

Progress on momentum transport predictions in NSTX

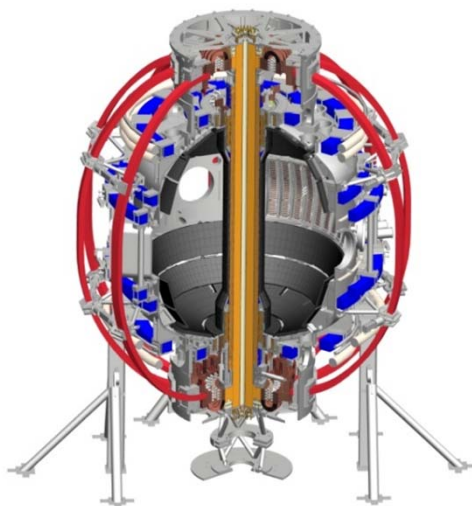
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Overview & Summary

- Quasilinear predictions are unable to account for experimental observations of strong momentum pinch in high beta NSTX H-modes
- Have begun investigating additional effects (nonlinear, $E \times B$ shear, finite ρ_*)
- Initially focused on a low beta L-mode unstable to ITG/TEM (easier to handle and interpret computationally):
 - 1) Nonlinear simulations, including $E \times B$ shear, give Pr and RV_ϕ/χ_ϕ similar to quasilinear analysis
 - IF this holds for H-modes, can not explain observed pinch
 - 2) Linear, global (finite ρ_*) simulations predict residual stress contributions comparable to pinch but directed outward; both smaller than diffusive flux
 - 3) Initial nonlinear, global simulations predict a strong inward momentum flux in the absence of flow or flow shear
 - Linear and nonlinear global simulations ongoing for high beta H-mode plasmas unstable to mix of microtearing (MT) and kinetic ballooning modes (KBM)

Background & Motivation

Interpretation of toroidal angular momentum transport often assumes diffusive and convective components

- Transport equation:
$$\frac{\partial}{\partial t} (n_i m_i \langle R^2 \rangle \Omega) + \nabla \cdot \Pi_\varphi = S_\Omega \rightarrow \sum_s (\dots)$$
- *Assumed* transport form:
$$\Pi_\varphi = -nmR\chi_\varphi (R\nabla\Omega) + nmV_\varphi (R\Omega)$$

Prandtl number

$$\text{Pr} = \frac{\chi_\varphi}{\chi_i}$$

Pinch parameter

$$\frac{RV_\varphi}{\chi_\varphi}$$

$$\hat{\Pi}_\varphi = \hat{\chi}_\varphi \left(\hat{u}' + \frac{RV_\varphi}{\chi_\varphi} \hat{u} \right)$$

$$\hat{u}' = \frac{-R^2 \nabla \Omega}{c_s} \quad \hat{u} = \frac{R\Omega}{c_s}$$

- Pinch expected due to Coriolis drift [Peeters, 2007], turbulent equipartition + thermoelectric force [Hahm, 2007]

Momentum transport is anomalous in NSTX, Prandtl numbers $\chi_\phi/\chi_i < 1$ for L- and H-modes

- $Pr = \chi_\phi/\chi_i \approx 0.3-1.0$ over many radii and discharges (assumes $V_\phi=0$)

- $\chi_\phi > \chi_{\phi,NC}$ for both L and H
In L-mode $\chi_i > \chi_{i,NC}$

$$Pr = \frac{\chi_\phi}{\chi_i} \approx \frac{\chi_{\phi,turb}}{\chi_{i,turb}}$$

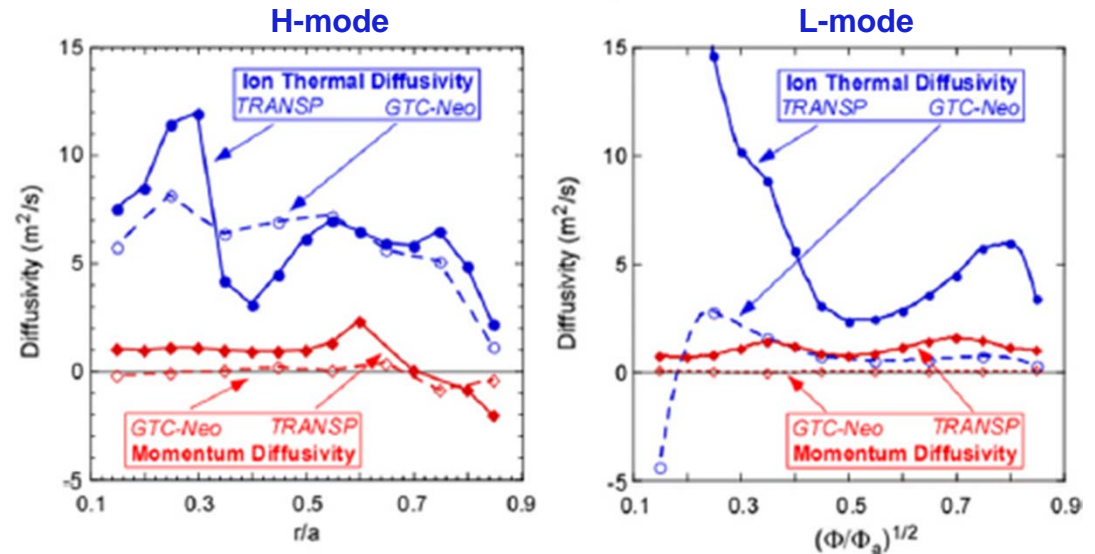
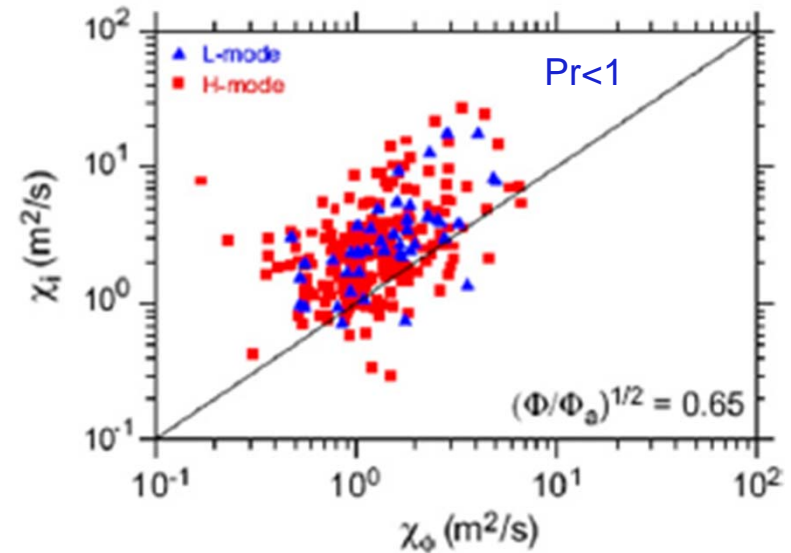
In H-mode $\chi_i \approx \chi_{i,NC}$

$$Pr = \frac{\chi_\phi}{\chi_i} = \frac{\chi_{\phi,turb}}{(\chi_{i,NC} + \chi_{i,turb})} \rightarrow \sim 0$$

⇒ Pr less useful in H-mode?

- RV_ϕ/χ_ϕ less ambiguous

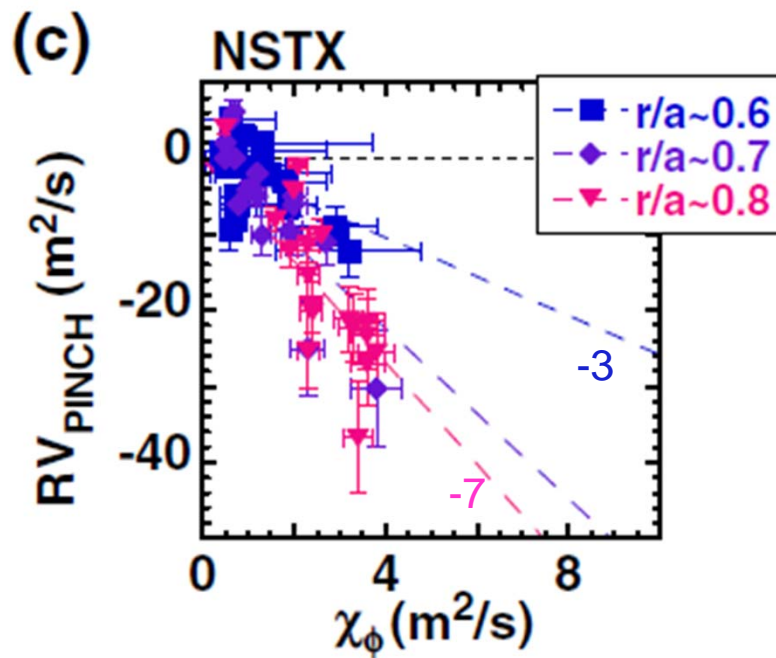
Steady state transport analysis (Kaye et al., 2009)



Perturbative H-mode experiments (using n=3 magnetic braking) indicate existence of an inward momentum pinch

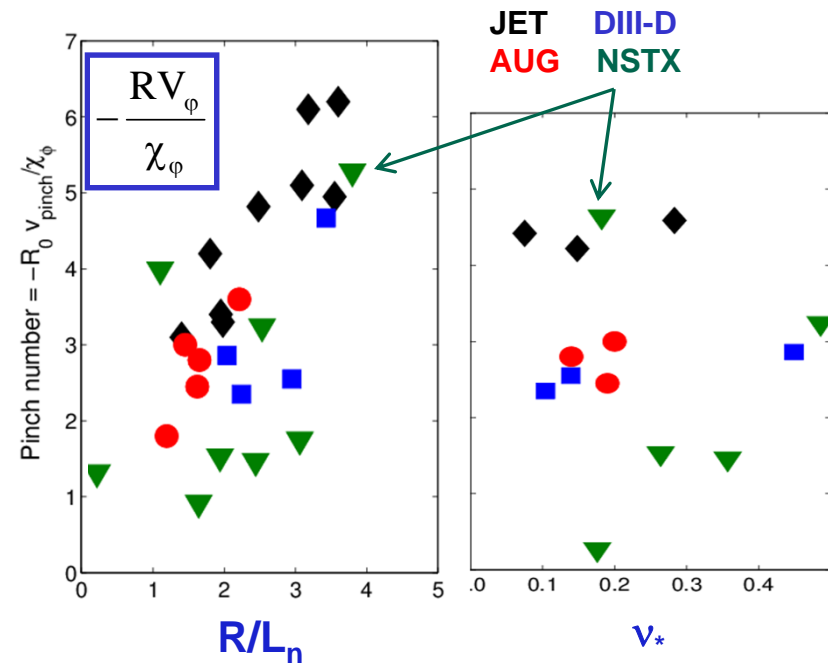
- $RV_{\phi}/\chi_{\phi} \approx -(1-7)$ for many NSTX discharges & radii

(Solomon et al., PRL 2008, PoP 2010; Yoshida et al., NF 2012)



- Possible dependence on density gradient (R/L_n), less clear with collisionality (ν^*)

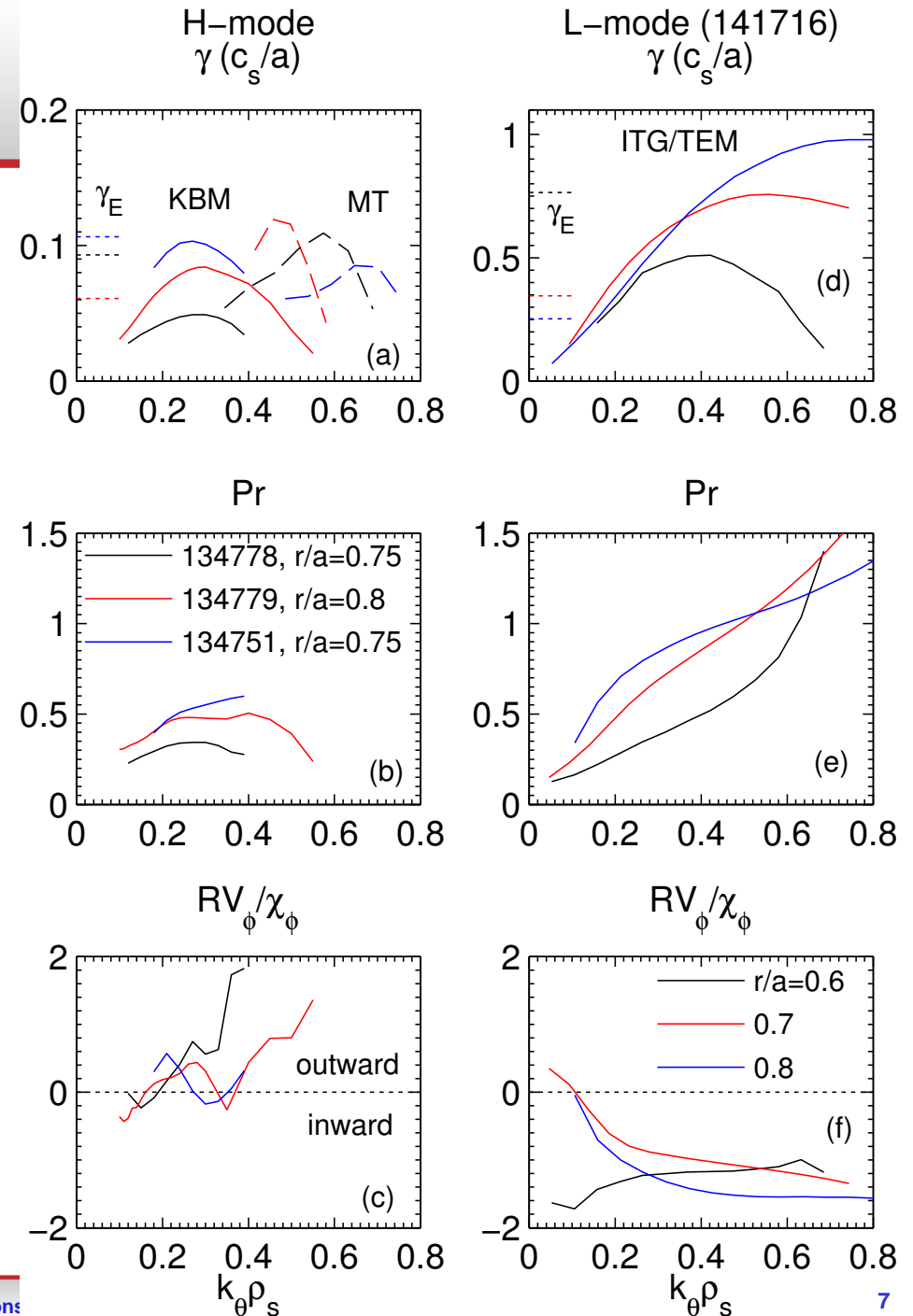
(Tala et al., IAEA 2012)



- Local, linear gyrokinetic simulations of ITG turbulence describe pinch and scaling in conventional tokamaks \Rightarrow **does this hold for NSTX?**

Local, linear sims unable to explain measured pinch

- Guttenfelder (TTF, 2013) showed gyrokinetic simulations (GYRO) predicting linear stability, Pr and pinch (following Peeters, 2007)
- In H-modes, mix of microtearing (MT) and KBM predicted unstable
- No momentum transport predicted for MT but KBM predicts:
 - Small $Pr \sim 0.3-0.5$
 - Small or outward convection, $RV_\phi/\chi_\phi \sim 0-2$
 - Pinch insensitive to parameter variations ($R/L_n, v^*, \dots$)
- In L-mode, ITG/TEM unstable:
 - Larger $Pr \leq 1$
 - Small inward pinch, $RV_\phi/\chi_\phi \sim -2-0$
 - Pinch insensitive to parameter variations ($R/L_n, v^*, \dots$)



Many theoretical mechanisms to consider for momentum transport

$$\Pi_{\varphi} = nmR(\chi_{\varphi} u' + \chi_{\varphi\perp} \gamma_E) + (nmRV_{\varphi} + mR\Gamma_p)u + C_{UD} + C_{\rho^*} + \dots$$

- More general expression for momentum transport (e.g., Peeters, NF 2011) includes contributions due to:
 - Perpendicular ($E \times B$) flow shear [Casson, 2010; Dominguez, 1993]
 - Particle convection (usually expected to be small)
 - Up-down asymmetry [Camenen, 2009]
 - Finite ρ^* /nonlocal effects (profile shearing, ...) [Camenen, 2011]
- Also, important to consider all mechanisms in fully developed nonlinear turbulence (i.e. not just quasi-linear)
- In the core of NSTX NBI plasmas, toroidal flow dominates radial force balance so that $u' = (qR/r) \cdot \gamma_E$ (i.e. negligible v_{pol} , ∇p_i contributions)
 - In theory and codes we can vary u' , γ_E , u , ρ^* independently to identify various physical mechanisms
- **Have begun to investigate nonlinear, $E \times B$ shear and finite ρ^* effects**

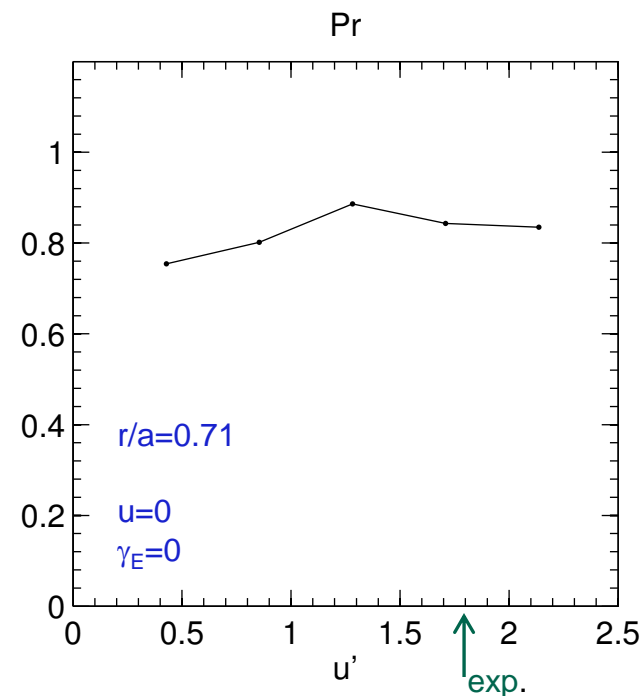
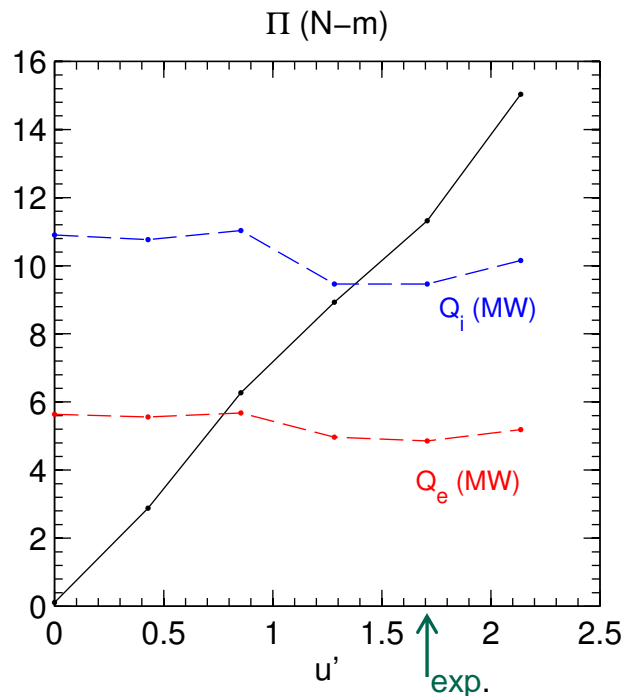
Local nonlinear L-mode predictions

- Using L-mode analyzed extensively in [Ren, NF 2013]
- Nonlinear simulations run with varying $E \times B$ shear, parallel flow shear and toroidal flow (following [Casson, 2009])
- Summary: Nonlinear results with $E \times B$ shear largely consistent with quasilinear results

Increasing u' drives diffusive transport

- Linear dependence $\Pi \sim u'$ as expected
- $Pr \sim 0.8$, consistent with quasilinear analysis using $k_{\theta} \rho_s \sim 0.35$ (nonlinear peak)
- Heat fluxes \sim constant, unaffected by parallel gradient drive

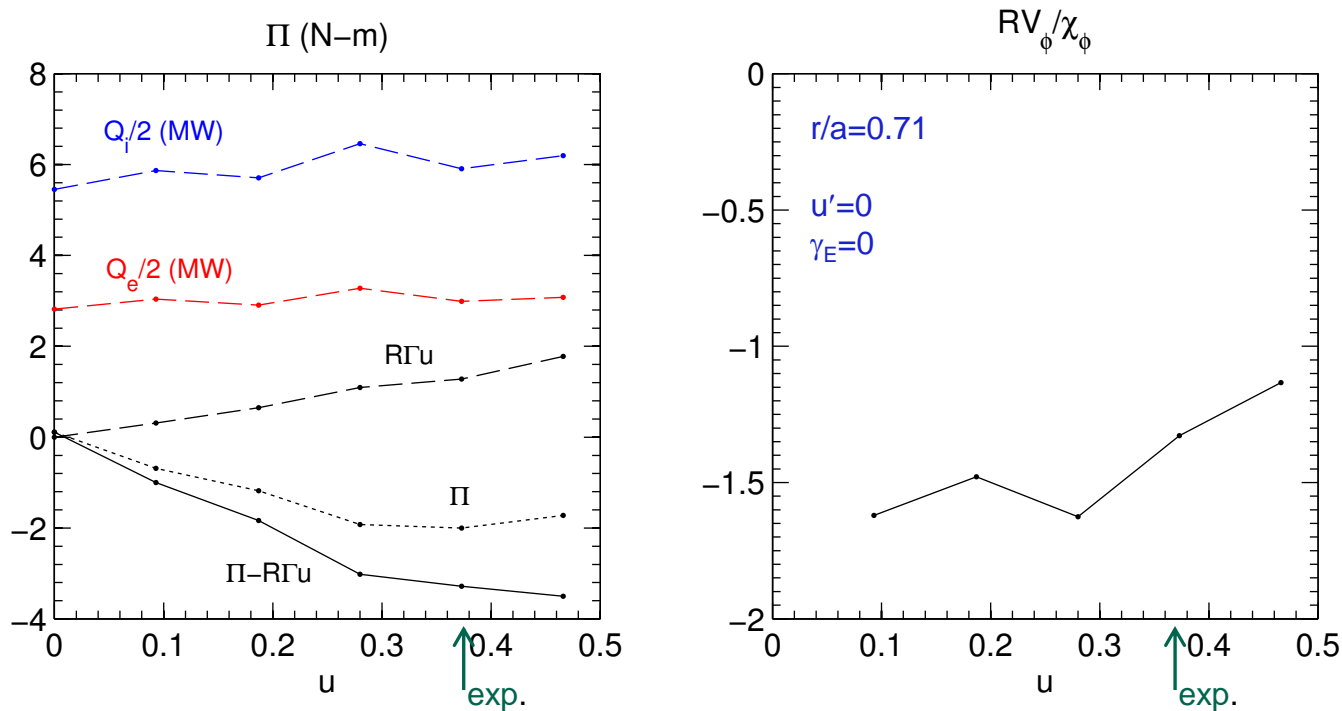
$$\Pi_{\phi} = nmR(\chi_{\phi} u' + \chi_{\phi\perp} \cancel{V_{\phi}}) + (nmR V_{\phi} + mR \Gamma_p) \cancel{u'} + \cancel{C_{UD}} + \cancel{C_{P*}} + \dots$$



Increasing u gives convective pinch

- Near linear dependence $\Pi \sim u$
- $RV_\phi/\chi_\phi \sim -1$, consistent with quasilinear analysis using $k_\theta \rho_s \sim 0.35$
 - Subtracting contribution from particle flux, $mR\Gamma_p u$
- Heat fluxes \sim constant, unaffected by toroidal rotation

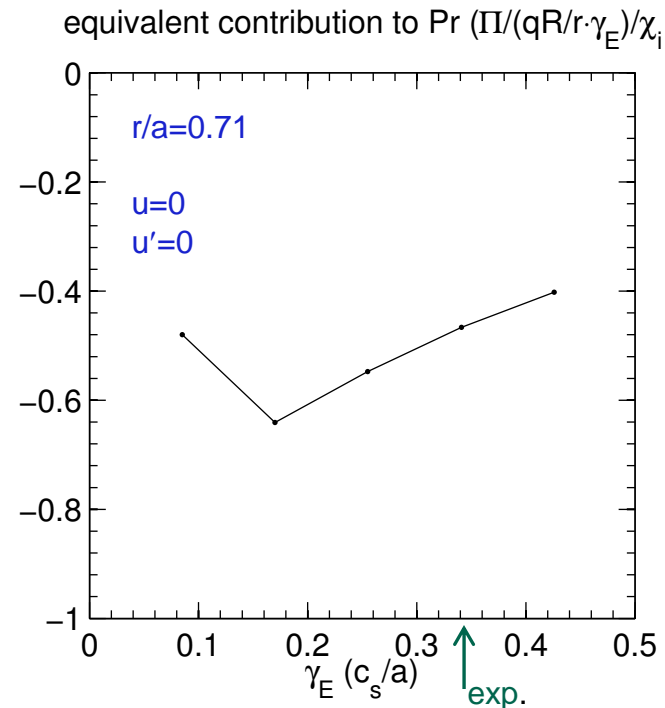
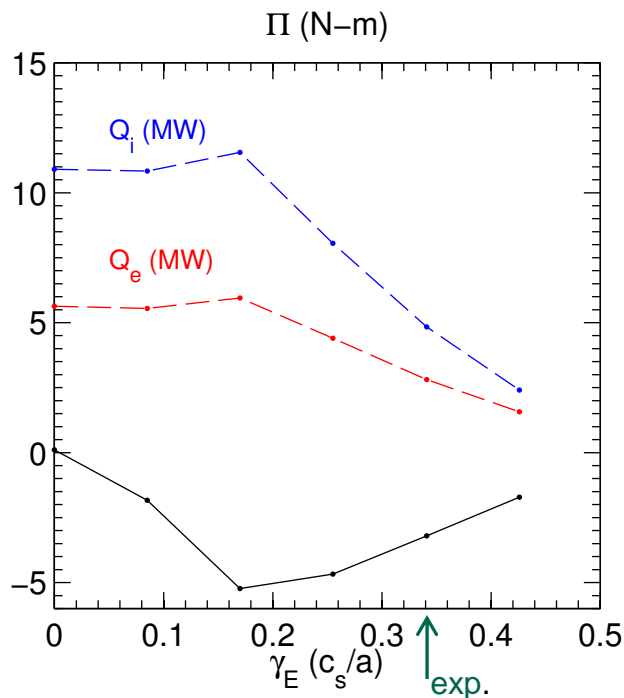
$$\Pi_\phi = nmR(\chi_\phi + \chi_{\phi\perp}) + \underline{(nmRV_\phi + mR\Gamma_p)u} + C_{\text{TD}} + C_{\text{p}^*} + \dots$$



$E \times B$ (perpendicular) shear drives strong inward convection but nonmonotonic

- Near linear dependence for small enough $E \times B$ shear
- Decreases above $\gamma_E > 0.15$ as $E \times B$ shear begins to suppress turbulence
 - Seen in all transport channels - Γ , Q , Π

$$\Pi_\phi = nmR(\chi_\phi \cancel{\gamma_E} + \chi_{\phi\perp} \gamma_E) + (nmRV_\phi \cancel{\gamma_E} + mR\Gamma_p)u + \cancel{C_{\text{UD}}} + \cancel{C_{\text{P}^*}} + \dots$$

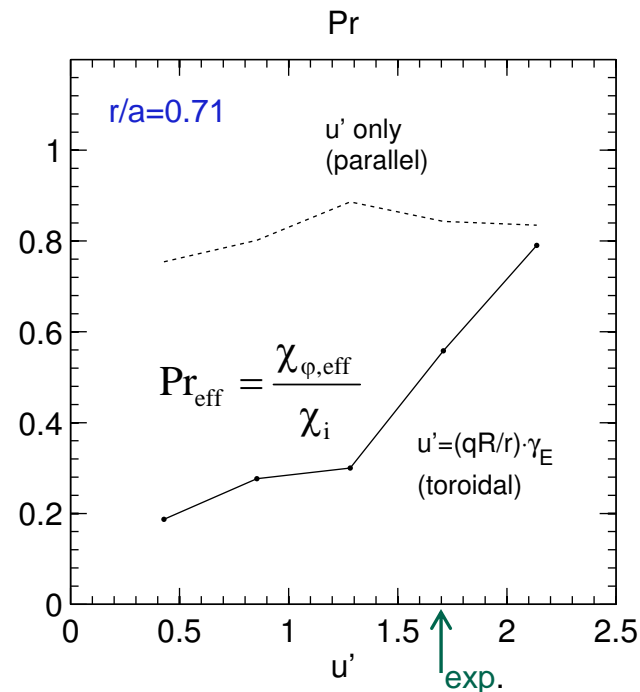
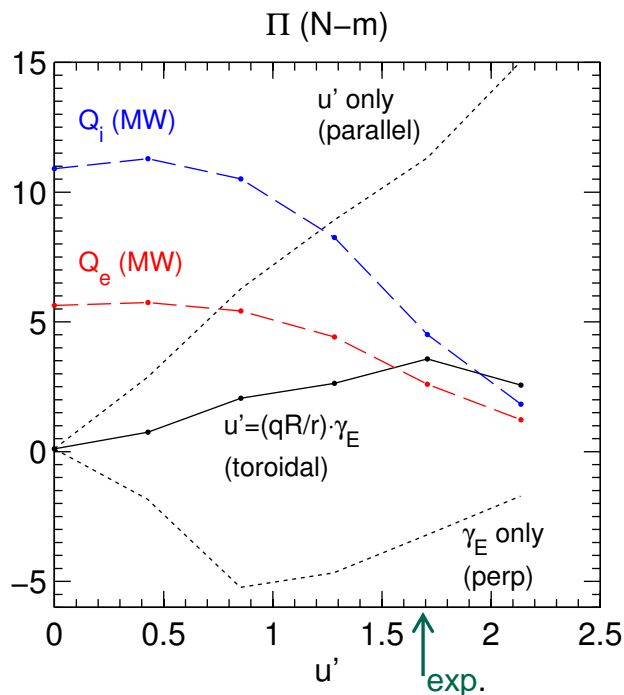


Using purely toroidal flow gives less transport, Pr approaches diffusive-only value at large flow shear

- Can define an effective momentum diffusion using purely toroidal flow, $\gamma_E = (r/qR) \cdot u'$ (appropriate for NBI driven core plasma in NSTX)

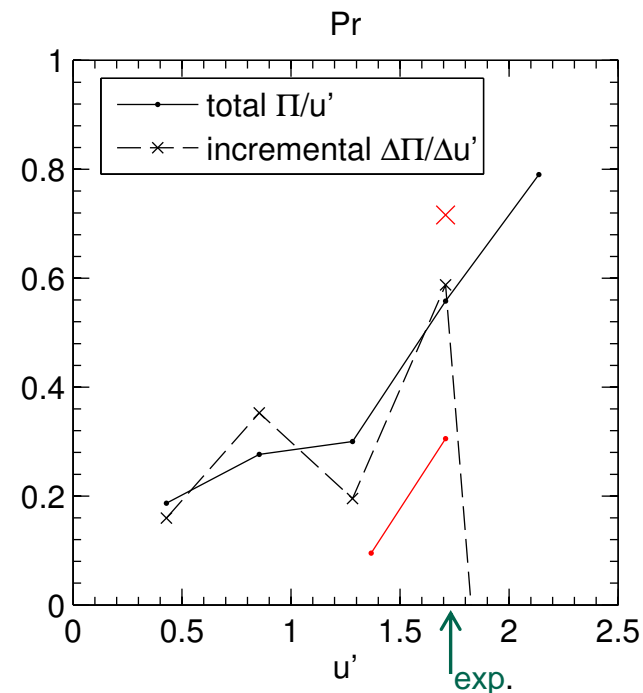
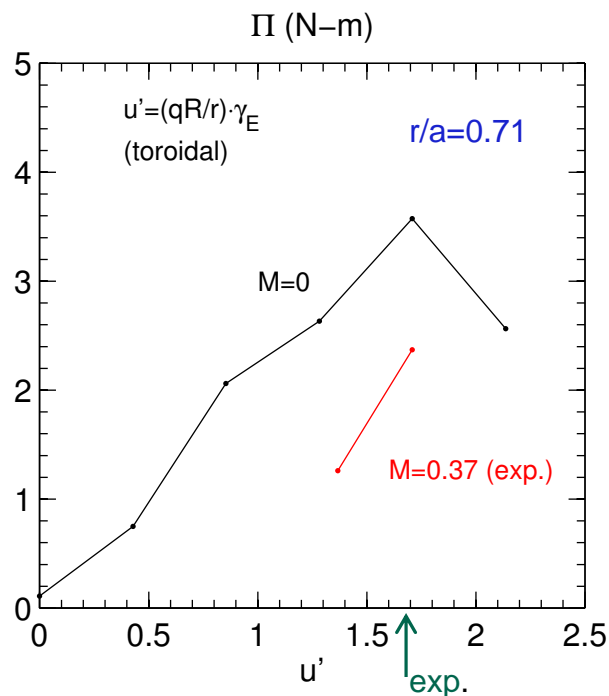
$$\Pi_\phi = nmR(\chi_\phi u' + \chi_{\phi\perp} \gamma_E) + (nmRV_\phi \times mR\Gamma_p)u + \cancel{C_{UD}} + \cancel{C_{P*}} + \dots$$

$$\left(\chi_\phi + \chi_{\phi\perp} \frac{r}{qR}\right)u' = \chi_{\phi,eff} u'$$



To better mimic experiment, calculating Pr and RV_ϕ/χ_ϕ using only 20% variation in u , $u' \sim \gamma_E$ around experimental values

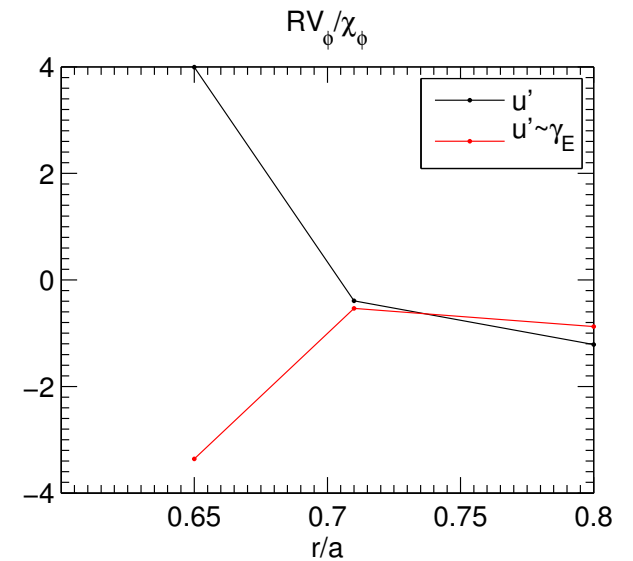
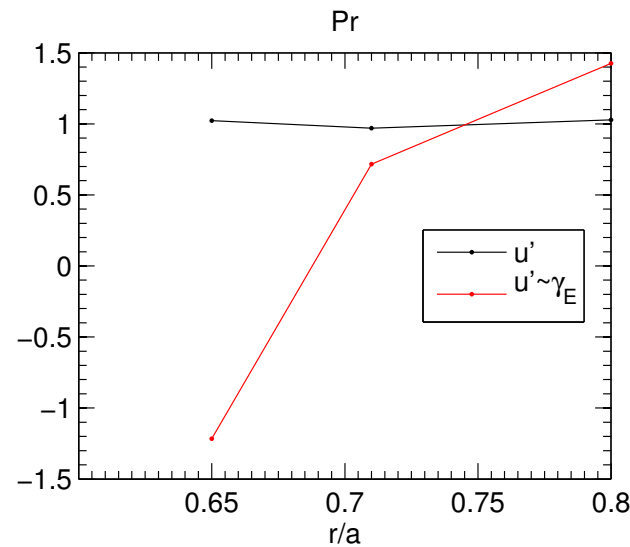
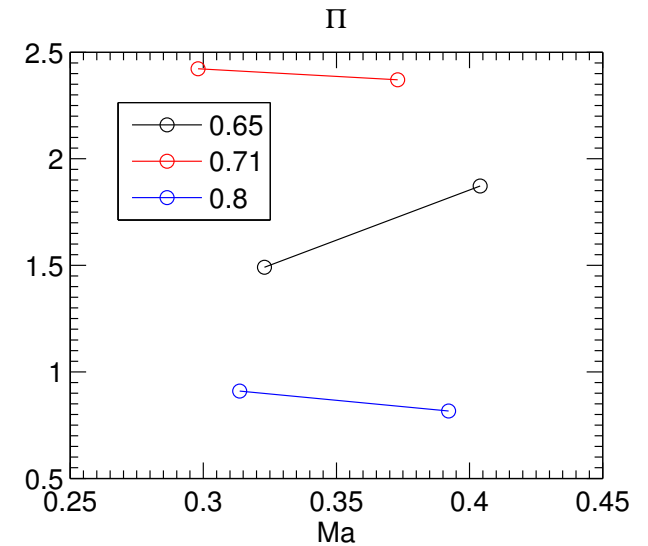
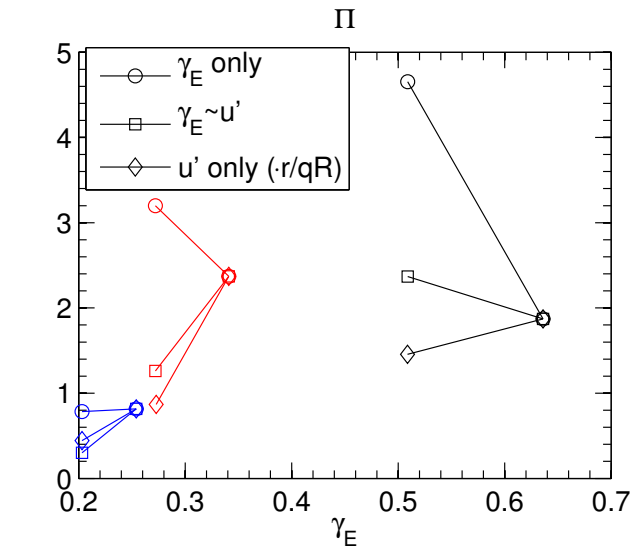
- Inferred Pr and RV_ϕ/χ_ϕ vary because of nonlinear dependencies
- Calculated two ways:
 - Using reference point at $u'=0$ (as above), e.g. $\Pi(u'_{\text{exp}}) - \Pi(0)$
 - Using incremental change $\Delta u'=20\%$, e.g. $\Pi(u'_{\text{exp}}) - \Pi(0.8 \cdot u'_{\text{exp}})$
- Incremental values ($Pr \sim 0.7$, $RV_\phi/\chi_\phi \sim -0.5$) similar to values above



Similar Pr , RV_ϕ/χ_ϕ found at $r/a=0.8$

- Odd results at $r/a=0.65$ because of much stronger $E \times B$ shear

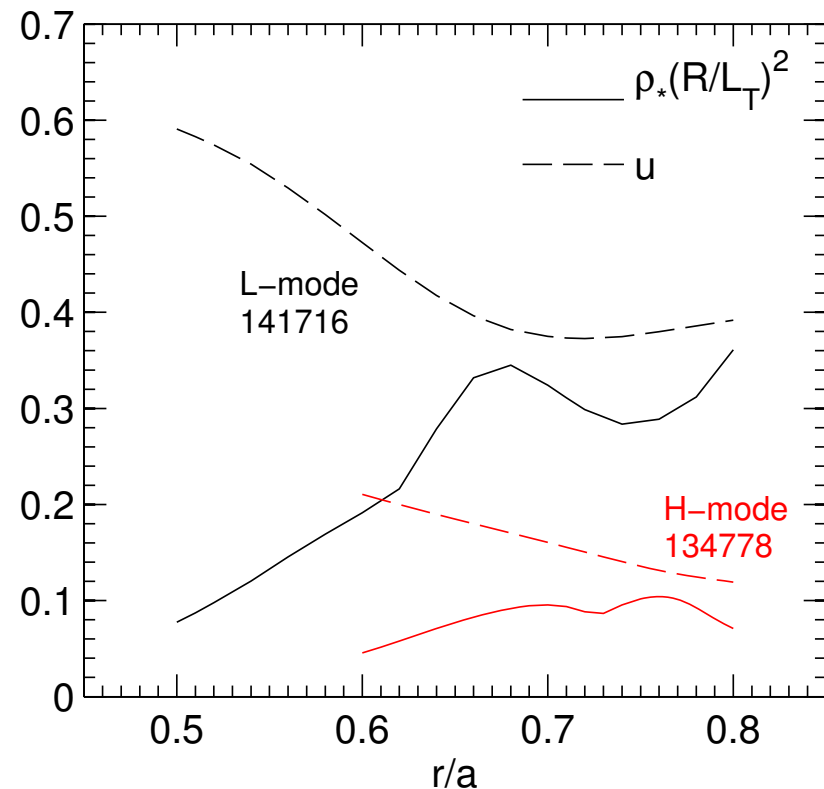
r/a	γ_E/γ_{lin}
0.65	0.8
0.71	0.45
0.8	0.25



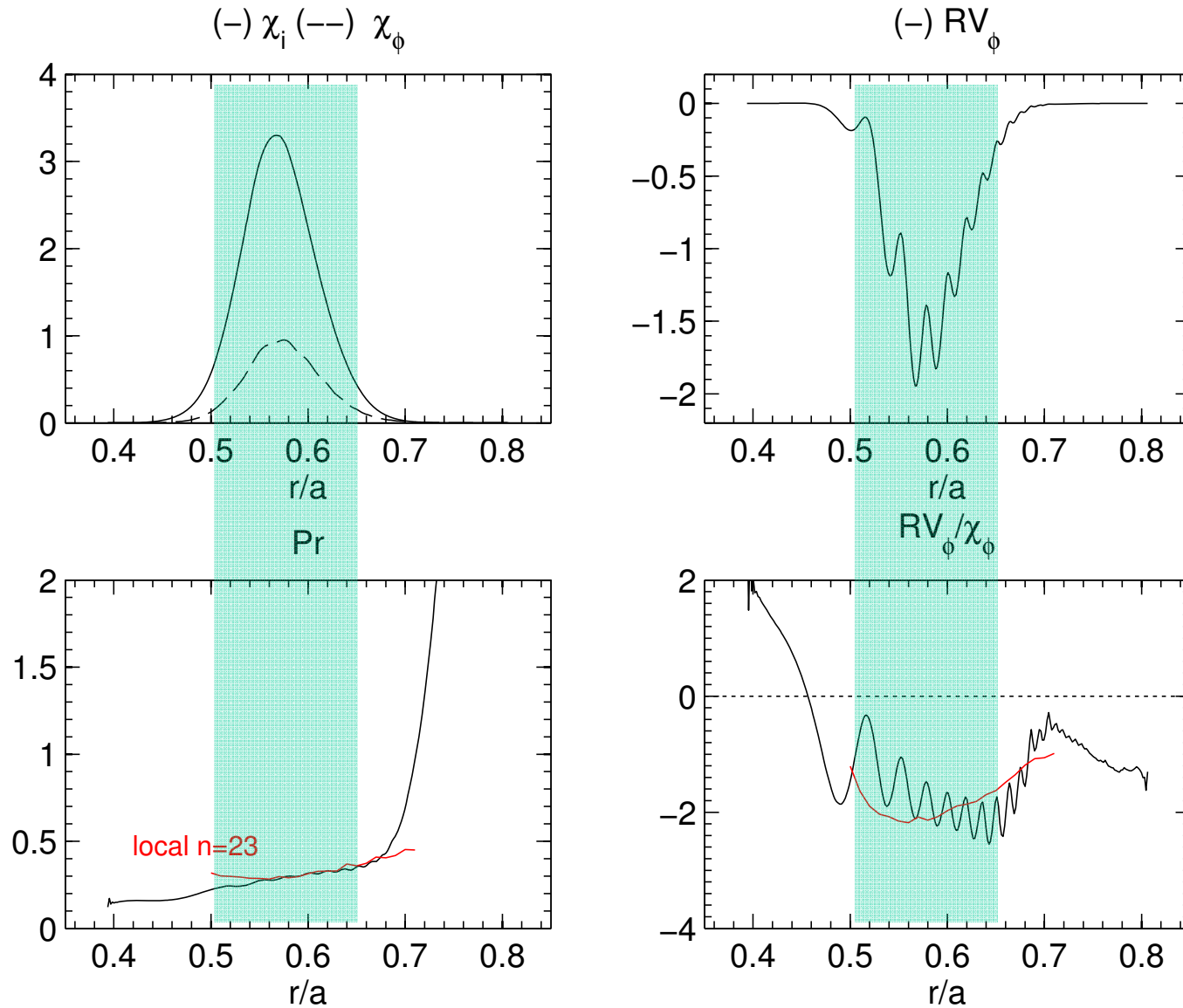
Global linear L-mode predictions

- Investigating Pr , RV_ϕ/χ_ϕ and residual stress (C_{ρ^*}/χ_ϕ) for linear, *global* simulation ($n=23$, $k_\theta\rho_s=0.3$ at $r/a=0.6$) (following Camenen, 2011)

- Rough criteria (Peeters, 201) - expect residual stress from profile shearing to be comparable to pinch if:
 $\rho_*(R/L_T)^2 > u$
 - Close for both L-mode and H-mode

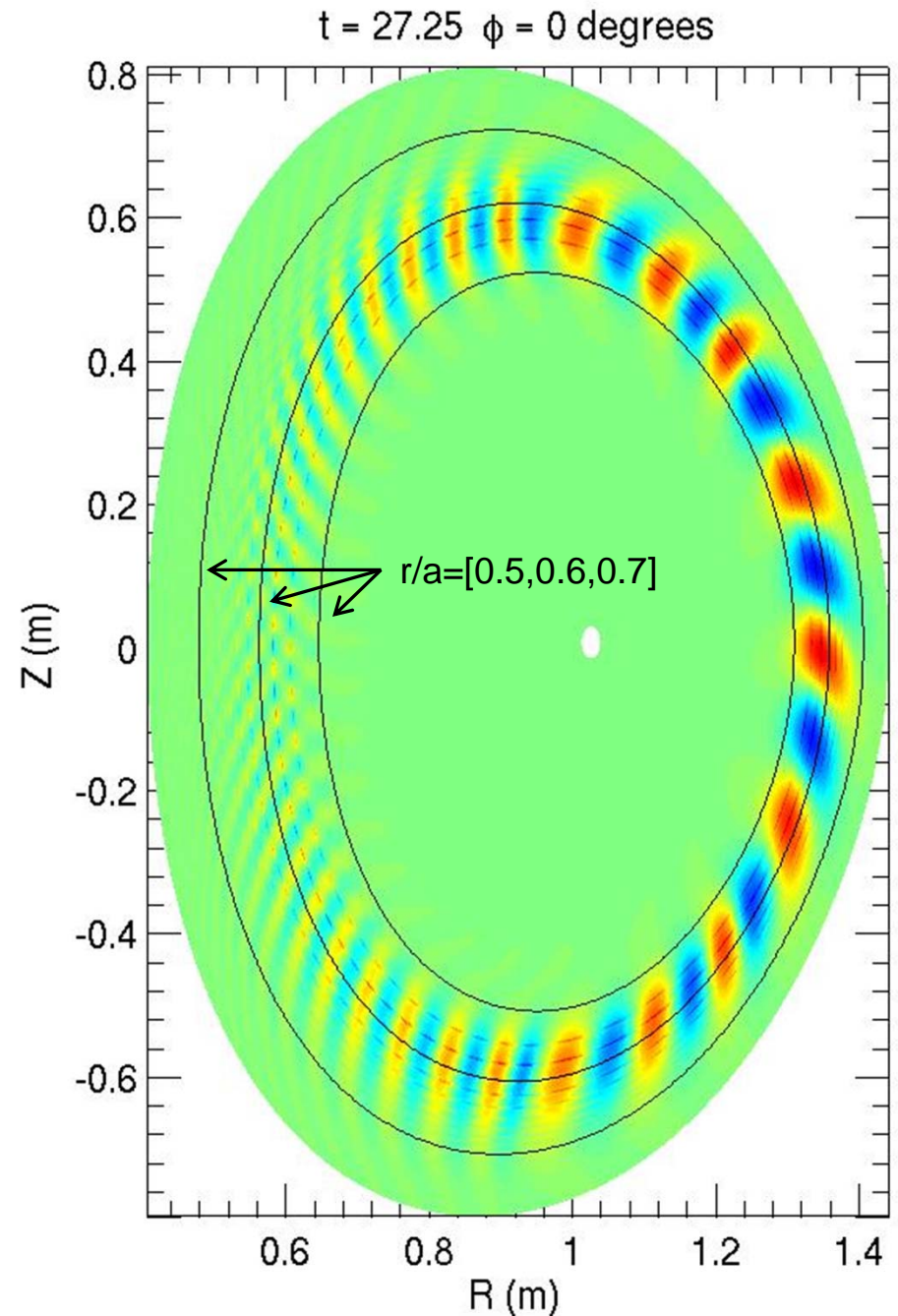
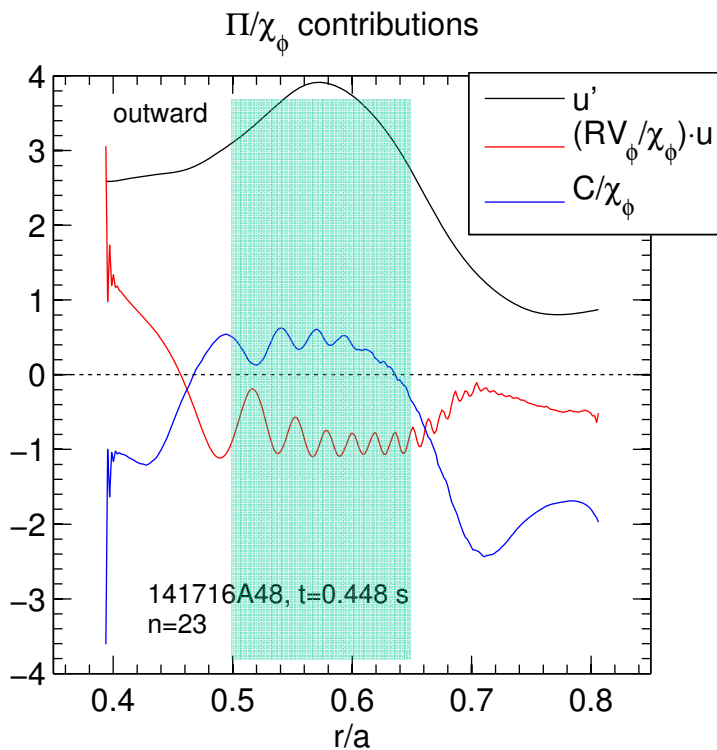


Quasilinear Pr and RV_ϕ/χ_ϕ from linear, global simulations in good agreement with local simulations



Diamagnetic profile shear tilts eddies, breaks symmetry

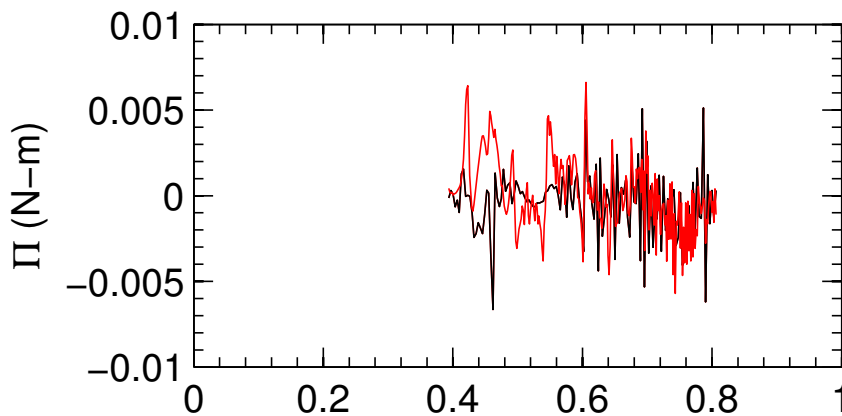
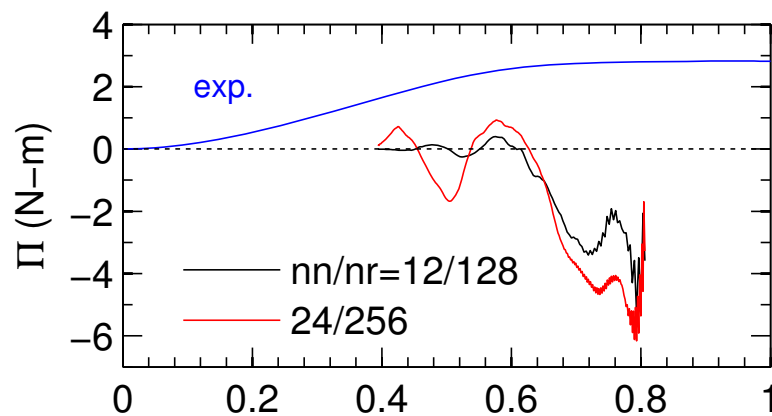
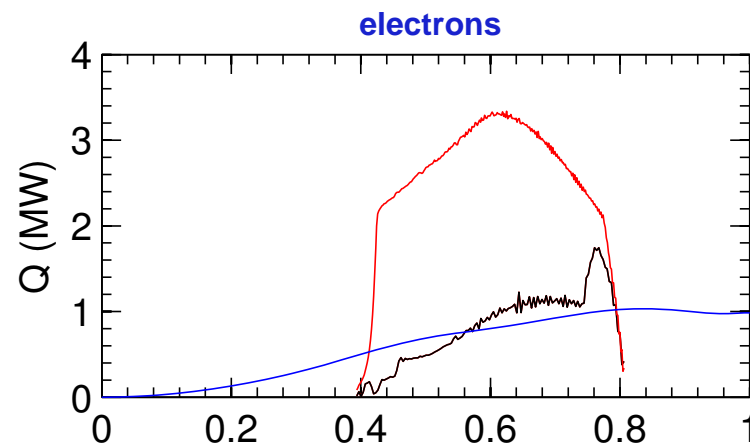
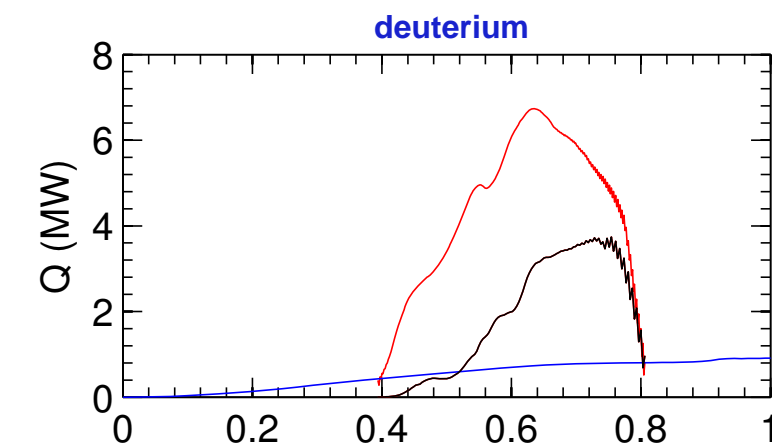
- Transport due to residual stress (C/χ_ϕ) is comparable to pinch transport ($(RV_\phi/\chi_\phi) \cdot u$), but directed outward (opposite pinch)
- Both small compared to diffusive component (u')



Global nonlinear L-mode predictions

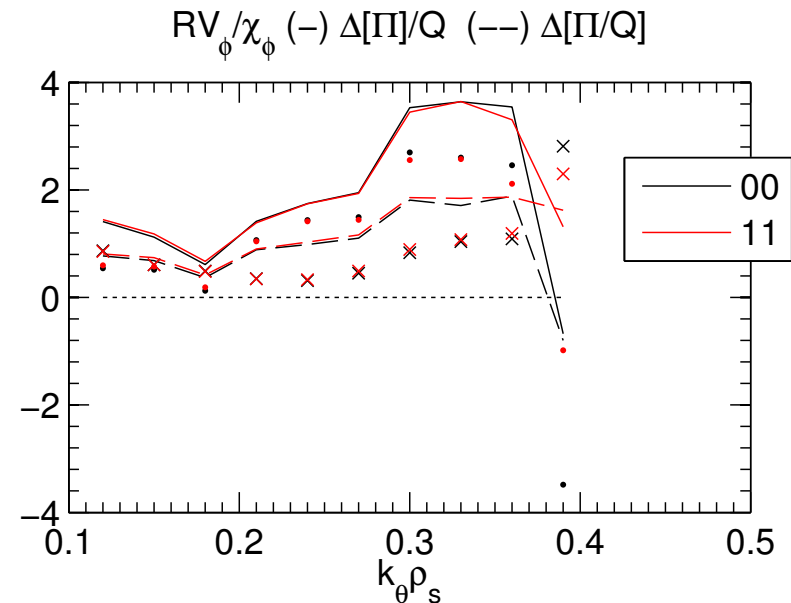
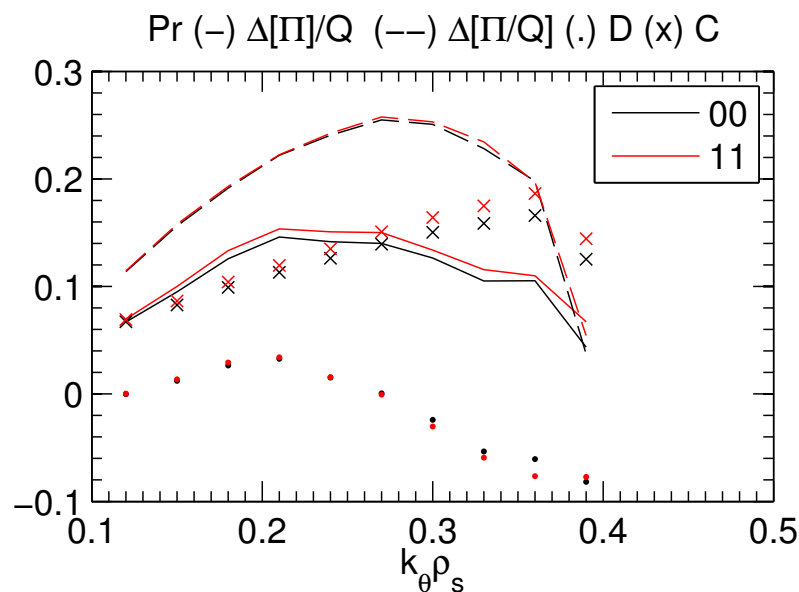
Global nonlinear simulation (no flow or flow shear)

- Significant inward directed momentum flux (no flow or flow shear)
- Transport peaks further out ($r/a \sim 0.8$) than linear $n=23$ eigenmode
 - More work required to investigate resolution and boundary conditions



Other considerations

- Up-down asymmetry weak in these plasmas (far from separatrix)
- Investigating influence of centrifugal effects using GKW (Buchholz, Hornsby, Peeters)
- Will investigate uncertainty in profiles and equilibrium reconstructions
- Interesting aside: D, C contribute differently to Pr and RV_ϕ/χ_ϕ (significant carbon impurity fraction in H-modes, $Z_{\text{eff}} \sim 3$)



Extra slides

Three flow terms in the strong flow limit (e.g., from GYRO Technical Guide, <https://fusion.gat.com/theory/Gyro>)

$$\frac{\partial h_a}{\partial t} + (v_{\parallel} \mathbf{b} + \mathbf{v}_d) \cdot \nabla H_a + \mathbf{v}_{E0} \cdot \nabla h_a + \delta \mathbf{v}_a \cdot \nabla h_a + \delta \mathbf{v}_a \cdot \left(\nabla f_{a0} + \frac{m_a v_{\parallel} f_{a0}}{T_a} \frac{I}{B} \nabla \omega_0 \right) = C_a^{GL} [H_a] . \quad (3.29)$$

$\mathbf{E} \times \mathbf{B}$ (perpendicular)
flow shear

$$\gamma_E \doteq - \frac{r}{q} \frac{\partial \omega_0}{\partial r} ,$$

Parallel/toroidal flow shear

$$\gamma_p \doteq - R_0 \frac{\partial \omega_0}{\partial r} .$$

$$\mathbf{v}_d \doteq \frac{v_{\parallel}^2 + \mu B}{\Omega_{ca} B} \mathbf{b} \times \nabla B + \frac{2v_{\parallel} \omega_0}{\Omega_{ca}} \mathbf{b} \times \mathbf{s} + \frac{4\pi v_{\parallel}^2}{\Omega_{ca} B^2} \mathbf{b} \times \nabla p$$

Toroidal flow

$$\omega_0 \doteq -c \frac{\partial \phi_{-1}}{\partial \psi}$$

$$M \doteq \frac{\omega_0 R_0}{c_s} ,$$

- Toroidal flow can lead to momentum pinch (e.g., Coriolis [1], TEP+thermoelectric [2])
- Parallel flow shear is a thermodynamic drive gradient, can cause/enhance instability [3] and drive momentum transport [4]
- $\mathbf{E} \times \mathbf{B}$ (perpendicular) flow shear can suppress instability and turbulent transport, can also cause momentum transport [5].
- For theoretical insight, can vary each term independently, although in the core of NSTX NBI plasmas, toroidal flow dominates radial force balance so that:

$$\gamma_E = \frac{r}{qR} \gamma_p$$

Method for predicting quasi-linear Prandtl (χ_ϕ/χ_i) and Pinch numbers (RV_ϕ/χ_ϕ)

- Local linear GYRO simulations run between $r/a=0.6-0.8$ ($\rho_{\text{tor}}\approx 0.5-0.7$), with
 - deuterium, carbon, electrons
 - $\phi, A_{\parallel}, B_{\parallel}$
 - numerical equilibrium (EFIT/LRDFIT)
 - n_e profiles from averaged inboard/outboard measurements (no centrifugal effects in GYRO)

- Pr and RV_ϕ/χ_ϕ determined using momentum

flux from different combinations of u, u' $\hat{\Pi}_\phi = \hat{\chi}_\phi \hat{u}' + (\hat{R}\hat{V}_\phi + \hat{R}\hat{\Gamma}_p)\hat{u} + \hat{\Pi}_{\phi,RS}$

$$\text{Pr} = \frac{\hat{\chi}_\phi}{\hat{\chi}_i} = \frac{\hat{\Pi}_\phi(0, u') - \hat{\Pi}_\phi(0, 0)}{\hat{u}'} \cdot \frac{a/L_{Ti}}{\hat{Q}_i}$$

Following Peeters et al.
PRL (2007)
Nucl. Fusion (2011)

$$\left(\frac{RV_\phi}{\chi_\phi} \right) = \left[\frac{\hat{\Pi}_\phi(u, 0) - \hat{\Pi}_\phi(0, 0)}{\hat{u}} - \frac{\hat{m}\hat{R}\hat{\Gamma}_p(u, 0)}{\hat{u}} \right] \cdot \frac{\hat{u}'}{\hat{\Pi}_\phi(0, u') - \hat{\Pi}_\phi(0, 0)}$$

- Subtracting particle convection contribution

Transport of toroidal angular momentum calculated from delta-f gyrokinetics (GYRO*)

- Transport calculated for **toroidal momentum** from correlation of **perturbed distribution function** and **effective radial drifts** from all EM fields

$$\delta f_s(\vec{x}) = -\frac{e\delta\phi(\vec{x})}{T_s} F_{s0} + H_s(\vec{R}) \quad (3.22)$$

$$\Pi_s = \oint_{\text{flux surface average}} \int d^3v H_s^*(\vec{R}) \left\langle \underbrace{[m_s \mathbf{R}(\vec{V}_0 + \vec{v}) \cdot \vec{e}_\phi]}_{\text{red line}} \underbrace{\frac{c}{B} \vec{b} \times \nabla}_{\text{blue line}} \left[\underbrace{\delta\phi(\vec{x})}_{\text{green line}} - \frac{1}{c} (\vec{V}_0 + \vec{v}) \cdot \delta\vec{A}(\vec{x}) \right] \cdot \nabla \mathbf{r} \right\rangle_{\text{gyro average}} \quad (3.55)$$

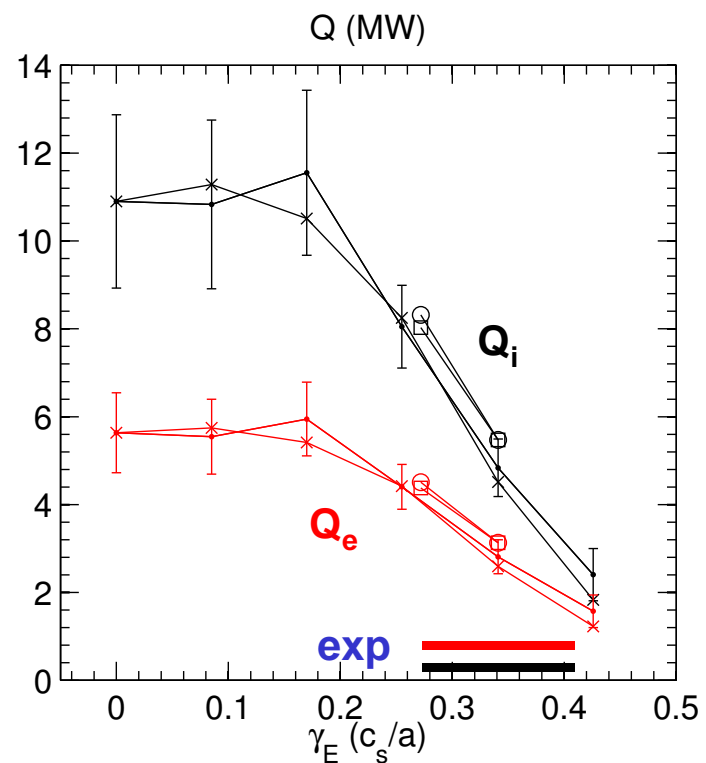
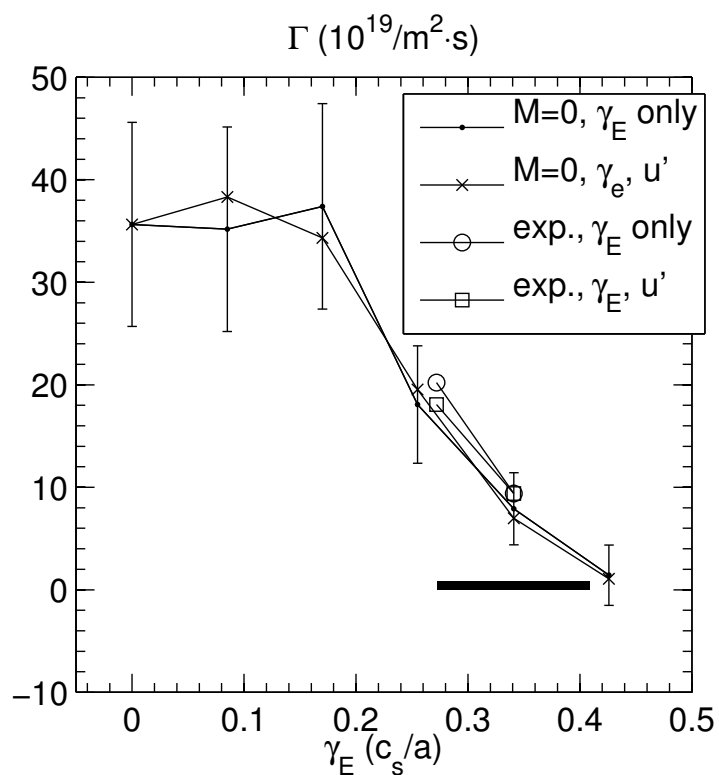
↓ Electrostatic E×B drift
↓ Drifts from shear ($v_{\parallel} \nabla A_{\parallel} \sim v_{\parallel} B_r$) and compressional ($v_{\perp} \nabla A_{\perp} \sim v_{\perp} B_{\parallel}$) magnetic perturbations

- EM contributions are important in NSTX H-modes*

*Candy & Belli, GYRO Technical Guide, <https://fusion.gat.com/theory/Gyro>

$E \times B$ shear significantly reduces predicted transport

- Predicted fluxes are larger than experiment, opposite ratio of heat fluxes (Q_e/Q_i)
- Including parallel flow shear and toroidal flow have negligible impact on particle and heat flux



Residual stress (Π/Q) compared to real frequency variation

