Homework 1

APAM 4990 (2010)

1) Derive the frequency and damping rate for the ion acoustic waves, i.e.,

$$\omega \approx \pm \frac{kc_s}{(1+k^2\lambda_{De}^2)^{1/2}} \left[\pm 1 - i\sqrt{\frac{\pi}{8}} \sqrt{\frac{m_e}{m_i}} \frac{1}{(1+k^2\lambda_{De}^2)^{3/2}} \right],$$

from the linearized Vlasov-Poisson system of

$$\frac{\partial \delta f_\alpha}{\partial t} + v\frac{\partial \delta f_\alpha}{\partial x} - \frac{q_\alpha}{m_\alpha}\frac{\partial \phi}{\partial x}\frac{\partial F_{\alpha 0}}{\partial v} = 0$$

and

$$\frac{\partial^2 \phi}{\partial x^2} = -4\pi e \int (\delta f_i - \delta f_e)\, dv,$$

where

$$F_{\alpha 0} = \frac{n_0}{\sqrt{2\pi} v_{t\alpha}} exp(-\frac{v^2}{2v_{t\alpha}^2}),$$

$v_{t\alpha} = \sqrt{T_\alpha/m_\alpha}$, $c_s = \sqrt{T_e/m_i}$, and α denotes species. From the fact that $m_e \ll m_i$, $\omega \approx kc_s$, and $v_{ti} \ll c_s \ll v_{te}$, the starting point is the warm electron response of

$$v_{te} \gg |\omega/k|,$$

which gives

$$\delta f_e = \frac{e}{T_e}\left[1 + i\pi\frac{\omega}{k}\delta(v - \omega/k)\right]\phi F_{e0},$$

and the cold ion response of

$$v_{ti} \ll |\omega/k|,$$

for

$$\delta f_i = -\frac{e}{T_i}\left[\frac{kv}{\omega} + \left(\frac{kv}{\omega}\right)^2\right]\phi F_{i0}.$$

Note that

$$\omega^2 = (\omega_R + i\omega_I)^2 \approx \omega_R^2(1 + 2i\omega_I/\omega_R)$$

for $|\omega_R| \gg |\omega_I|$.

2) Use the cold electron response of $v_{te} \ll |\omega/k|$ with no contributions from the ions to obtain the frequency and damping rate for the plasmas waves

$$\omega \approx \omega_{pe}\sqrt{1 + 3k^2\lambda_{De}^2}\left[\pm 1 - i\sqrt{\frac{\pi}{8}}\sqrt{1 + 3k^2\lambda_{De}^2}\frac{\exp(-1/2k^2\lambda_{De}^2)}{(k\lambda_{De})^3}\right].$$