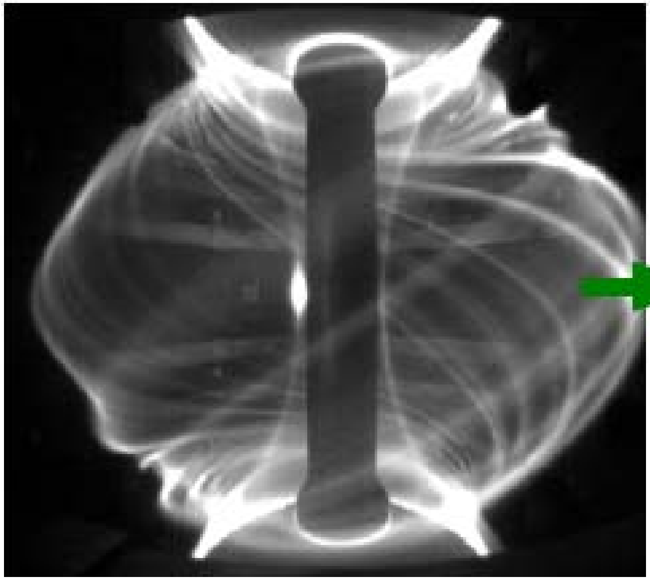
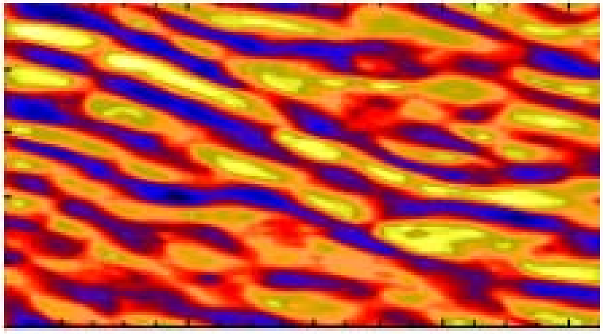
centre for fusion, space and astrophysics

THE UNIVERSITY OF WARWICK

# Micro-instabilities and Turbulence In Toroidal Magnetically Confined Plasmas



Power loss



Power loss


Walter Guttenfelder


Postdoc Seminar

December 15, 2009

# Motivation

- Idealistic goal of magnetic fusion energy (MFE) – create a self-sustaining “burning” plasma with ~1 GWe of power

- Requires sufficient pressure & energy confinement to obtain fusion power > power loss

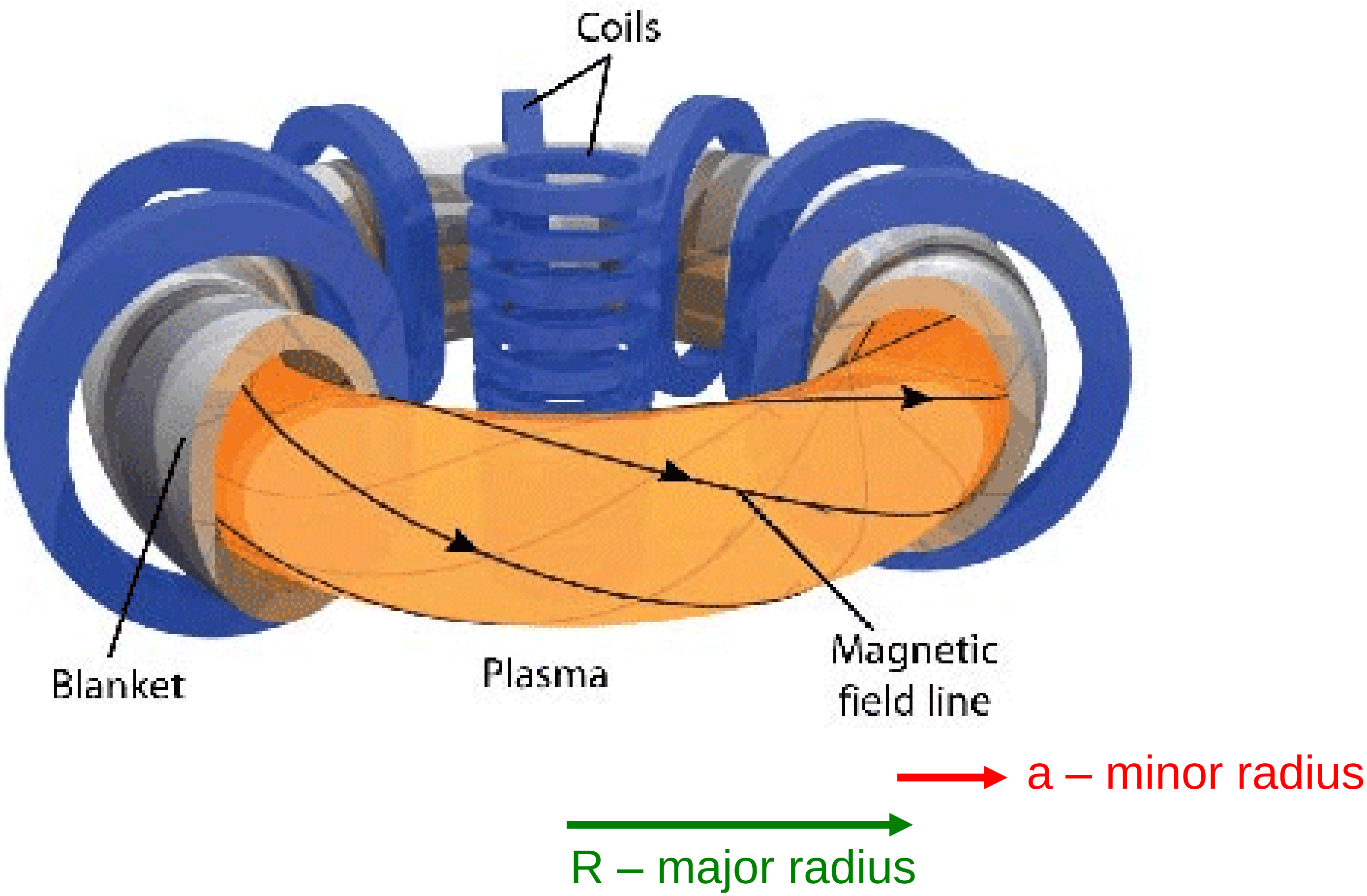
$$nT\tau_E > 5\times10^{21}\ \mathrm{m^{-3}\cdot keV\cdot s}$$

- Confinement ($\tau_E$) dominated by turbulent transport (much greater than collisional diffusion)

$$\chi_{turb} >> \chi_{collisions} \qquad \left(q = -n\chi\nabla T\right)$$

$\Rightarrow$ Want to understand (reduce?) turbulent transport in MFE

# Toroidal Magnetic Geometry For Plasma Confinement - Tokamaks




- Helical magnetic field provides nested flux surfaces to confine energetic (~100 million °C, 10 keV) plasma

$$\text{safety factor} = q = \frac{\text{toroidal transits}}{\text{poloidal transits}}$$

# Gyromotion In A Magnetic Field

- Particles free to move parallel to B
- Strong magnetic field (B=5T) leads to a deuterium gyroradius (~10 keV)

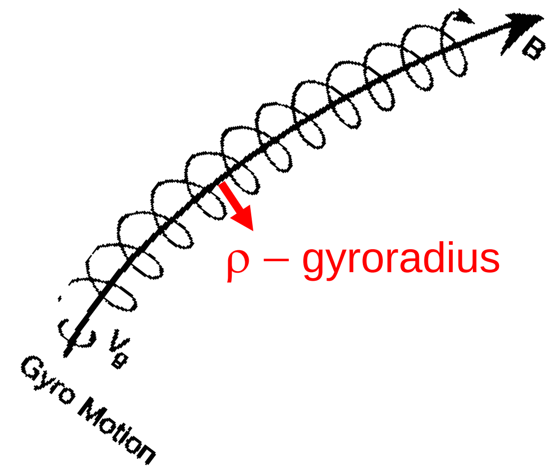
  $\rho_i \approx 3.7$ mm << 1-5 meter device size

- If transport was due only to collisional diffusivity

$$\chi_{coll} \sim \frac{\Delta x^2}{\Delta t} \sim \rho_i^2 \cdot \nu_i$$

confinement times would be sufficiently long, also increasing with temperature

$$\tau_{E,coll} \sim \frac{a^2}{\chi_{coll}} \sim \left(\frac{a}{\rho_i}\right)^2 \frac{1}{\nu_i} \sim T_i^{1/2}$$




$$m\frac{d\vec{v}}{dt} = q\vec{v} \times \vec{B}$$

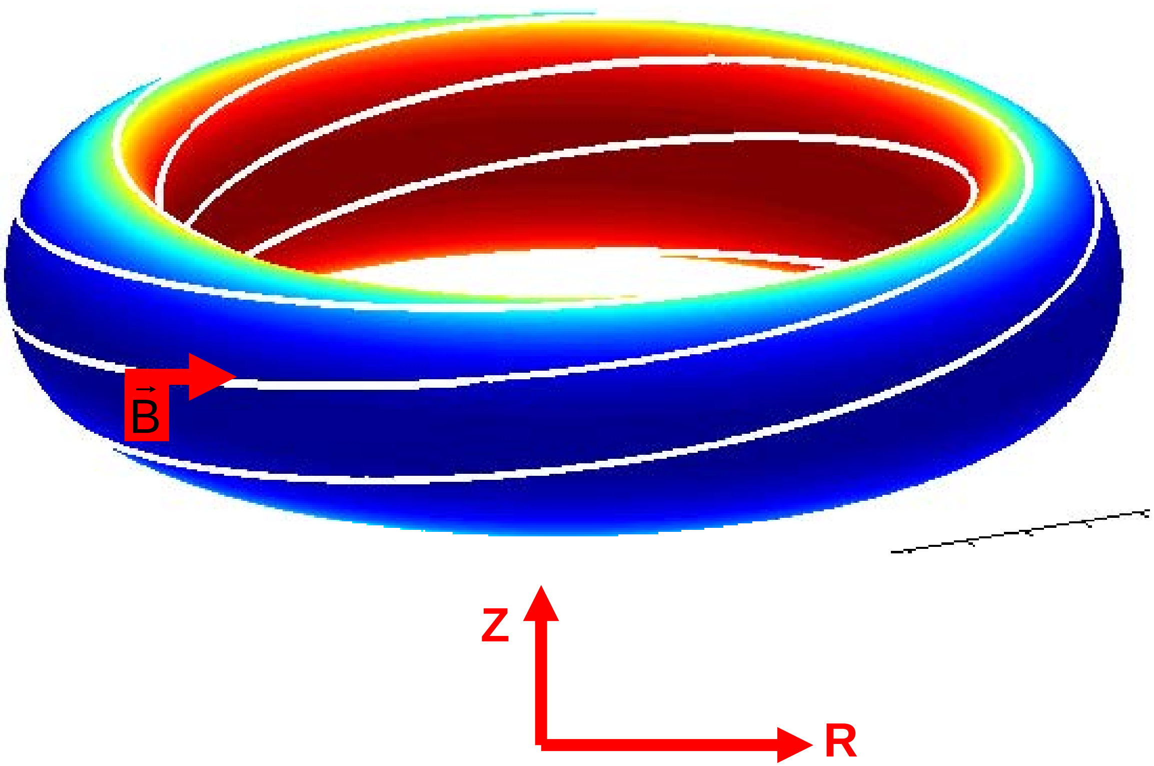
$$\dot{\vec{v}} = \Omega\vec{v} \times \hat{b}$$

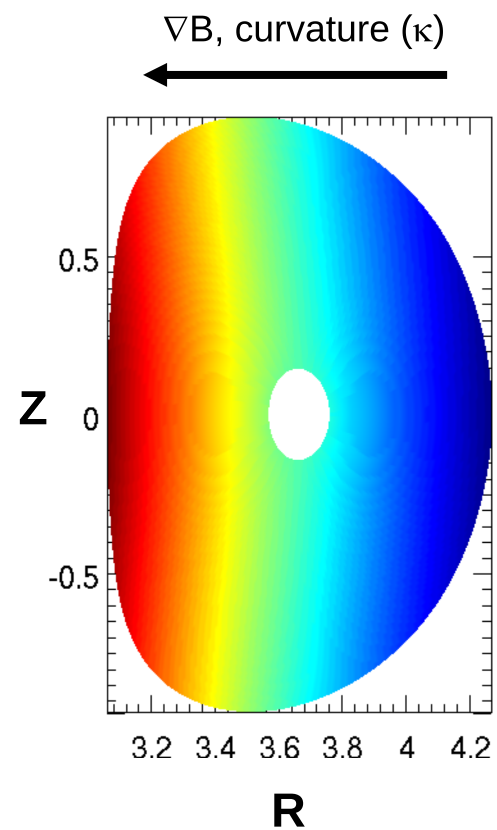
$$\Omega = \frac{qB}{m}$$

$$\rho = \frac{v}{\Omega} = \frac{\sqrt{mT}}{qB}$$

# Toroidicity Leads To Inhomogeneity in |B|

- Magnetic field strength varies as B ~ 1/R, weaker on the outboard side
- ∇B and curvature (κ) point towards symmetry axis, leads to additional perpendicular drifts



# $\nabla$B & Curvature Lead To Perpendicular Drifts

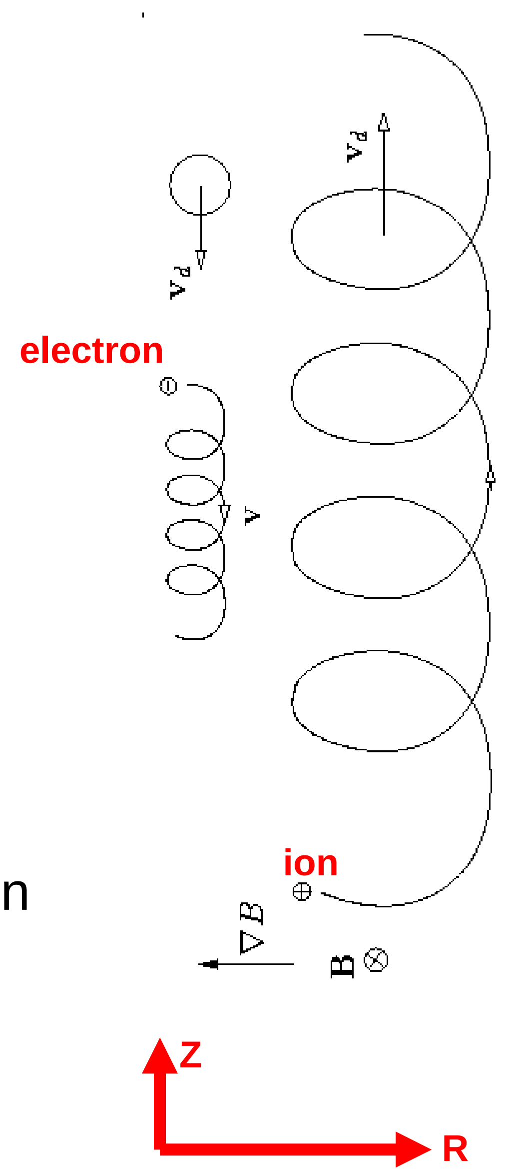
Assuming $\rho \cdot \nabla B/B = \rho/L_B << 1$

$$\vec{v}_\kappa = m v_{||}^2 \frac{\hat{b} \times \vec{\kappa}}{qB}$$

$$\vec{v}_{\nabla B} = \frac{m v_\perp^2}{2} \frac{\hat{b} \times \nabla B/B}{qB}$$

If $\beta = nT \cdot 2\mu_0/B^2 << 1$

$\nabla B/B \approx \kappa \approx 1/R$

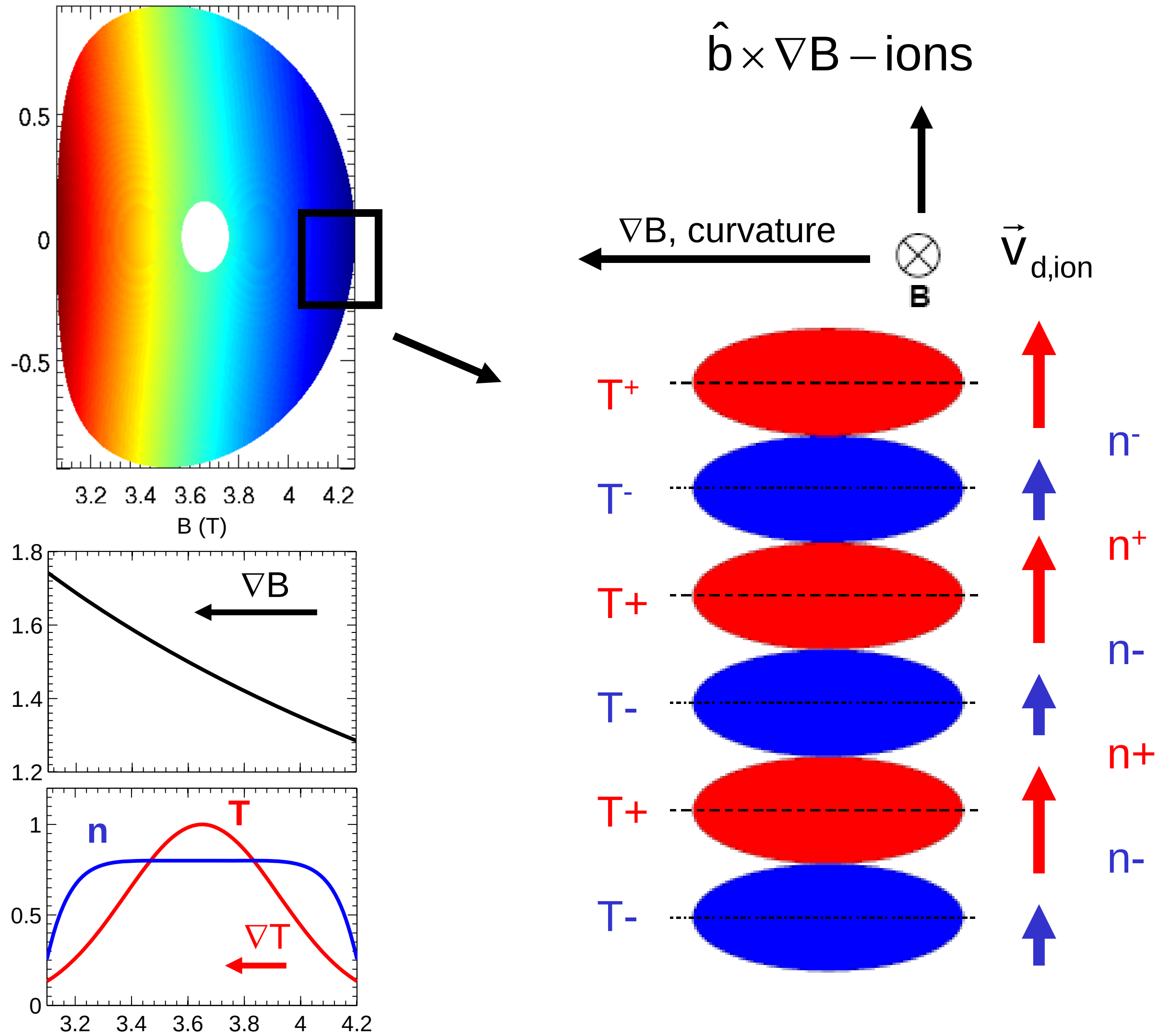



- Drifts are mostly vertical (Z direction)
- Drift off flux surface leads to enhancement of collisional transport

- Dependent on particle energy ($v_{||}^2$, $v_\perp^2$) ~ ($T_{||}$, $T_\perp$)
- What happens when there are small perturbations in $T_{||}$, $T_\perp$?

$\Rightarrow$ Linear stability analysis…

# Cartoon of Temperature Gradient Driven Instabilities




- Fourier decompose perturbations in space, assume small $\delta T$ perturbation
- Spatial variation in $T(\theta)$ leads to variation in toroidal drifts
- Resulting compression ($\nabla \cdot v_{di}$) causes a density perturbation

# Dynamics Must Satisfy Quasi-neutrality

- Quasi-neutrality (Poisson equation, $k_\perp^2 \lambda_D^2 << 1$) requires

$$-\nabla^2 \tilde{\varphi} = \frac{1}{\varepsilon_0} \sum_s e Z_s \int d^3 v f_s$$

$$\left(k_\perp^2 \lambda_D^2\right) \frac{\tilde{\varphi}}{T} = \frac{\tilde{n}_i - \tilde{n}_e}{n_0}$$

$$\boxed{\tilde{n}_i = \tilde{n}_e}$$

- For this ion drift wave instability, parallel electron motion is very rapid

$$\omega < k_{||} v_{Te}$$

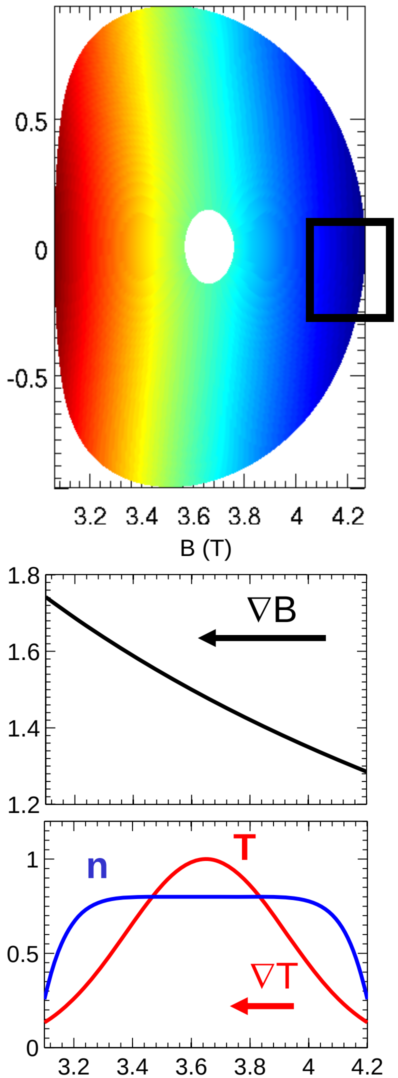
$\Rightarrow$ Electrons (approximately) maintain a Boltzmann distribution

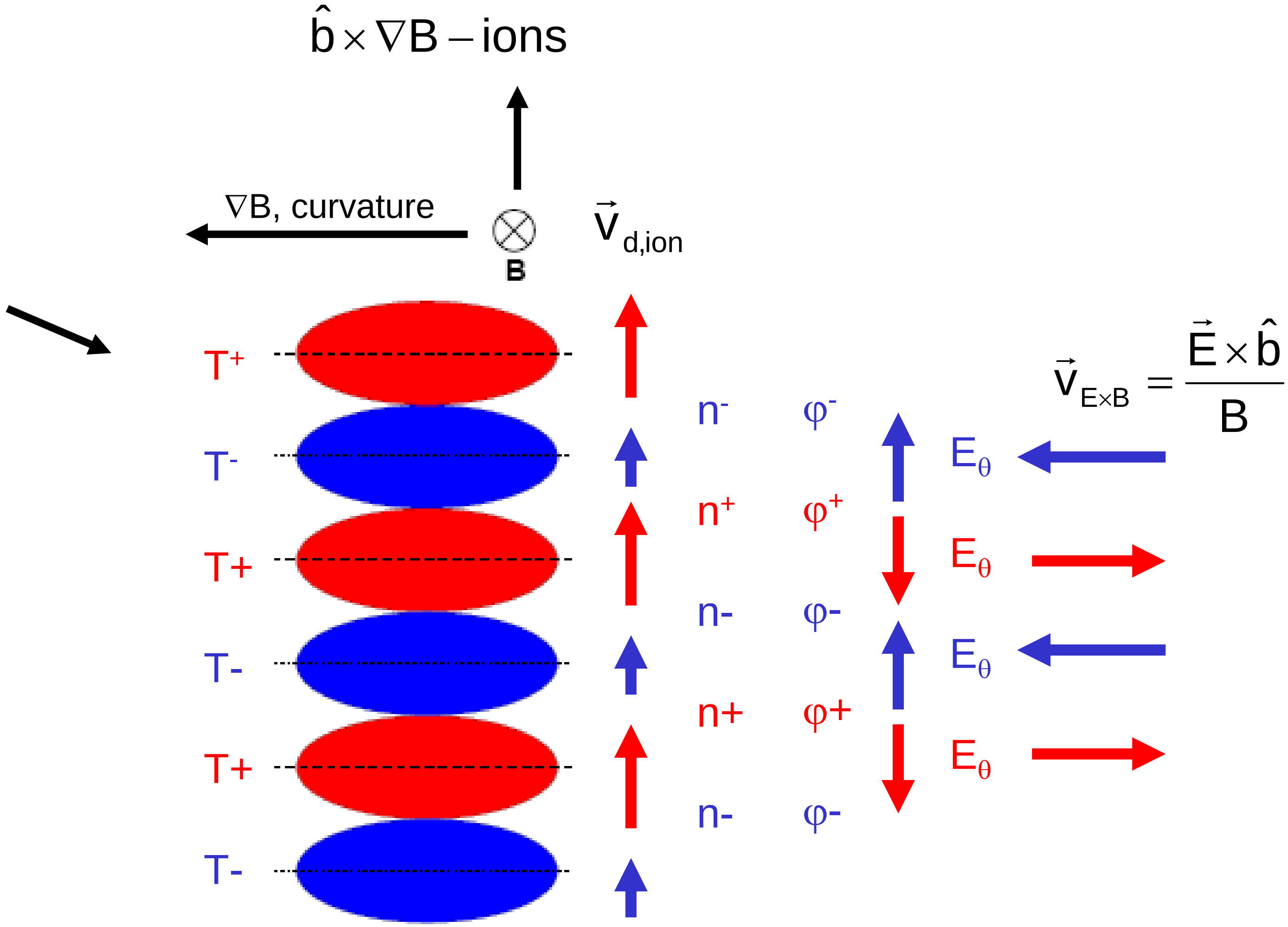
$$\left(n_0 + \tilde{n}_e\right) = n_0 \exp\left(-e\tilde{\varphi}/T_e\right)$$

$$\tilde{n}_e \approx n_0 e\tilde{\varphi}/T_e \Rightarrow \underline{\tilde{n}_e \approx \tilde{\varphi}}$$

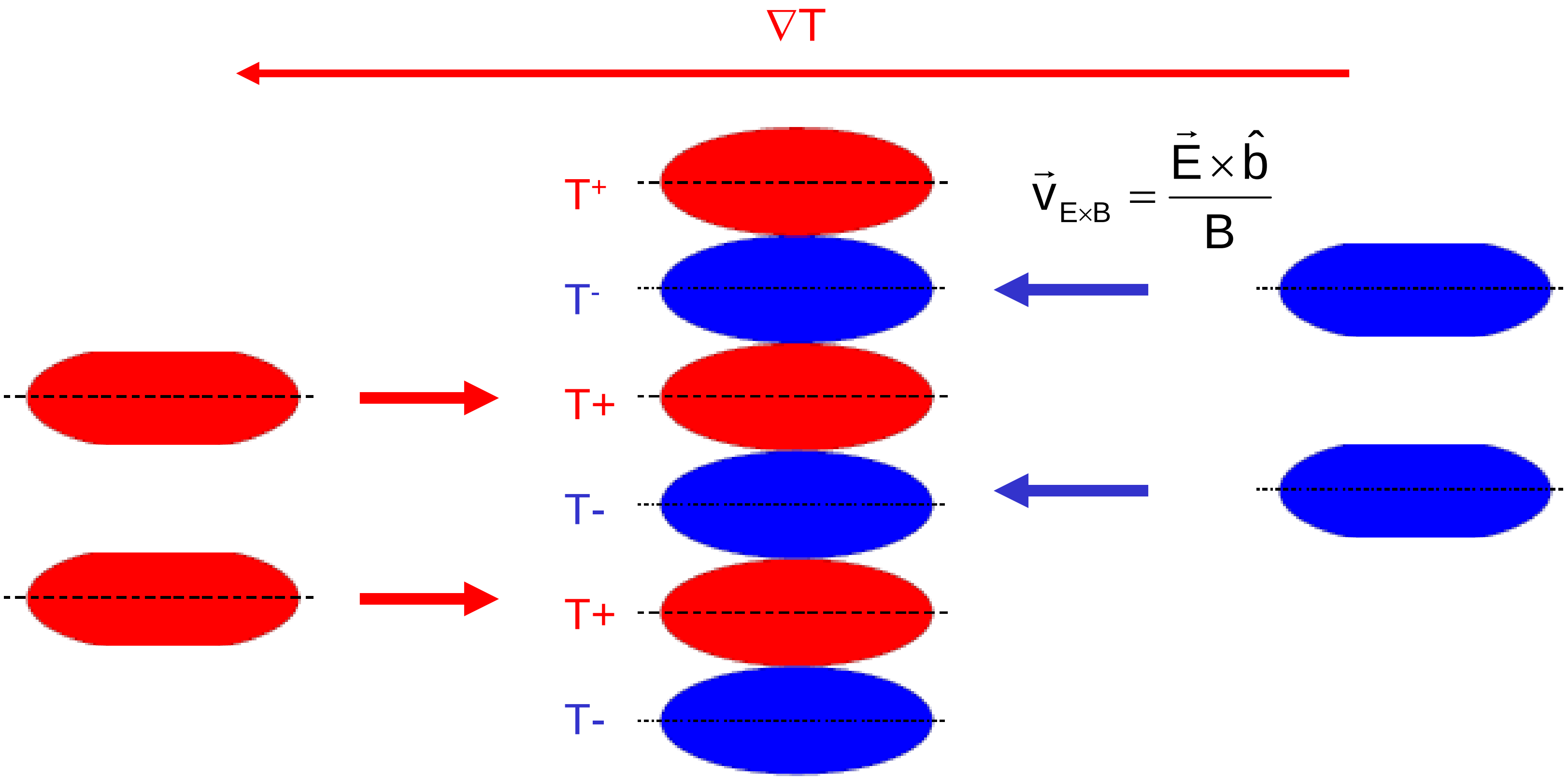
# Perturbed Potential Creates E×B Advection

- Advection occurs in the radial direction



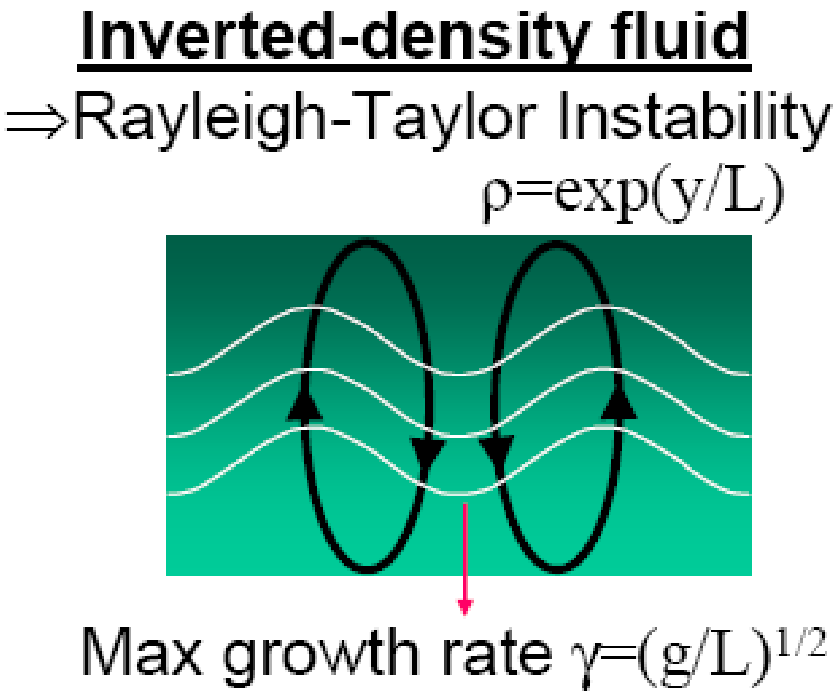



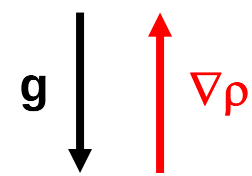
# Background Temperature Gradient Reinforces Perturbation ⇒ Instability



# Simple Analogy to Rayleigh-Taylor (Rayleigh-Benard) Instabilities

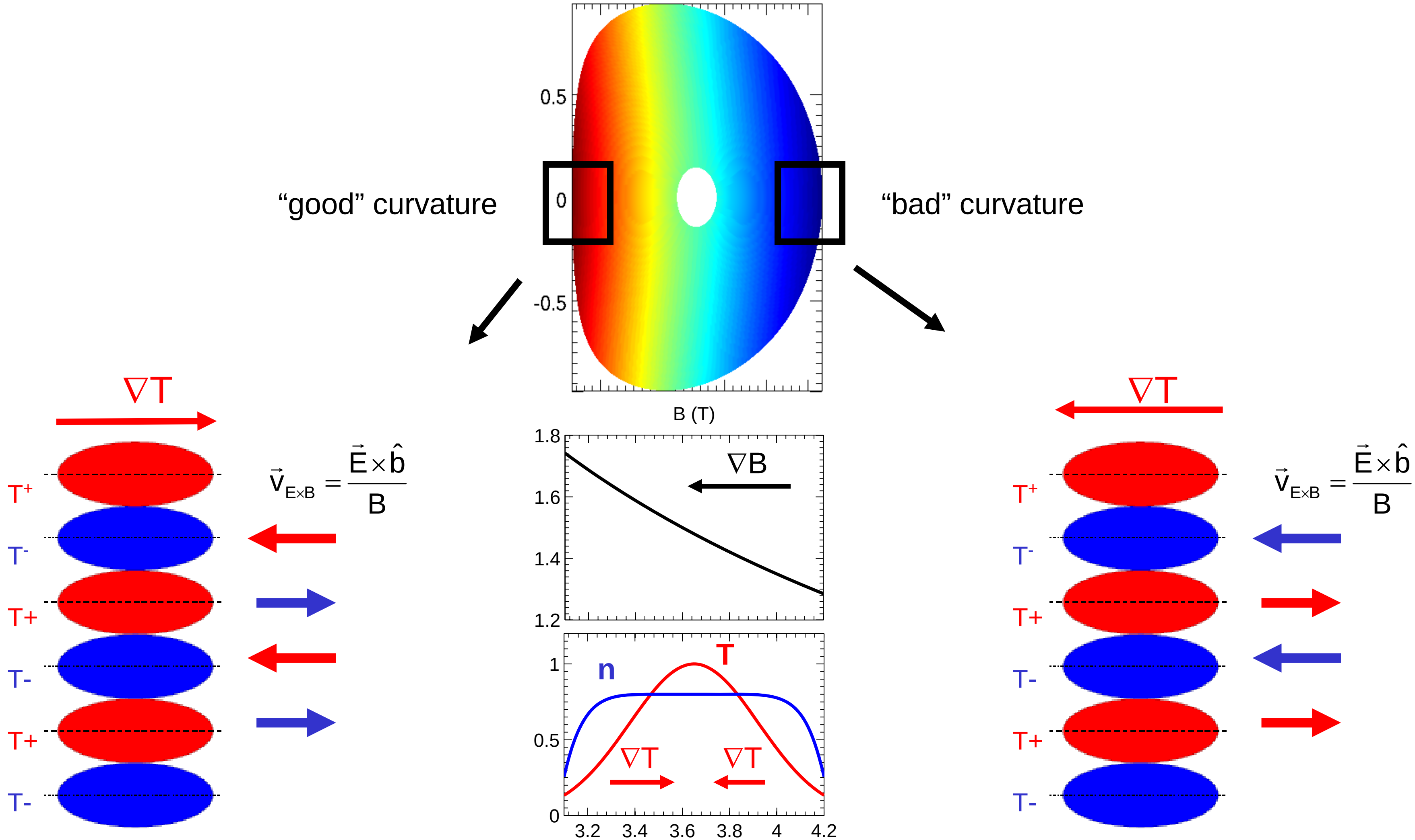
- Instability due to alignment of gravity force with density gradient force



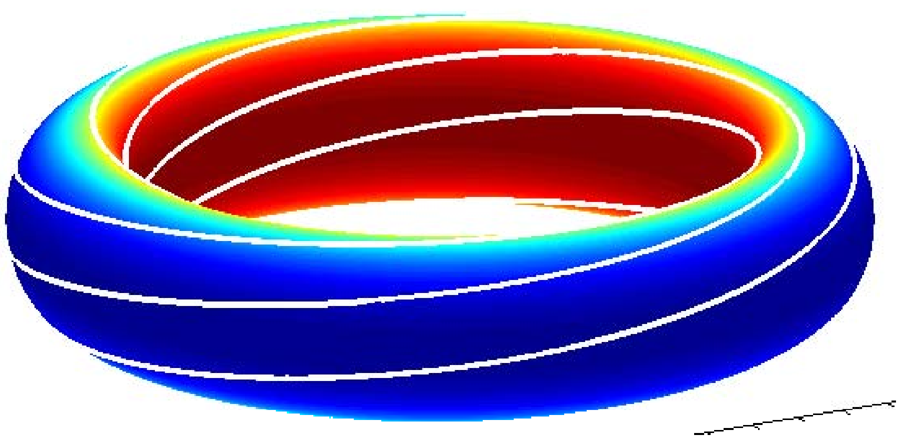
# Same Dynamics Occur On Inboard Side But Now Temperature Gradient Is Stabilizing

- Advection with ∇T counteracts perturbations on inboard side – “good” curvature region



# Fast Parallel Motion Along Helical Field Line Connects Good & Bad Curvature Regions

- Approximate growth rate on outboard side $\gamma_{instability} \sim \frac{v_{th}}{\sqrt{RL_T}} \quad 1/L_T = -1/T \cdot \nabla T$
- Parallel transit time $\gamma_{parallel} \sim \frac{v_{th}}{qR}$



- Expect instability if $\gamma_{instability} > \gamma_{parallel}$ , or $\left(\frac{R}{L_T}\right)_{threshold} \approx \frac{1}{q^2}$
- Threshold gradient for temperature gradient driven instabilities have been characterized over parameter space with more accurate calculations…

# Stability Calculated With Gyrokinetic Codes

- Evolving 5D “gyro-averaged” distribution function

$$f(\vec{x}, \vec{v}, t) \xrightarrow{\text{gyroaverage}} f(\vec{R}, v_{||}, v_{\perp}, t)$$

$$f = F_M + \delta f$$

$$\frac{\partial(\delta f)}{\partial t} + v_{||}\hat{b} \cdot \nabla \delta f + \vec{v}_d \cdot \nabla \delta f + \delta \vec{v}_E \cdot \nabla F_M + \delta \vec{v}_E \cdot \nabla \delta f = 0$$

Fast parallel motion

Slow perpendicular toroidal drifts

Advection across equilibrium gradients ($\nabla T$, $\nabla n$)

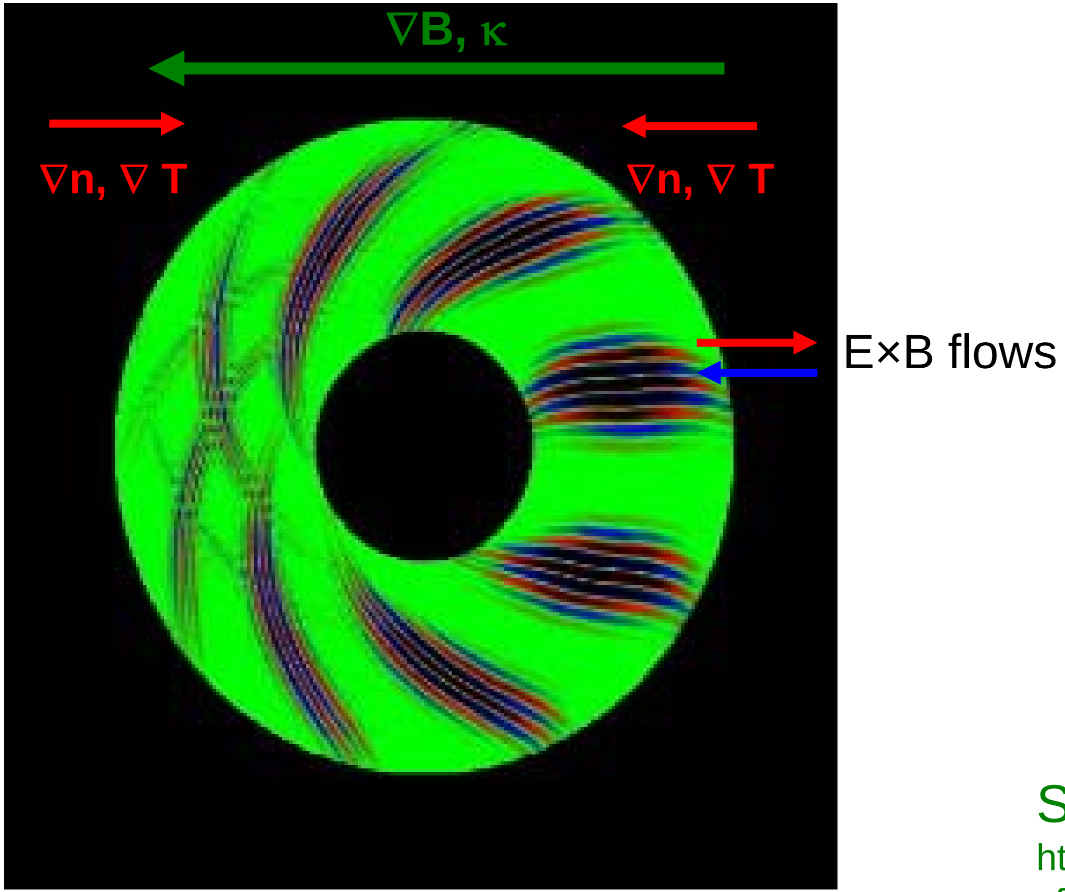
Perpendicular non-linearity

- Also must solve gyrokinetic-Maxwell equations to obtain perturbed fields $\tilde{\varphi}$

# Example From Gyrokinetics

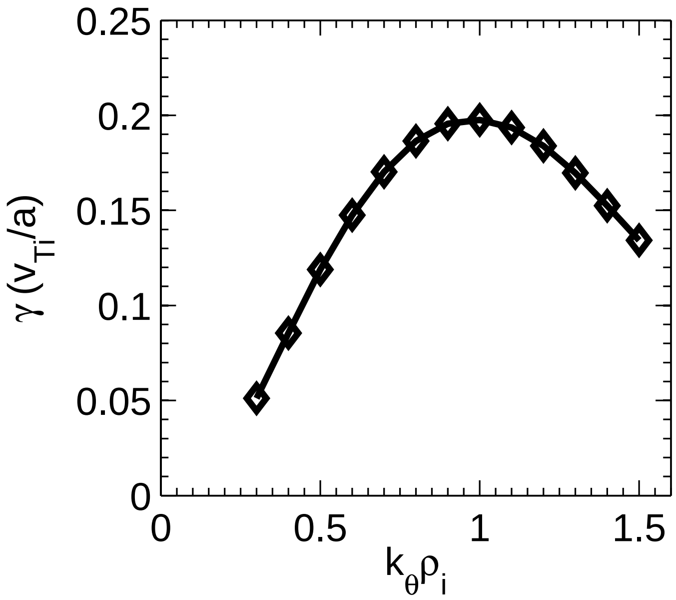
- Using fixed profile gradients in tokamak geometry
- Many poloidal modes unstable

Snapshot of density perturbations in linear phase




GYRO simulation (http://fusion.gat.com/theory/Gyro)

Linear growth rate spectra




SEE ALSO GKW WEBPAGES:

http://www2.warwick.ac.uk/fac/sci/physics/research/cfsa/people/hornsby/

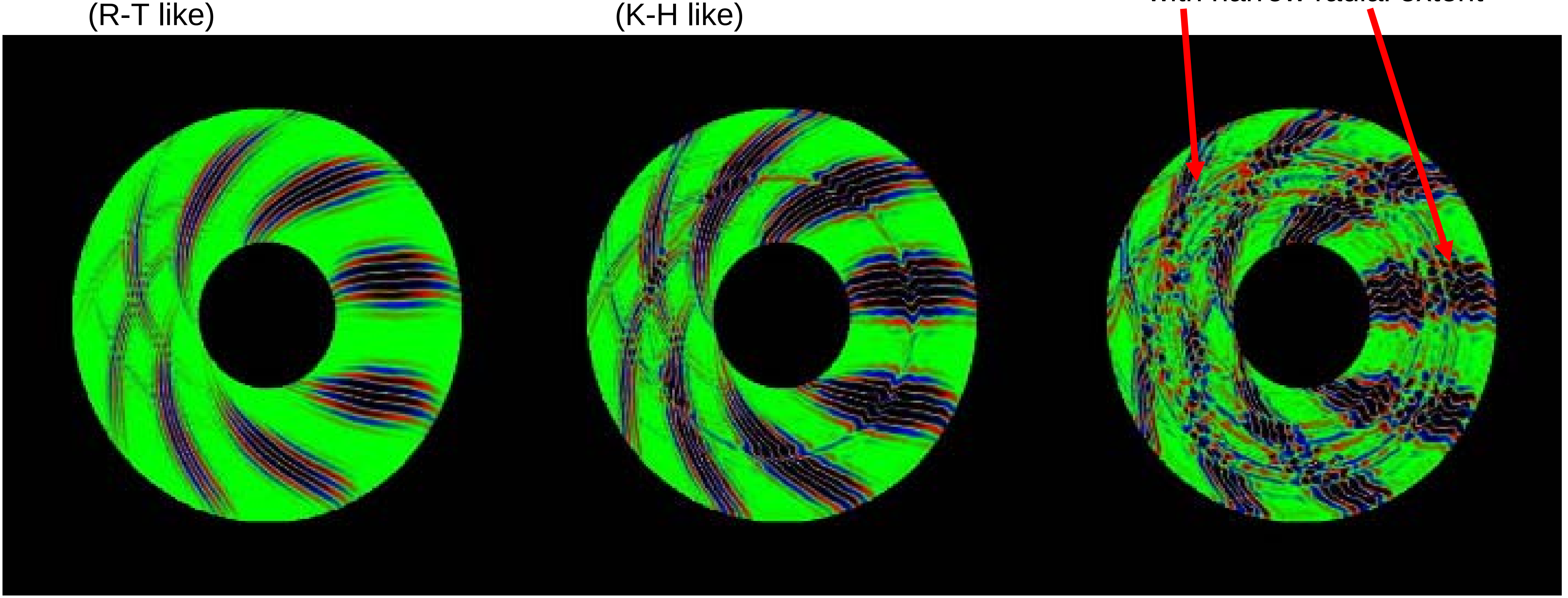
# Perpendicular Non-Linear Interactions Provide Saturation

$$\frac{\partial(\delta f)}{\partial t} = \delta\vec{v}_E \cdot \nabla\delta f + \ldots$$

Late linear stage demonstrates structure of fastest growing modes (R-T like)

Large shear flows from primary instabilities cause zonal flows to develop (K-H like)

Zonal flows develop uniformly on flux surfaces, with narrow radial extent

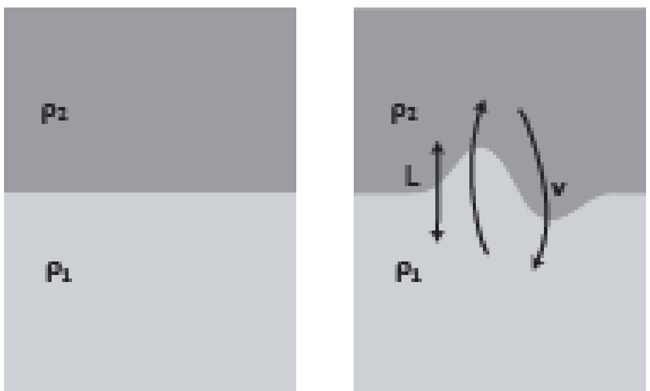


GYRO simulation (http://fusion.gat.com/theory/Gyro)

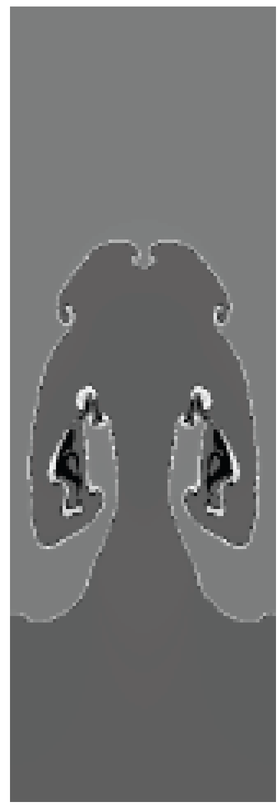
# Again, Simple Analogy with R-T and Kelvin-Helmotz Instability

- Linearly growing sheared flow field results in Kelvin-Helmholtz instability

RT - Linear




RT Non-Linear



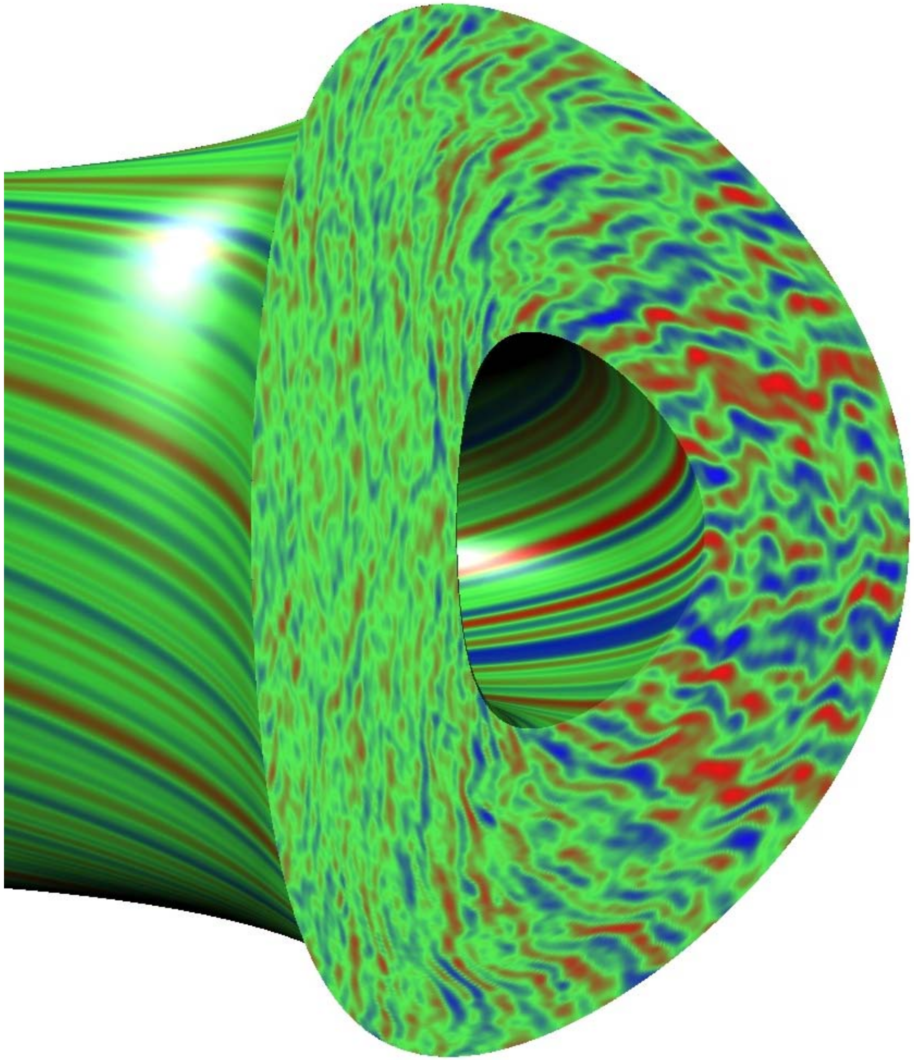
- Flows driven by RT primary excite KH instability, leading to generation of smaller scale structure
- Secondary instability mechanism important with dominant single-mode primary

# Fully Developed Turbulence Becomes Isotropic (in 2D)

- Highly elongated along the field line (fast parallel motion)
- Roughly isotropic in perpendicular directions (non-linear interactions)

$\Rightarrow$ Quasi-2D

- Stronger fluctuations on the outboard side (“bad” curvature)



# Spatial Correlation On The Order Of Gyroradii

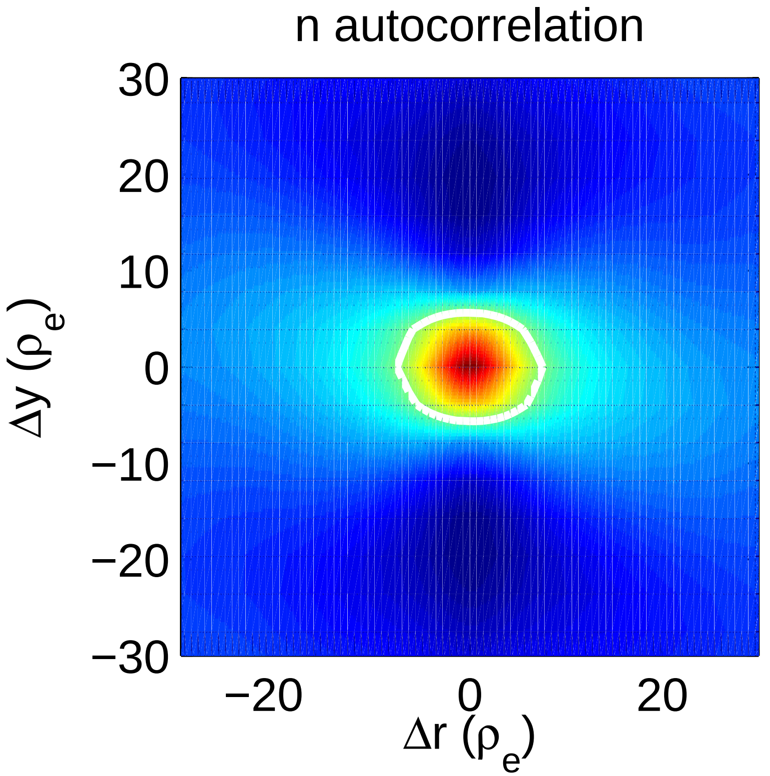
- Perpendicular correlation $L_r \approx L_\theta \sim 7\rho_i$
- Decorrelation time $\tau \sim 10\, L_T/v_{Ti}$

- Turbulent thermal diffusivity

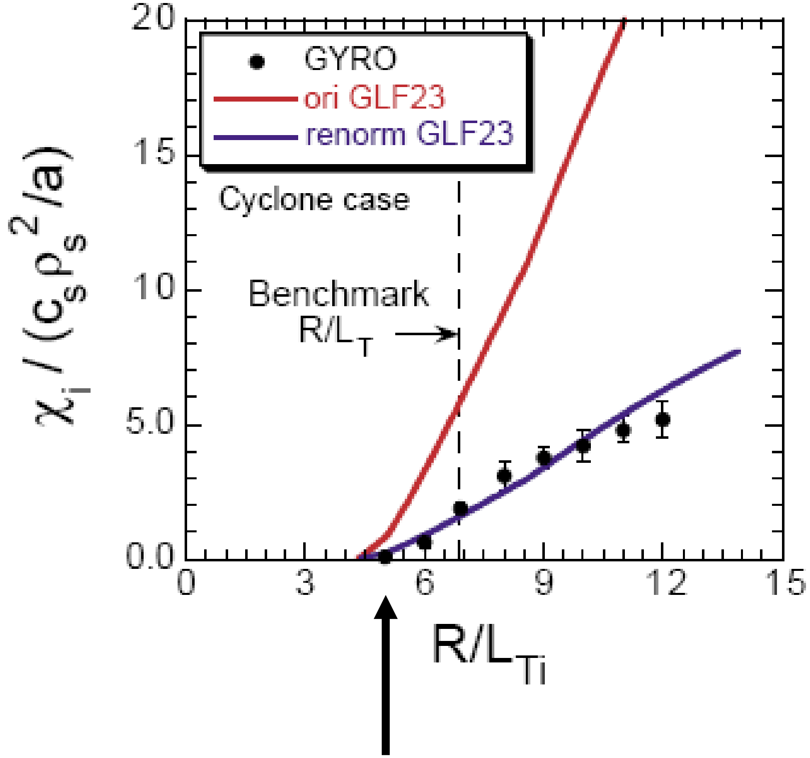
$$\chi_i \leq 1.0 \cdot \frac{\rho_i^2 v_{Ti}}{L_{Ti}} \quad \left( \sim \frac{L_r^2}{\tau} \right)$$

- Intensity levels

$$\frac{\tilde{T}}{T_0} \leq 1\%$$



# Transport Very Non-Linear With Gradients

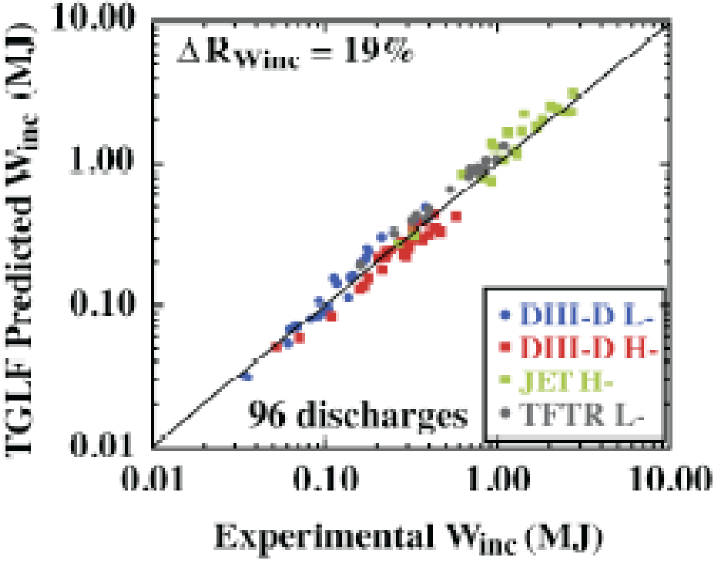


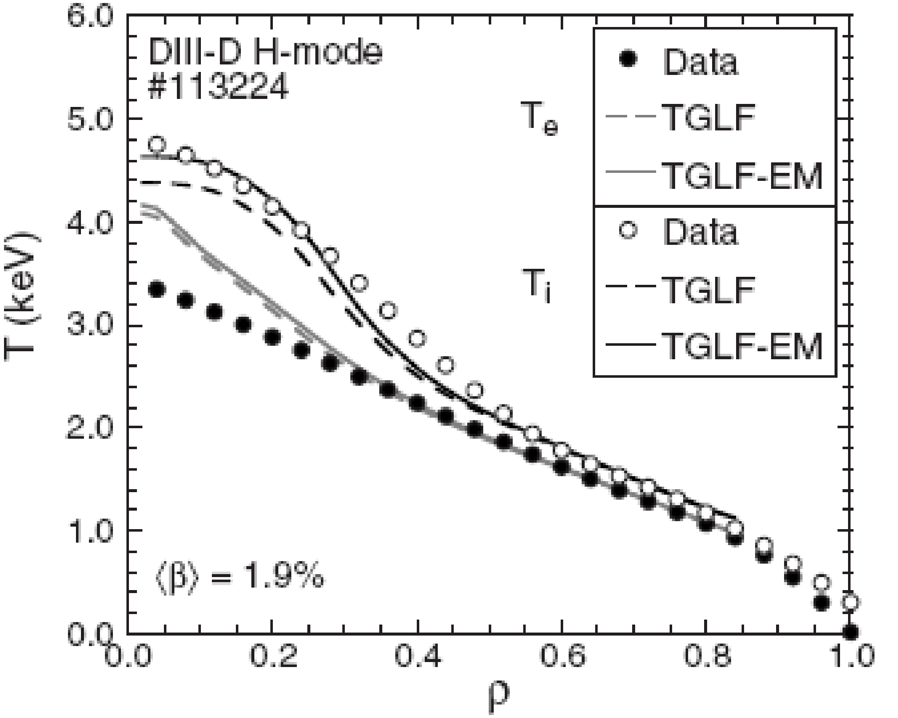

Effective threshold

# Do We Understand Plasma Turbulence?

- A little bit - *quantitative* predictions of energy confinement times *and* plasma profiles based on comprehensive turbulence codes are becoming plausible

Predicted energy content



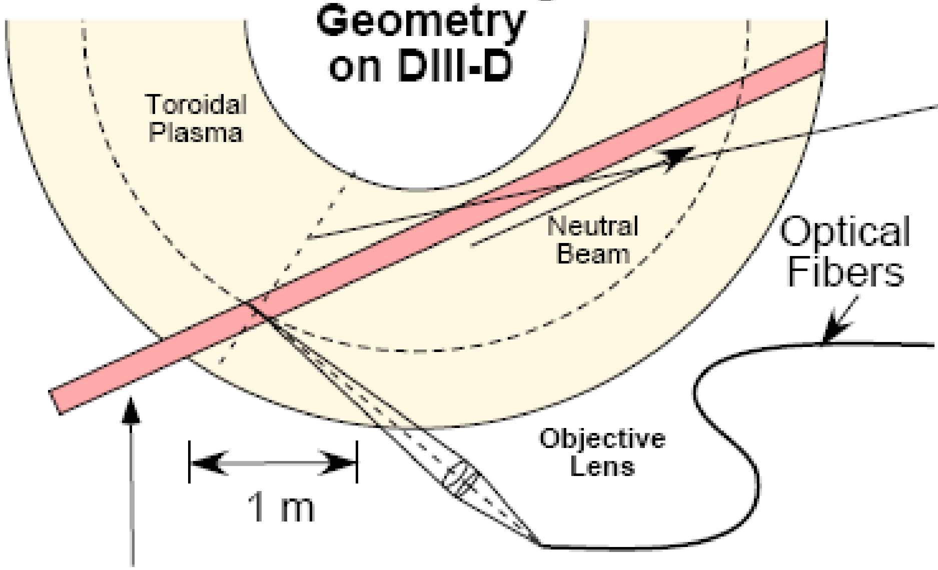

- ANY QUESTIONS?

# Beam Emission Spectroscopy Measurement of Localized, Long-Wavelength ($k_\perp \rho_I < 1$) Density Fluctuations

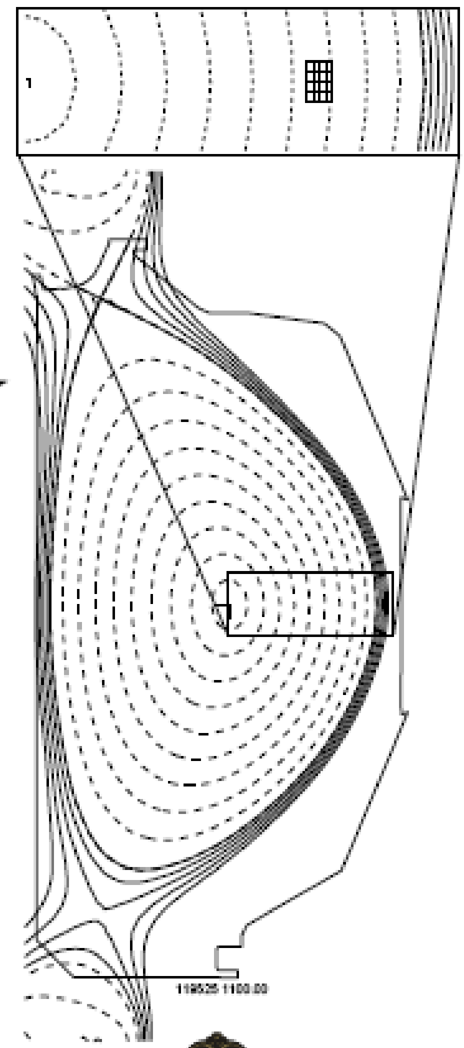
**Collisionally-excited, Doppler-shifted neutral beam fluorescence**

$$D^o + e,i \rightarrow \left(D^o\right)^* \rightarrow D^o + \gamma(n = 3 \rightarrow 2, \lambda_o = 656.1\ \mathrm{nm})$$

**BES Viewing Geometry on DIII-D**




**75 KeV $D^O$ Neutral Beam (150 L (R))**



$$\frac{\tilde{I}}{I} \mu \frac{\tilde{n}}{n}$$

DIII-D NATIONAL FUSION FACILITY

THE UNIVERSITY of WISCONSIN MADISON

# Example Sequence of Time-Resolved 2D Turbulence Flow Field

**Analysis Provides:**

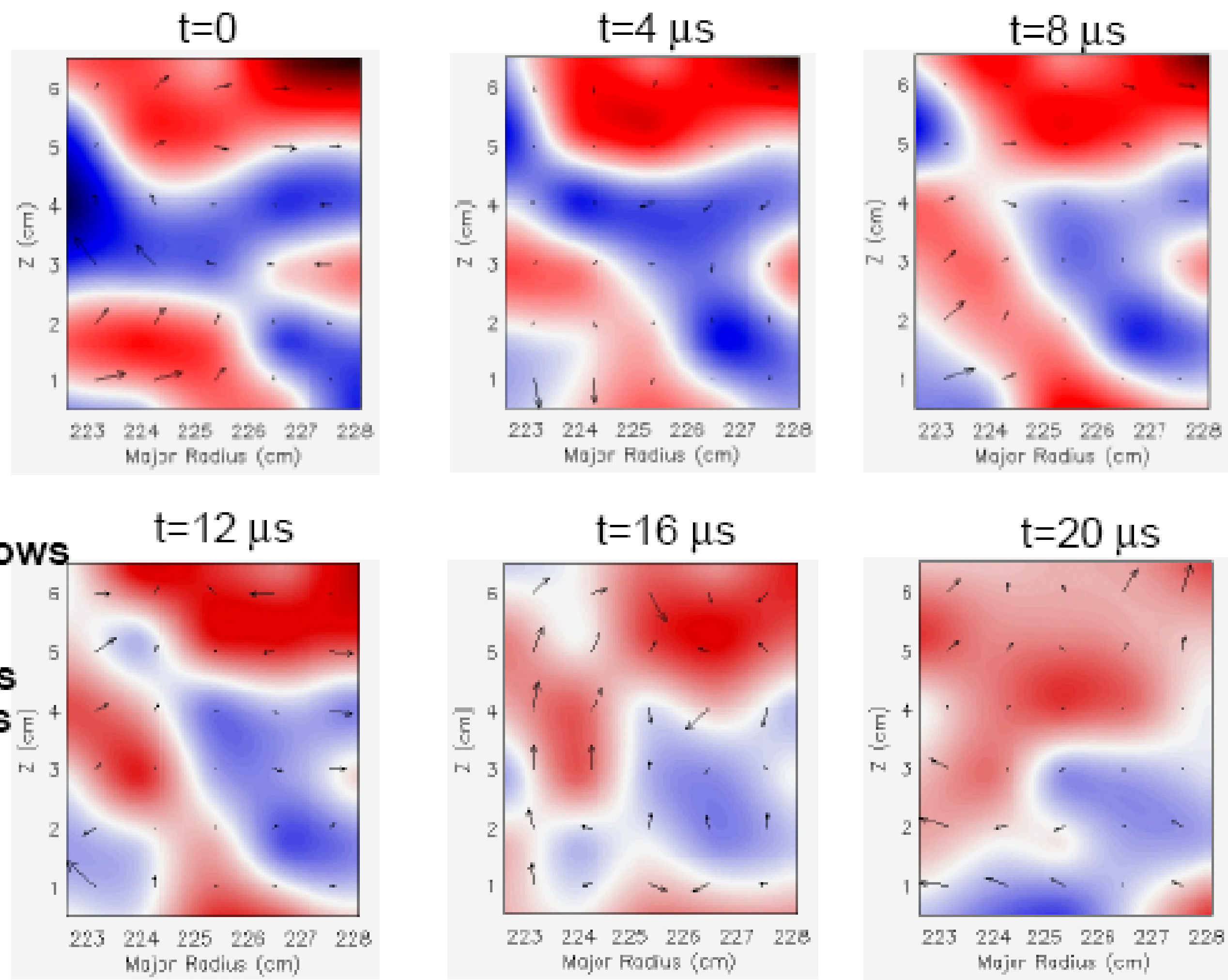
**$v_r$ (R, Z, t)**

- **Flux**

**$v_\theta$(R, Z, t)**

- **Zonal Flows**

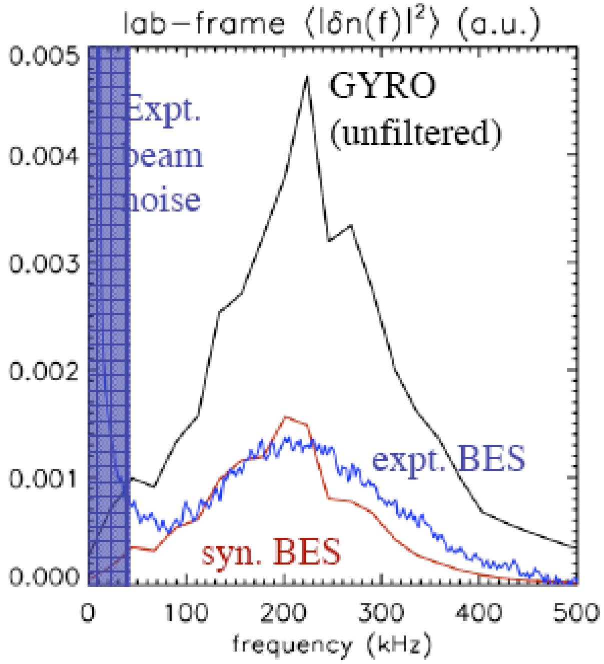
**R - 5 channels**
**Z - 6 channels**
**t - 1000 μs**

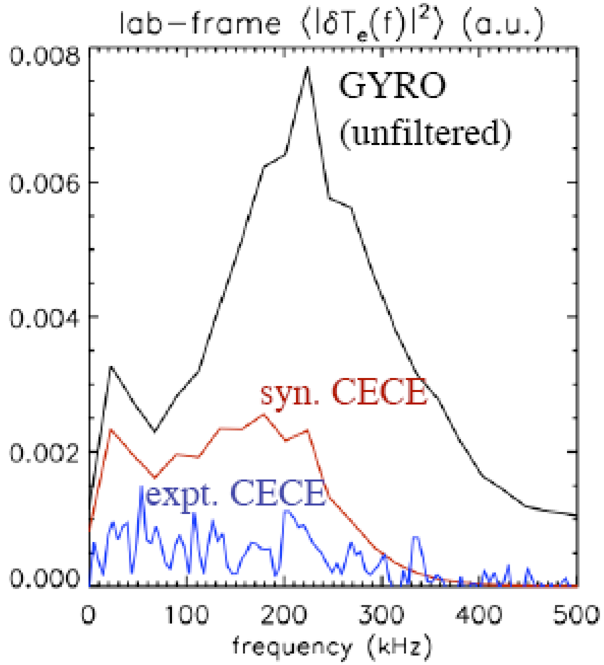



**Vectors represent local velocity field (scaled by image)**

# Direct Comparison Of Measured Fluctuation Spectra With Simulated Spectra

- Simulation output is processed in a manner representative of the actual diagnostic (“synthetic diagnostics”)
- Agreement in core plasmas are not far off



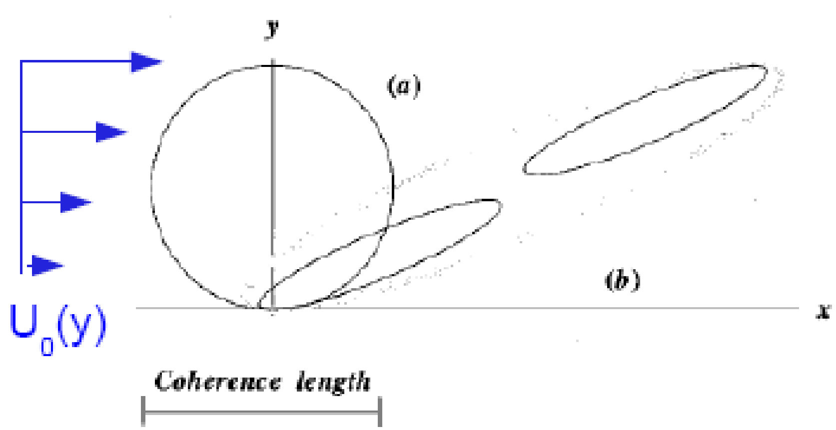



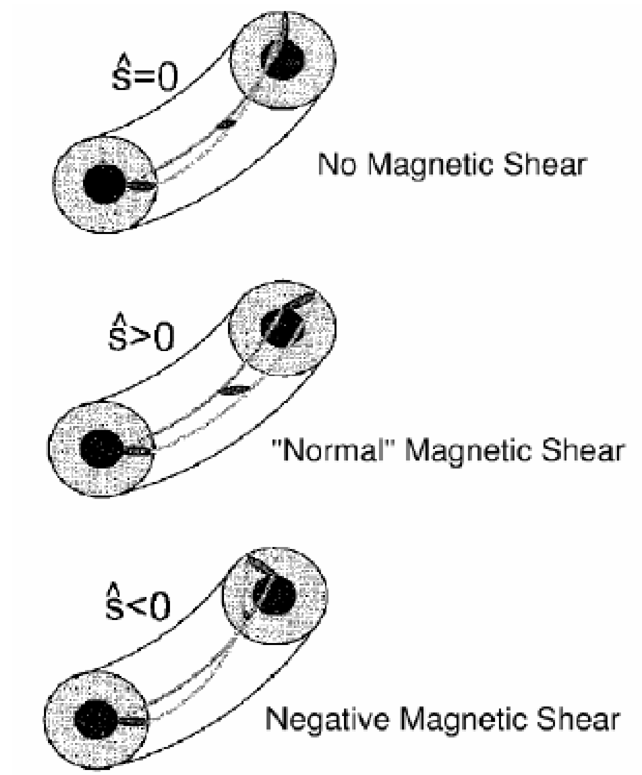
# Additional Physics Often Important

- Particles trapped in the inhomogenous field can add to instability
- Collisions can stabilize these trapped particle influences
- Electromagnetic perturbations at higher $\beta = nT \cdot 2\mu_0 / B^2$ can become important

Magnetic shear can also influence stability and transport significantly

- Physically, turbulent transport is expected to be reduced as the shear rate ($\omega_s \sim dU_0/dy$) approaches the turbulence decorrelation rate ($\Delta\omega_D$) (Biglari, Diamond, Terry, 1990)

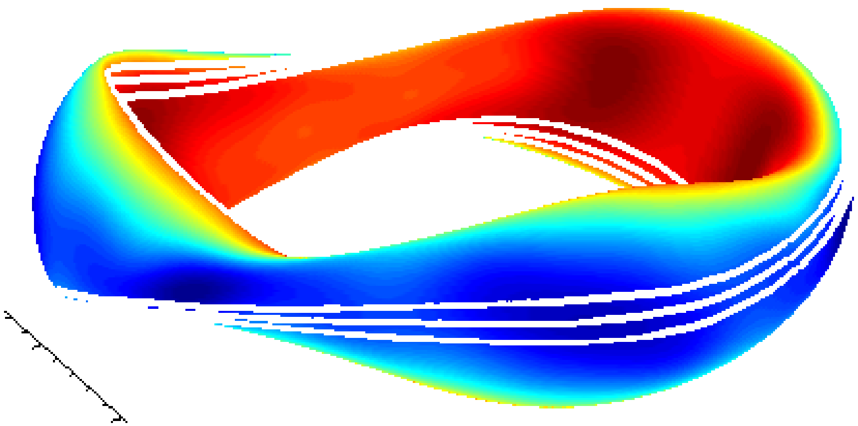


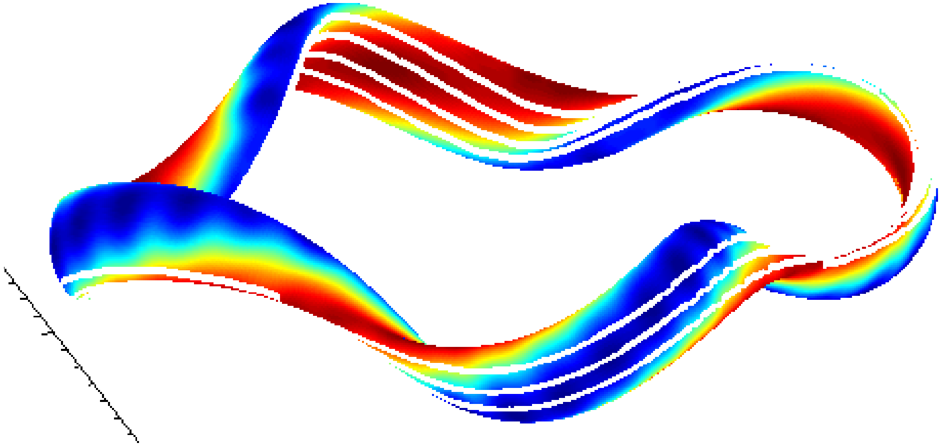



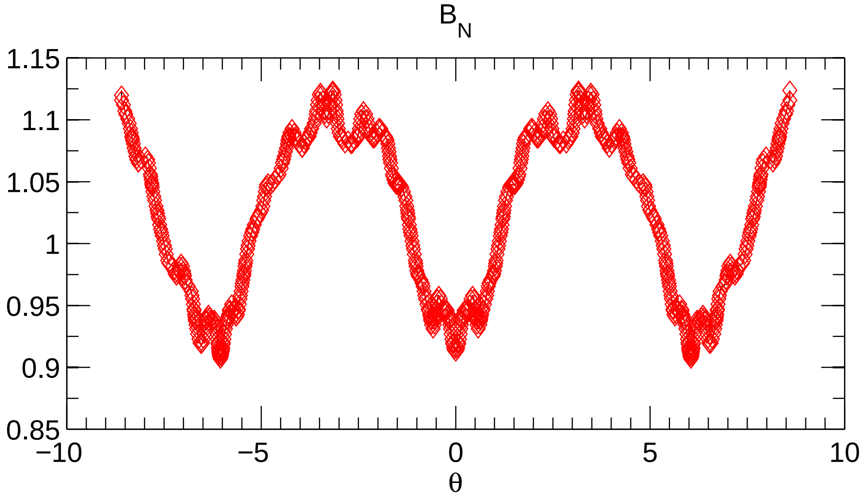
# Can Generalize To Arbitrary 3D Topology - Stellarators

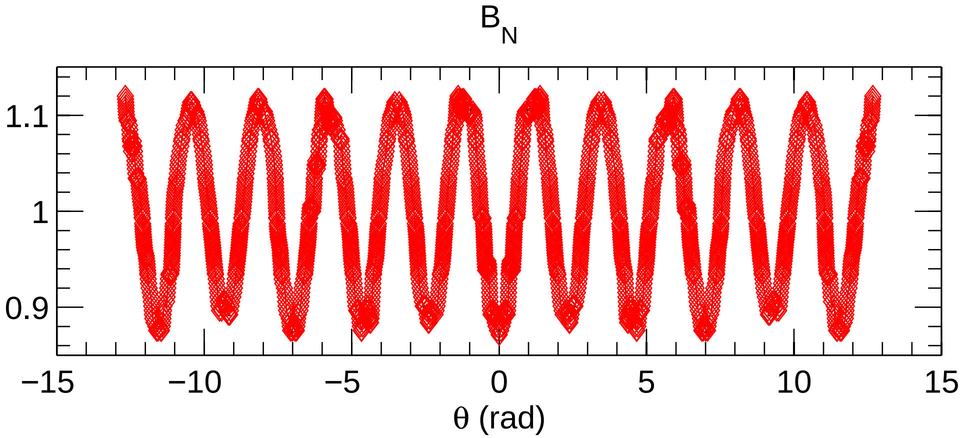
NCSX (National Compact Stellarator Experiment, Princeton, NJ USA)

HSX (Helically Symmetric Experiment, Madison, WI USA)







# HSX - Helically Symmetric Experiment