Microinstabilities

- Linear properties
  - drift waves and ion temperature gradient (ITG) instabilities

- Numerical Schemes

- Nonlinear Saturation

- Hasegawa-Mima Equation

- Generalized Gyrokinetic Poisson's equation
  -- including both ion and electron polarization responses

• Lowest Order Gyrokinetic-Vlasov Equations

$$\rho_i \longrightarrow 0 \qquad \rho_s \neq 0$$

$$\boxed{\begin{gathered}\frac{dF}{dt} \equiv \frac{\partial F}{\partial t} + (v_\parallel \hat{\mathbf{b}} + \frac{c}{B}\mathbf{E}\times\hat{\mathbf{b}})\cdot\frac{\partial F}{\partial \mathbf{x}} + \frac{q}{m}\mathbf{E}\cdot\hat{\mathbf{b}}\frac{\partial F}{\partial v_\parallel} = 0 \\ \frac{\rho_s^2}{\lambda_{De}^2}\nabla_\perp^2\phi = -4\pi e\int(F_i - F_e)dv_\parallel\end{gathered}}$$

• Drift Waves

$$F = F_0 + \delta f \qquad F_0 = \frac{n_0(x)}{\sqrt{2\pi}v_t}exp(-\frac{v_\parallel^2}{2v_t^2}) \qquad n_0 \propto e^{-\kappa_n x}$$

$$\frac{d\delta f}{dt} = -\frac{dF_0}{dt}$$

$$= -\frac{c}{B}\mathbf{E}\times\hat{\mathbf{b}}\frac{\partial F_0}{\partial x} - \frac{q}{m}\mathbf{E}\cdot\hat{\mathbf{b}}\frac{\partial F_0}{\partial v_\parallel} = \frac{cT_e}{eB}\frac{eE_y}{T_e}\kappa_n F_0 + \frac{q}{T}E_\parallel v_\parallel F_0$$

• Let $\delta f \propto e^{-i\omega t + i\mathbf{k}\cdot\mathbf{x}}$

$$[(k_\parallel\rho_s)(\frac{v_\parallel}{c_s}) - (\frac{\omega}{\Omega_i})]\delta f = -(k_y\rho_s)(\kappa_n\rho_s)(\frac{e\phi}{T_e})F_0 - (\frac{q\phi}{T})(k_\parallel\rho_s)(\frac{v_\parallel}{c_s})F_0$$

• In the gyrokinetic units, we have: $\omega_* \equiv k_y\kappa_n$

$$(k_\parallel v_\parallel - \omega)\delta f = -k_y\kappa_n\phi F_0 - (\frac{q}{e}\frac{T_e}{T})\phi k_\parallel v_\parallel F_0 \qquad k_\perp^2\phi = \frac{1}{n_0}\int(\delta f_i - \delta f_e)dv_\parallel$$

• Perturbed distributions

$$\delta f_e = \left[1 - \frac{\omega_* - \omega}{k_\parallel v_\parallel - \omega}\right]\phi F_0 \qquad\qquad \delta f_i = -\frac{\omega_*}{\omega}\phi F_0 \qquad \text{--} \qquad \omega/k_\parallel v_{ti} \gg 1$$

• Dispersion relation: $\omega/k_\parallel v_{te} \ll 1$

$$k_\perp^2 - \frac{\omega_*}{\omega} + 1 - i\sqrt{\frac{\pi}{2}}\frac{\omega_* - \omega}{k_\parallel v_{te}} = 0$$

• Drift waves

$$\omega_R = \frac{\omega_*}{1 + k_\perp^2}$$

$$\frac{\omega_I}{\omega_R} = \sqrt{\frac{\pi}{2}}\frac{\omega_* - \omega_R}{k_\parallel v_{te}}\frac{1}{1 + k_\perp^2}$$

unstable for $\omega_* > \omega_R$

• Perturbative simulation scheme

$$\frac{d\mathbf{x}}{dt} = v_\parallel \hat{\mathbf{b}} + \frac{c}{B}\mathbf{E} \times \hat{\mathbf{b}}$$

$$\frac{dv_\parallel}{dt} = \frac{q}{m}\mathbf{E} \cdot \hat{\mathbf{b}}$$

$$\frac{dw}{dt} = (1 - w)\left[\frac{c}{B}\mathbf{E} \times \hat{\mathbf{b}} \cdot \kappa_n \hat{\mathbf{x}} + \frac{q}{T}E_\parallel v_\parallel\right]$$

• Numerical properties can be obtained by using ρ_s to replace λ_{De} for unmagnetized plasmas

• Ion Temperature Gradient (ITG) Modes

$$F_0 = \frac{n_0(x)}{\sqrt{2\pi T_0(x)/m}} exp\left[-\frac{mv_\parallel^2}{2T_0(x)}\right]$$

$$\frac{1}{F_0}\frac{dF_0}{dx} = -\kappa_n + \frac{1}{2}\kappa_T - \frac{1}{2}\frac{v_\parallel^2}{v_t^2}\kappa_T$$

$$(k_\parallel v_\parallel - \omega)\delta f = -k_y(\kappa_n - \frac{1}{2}\kappa_T + \frac{1}{2}\frac{v_\parallel^2}{v_t^2}\kappa_T)\phi F_0 - (\frac{q}{e}\frac{T_e}{T})\phi k_\parallel v_\parallel F_0$$

- Warm electrons and cold ions: $\delta f_e = \phi F_0$ -- adiabatic response, $|k_\parallel v_{ti}/\omega| \ll 1$ -- cold ions

$$1 + k_\perp^2 = (\tau + \frac{\omega_{*Ti}}{\omega})(\frac{k_\parallel v_{ti}}{\omega})^2 + \frac{\omega_*}{\omega} \qquad \omega_{*Ti} \equiv k_y \kappa_{Ti}$$

- For $k_\perp^2 \ll 1$ $\omega_* = 0$ $1 = (\tau + \frac{\omega_{*Ti}}{\omega})(\frac{k_\parallel v_{ti}}{\omega})^2$ $\frac{\omega}{\omega_*} = \frac{-1 + i\sqrt{3}}{2}\left(\frac{k_\parallel v_{ti}}{\omega_{*Ti}}\right)^{2/3}$

- Perturbative particle pushing schemes

  -- delta-f scheme [Dimits and Lee, JCP '93; Parker and Lee, PF '93]

  -- split-weight scheme [Manuilskiy and Lee, PoP '00]

- Nonlinear saturation mechanisms [for example]

  -- nonlinear E x B trapping of the resonant particles in 2D [Lee, Krommes, Oberman and Smith, PF '84]

  -- nonlinear E x B generated zero-frequency response for the ion parallel momentum and pressure is responsible for nonlinear saturation [Lee and Tang, PF '88]

  -- Velocity-space trapping of the resonant particles in 1D [Parker and Lee, PF '93]

  -- Zonal flows as the result of the nonlinear E x B generated response [Lin et al., Science '98] as well as the velocity space nonlinearity [Lee et al., Computational Science and Discovery, '08] are dominant saturation mechanisms.

• Hasegawa-Mima Equation [PRL '77; PF '78]

$$\frac{dF}{dt} \equiv \frac{\partial F}{\partial t} + (v_\parallel \hat{\mathbf{b}} + \frac{c}{B}\mathbf{E} \times \hat{\mathbf{b}}) \cdot \frac{\partial F}{\partial \mathbf{x}} + \frac{q}{m}\mathbf{E} \cdot \hat{\mathbf{b}} \frac{\partial F}{\partial v_\parallel} = 0$$

$$\frac{\rho_s^2}{\lambda_{De}^2} \nabla_\perp^2 \phi = -4\pi e \int (F_i - F_e) dv_\parallel$$

$$F = F_0 + \delta f$$

$$\frac{d\delta f}{dt} = \frac{c}{B} E_y \kappa_n F_0 + \frac{q}{T} E_\parallel v_\parallel F_0$$

$$\frac{\partial}{\partial x_\parallel} \to 0 \qquad\qquad \frac{\delta n_e}{n_0} \approx \frac{e\phi}{T_e}$$

$$\boxed{\frac{\partial}{\partial \Omega_i t}[1 - \rho_s^2 \nabla_\perp^2] \frac{e\phi}{T_e} + \rho_s \nabla \frac{e\phi}{T_e} \times \hat{b} \cdot \rho_s \nabla [\rho_s^2 \nabla_\perp^2 \frac{e\phi}{T_e}] = -\rho_s \frac{\partial}{\partial y} \frac{e\phi}{T_e} \kappa_n \rho_s}$$ H-M equation

• Dual cascade property [Hasegawa and Kodoma '78]

• A more complete gyrokinetic Poisson's equation

- including both ion and electron polarization responses

$$\nabla^2\Phi - (\Phi - \tilde{\Phi}_i)/\lambda_{Di}^2 - (\Phi - \tilde{\Phi}_e)/\lambda_{De}^2 = -4\pi\rho$$

$$\tilde{\Phi}_i(\mathbf{x}) = \sum_k \Phi(\mathbf{k})\Gamma_0(k_\perp^2\rho_i^2)exp(i\mathbf{k}\cdot\mathbf{x})$$

$$\tilde{\Phi}_e(\mathbf{x}) = \sum_k \Phi(\mathbf{k})\Gamma_0(k_\perp^2\rho_e^2)exp(i\mathbf{k}\cdot\mathbf{x})$$

ITG: $k_\perp^2\rho_e^2 \rightarrow 0$ $\quad k_\perp^2\rho_i^2 \ll 1$

$$\nabla^2\phi + \frac{\rho_i^2}{\lambda_{Di}^2}\nabla_\perp^2\phi - \frac{1}{\lambda_{De}^2}\phi = -4\pi e(n_i - n_0)$$

ETG: $k_\perp^2\rho_e^2 \ll 1$ $\quad k_\perp^2\rho_i^2 \gg 1$

$$\nabla^2\phi + \frac{\rho_e^2}{\lambda_{De}^2}\nabla_\perp^2\phi - \frac{1}{\lambda_{Di}^2}\phi = -4\pi e(n_0 - n_e)$$