependent thermal force (TDTF) *when treated kinetically*

$$R_T = -\alpha_T \cdot n_e \nabla T_e$$

- Fully kinetic (Hazeltine et al., 1975; Gladd et al., 1980; D'Ippolito et al., 1980; Rosenberg et al., 1980)

  $0<\omega/\nu_e<\infty$, $k_{||}\lambda_{mfp}<<1$

$$\alpha_T(\omega) = 0.8\frac{1+i\cdot 0.54(\omega/\nu_e)}{1+0.29(\omega/\nu_e)^2}$$

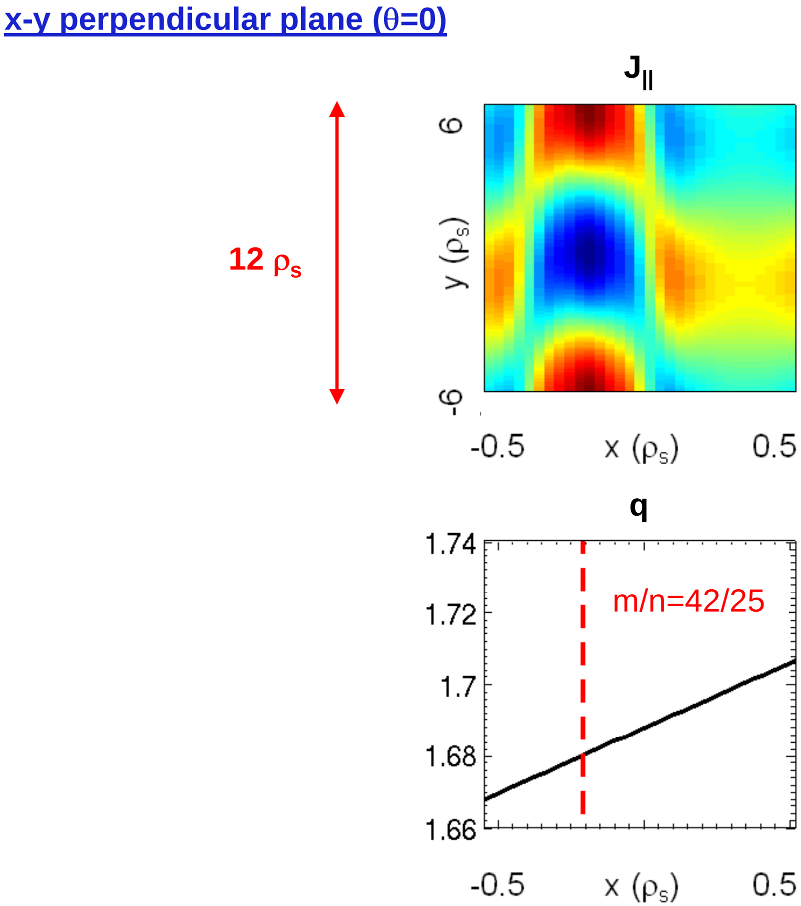
- Makes assumptions on inner layer width ($\Delta>r_s$) and mean free path ($k_{||}\lambda_{mfp}<<1$) that are typically violated in hot core of tokamaks

- In addition, other "flavors" of microtearing modes exist such as purely collisionless often near the plasma edge, possibly driven by $\nabla B/\kappa$ drifts [Canik, IAEA 2012; Dickinson, arXiv; Predebon, arXiv; Carmody, arXiv], or stable microtearing modes driven non-linearly [Hatch, 2012]

$\Rightarrow$ Missing unified analytic (or semi-analytic) theory for understanding and modeling

Linear mode structure in perpendicular plane illustrates key microtearing mode features

- Narrow resonant current channel ($\approx 0.3\rho_s \approx 1.4$ mm) centered on rational surface



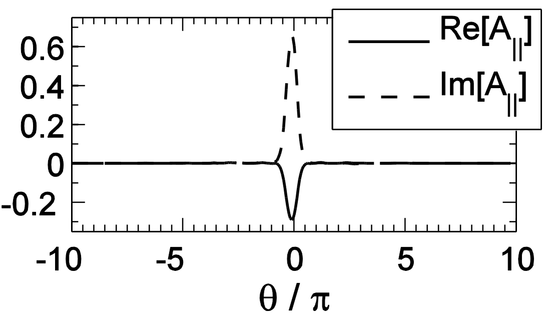
Linear mode structure in perpendicular plane illustrates key microtearing mode features

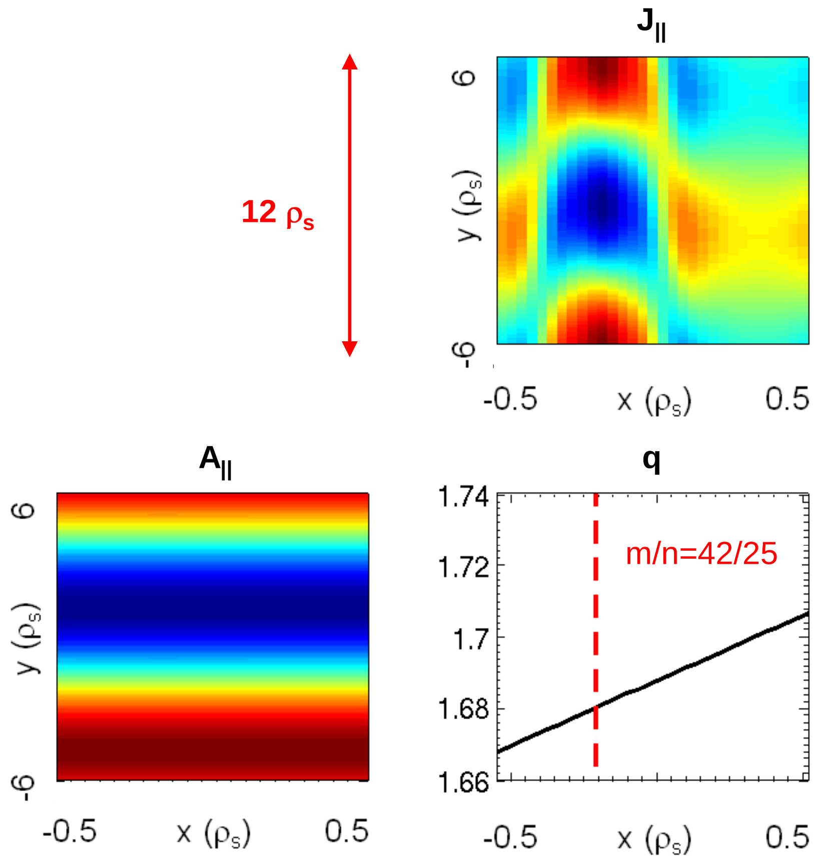
- Narrow resonant current channel (≈$0.3\rho_s$≈1.4 mm) centered on rational surface
- Finite $\langle A_{||}\rangle_\theta$ (resonant tearing parity), strongly ballooning

x-y perpendicular plane (θ=0)

"ballooning" space

$$k_r(\theta) = \hat{s} k_\theta (\theta - \theta_0)$$



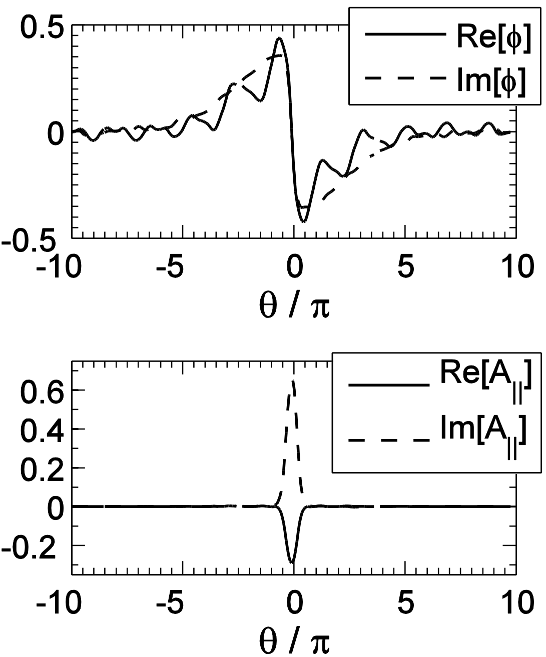


Linear mode structure in perpendicular plane illustrates key microtearing mode features

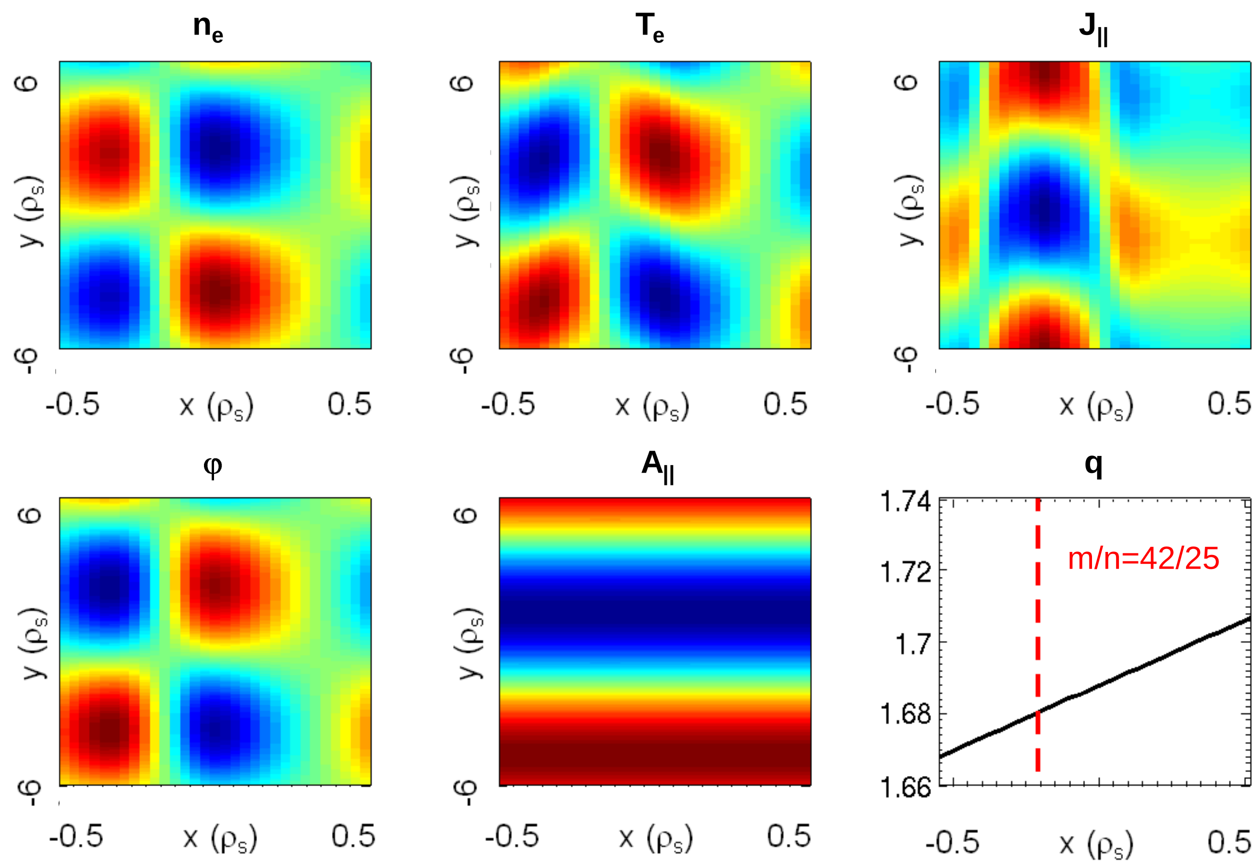
- Narrow resonant current channel ($\approx 0.3\rho_s \approx 1.4$ mm) centered on rational surface
- Finite $\langle A_{||}\rangle_\theta$ (resonant tearing parity), strongly ballooning
- Narrow n_e & T_e perturbations
- Nearly unmagnetized/adiabatic ion response $\Rightarrow \frac{\tilde{n}}{n_0} \approx -Z_{eff}\left(\frac{e\tilde{\varphi}}{T_i}\right)$

"ballooning" space

$$k_r(\theta) = \hat{s}k_\theta(\theta - \theta_0)$$

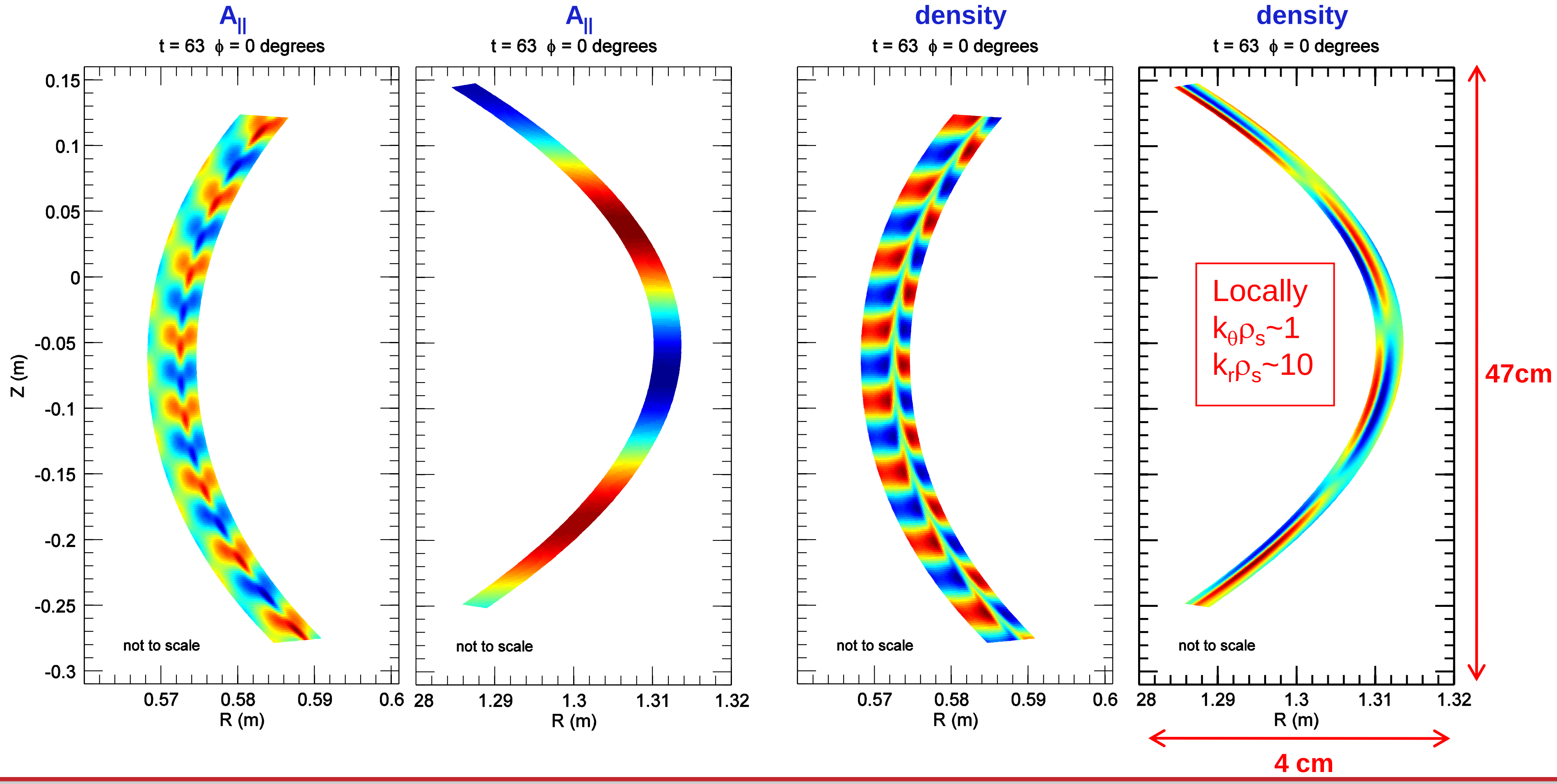


x-y perpendicular plane (θ=0)

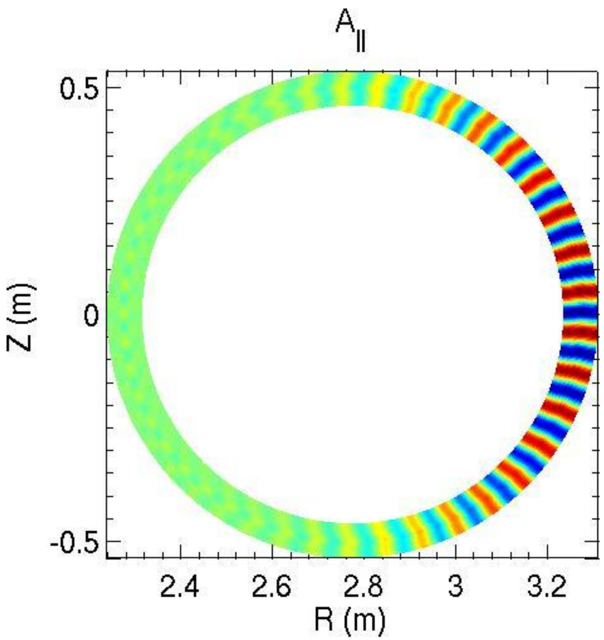


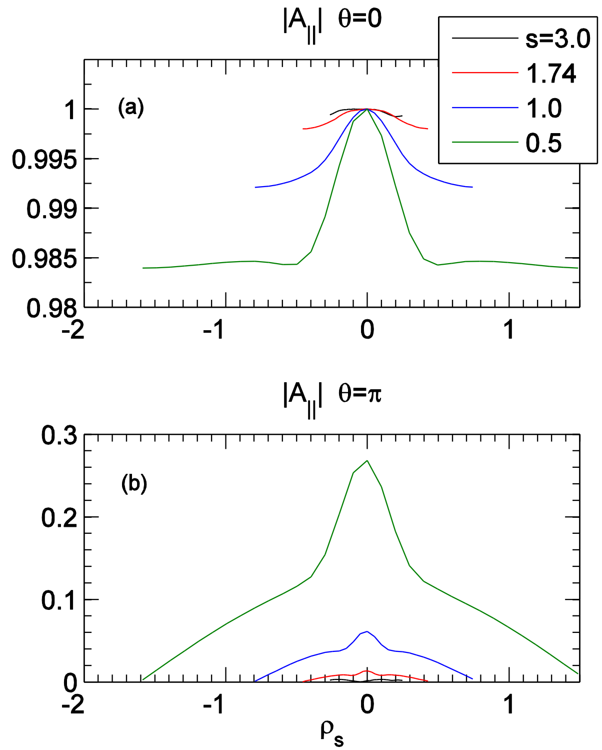
Linear mode structure in NSTX toroidal (R,Z) plane

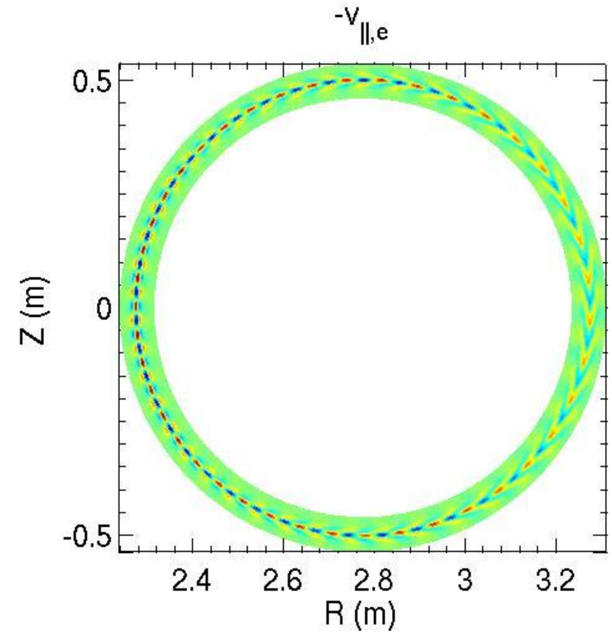
- Nonuniform poloidal structure (comparing inboard and outboard perturbations)
- Density perturbations radially narrow, extended vertically on outboard side

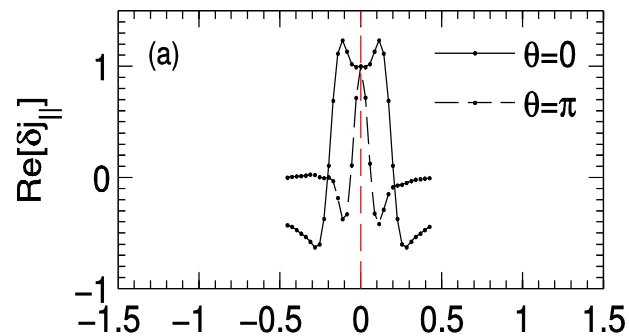


$A_{||}$ much stronger on outboard side (ballooning), $j_{||}$ stronger and narrower on inboard side









- Analytic theories know nothing of these poloidal variations

Field line integration used to identify island growth

- δB_r in linear run (arbitrary) determines $w_{island} \sim 0.4\ \rho_s$

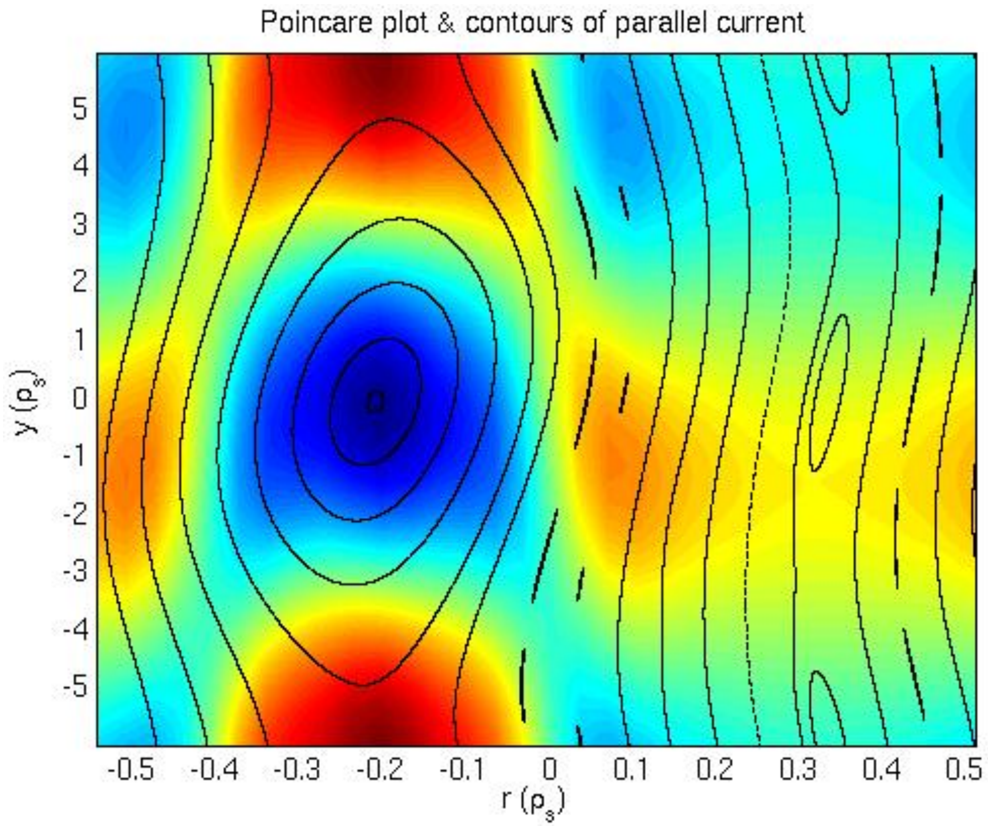
- Estimate using rms δB_r

$$\left| \left\langle \frac{\delta B_r^2}{B^2} \right\rangle_{\alpha,\theta} \right|^{1/2} = 2.5 \cdot 10^{-5}$$

$$w = 4 \cdot \left[\left(\frac{\delta B_r}{B} \right)_{rms} \frac{rR}{n\hat{s}} \right]^{1/2} = 0.39\rho_s$$

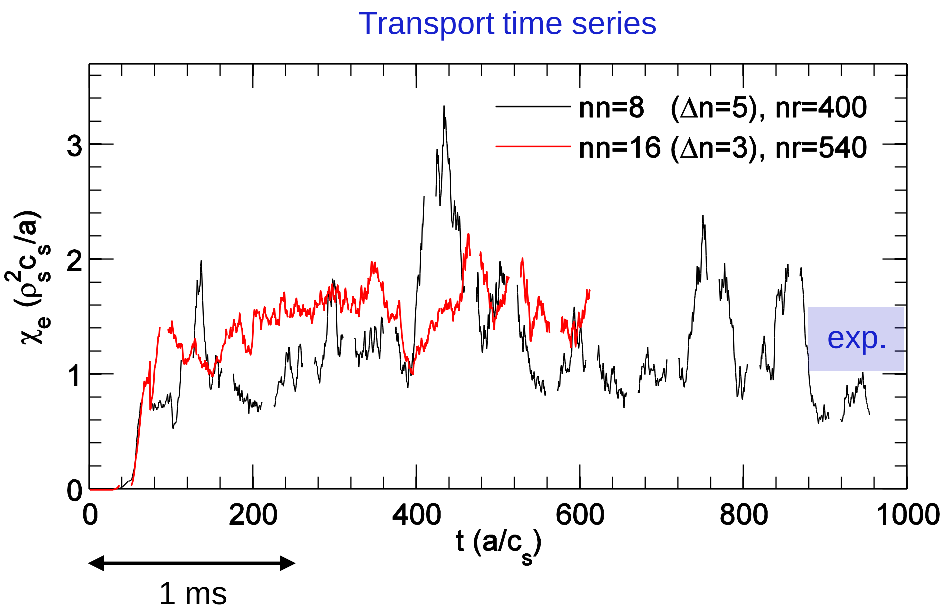
- $w_{island}/L_{Te} \approx 8 \cdot 10^{-3}$ but $\max(\delta T_e/T_e) \approx 4.5 \cdot 10^{-4}$

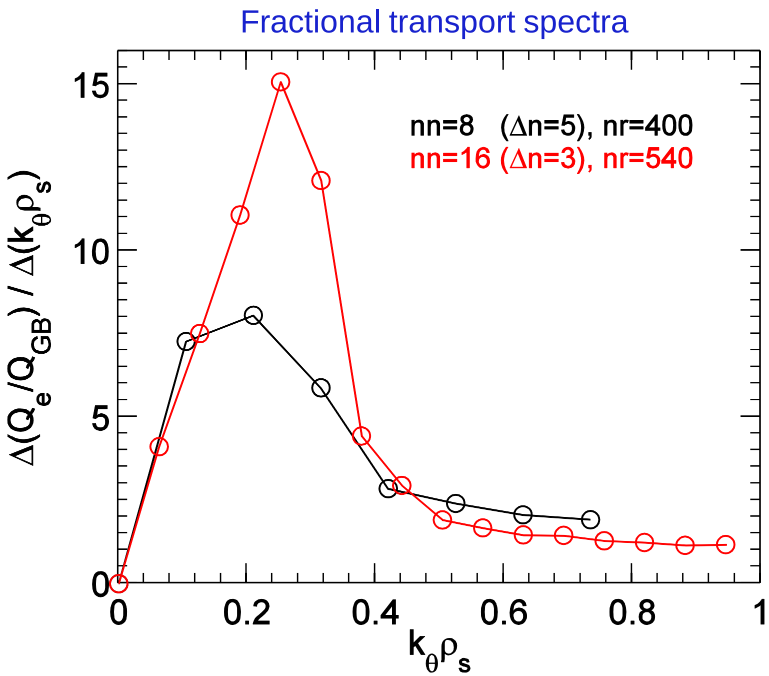
$\Rightarrow$ Additional drift dynamics important



Predicted electron thermal transport comparable to experiment

- Simulated transport (1.2 $\rho_s^2 c_s/a$, 6 m^2/s) comparable to experimental transport (1.0-1.6 $\rho_s^2 c_s/a$)
- Well defined peak in transport spectra ($k_\theta \rho_s \approx 0.2$), downshifted from maximum γ_{lin} ($k_\theta \rho_s \approx 0.6$)





$$k_\theta = \frac{nq}{r}$$

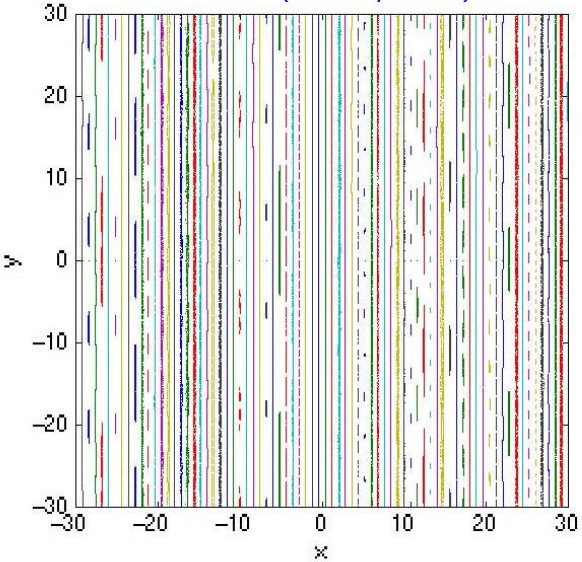
equivalent toroidal mode numbers n=[0,3,6,9…45]

- Negligible particle, momentum, or ion thermal transport

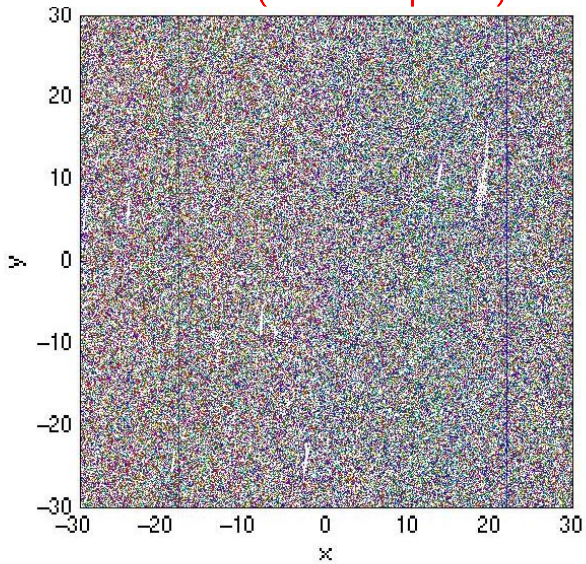
~98% of transport due to magnetic "flutter" contribution

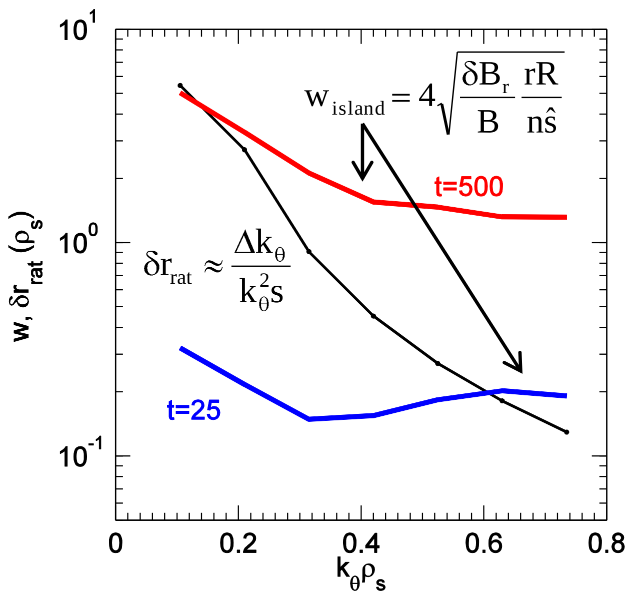
- Flux surfaces become distorted in linear phase (t=25)
- Globally stochastic in saturated phase, complete island overlap $w_{island}(n) > \delta r_{rat}(n)$

t=25 (linear phase)



t=500 (saturated phase)



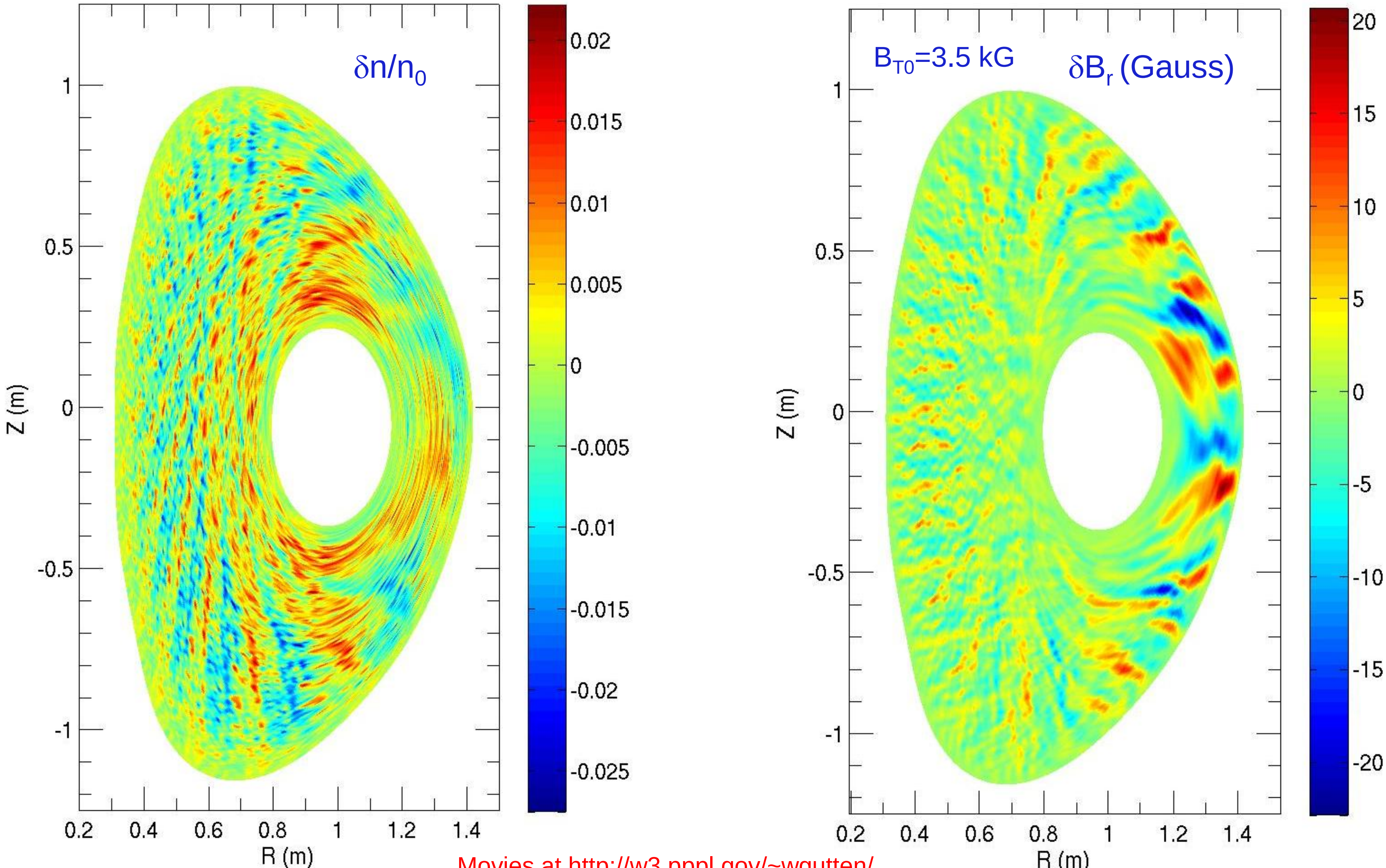


- $\chi_{e,EM}$ close to *collisionless* Rechester-Rosenbluth* (λ_{mfp}=12 m, $L_c \approx$2.5 m)

$$D_{st} = \lim_{s\to\infty} \frac{\langle [r_i(s) - r_i(0)]^2 \rangle}{2s} \qquad \chi^{RR} \approx 2\left(\frac{2}{\pi}\right)^{1/2} D_{st} v_{Te} \approx 0.9\left(\frac{\rho_s^2 c_s}{a}\right)$$

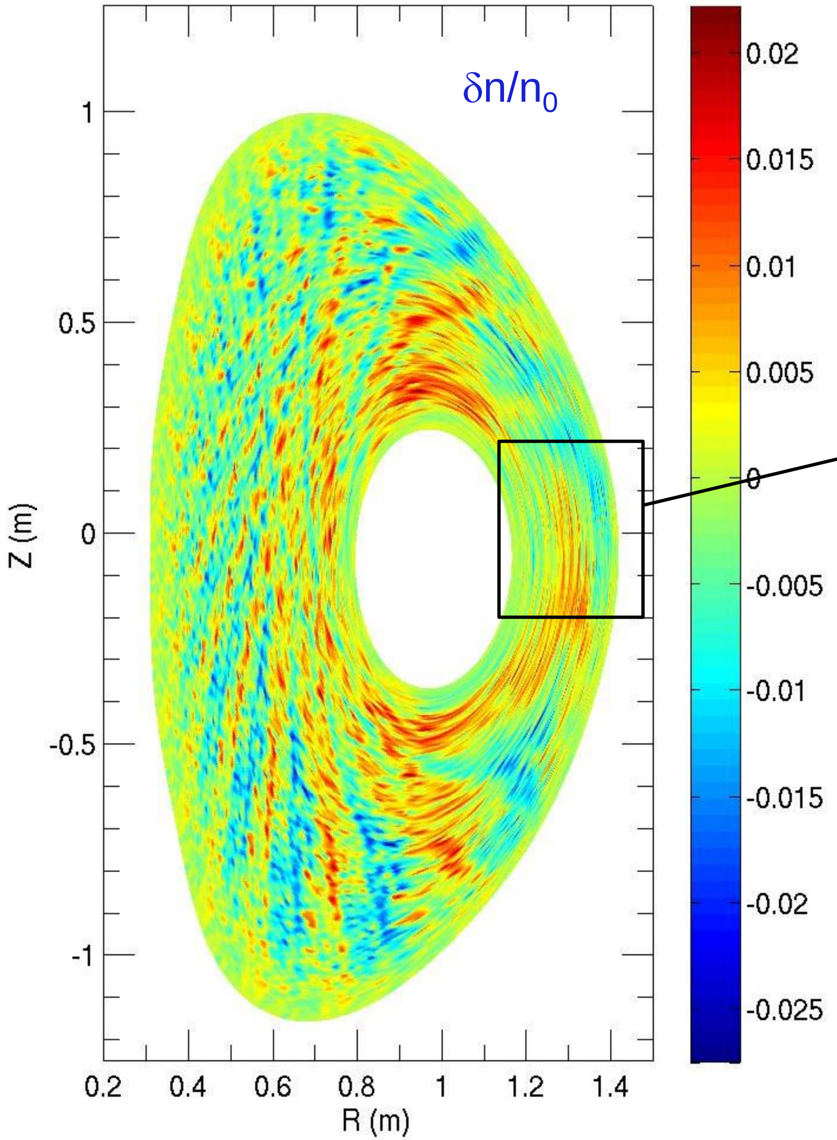
- Unclear what sets overall saturation and scaling of $\delta B_r/B_0$
- Also of interest, what determines minimum numerical resolution Δx requirements?

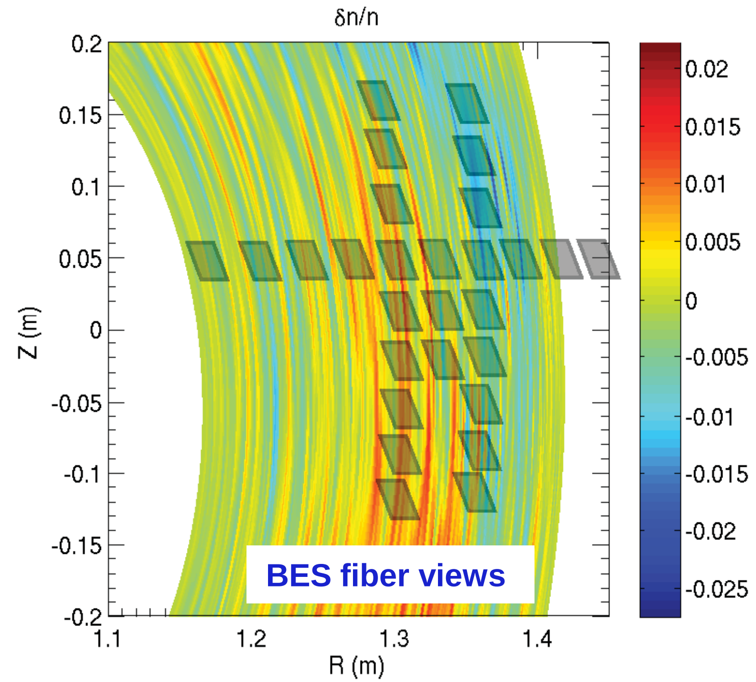
How can we experimentally identify microtearing modes?



Movies at http://w3.pppl.gov/~wgutten/

BES for density fluctuations

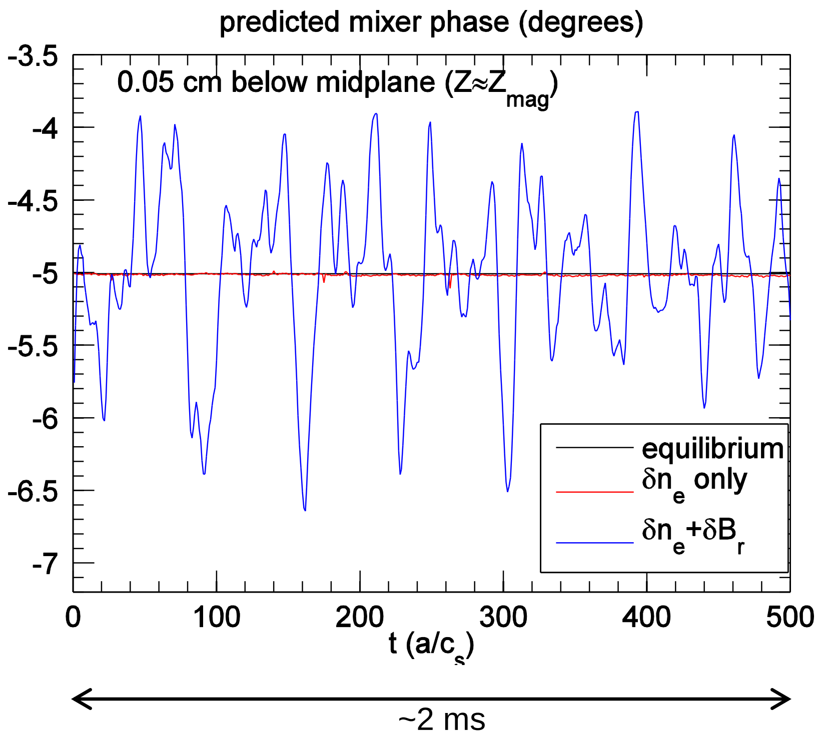


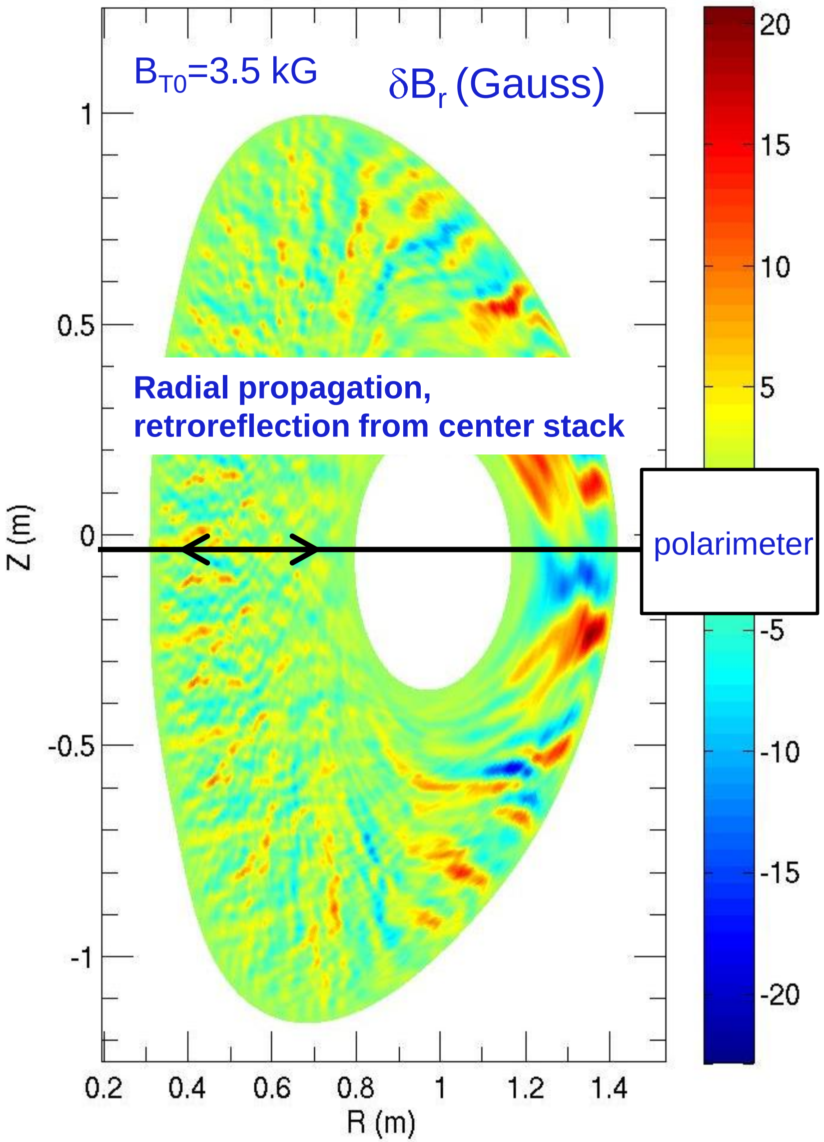


- BES suitable for long poloidal scale (U-Wisconsin, Smith et al., RSI 2010)
- May average over narrow radial scale – requires synthetic diagnostic and instrument function (D. Smith, BO4.2)

Polarimetry for magnetic field fluctuations

- New UCLA polarimetry system (Zhang, 2013)
- Simulations suggest $(\delta B/B)_{internal} \leq 0.1\%$ may be detectable ($1\text{-}2^0$ or $\sim 0.3^0$ rms mixer phase)





⇒ **Useful to research other potential diagnostics to infer δB_r (e.g. MSE, external magnetics, …?)**

Summary

- Microtearing modes predicted in many devices NSTX, MAST, ASDEX-UG, DIII-D, JET, RFX, MST
- Initial nonlinear simulations suggest they could cause significant electron thermal transport
- Still lack of understanding in thresholds (a/L_{Te}, β_e), scaling with other parameters of interest (ν_e, s), distinction of different instability drives (time-depedent thermal force, curvature/grad-B drifts), saturation and non-linear scaling, …

<u>Possible future work:</u>

1. Analytic theory improvements including:
   (i) arbitrary ω/ν_e and magnetic shear
   (ii) toroidal effects, curv/grad-B, strong ballooning, trapped particles
   (iii) influence of potential
   (iv) prediction of (a/L_{Te}, β_e) thresholds as a function of relevant parameters (R/a, nu, s, q)
2. Improve understanding in saturation model ($\rightarrow$improve transport models, e.g. TGLF)
   – What sets nonlinear $\delta B_r/B$ and how does it scale?
3. Develop & improve diagnostic measurement and interpretation
   – How can we distinguish microtearing turbulence from others?